

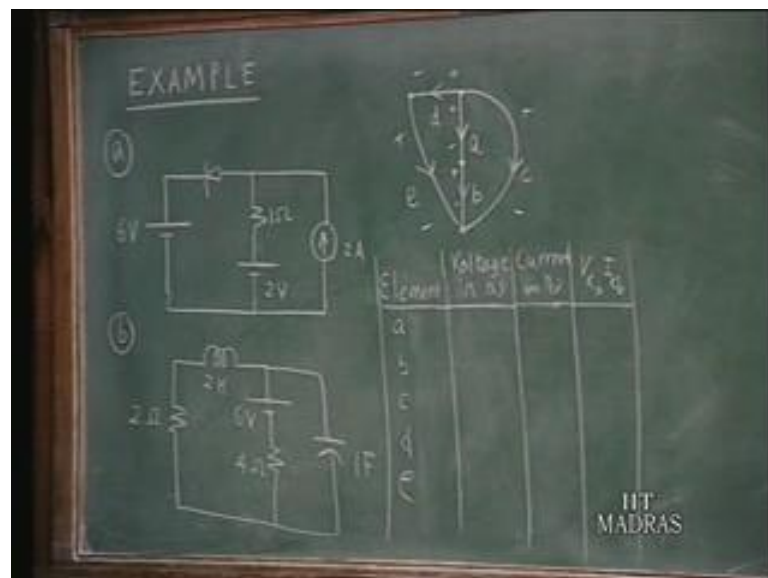
**Networks and Systems**  
**Prof V G K Murti**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Madras**

**Lecture -37**  
**Network Theorems (4)**  
**Examples**  
**Exercise 7**

In the last lecture, we had discussed the statement of Tellegen's theorem and also constructed a proof of this. And we mention that, if we had 2 networks of identical topology, then we can take the current variables in 1 network, multiply by the corresponding voltage variables in other network and the sum of those products will add upto, 0 irrespective of the nature of the elements and irrespective of the nature of voltage and current variables that you are using, provided to that the voltage variables satisfy Kirchhoff's voltage law in the particular network and the current variable satisfy Kirchhoff's current law in the relevant network.

We will illustrate this with an example to start with today.

(Refer Slide Time: 02:02)

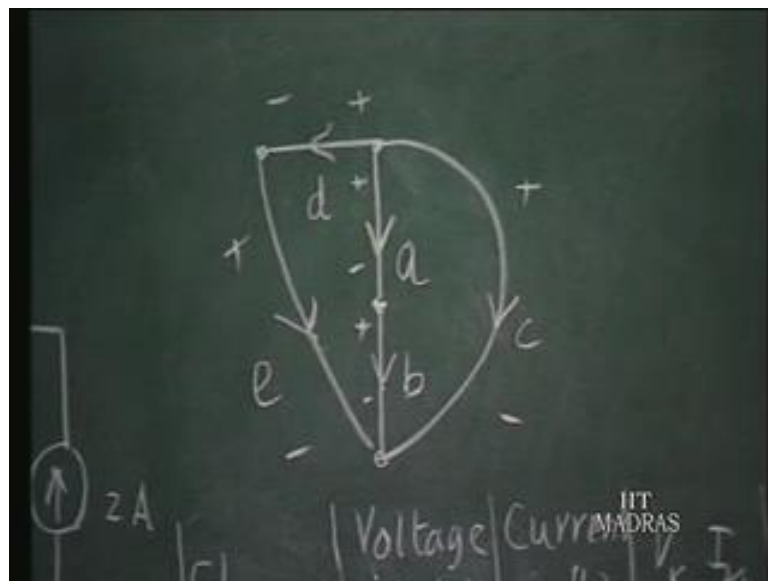


We have here a 2 networks a and b have identical geometry in the sense that, we can put each element here into 1 to 1 correspondence with the other elements here and the way they are interconnected are 1 at the same. That means: if you represent each 1 of these

elements by a straight line or a line a straight or curved line and the you can represent both these diagrams by means of a structure like this, where this element a represents this 1 ohm resistance element b in this network and this 6 volts source in the another network. Element b stands for this 2 volts source or this 4 volt source 4 ohm resistor, elements c stands for the current here or the capacitor element e stands for this 2 ohm resistor or the 6 volt source and likewise element d is at the diode or the inductor. So, this is what is called the topology of this network.

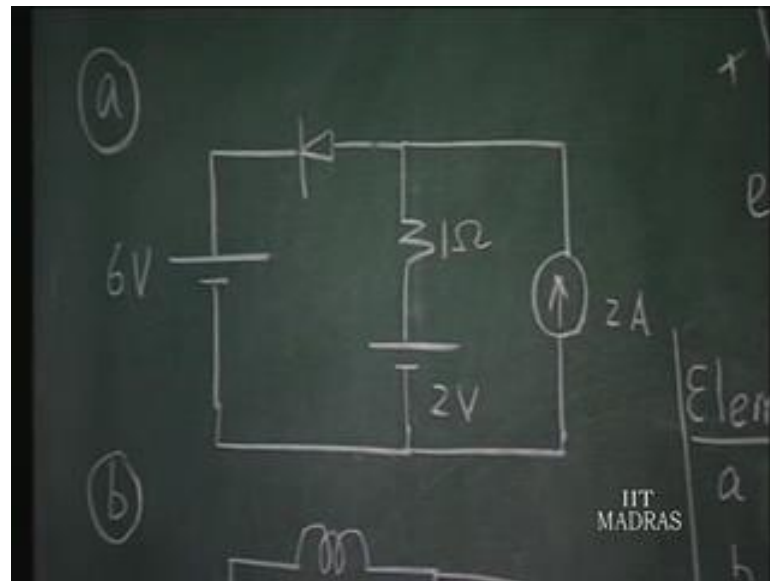
Topology is the term used to represent the geometry of the network the way which the elements are interconnected. Therefore, these 2 networks have identical topology. And 5 elements are there and you have 5 lines here, 5 curved lines here or straight lines whatever it is 1 2 3 4 5.

(Refer Slide Time 03:23)



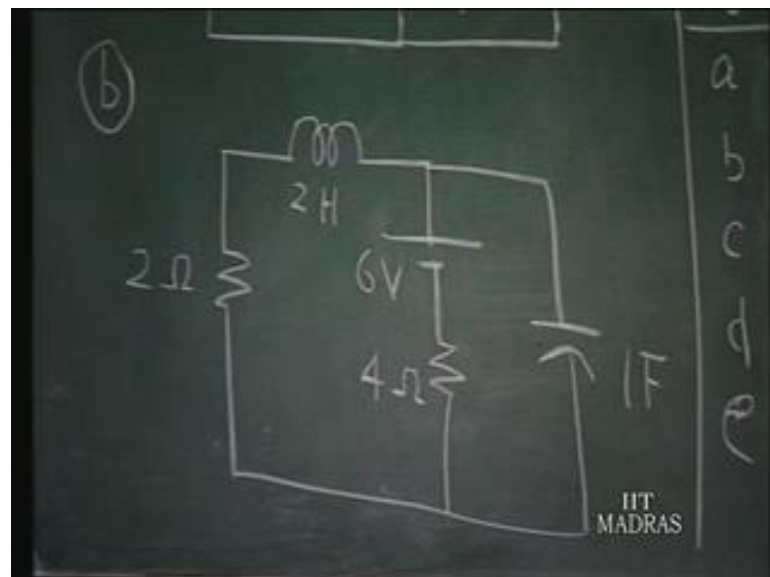
Now, we would like to illustrate the Tellegen's theorem here with respect to this network, taking the voltages in network a and currents in network b. So, if you find the voltages in network a, what happens is; when what is the diode conducting or not.

(Refer Slide Time 03:36)



Suppose you open circuited this point, then this 2 amperes goes through this so, developing a 2 volts here 2 volts here therefore, this 4 volts developed across this and this is 6 volts. That means the diode is reverse biased and therefore, it does not conduct. Therefore, the voltages in first network can be obtained.

