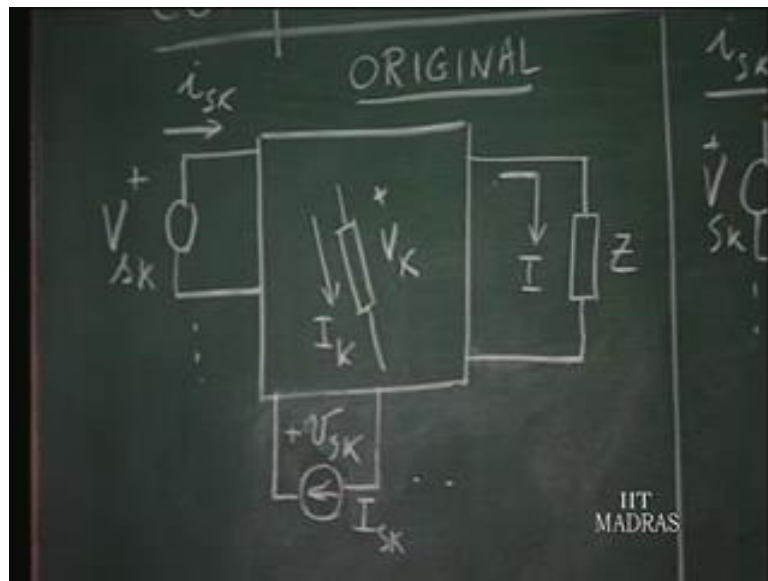


Networks and Systems
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Lecture - 36
Network Theorems (3)
Compensation Theorem (contd.)
Tellegen's Theorem

In the last lecture, we looked at the statement of the compensation theorem. And worked out a simple example to illustrate, the application of the compensation theorem. We promised ourselves at that time. That, we will look at a justification or the proof of the compensation theorem in the next lecture. And that is what we are going to do presently.

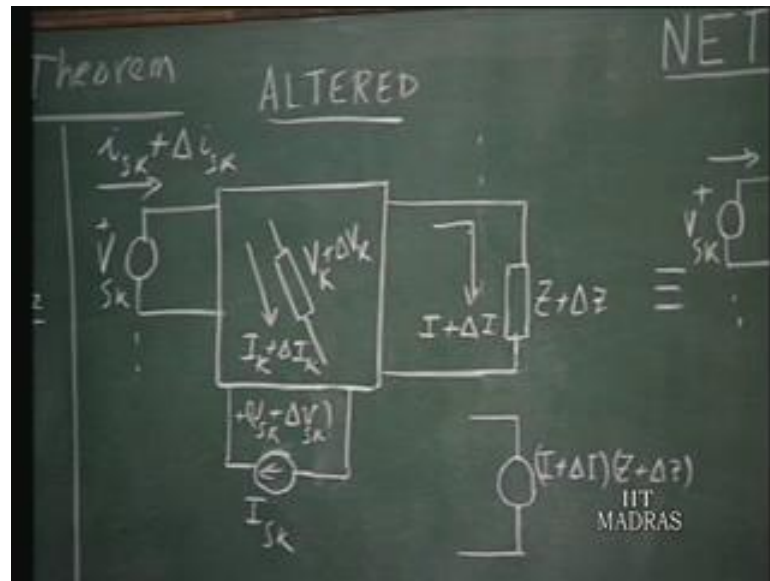
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We have a network comprising various linear elements. A typical element two terminal element is indicated here, carrying a voltage V_k carrying current I_k . We have a number of independent sources. They represent the source the k 'th source is indicated here. Number of current sources k 'th current source is indicated here. The voltage across current in the voltage source is indicated by i_{sk} .

We do not want to use the capital letter because that has been reserved for this. And the voltage across the current source is given a small v_{sk} . And there is an impedance Z in the network which carrying a current I . Now, the question which you would like to ask is what are the changes produced in the network, when Z is changed to Z plus ΔZ ? So, this is the original network and this is the altered network.

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In the altered network, we have the impedance changed to z plus Δz . Otherwise the parameters are the same. However, the variables in the network undergo changes, the voltage are the independent voltage source cannot change. Because that is a independent voltage source. But, the current changes from i_{sk} to Δi_{sk} , the voltage across the current source changes from v_{sk} to v_{sk} plus Δv_{sk} . And this general voltage of an element changes from V_k to V_k plus ΔV_k and the current in this changes from I_k to ΔI_k .

All this variables in the original network and here are functions of S . We take them to the Laplace transform variables. And the impedances or functions of a generalized impedances functions. So, we would like to analyze this to find out the alterations in the various variables in the network. To do that, since a current I plus ΔI , flowing through Z plus ΔZ . By substitution theorem we can replace this impedance by a voltage source, which has got this strength.

A current in the voltage developed across this is $I + \Delta I$ times $Z + \Delta Z$. So, this is the voltage of this voltage source, which can be used in the place of this $Z + \Delta Z$. And the rest of the network will not see any change. That is the principle and the substitution theorem.

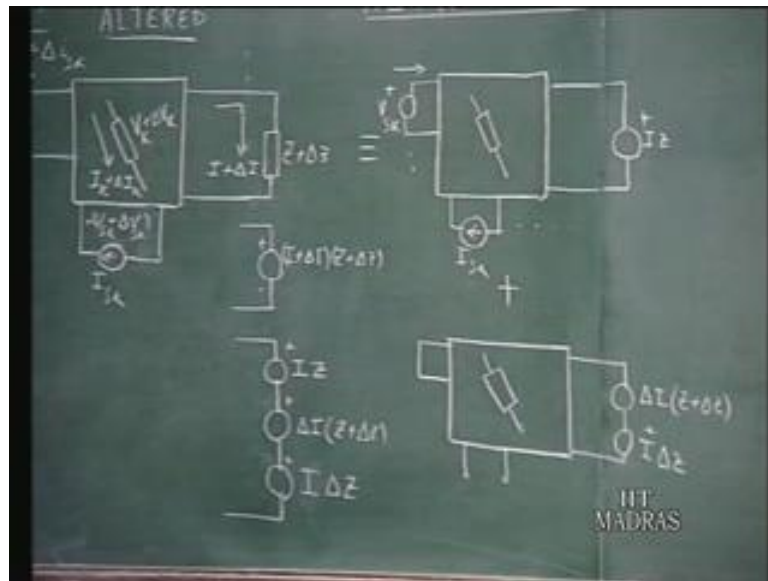
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Now this, a composite voltage source, can be split up in this fashion, which the purpose for that will be evident later. But, let us say $I + \Delta I$ times $Z + \Delta Z$ is broken up into three sources their sum will add up to this. $I Z$ I times Z that is one source, ΔI times $Z + \Delta Z$ that is one source. And I times ΔZ that is third source. So, sum of these sources will add up to this value. So, instead of this impedance ((Refer Time: 04:28)) we can replace the series combination of these three sources. And say that the conditions in the network will not change.

Now, so we have as far as this altered network is concerned. We have the various voltage sources various current sources. And these three independent voltage sources standing for the impedance $Z + \Delta Z$. Now, to find out the conditions in this network under these excitations. I use the principle of superposition. So, I take all the voltage sources and the current sources origin originally existing plus this single source.

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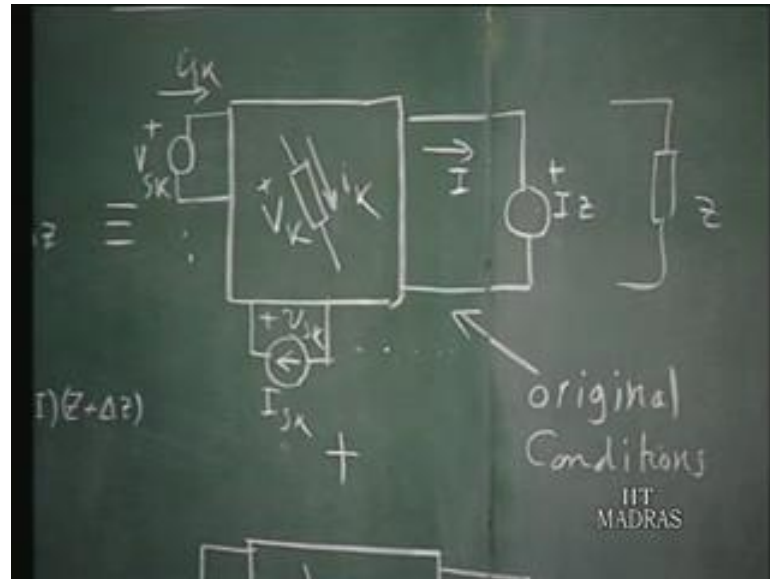


So, V_s is I_s and this I_z , that is I consider the effect of all those sources once. And when you do that these two sources must be short circuited; therefore, that is they are not present here. And then I would like to find the effect of these two sources alone keeping the other sources as deactivated. Therefore, I write ΔI times Z plus ΔZ and I times ΔZ , these are the two sources that are acting.

And all the other sources are deactivated. Therefore, this voltage source is short circuited, this current source is open circuited. And similarly all other current sources and all other voltage sources. Now, let us look at this network. In this network we have essentially the same conditions as in the original network. Because, in the original network if you have I_z ((Refer Time: 05:53)) I can certainly replace this by a voltage source I times Z .

And the network will not change will not all the parameters, will not all the variables in the network will remain the same. So, in the original network instead of the impedance Z carrying a current I , I could as well replaced it by a voltage source I times Z .

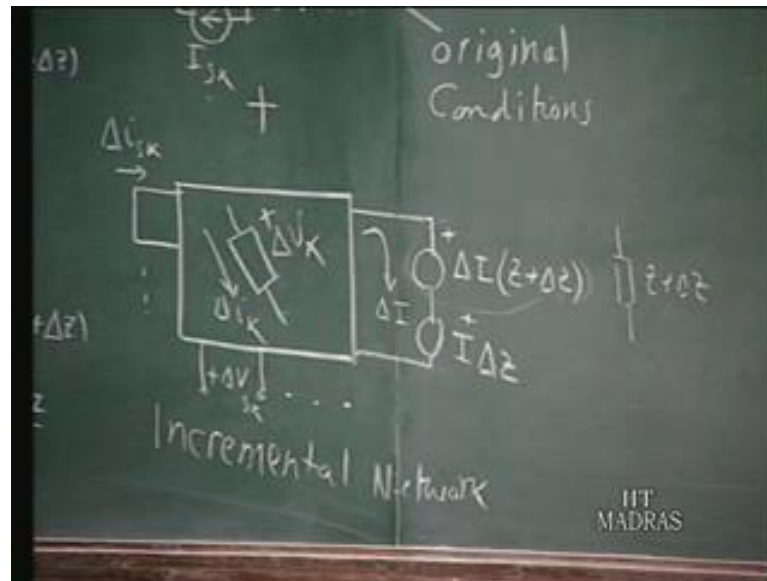
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Therefore, that is exactly what we have here, therefore, the variables inside here for this network is the same as the original network. So, this is the original network, original conditions. Because, this is same as the original conditions; that means, this I_z is equivalent simply an impedance Z . And the current here will remain as I , this is V_k and the current here is I_k , this is I_{sk} , this will be V_{sk} and this will be I_{sk} .

So, this is the all the conditions in this network are the original conditions. Consequently, the sum of the conditions here and conditions here must be the conditions in the altered network ((Refer Time: 06:57)). Therefore this network will tell us the changes that are brought about in the various voltage and currents. So, this we can call this the incremental network. And in the altered network the total current here is I plus ΔI

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This is I therefore, this has got to be ΔI by the principle of superposition. Because it is a linear network, the sum of these quantities must be equal to the quantity here. Therefore, if this is I this must be ΔI . And this voltage this is V_k and this V_k plus ΔV_k ((Refer Time: 07:40)). Therefore, this must be ΔV_k and the current here like wise is Δi_k . And this in the shorted lead here this is $I_{s k}$ this is $I_{s k}$ plus $\Delta I_{s k}$. Therefore, this must be $\Delta I_{s k}$ and the voltage here must be $\Delta V_{s k}$.