(Refer Slide Time: 03:57)



And similarly as far the current network is concerned, second network is concerned it is a DC network. We want to illustrate this under steady state conditions. The capacitor is open circuited therefore, this 6 volts drives a current through this loop and this current

will be 1 ampere because, 6 volts divided by 6 ohms is 1 ampere. And as far as this is concerned the under steady state conditions 2 volts is developed here and the voltage across this is 2 volts.

So, let us take a element by element, find out the voltage in the a network, current in the b network and take their product.

(Refer Slide Time: 04:34)

Element	Voltage in (a)	Current in (b)	$V \cdot I$
a	2V	-1A	-2
b	2V	-1A	-2
c	4V	0	0
d	2V	1A	2
e	6V	1A	6

So, I have written here a b c d e element, p k a; the voltage of this element in the a network, a element is this so, this is 2 volts. And the current in this is in this direction; that is 1 ampere. But in application of Tellegen's theorem, we must have a uniform convention; the current in an element must be normally the drop must be in the same direction as drop in voltage from positive to negative. Therefore, if you take this as a positive reference direction for current, the current here is upwards therefore, I must write this as minus 1 ampere therefore, the product is minus 2. That is how it will be.

So, it is important to when you apply Tellegen's theorem to have this convention, it may be consistent. The current in a particular element must be from positive reference sign to negative reference sign in all the elements, the reference direction for current or in all elements it should be the opposite, but here you have taken the current direction, reference direction from the current to be from the positive to the negative reference direction for the voltages like this.

Then take element b, the voltage here is 2 volts, the current here is once again is in the opposite direction therefore, minus 1 ampere therefore, the product is minus 2. Take element c, that is, this 1. The current reference direction is like this, the actual current is opposite, but anyway that we are not worried about, but what we are interest in voltage across these 2 points; that is equal to 2 plus 2 4; 4 volts.

The current here is 0, because on the steady state the capacitor does not have any current therefore, 0 the product is 0 the product is the product is 0. Take element d, that is, this 1 this is the reference direction for the voltage is 2 volts, the current here is 1 ampere, and therefore the product is 2. Take element e the voltage is 6 volts and the current is 1 ampere therefore, the product is 6.

(Refer Slide Time: 07:00)

Element	Voltage in (v)	Current in (A)	$V_k I_k$
a	2V	-1A	-2
b	2V	-1A	-2
c	4V	0	0
d	-2V	1A	-2
e	6V	1A	6

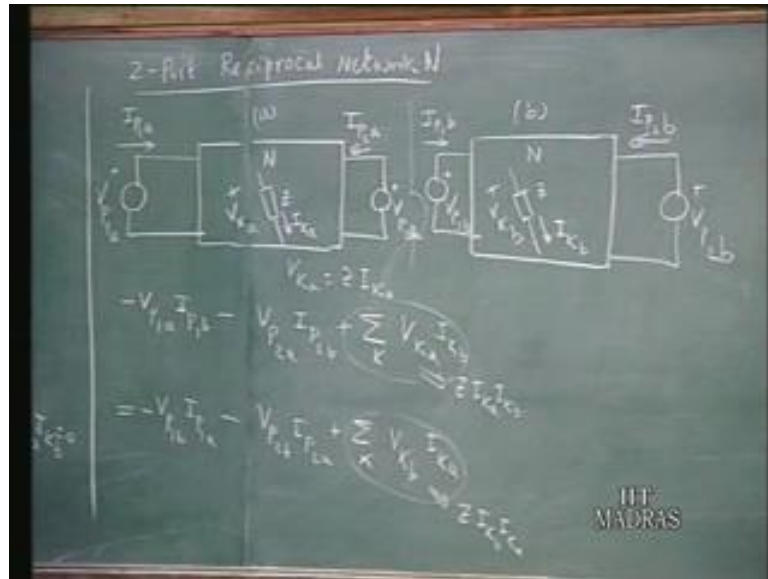
$\sum V_k I_k = 0$

IIT MADRAS

Now, you observe that as far as element d is concerned, this is positive with reference to this, but this is positive with reference to this therefore, you must write this as minus 2 volts. So, if you add up all these sigma  $V_k I_k$  is 0. So, the sum of all this impedance is 0. This is just an illustration of Tellegen's theorem, applicable to 2 networks with identical topology, but different elements.

Now, we would like to apply Tellegen's theorem to a 2 port reciprocal network.

(Refer Slide Time: 07:39)



Let us consider the same network containing reciprocal elements under 2 different excitation conditions. So, this is under condition a, this is under condition b. So, I put  $V_{p1a}$ , that is, port voltage 1 under network a condition a and the corresponding current  $I_{p1a}$ , this is  $V_{p2a}$  port 2 means port 2 a and the corresponding current  $I_{p2a}$ . And there is general elements here inside the network; let us say impedance  $z$ . It has got a voltage  $V_{ka}$  and a current  $I_{ka}$ .

Let us have the same network with a different set up excitation conditions. So, I have  $V_{p1b}$  and corresponding port current I will call that  $V_{p1b}$ . Similarly, I have  $V_{p2b}$  and the corresponding current  $I_{p2b}$  in the port. And in this element here same impedance  $z$ , I have  $V_{kb}$  and the corresponding current  $I_{kb}$  that is the; that means, you have taken the same network  $Z_n$  and have 2 different terminal conditions. In 1 set I call a, the other set I call with subscript b.

Now let me apply Tellegen's theorem taking the voltages here multiplied by the currents here. Take this element; you have the voltage here and the current here. Now, note this that, as far as this element is concerned the voltage dropped here, but the current is from minus to negative minus to positive. Therefore, I can write this as minus  $V_{p1a}$  times  $I_{p1b}$  because, they are not in the conventional sense the current is not from positive to negative reference signs of the voltage.

Similarly, take this element and this current here, likewise  $V_{p2a}$  multiplied by  $I_{p2b}$  plus summation of all such voltages in the internal structure of network  $N$  in voltage there is  $V_{Ka}$  and the current here is  $I_{Kb}$ , summed on all  $K$ ; they all are internal elements. This of course, is 0 this also equal to take the voltages here multiply by the currents here. So, here  $V_{p1b}$  minus  $V_{p1a}$  again same reason because, the voltage here and the current here are according to contrary to the conventional reference directions. Therefore,  $V_{p1b}$  times  $I_{p1a}$  minus again this voltage  $V_{p2b}$  times this current  $I_{p2a}$  plus summed on all internal elements  $V_{Kb}$  times  $I_{Ka}$ .

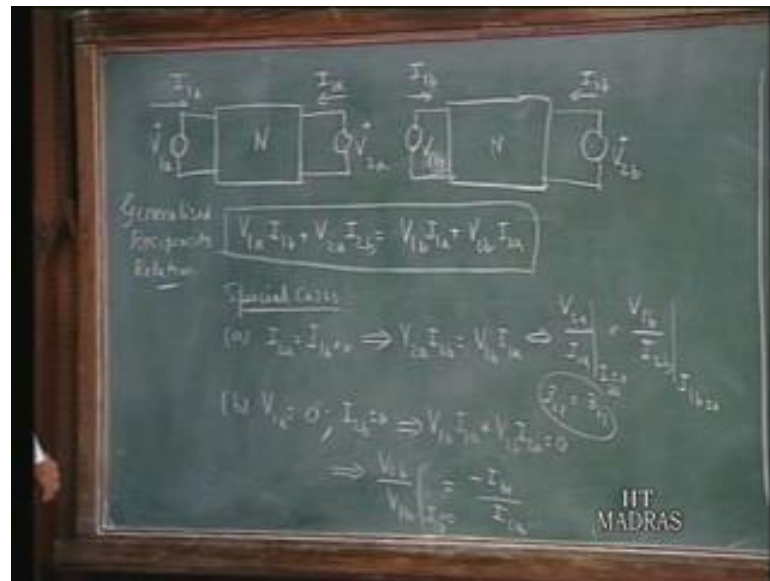
So, these 2 both are equal of course, this whole group of terms add up to 0, this whole group of terms add up to zero, but that is not interest to us, all we want to see is this 2 are equal. Now, if this are all reciprocal linear elements that you are having,  $V_{Ka}$  equals  $z$  times  $I_{Kb}$ . So, this can be written as  $z$  times  $I_{Kb}$   $I_{Ka}$  and this can be written likewise as  $z$  times  $I_{Ka}$  times  $I_{Kb}$ .