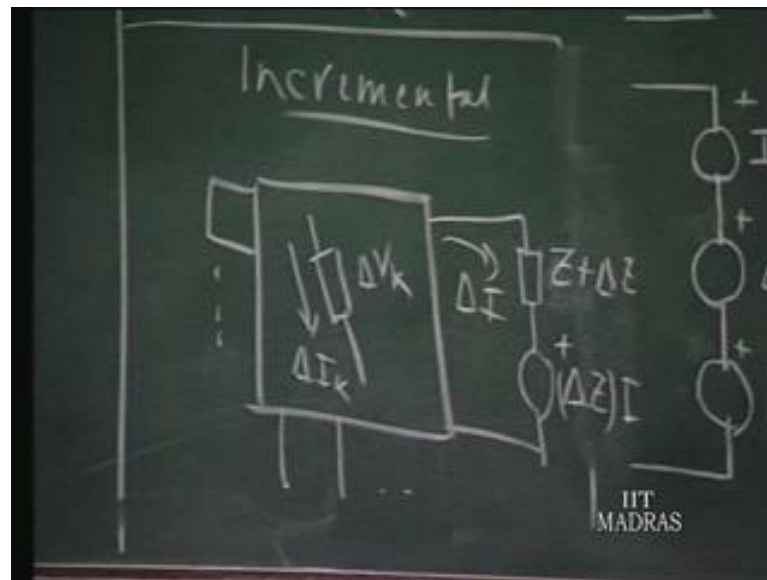
So, if you solve this network, you will be able to find out all the quantities, all the changes that are brought about as a result of change in impedance. That we do not know what ΔI is, how do we find that out, I mean that is exactly what we want. So, you do not know this strength of this voltage source. But, then you observe a ΔI passing through an impedance Z plus ΔZ produces just that much voltage.

So, again using the substitution theorem in a reverse manner you can replace this by means of an impedance, which is Z plus ΔZ . So, if you replace this voltage source by a impedance Z plus ΔZ , ΔI flowing through that will produce that much voltage source. So, we use the substitution theorem in reverse fashion. And say replace this portion by an impedance Z plus ΔZ for this. So, ΔI flowing through that will produce that much voltage.

So, as far the incremental network is concerned all we have to do is deactivate all the original independent sources. Introduce a voltage source I plus I times ΔZ in series

with altered impedance. And put the altered impedance in position Z plus ΔZ . And this source working into this network will establish currents and voltages in this network, which are the increments that are caused as a result to the change in impedance. That means, the incremental network.

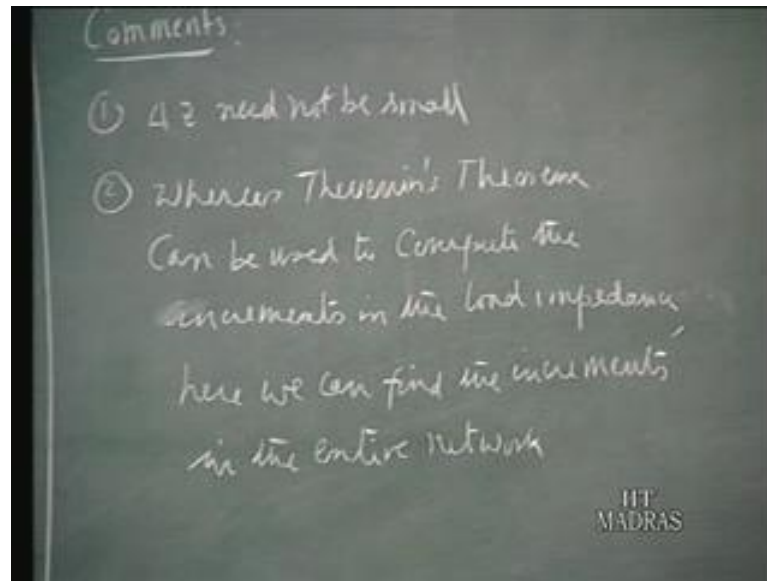
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So, we will write here. So, the incremental network will be simply one in which all the voltage sources are short circuited. All the current sources are open circuited. And you have the altered impedance Z plus ΔZ . And you have a source voltage source, which is the change in impedance ΔZ multiplied by the current, which was originally flowing in that element.

So, whatever this voltage source produces, those will be the changes in the network ΔV_k ΔI_k . So, this is what we said is the statement in the compensation theorem in the last lecture and this is the proof of that.

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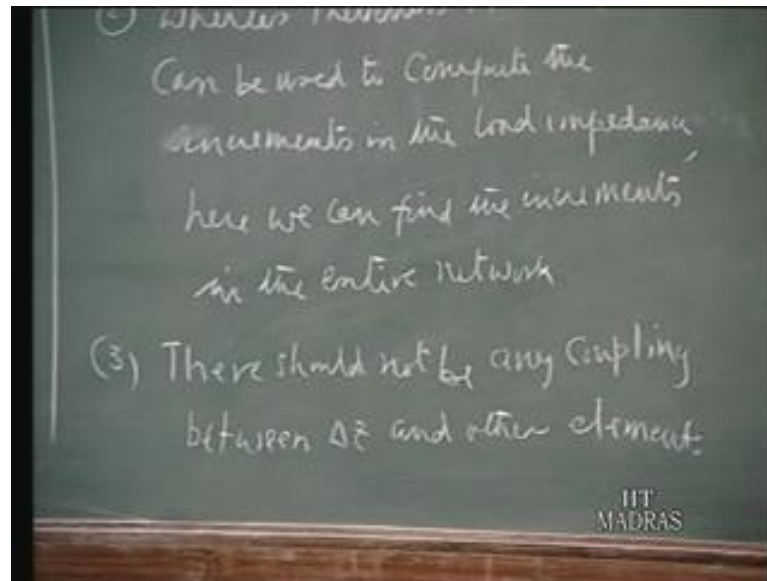


A few comments on this, first of all ΔZ is only a change need not be small. You have no, where used the fact that ΔZ is a small increment could be quite large. It could be 100 percent, 200 percent, no problem at all. Now, secondly if you compare this statement of the theorem with the Thevenin's theorem. If you replace the original network by Thevenin's theorem and then change the Z from Z plus ΔZ .

You can calculate the increment that are produced in the load impedance only. Whereas here if you find out the changes in all the elements. Whereas Thevenin's theorem can be used to compute the increments in the load impedance, as a results of a change.

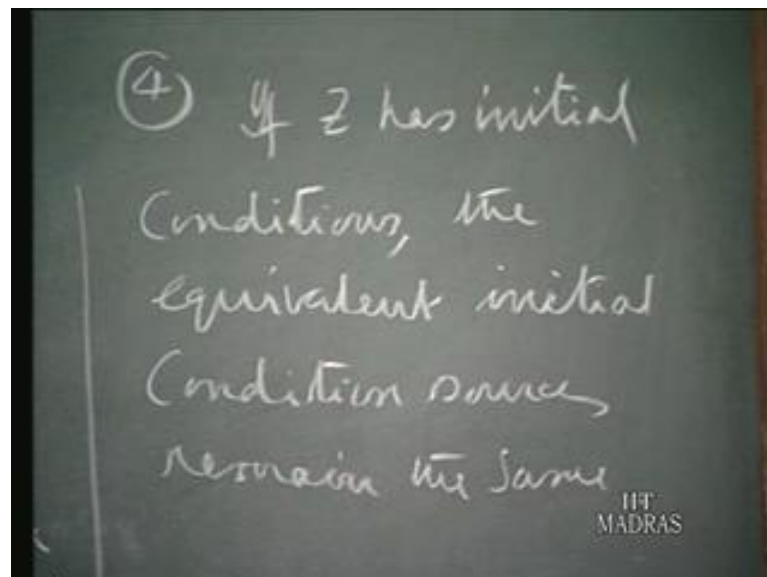
Here we can find the increments in the entire network. So, this is a advantage. Now, we what we have to see another thing is.

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There should not be any coupling between ΔZ and other elements in the network. Because when we change this, because a coupling element is destroyed. Therefore, we should not have a mutual coupling or coupling through controlled sources between the ΔZ and other elements of the network.

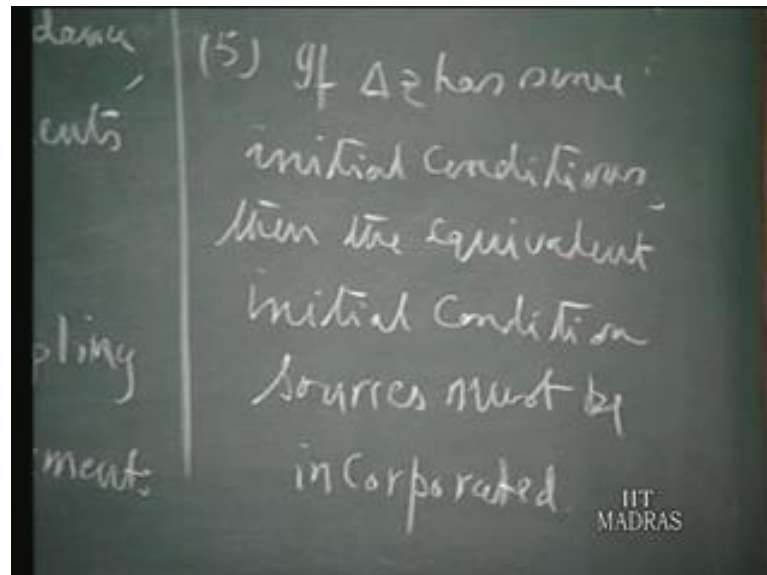
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The fourth point is if the original impedance Z has initial conditions. The equivalent initial condition sources remain the same. That is if originally we had in this impedance some inductance carrying a current or a capacitance carrying a voltage and you have to replace

this by initial volt sources here. They continue to be the same in this network there is no problem about that.

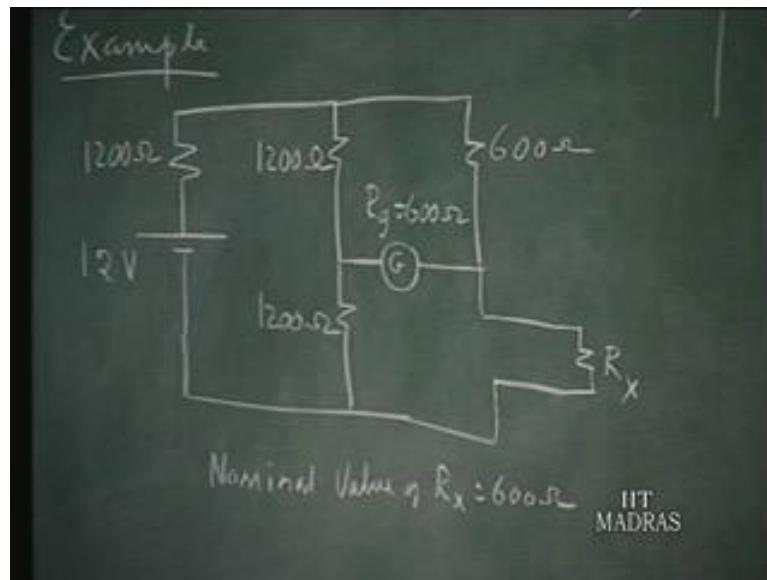
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However, if ΔZ has some initial conditions, what we mean is an inductor carrying a current or capacitor having a voltage. Then the equivalent initial condition sources must be incorporated. That is in addition to ΔZ times I we must also have in addition source. Usually you have voltage source in series with that to represent the initial conditions that are present in ΔZ .