So, in other words this whole summation is equal to this whole summation. So, in this equality this group of terms can be cancelled on both sides. This is because we have linear elements and reciprocal elements, what we have shown is here only for 2 terminal elements. If these are also having mutual inductances which are reciprocal 2 port elements, then also this particular equality will be valid, we do not take time to show that, it can be demonstrated to take it for granted that this will be true even if you had 2 terminal pair elements, which are reciprocal inside the network  $m$ .

So, these 2 are equal; that means, if you take this port voltage multiplied by this port current, this port voltage multiplied by this port current; that sum is equal to the sum that is obtained by taking this port voltage multiplied by this port current plus this port voltage multiplied by this port current. So, that is what you are having. You can cancel the negative signs on both sides so, you can write  $V_{p1b}$  times  $I_{p1a}$  plus  $V_{p2b}$  times  $I_{p2a}$  minus  $V_{p1a}$  times  $I_{p1b}$  plus  $V_{p2a}$  times  $I_{p2b}$  equals  $V_{p1b}$  times  $I_{p1a}$  plus  $V_{p2b}$  times  $I_{p2a}$ .

This whole thing can be put in a more compact fashion like this.

(Refer Slide Time: 13:48)



So, we have a 2 port network under 2 different excitation conditions. So, we have N here, you can write I will remove the subscript p here so, I will write here V 1 a and the corresponding current I 1 a, V 2 a and the corresponding current I 2 a. This is at condition a. And the network same network with a different terminal conditions V p 1 b I no longer p is required V 1 b and this is I 1 b and this is V 2 b and this is I 2 b, then we can say that V 1 a times I 1 b plus V 2 a times I 2 b equals V 1 b times I 1 a plus V 2 b times I 2 a.

So, this particular relation is valid for a general reciprocal 2 port network and this is called generalized reciprocity relation. Generalized reciprocity relation for a 2 port network and this can be used to demonstrate several results that we know. For example, special cases; a: take I 2 a 0 and I 1 b 0, suppose you take I 2 a 0 and I 2 a 0 and I 1 b 0, then this leads to V 2 I 1 b is 0 and I 2 a is 0 therefore, it will lead to V 2 a times I 2 b V 2 a times I 2 b, since I 2 a is made equal to 0 we have only this term V 1 b times I 1 a. What this means is; V 2 a divided by I 1 a equals V 1 b divided by I 2 b. Under what conditions; you are measuring V 2 a by I 2 a I 1 a, under the condition I 2 a is 0 I 2 a is 0 therefore, this is measured under I 2 a equal to 0 under those conditions. You are measuring this voltage, the ratio of this voltage to this current when I 2 a is 0.

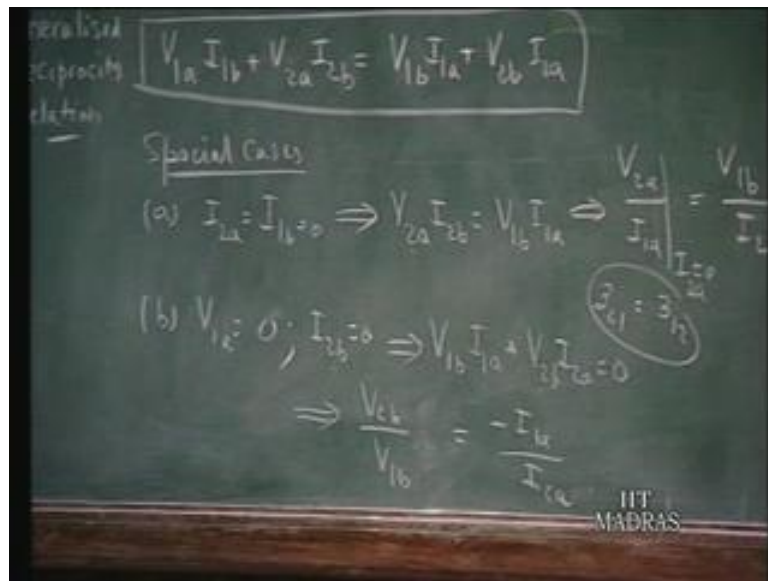
Similarly, here you are measuring V 1 b to I 2 b when I 1 b is 0. We know this is true V 2 a by I 1 a is z 2 1 and this is z 1 2. So, z 2 1 equals z 1 2 follows as a special relation of



this. You can likewise put special conditions here and identify the reciprocity relation that we already know. Let me check 1 more example.

Let me make  $V_1 = 0$  and  $I_2 = 0$ . So, if  $V_1 = I_2 = 0$ , I have  $V_1 = I_1 a$ , this  $I_1$  is 0 and  $I_2 = 0$ . That means, these 2 terms as 0, therefore I have  $V_1 = I_1 a$  plus  $V_2 = I_2 b$  equals 0.

(Refer Slide Time: 18:10)



What this would mean further is;  $V_2 = b$  divided by  $V_1 = b$  equals minus of  $I_1 = a$   $V_2 = b$  by  $I_1 = a$  equals minus of  $I_1 = a$  by  $I_2 = a$ .

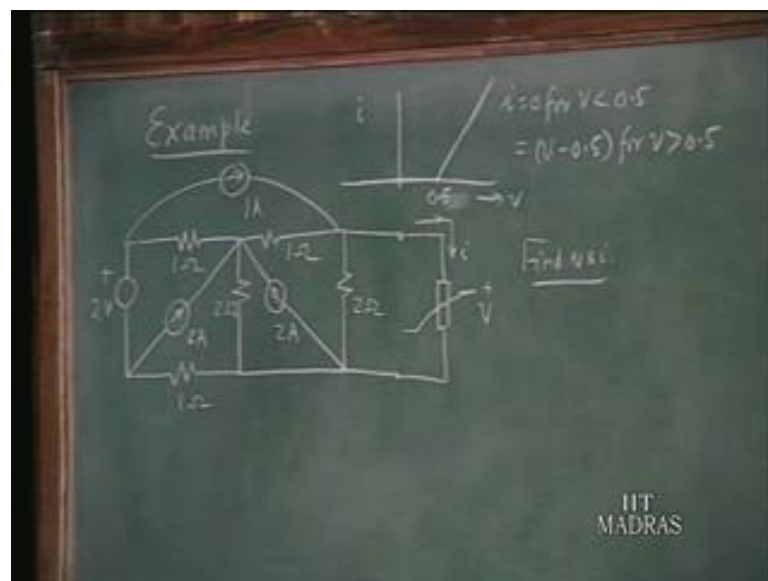
(Refer Slide Time: 18:38)



Now, under what conditions; you are measuring  $V_2$  divided by  $V_1$  under the condition  $I_2$  is 0  $I_2$  equals 0 and you are measuring  $I_1$  by  $I_2$  under the condition that  $V_1$  is 0. So, what we are having here;  $V_2$  divided by  $I_1$  when this is 0 that is the open circuit voltage transfer function  $g_{21}$  of  $s$ ,  $I_1$  divided by  $I_2$  when this is short circuited that the short circuit transfer current ratios so, minus  $\alpha_{12}$  of  $s$ . So, this is the relation we obtain here. Likewise you can also use this take some special cases and show that  $y_{12}$  is equal to  $y_{21}$  and  $g_{12}$  of  $s$  is equal to minus  $\alpha_{21}$  of  $s$ , all the relation that you know can be deduced as the special case of the generalized reciprocity relation, which we have deduced as an application of Tellegen's theorem.

Now, let us work out some more examples to illustrate the various networks theorem that we have discussed so far.