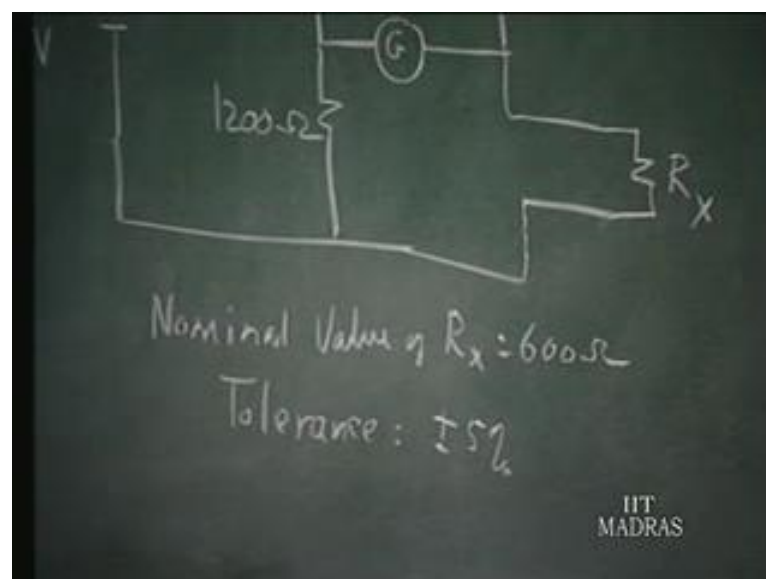
So, this is something which we have to pay attention to. Apart from this you must also keep in mind the polarity of this incremental network ((Refer Time: 15:02)) this is the incremental network, original current was in this direction. So, ΔZ times I if ΔZ is positive. Then the current must be in the up in the source should be such as to drive the current in the opposite direction, this is something which we have already mentioned.

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So, let us work out an example to illustrate this idea. We have here a Winston bridge network, which is used to test a resistor R_x . So, these are the values of the resistors in the fixed arms. And the resistance of the galvanometer is also 600 ohms. Now, the nominal value of the resistance R_x is 600 ohms. So, if this is 600 ohms the bridge is balanced 600, 600, 1200, 1200, galvanometer current is 0. So, the galvanometer current is 0, when the R_x happens to be 600 ohms.

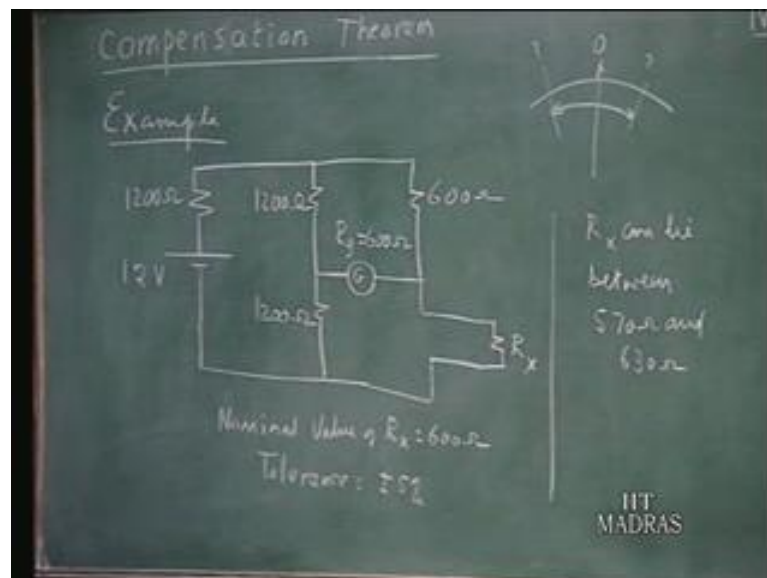
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So, the nominal value of R_x equals 600 ohms. However, this bridge is used to test a whole lot of resistors and the tolerance allowed for the resistor is plus or minus 5 percent. So, we would like to test a whole batch of resistors. And as you put each resistor we should like to define the range of current, which is acceptable if the R_x is to be within the tolerance. That means, if the resistance is can change from 570 to 630. So, depending upon the value of the resistance the galvanometer current centre.

Zero galvanometer current will not be here, it will be deviate one way or the other. And we would like to set up the what is the lower limit of the current; and what is the upper limit of the current. If the resistance is to be within the tolerance limits. So, that is the problem.

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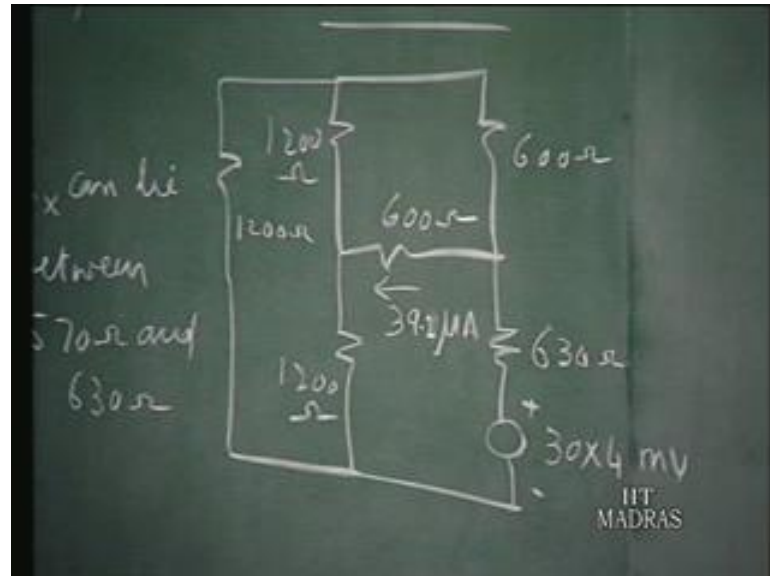


So, R_x can change R_x can lie between 570 ohms and 630 ohms and we would like to find out the galvanometer current for that. Now, if R_x was 600 ohms the galvanometer current is 0. So, if from 600 ohms it deviates to 630 ohms what is the galvanometer current we would like to establish. Similarly, from 600 ohms it changes to 570 ohms, what is the galvanometer current that is you should establish.

Since the original galvanometer current I_g is 0, what we can do is, we can when we can use the can use the compensation theorem conveniently. And when R_x changes from 600 to 630, what is the incremental current. That is produced in the galvanometer could be indeed the total current. Because, the original current was 0.

So, using compensation theorem, we will set up the incremental network for when R_x goes to 630 ohms.

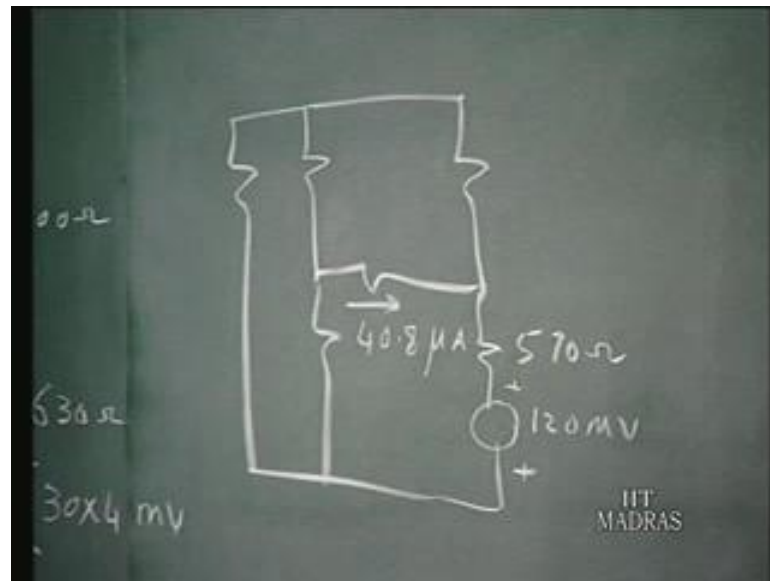
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So, you have the original network 1200, 1200 and the galvanometer current galvanometer 600 ohms, this is 1200, this is 600 ohms. And the source resistance is 1200 ohms and the independent sources are deactivated and here in the galvanometer this is 1200. And in the galvanometer branch you have the altered resistance, which is 630 ohms. And you have a DC source, which is equal to the change in resistance, which is 30 times the original current.

The original current here can be computed and it will be equal to 4 milli amperes. So, the current here is 4 milli amperes in the original network. So, 30 into 4 milli volts. So, with this voltage, what is the current in the galvanometer. That is what we should calculate, it can be shown you can use this network. And then calculate the current here in the galvanometer and it can be shown to be 39.2 micro amperes.

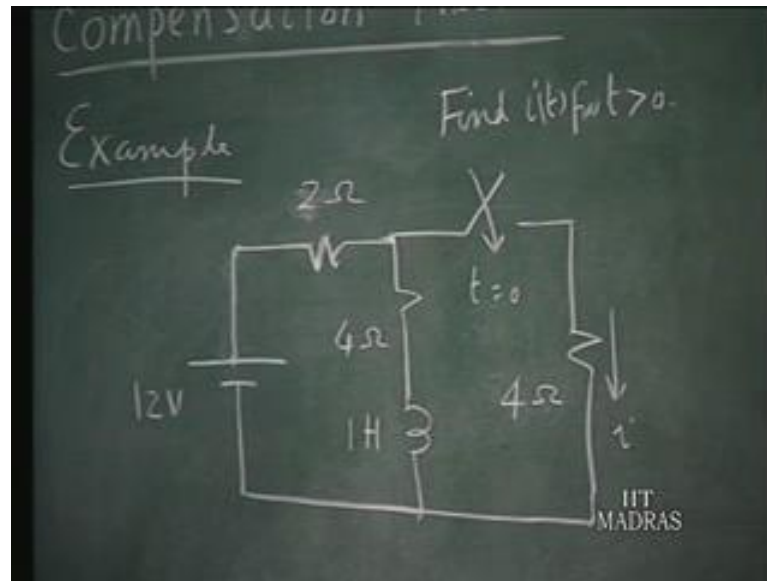
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In a similar fashion you also value calculate the current in the galvanometer, when the resistance changes to 570 ohms. And now the change in the resistance is minus 30 ohms and the original current is 4 milli amperes. That means, the voltage will be 120 milli volts in this direction. Because ΔZ is negative therefore, it should be in the opposite direction. So, with this you can calculate the current and this can be shown to be equal to 40.8 micro amperes.

So, we can now say that the galvanometer current must be from 30.8 or the 139 point in one direction to 40.8 micro amperes in other direction. As long as the current lies within that margin the resistance is 570 to 630 ohms. If it exceeds that margin on either side, then the resistance goes outside the tolerance limits. So, this is an example of illustration of use of compensation theorem in situation like this. Let us work out a second example this is a slightly more complicated example, where Laplace transform method is used. Let us see, what it is.

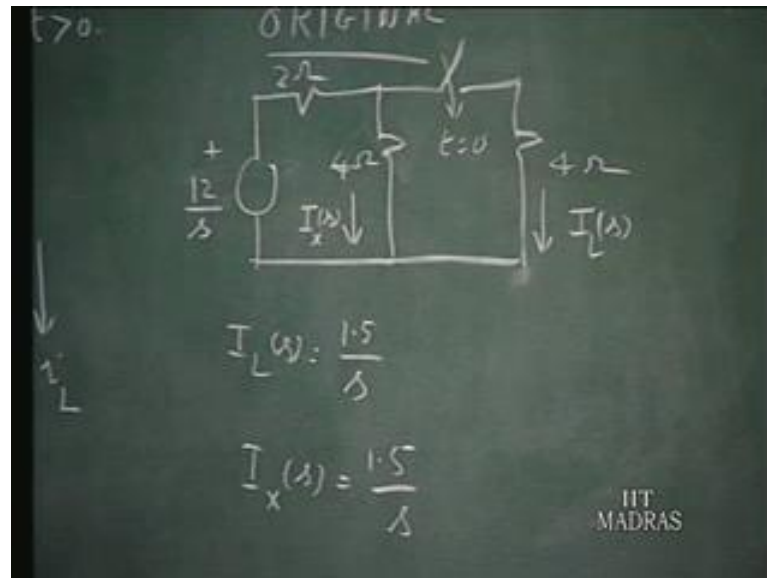
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You have a 12 volt source DC source operating in a circuit, which has one is the inductor. And then the switch is closed at t equals 0 connecting a 4 ohm resistor. Find I of t for t greater than 0. So, that is the straight forward problem. We can do it is a loop kind method of analysis or Thevenin's equivalent and so on, but we would like to use compensation theorem for that.

What we propose to do is, we would like to solve a resistive network in removing this inductance. So, then it will be pure resistance, we solve that and we consider that as original network. Now, we introduce this inductance here as the additional impedance introduced into this circuit. And then use the compensation theorem to find out the effect that are produced.