(Refer Slide Time: 19:56)



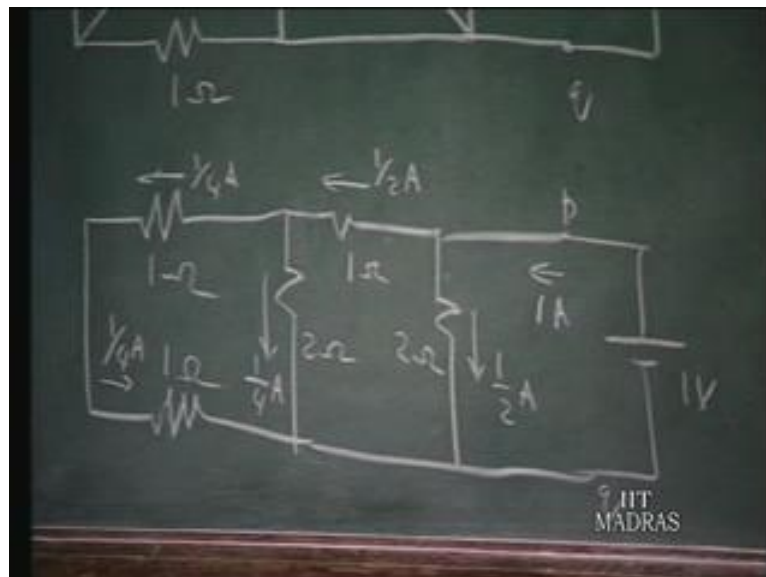
Let me now work out an example to illustrate some of the network theorems that we have studied so far. We have here a network with various resistors and independent sources, DC sources along with an element here which is non-linear. This particular element here has got the VI relations sketched here and described by this equation. The current is 0 for  $V$  less than 0.5; that means, this is the characteristics and as once voltage exceeds 0.5 current linearly increases according to the ratio according to the formula  $V$  minus 0.5.

So, this non-linear element is connected in this network, containing resistive network with a lot of independent sources; 3 current sources and 1 voltage source. We are asked

to find out the values of V and I existing in this network. So, networks of this type can be analyzed because, 1 non-linear element is involved and it is convenient for us to calculate the V and I if, we replace this entire network at these 2 terminals by its equivalent Thevenin's network. Then we have 1 source and 1 resistance in series with this. Then we can easily manipulate and find out the values of V and I.

So, this is particular example, specifies a type of problem where Thevenin's equivalent is quite convenient. So, let us try to find out the Thevenin's equivalent of this network at the terminals p q of the network to the left of terminals p and q. So, to do this, let me find out first of all the Thevenin's impedance the equivalent impedance as seen from p q.

(Refer Slide Time: 21:42)



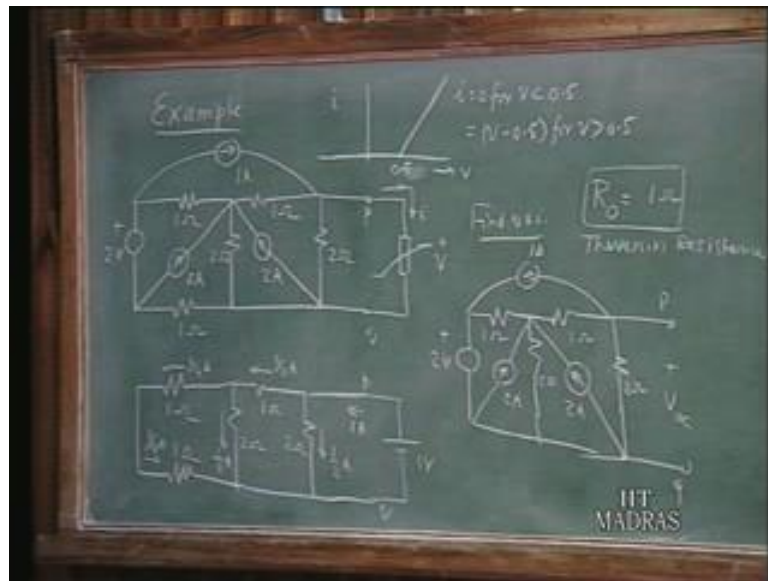
So, for that purpose we have deactivated all sources. So, we have only these resistances and then these are the terminals p q. So, we have to find out the equivalent resistance of this effective resistance of this 1 port network, 2 ohms 1 ohm 2 ohms. So, for this purpose let me say, I connect a 1 volt source and try to find out the current and that voltage divide by the current will be the equivalent effective impedance. Now, this 2 1 ohms will constitute in series with this 2 ohms, 2 ohms in parallel with 2 ohms is 1 ohm and 1 ohm will come in series with 1 ohm, that is, 2 ohm, 2 ohms in parallel with 2 ohms is 1 ohm.

Therefore, as far as we see from p q, the Thevenin's resistance or effective resistance is 1 ohm therefore, this current is 1 ampere. And this current is 1 ampere; that 1 ampere

divides into 2 half equal half because, this is 2 ohms and the resistance in the network to the left is also 2 ohms. So, therefore, this current is half an ampere and this current is also half an ampere. And again this is 2 ohms this is 2 ohms therefore, this is one-fourth of an ampere and this current is one-fourth of an ampere and that current returns here one-fourth of an ampere.

So, this is the analysis of the circuit after deactivating all sources.

(Refer Slide Time: 23:16)



So, we have in the Thevenin's equivalent R not because, after all this is resistive network we do not have to calculate z not, R not equals 1 ohm Thevenin's resistance. Now we have to find the open circuit voltage across the terminals p q, when all this sources are acting are acting at the same time. So, we have we have a network like this; we have a 2 ampere source, we have a 2 volts voltage source, then we are having another source here, that is, this 1 2 ampere and we have a 1 ampere source. And finally, here you have your p q.

So, we want to find out the open circuit voltage. Now we can analyze this network with all the resistor that are given to us; 1 ohm 1 ohm 2 ohms and this also of course, 2 ohms, you can do that. But what I would like to demonstrate is that, we can calculate V o c in a fairly simple fashion, without having to analyze this network by using the data that is available with this analysis and using the reciprocity principle.

We would like to find out  $V_{oc}$  using the reciprocity theorem. How do we do that? Now we would like to after all this  $V_{oc}$  is contributed by these 4 sources because, without these sources  $V_{oc}$  in the circuit is dead, there will be no voltage. So, each of these sources will contribute to  $V_{oc}$ . So, we would like to find out the contribution of  $V_{oc}$  due to each 1 of these in turn. So, let me do that.

(Refer Slide Time: 25:32)

The image shows handwritten notes on a chalkboard. On the left side, there are vertical labels:  $V_{oc}$ ,  $P$ ,  $Q$ ,  $V_{PQ}$ , and  $V_{oc}$ . The main text is written in two sections:

Due to 2V source  

$$\frac{V_{PQ}}{2} = \frac{0.25}{1} \Rightarrow V_{PQ} = 0.5V$$

Due to 1A current source  

$$\frac{V_{PQ}}{1} = \frac{0.75}{1} \Rightarrow V_{PQ} = 0.75V$$

In the bottom right corner of the chalkboard, there is a logo that reads "IIT MADRAS".

I will from here contribution to  $V_{oc}$  so, due to 2 volts voltage source. Now what we are looking for is; if this 2 volts source is only acting all the current sources are removed, what is the voltage here. So, you can say you are exciting this, this is the excitation, this is the response. The voltage here divided by this voltage is what we are after. But we observe here that in this circuit after all once is everything is deactivated, suppose you are exciting this by a current source of 1 ampere, the current here is one-fourth of an ampere.

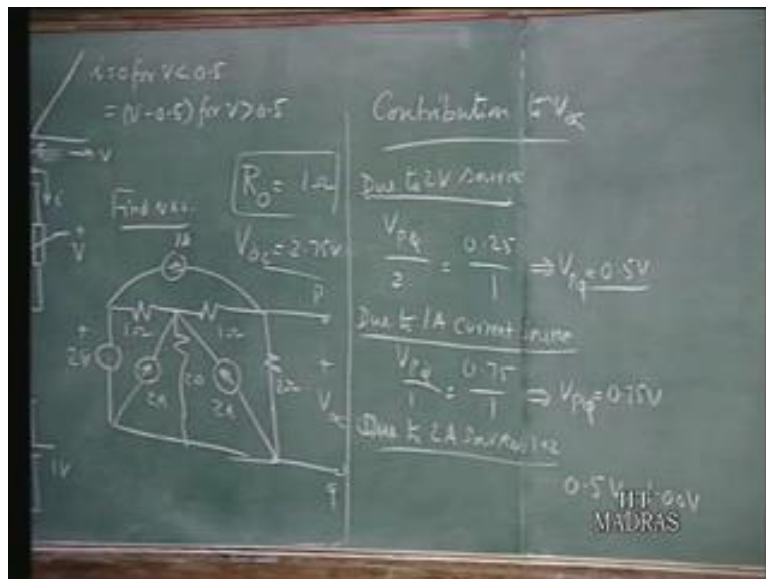
So, it is the 2 branches; the original network is when this source is removed; that means, this is short circuited you introducing a 1 ampere current source and you are measuring here the current as one-fourth. That means the current ratio short circuit this is a 2 port network the short circuit current ratio in 1 direction equals the open circuit voltage ratio in the other direction with a negative sign. And so, we can see that we know the current and this current so, this current ratio will tell us also this open circuit voltage ratio here. Therefore, due to that we can say  $V_{oc} V_{PQ}$  divided by 2 must be equal to one-fourth

by 1 ampere. And the current direction on normally when you take a 2 port the current direction is in this sense therefore, we take open circuit voltage ratio in 1 direction; is a negative of the short circuit current ratio in the opposite direction with this reference direction for current.

Now, the reference direction for current is the other way round therefore, we do not have to take the negative sign, this is equal to 0.25 by 1; this will give you the contribution V P Q to this 2 volt source will turn out to be 0.5. So, due to let us say 1 ampere current source: so, we apply a current at this pair of terminals 1 ampere and we would like to find out the open circuit voltage here. Now we have the result already available to us in the sense; if you apply 1 ampere across these 2 terminals we must know what is the voltage between these 2 points, the voltage between these 2 points is half plus one-fourth; half ampere that is half a volt plus one-fourth volt. So, the voltage between these 2 points is 0.75 volts.

So, when you apply 1 ampere here you are getting 0.75 volts here. If you apply 1 ampere here therefore, you must also get 0.75 volts there. Therefore, you can say V P Q divided by 1 ampere is also equal to whatever voltage you are getting here 0.75 divided by 1; that means, your V P Q due to this is 0.75 volts.

(Refer Slide Time: 29:05)

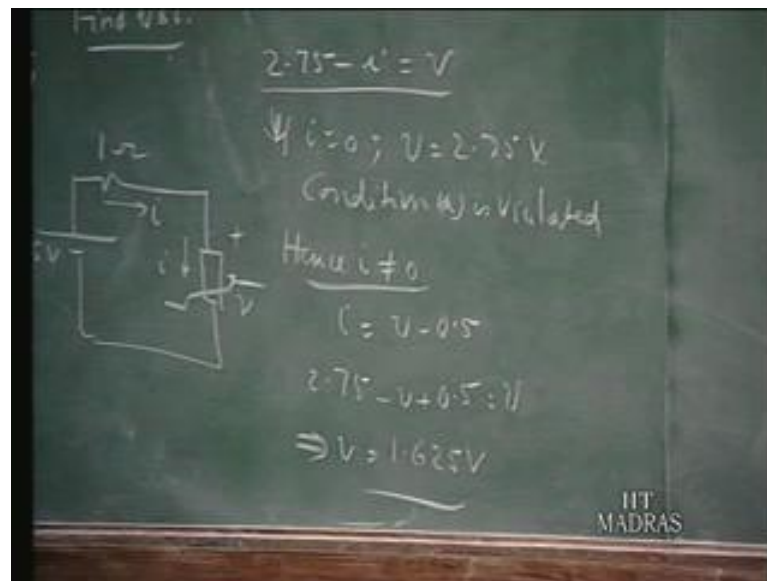


Like this, you can I would leave this as an exercise for you to calculate the contribution to this 2 ampere source and this 2 ampere source. 1 and 2 there are 2 such sources and

you can calculate them to be 0.5 and 0.1 volts So, the total contribution to all this put together will turn out to be  $V_{oc}$  is the sum of all this so, 1 plus 1 0.5 plus 2.75 volts. So, you can complete this work and show that the total voltage is 2.75 volts.

So, once you have that information now.

(Refer Slide Time: 29:48)



You have the Thevenin's equivalent as far the terminals are concerned, you have 2.75 volts and Thevenin's resistance is 1 ohm. And this is now connected to the non-linear resistance here, which has got this  $V-i$  relation. So, you can say that, you can solve this problem using the load line approach which you are familiar with electronic circuits or alternately you can do it analytically. If this is current is  $i$  2.75 minus  $i$  times 1 minus  $i$ ; that is the voltage across this a non-linear element.

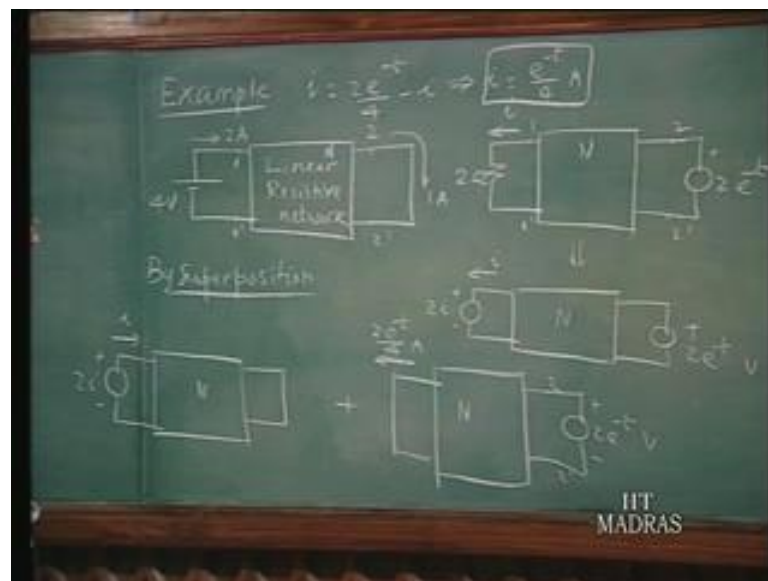
Therefore we must find out  $V$  and  $i$  are also constrained in this manner. So, simultaneously this satisfaction of this 2 whatever set of values pair of values  $V$  and  $i$ , you get should satisfy will be the actual voltage and current. So, this is the relation. If  $i$  is 0 let us say see whether this condition is valid, if  $i$  equal 0 you have 2.75,  $V$  equals 2.75 volts. But then, if  $i$  equals 0  $V$  must be less than 0.5. So, condition a is violated. If  $i$  is equal to 0  $V$  must be less than 0.5. If  $i$  is 0 according to this equation  $V$  equal to 7.5 volts therefore, this is not a proper condition.



Hence  $i$  is not equal to zero. So, it is this relation which is important for us, therefore,  $i$  equals  $V$  minus  $0.5$ . So, when you substitute over this, you have  $2.75$  minus  $V$  plus  $0.5$  equals  $V$ . So, the solution for that will give you  $V$  equals  $1.625$  volts. So, that is the solution and the corresponding current can be found out as  $1.625$  minus  $0.5$ ;  $1.125$  amperes. So, in this example it gives you a nice illustration of various principle that is involved, various network theorems, we used for example, the Thevenin's network theorem we have used and then we have used the series parallel reduction technique for calculating the currents here. We use the principle of superposition because, we found out the effect of various sources on the open circuit voltage, we use reciprocity theorems.