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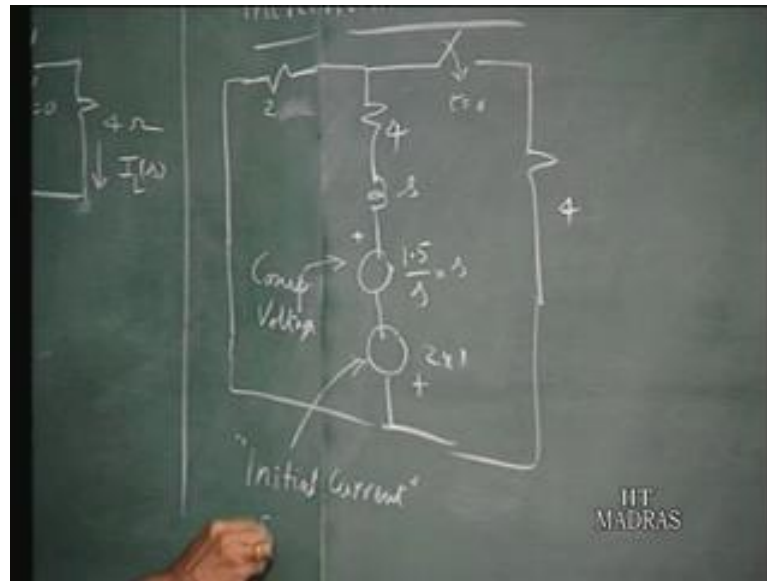


So, we can say that we have the original network, which has a purely resistive affair 2 ohms, 4 ohms. And then another 4 ohms and then this is 12 volts. I can we will use in Laplace transform domain. So, I will use this as 12 by s. And in this network if the switch is closed at t equals 0, what are the various currents that are produced.

So, there will be a current here and this current. So, the current here will be of course, this I will call this I_L . So, we will say that is the load current I_L , I_L of s and I will call this I_X of s. So, this is original network we can solve for that quite straight forward fashion.

The load current I_L of s will turn out to be 1.5 amperes. Therefore, 1.5 by s is that current because after all in the DC network 4 2 4 ohms in parallel with the 2 ohms 2 ohms in series with 2 ohms is a 4 ohms. Therefore the current here is 3 amperes therefore, the 1.5 flows here, 1.5 amperes flows there. So, I_X of s is also 1.5 by s, this is the original network. Now, we want to introduce an inductance in this circuit and find out, what are the alteration that are produced here.

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So, in the incremental network, we have this 4 ohms and in the inductance is introduced of impedance function is s . And this original source is deactivated that is the it is replaced by a short circuit. And here again the switch is closed of course, t equals 0 and then a 4 ohms this is also, this is also 2 this is 4, this is 4.

Now, the change in the impedance is s and the original current in the inductance branch is $1.5/s$. So, the compensation voltage that you get here is 1.5 by the by s that original current multiplied by the change in impedance which is s . This is the compensation voltage right. But, that is not all because, this inductance here in the network has also a initial current, before the switch was closed the inductance is carrying a current.

And what is that current 12 volts divided by when the switch is open here, 12 volts divided drives a current of 2 amperes to this 2 ohms and 4 ohms in series. Therefore, a 2 ampere current was flowing through that. Therefore we have a two times inductance is one. So, that voltage is also should be there this is the initial current. So, the source which represent the voltage source, which represent the initial current in the inductor that also must be put in place.

So, this is the incremental network. And you can use this incremental network to calculate the current here. When the switch is closed and that current plus this current will be the total current. Therefore you have this current can be shown to be I_L dash s in

this you can calculate this. And it can be shown to be or we can say delta I L may be likewise better you can use after this is I L we can call it delta I L of s.

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Handwritten equations on a chalkboard:

$$\Delta I_L(s) = \frac{-0.5}{3s + 16}$$

$$\Delta i_L(t) = -\frac{1}{6} e^{-\frac{16t}{3}}$$

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You can analyze this network and calculate delta I L of s it will be point minus 0.5 divided by 3 s plus 16 or it can be shown that delta I L of t is minus 1 6th e to the power of minus 16 t by 3. So, I L of t here is 1.5

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Handwritten circuit diagram and equation on a chalkboard:

The diagram shows a circuit with a 4Ω resistor and a current source. The current through the resistor is labeled i_L . The total current is labeled I_L .

$$i_L = 1.5 - \frac{1}{6} e^{-\frac{16t}{3}}$$

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Therefore, the total current in this altered network original network is I L will be 1.5 minus 1 6th e to the power of 16 t by 3. So, this is the answer for this problem. So, this

example illustrates an interesting application of compensation theorem, where we have the reactive element completely removed in the original network. Introduce the reactive element as an additional impedance. And calculate the increment that are produced with in this network.

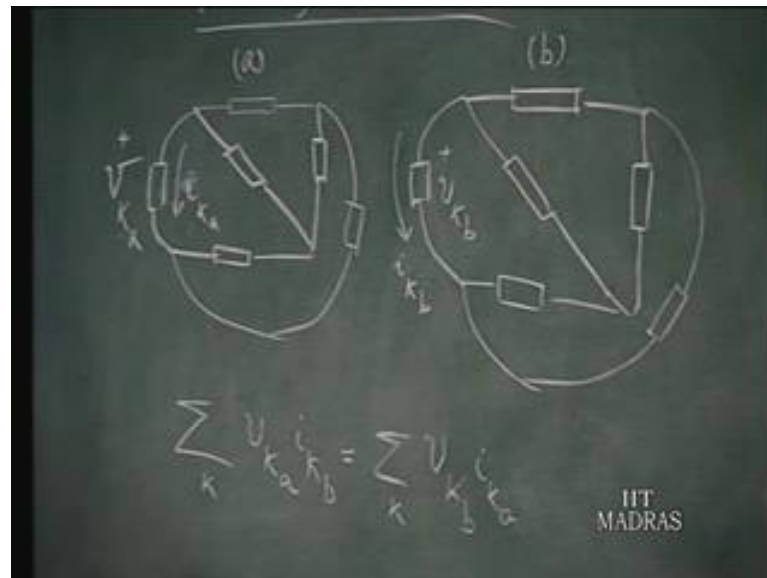
We shall now discuss a new network theorem, which has the name Tellegen's theorem. BDH Tellegen was a research scientist with a Philips laboratories in Netherlands. And he proposed this theorem around 1952 or so on. And he did not receive the attention world wide at that time 15, 20 years later it was received world wide attention, because of its generality. And the fact that the whole lot of network theorems can be derived from Tellegen's theorem.