So, we have used reciprocity theorem, Thevenin's theorem, and superposition theorem, we calculate this answer. And we also have observed that Thevenin's theorem can be applied when the load network is non-linear, is this is it this is an illustration of that particular property. Now, let me take another example.

(Refer Slide Time: 32:54)



We have a 2 port network which is a linear and consists of purely resistances; linear resistive network. Suppose we have a 4 volts DC source here and the second port is short circuited, then it is given the current here is 2 amperes and the current here is 1 ampere. This is  $2 \text{ A}$ ,  $1 \text{ A}$ ; this is a first port. Given this information, you are asked to find out the current that exist in port 1, when this is terminated in a 2 ohm resistor and you are asked to find this current  $I$  when, the second port is connected to a voltage source



which is  $2e - t$  volts. So, the same network  $N$ . Given this information, you are asked to find out the current here in this second network.

We would like to apply network theorems to solve this problem. So, what we can do is; if current  $i$  flowing through  $2\ \Omega$  resistor, develops a voltage which is  $2i$ . So, using substitution theorem, we can say this is equivalent to another 2 port network, in which we have a voltage source which is  $2i$ , whatever that voltage source is, whatever that current  $i$  is and then here we have a voltage source which is  $2e - t$  volts. And even in this case, the current here continues to be  $i$ . By using substitution theorem, we have replaced the same element  $2\ \Omega$  resistor by an equivalent voltage source.

Now, we have in this 2 port network acted upon by 2 sources at port 1 and port 2. We can use the superposition theorem and find out the contribution to this current, from this voltage source and this voltage source in turn. So, the by superposition theorem or by superposition principle, we have the conditions in this second network would be obtained by considering this source alone  $2i$  volts and short circuiting the second source, plus short circuiting the first source, that is, at port 1 and applying the voltage  $2e - t$  volts at port 2.

Now, the contribution of these 2 sources to the current here can be superposed and the from that we can get this  $i$ . Now you observe that, in the original network 4 volts applied at port 1 port 2 shorted will give me a voltage of 2 amperes at port 1. So, 4 volts applied to this network shorted at port 2 will give me 2 amperes. So,  $2i$  volts applied to this will give me a current which is one-half of  $2i$ ; that means, the current strength here is  $i$ , that is, using your you are using the proportionality principle from the original network. This is  $N$  of course.

Now as far as this is concerned, when you apply 4 volts here, a short circuited current in the other port is 1 ampere. So, 4 volts applied here leads to 1 ampere; that means, the transfer admittance here is minus one-fourth. But since, the current direction is in this direction therefore, this current divided by this voltage is equal to 1 by fourth. Now you are applying  $2e - t$  volts. So, this is a reciprocal network purely linear resistive network reciprocal network, so if you are applying  $2e - t$  volts here, the current here would be  $2e - t$  divided by 4 amperes. So, we have the

current due to this source is  $2 e^{-t} / 4$ , the current due to this source is  $i$ . The sum of these 2 current must indeed be the current that you are looking for.

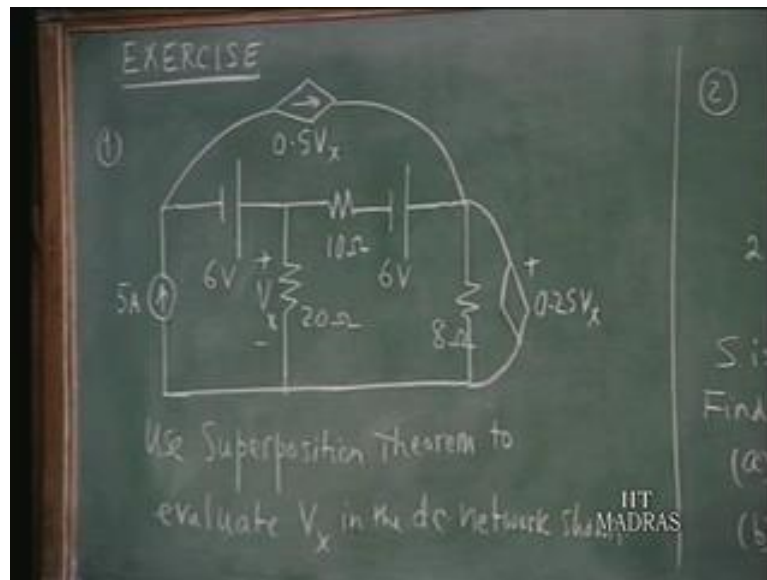
So, from this we equate the sum of these 2 this. So, the current flowing in this direction this  $i$  equals the current flowing in this direction is  $2 e^{-t} / 4$  that is the current here plus the current here, but the current is now in the other direction therefore, minus  $i$ . So, this is the equation the sum of superposing these 2 currents you must get this and this leads me a result  $i$  equals  $e^{-t} / 4$  amperes. So, that is the answer for this question.

Now since this is a purely resistive network, there is a proportionality relationship in time domain between the current and this voltage. So, we did not use any Laplace transformation technique here because, this is not dynamic network; resistive network. The current and the voltage are proportional to each other in the time domain whereas, in a dynamic network such proportionality arises only in Laplace transform domain. But here it is purely resistive network, so we did not use Laplace transformation technique. We know the current here is proportional to this voltage in time domain itself therefore, we did not use Laplace transform. So, finally, the answer is  $i$  equals  $e^{-t} / 4$ .

So, in this example we have used superposition principle and the reciprocity principle. So, this is an illustration of how the network theorems can be applied, to find out required results in a particular case like this. In the last few lectures, we have considered various network theorems and we discussed the various the characteristics of the various theorems and illustrated them with examples here and there.

Now, it is time to work out an exercise and I will give set of problems which you may workout and these problems illustrate the various network theorems that we have studied so far.

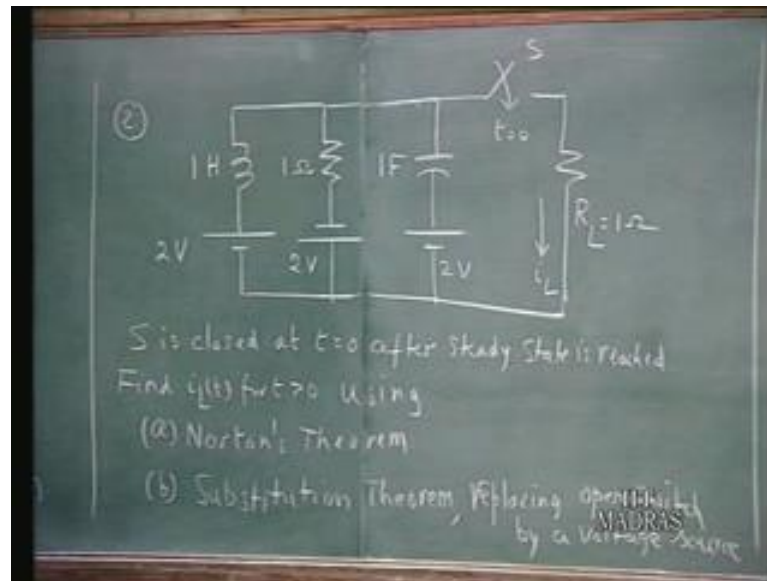
(Refer Slide Time: 39:34)



Problem number 1: we have essentially a DC network, in which there are 2 independent 3 independent sources; 2 voltage source and a 1 current source and we have 2 dependent sources; 1 voltage source and a current source both dependent on the control voltage  $V_x$ . So, you take down this particular figure and use superposition theorem to evaluate  $V_x$  in the DC network shown.

So, in this DC network, we have 5 amperes current source, 6 volts voltage source,  $V_x$  is the control voltage which controls this voltage dependent current source and this voltage dependent voltage source. And we have 20 ohms resistors 10 ohm resistors 6 volts voltage source and of course, 8 ohms resistor. So, use superposition theorem to evaluate  $V_x$  in the DC network shown. And when we working this out, we should keep in mind when you are deactivating the sources you should deactivate only the independent sources; that means, you must analyze this network with 5 amperes working alone with the dependent sources intact, 6 volts acting alone with this and other 1 deactivated and the other 6 volts. That means 3 repeated analyses you have to make and superpose this effects to find out the total voltage  $V_x$  and as a contribution for the 3 individual sources. This is problem number 1.

(Refer Slide Time: 41:07)



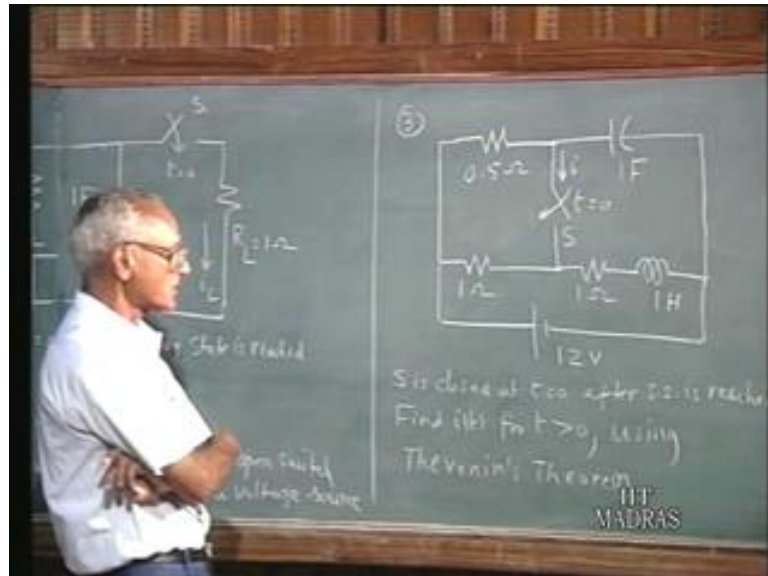
In problem number 2, you have essentially a parallel connection of 4 branches when the switch is closed; the switch  $s$  is closed at  $t$  equals 0 after steady state is reached prior to the closure of the switch. So, when the switch is open there will be there may or may not be a current in the inductor, there is a possibly a voltage across the capacitor. So, you have to use the initial conditions. And once the switch is closed, 4 branches come in parallel. So, you can use the transform diagram approach for solving this problem, work out in the Laplace transform domain. You find out  $I_L$  of  $t$  in the load resistance  $R_L$  using Norton's theorem; that is 1 method of solution.

The second method of solution is; when the switch is open, there will be a open circuit voltage across the switch. So, you replace this by a voltage source using substitution theorem. And when the switch is closed, you have to put another voltage source which is the same magnitude with the opposite side in series with that and use the superposition effects and find out the current  $I_L$  of  $t$  again using sub again by superposing the effect of each 1 set of sources with another voltage source.

So, this is again an illustration or the application of a network theorem. So, we find out  $I_L$  of  $t$  using both these methods and compare the results. And this should be worked out again in transform domain. Again let me say; this is 2 volts 1 Henry 2 volts 1 ohm 2 volts 1 Farad capacitor, this switch is kept open for a long time till steady state is reached

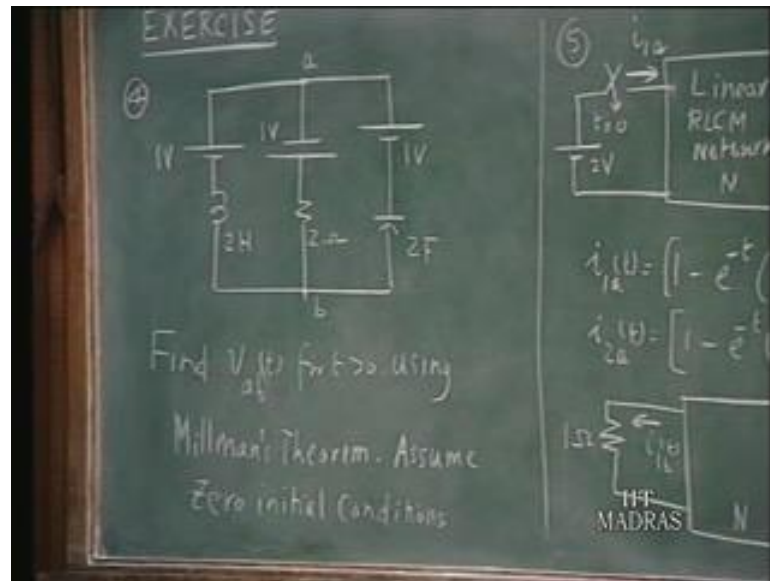
and then it is closed at  $t$  equals 0. You have a load resistance  $R_L$  as 1 ohm. So, you have to find out the current  $I_L$  here.

(Refer Slide Time: 42:56)



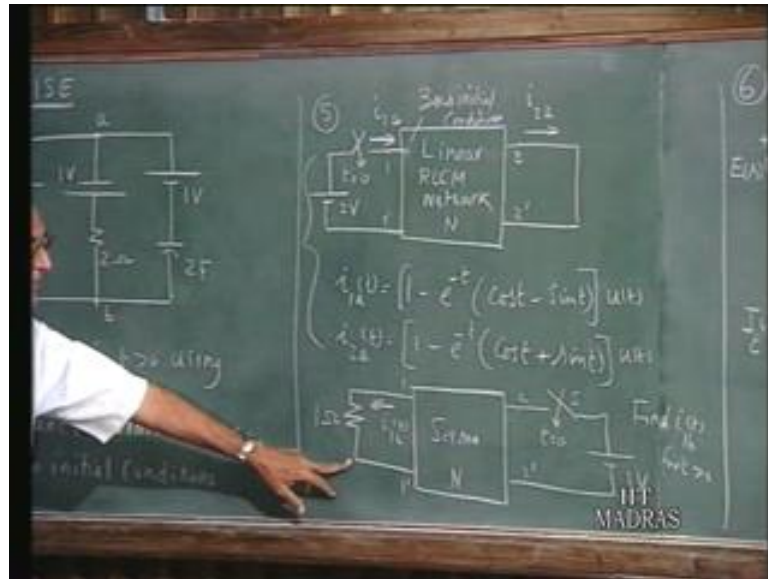
The third problem, again this is a analysis of the transient solution in a particular network.  $s$  is closed at  $t$  equals 0 in this network of a steady state has been previously reached. And you are asked to find the  $i$  of  $t$  in this shorted switch and you have to use Thevenin's theorem for that purpose. The source is 12 volts 1 ohm 1 ohm 1 Henry 0.5 ohm 1 Farad capacitor. And so, you have to calculate the initial currents in the inductor and the initial charge in the capacitor use them and then find out the open circuit voltage here, then looking in impedance, set up the Thevenin's theorem, Thevenin's equivalent circuit and find out the current in this shorted branch. That is the third problem.

(Refer Slide Time: 43:47)



Problem number 4: copy this circuit, it has 3 parallel branches, 3 1 volt sources, this polarity is of course, reversed and you have a 2 Henry inductor 2 ohm resistor and 2 Farad capacitor. Assume 0 initial conditions in the circuit and find out the voltage across these 2 nodes, find  $V_{ab}(t)$  for  $t > 0$  using Millman's theorem because, you have this is a tailor made for a Millman's theorem application, your 3 parallel branches with independent sources and you also put initial conditions sources whatever they might be and find out  $V_{ab}(s)$  and from that  $V_{ab}(t)$ . Assume 0 initial conditions of course initial conditions are absent because, the 0 initial conditions; that means, capacitor inductor are not charged to start with. That is problem number 4.

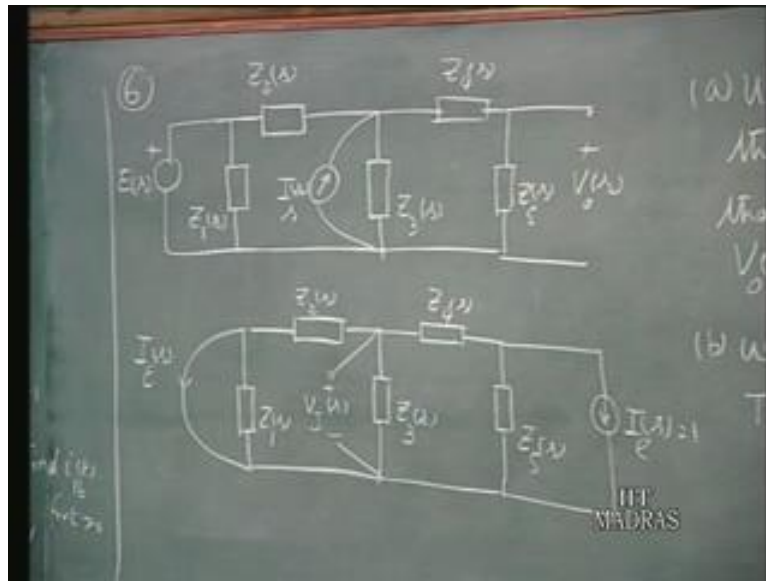
(Refer Slide Time: 44:46)



Problem number 5: you have a linear reciprocal network N with 0 initial conditions. So, when port 2 is short circuit it and you apply 2 volts at port 1M close this switch at t equals 0, current  $i_1$  a and current  $i_2$  a at the 2 ports are given by these expressions;  $i_1$  a of t is 1 minus e to the power of minus times cos t minus sin t whole thing of course, multiplied by u of t,  $i_2$  a t is likewise 1 minus e to the power of minus t cos t plus sin t multiplied by u of t. These are the 2 terms that are given here this.

So, from these conditions, we contemplate this network same network N, is now excited by a 1 volt source at port 2, with a 1 volt source which is closed at t equals 0. No initial conditions in the network N once again. And port 1 you shorted through a 1 ohm resistance. So, you find  $i_1$  b here using the data that is available in the first set of excitations, this is all 1 set of excitation. Using these in the second set of conditions find  $i_1$  b for t greater than 0 again, using possibly reciprocity theorem, possibly substitution theorem, whatever and superposition principle. So, find  $i_1$  b of t in the second set of excitation conditions using the data in the first set.

(Refer Slide Time: 46:39)



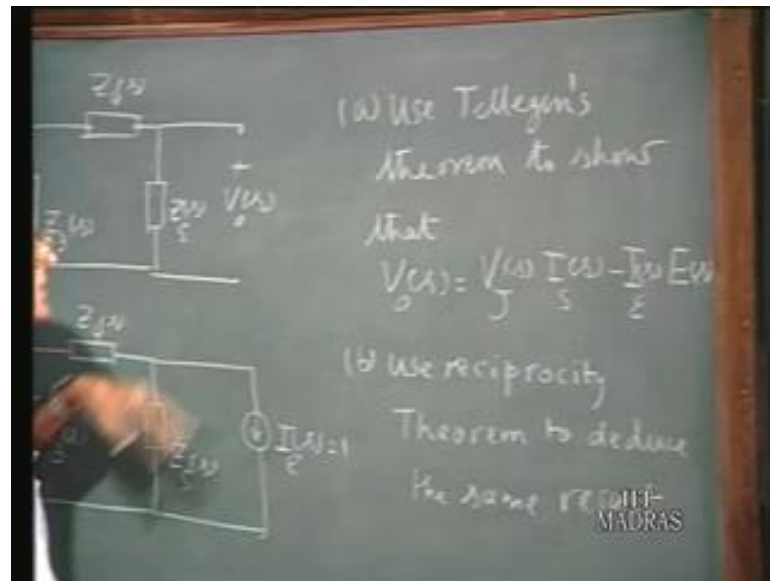
Sixth problem: you have a network comprising  $z_1 z_2 z_3 z_4 z_5$  linear elements. And they are excited by that network is excited by 2 sources; a voltage source  $E(s)$  and current  $I(s)$ . So, these are the sources in the transform domain. You are asked to find you have to find out the open circuit voltage at these 2 terminals. Now, 1 way of arriving at the open circuit voltage  $V_0(s)$  would be not to analyze this circuit as such, but to take another possible circuit like this. The passive portion is same,  $z_1$  of  $z_2$  of  $z_3$  of  $z_4$  of  $z_5$ , but you excite these by a current source  $I(s) = 1$ .

So, in the transform domain that  $I(s)$  is equal to 1. So, if you use that current source and measure these 2 this voltage  $V_j(s)$  here, that is, where ever current source must present earlier you measure this voltage  $V_j(s)$  calculate this. And calculate the current because of this voltage source is replaced by a short circuit calculate  $I(s)$ . So, you calculate  $V_j(s)$ , the open circuit voltage across the terminals of the whole current source and the short circuit current through the whole voltage source location.

So, you do analyze this network and find out these quantities.



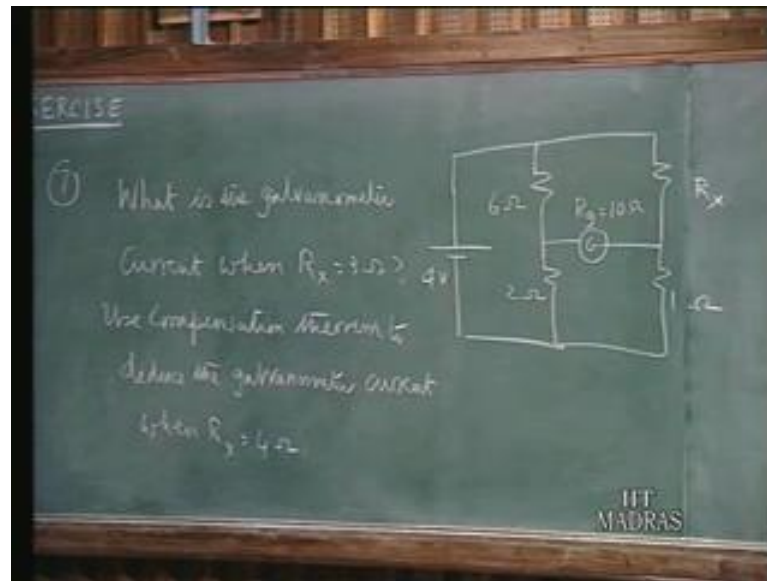
(Refer Slide Time: 48:11)



And now you are asked to show use Tellegen's theorem to both these networks to show that  $V_0$  of  $s$  that is what we are interested is obtained by  $V_J$  of  $s$  times  $I_S$  of  $s$  minus  $I_E$  of  $s$  times  $E$  of  $s$ .  $V_J$  of  $s$  is this voltage,  $I_E$  of  $s$  is current. So, since you have calculated these 2 quantities and you know the strengths of these sources you can calculate  $V_0$  of  $s$ . So, instead of analyzing this network, we should be able to calculate  $V_0$  of  $s$  from this information. Now use reciprocity theorem to deduce the same result. So, derive this result using Tellegen's theorem on 1 hand and reciprocity theorem on the other hand.

So, these are the 2 parts of this question. So, take down these figures once. 1 more problem.

(Refer Slide Time: 49:06)



Problem number 7; what is the galvanometer current when  $R_x$  equals 3 ohms. The circuit is; you have a 4 volt source here, 6 ohms resistor, another 2 ohms resistor and you have a resistance  $R_x$  unknown resistance, a galvanometer whose resistance is 10 ohms and a 1 ohm resistor. So, find out the galvanometer current when  $R_x$  is equal to 3 ohms. Use compensation theorem to deduce the galvanometer current when  $R_x$  equals 4 ohms. So, if the  $R_x$  value is changed from 3 ohms to 4 ohms, what would be the new galvanometer current. So, this is an illustration of the application of the compensation theorem; the similar example, which we worked out in our lecture earlier.

So, this set of 7 problems in this exercise, will provide you enough scope for having some practice in the applying the various network theorems that we have discussed in our 2 3 lectures on network theorems, which you have covered so far, that have been covered so, far.