You would find in most textbooks it in or circuits theory written prior to 1969 or there about very rarely contains a reference to Tellegen's theorem. But, now-a-days it would not be should be difficult for us to spot a textbook a modern textbook, which does not contain a reference to Tellegen's theorem. That means, its importance has been boosted up.

This is as a result of the discovery that Tellegen's theorem can be employed in studies of sensitivities of networks. And similar other situations to a great advantage. In fact, the generality of Tellegen's theorem gives it a unique place for circuit theory.

It has all the same foot in as Kirchhoff's law; that means, it has as fundamental to network theory as Kirchhoff's laws in a sense. Let us see what Tellegen's theorem is statement of Tellegen theorem is. And we would like to illustrate this by an example. And use this for one or two application to derive other network theorems. So, we will take up first of all the statement of Tellegen's theorem.

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Let us consider two networks of identical topology. So, we have a network like this, another which has got the same geometry, same manner of connection of elements. But these two networks are not the same they have. However, the same geometry the same method of interconnection of the various elements. We will call this network a, call this network b.

And we have not made any specific commitment about the nature of the elements, the elements here quite arbitrarily. The elements here also quite arbitrarily and this element need not be the same as this element. They may be voltage sources, current sources, whatever element we have. Now, suppose you take the kth element here. That means, each element kth element is a one to one correspondence with kth element here.

Because the geometry is the same, suppose we say the voltage across this is v_k in the a network and current through that is i_k . We will say small v_k , i_k . Similarly here let us say this is v'_k and the current here is i'_k that the kth element and like that we have a lot of elements.

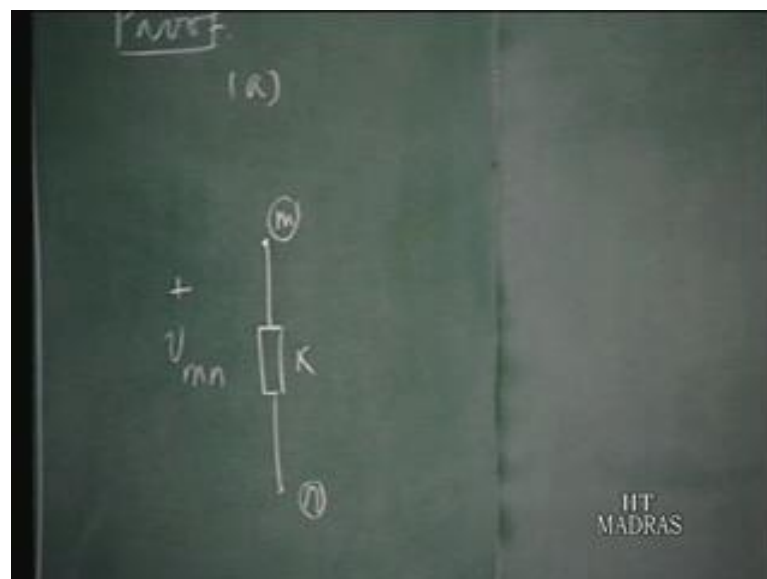
Now, suppose I take this voltage of the kth element here multiplied by the current in the kth element here. And sum them up over all elements I will get v_k multiplied by i_k summed on k over all elements in the network.

Now, let us let me do the other reverse operation take the voltage of the k 'th element in the b 'th network multiplied by the current in k 'th element in the a network. Then I will get $v_k b$ times $i_k a$ Tellegen's theorem says, both these are equal this is equality.

You take the k 'th element here voltage of k 'th element here multiply with a current here. Sum of over all possible elements in the network. Similarly, take the currents voltage in the k 'th element here multiply by the current in the k 'th element in the a network. This is the summation that you get and both these are equal. And it does not make any difference what the elements, the elements could be quite arbitrary. So, this is the statement of the Tellegen's theorem.

All we demand is that the two networks have identical geometry. We call that identical topology; that means, there must be one to one correspondence between the two an element here and other element there; from the point of view of interconnection with the other elements.

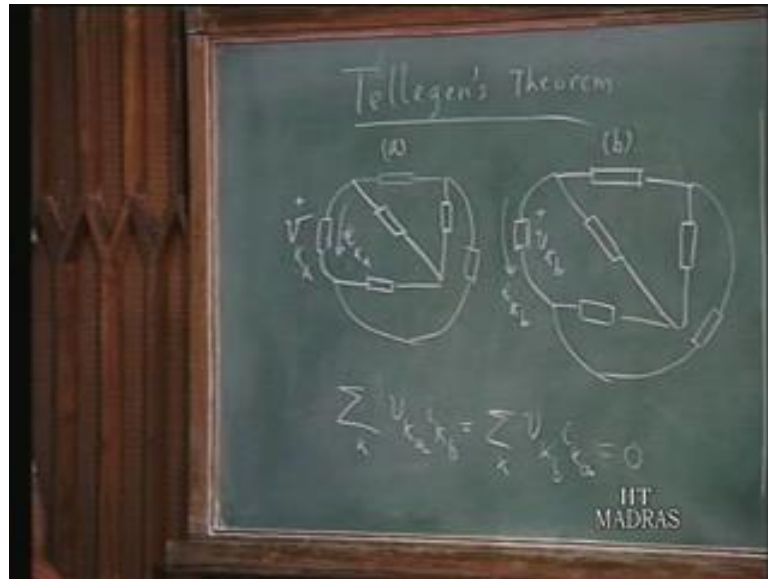
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Now, proof of this can be given in this manner. Suppose in the a network you have the k 'th element like this. And let us say the nodes of the k 'th element are m and n . So, you have the k 'th element, this the k 'th element you have a voltage $V_m n$ in the a network.

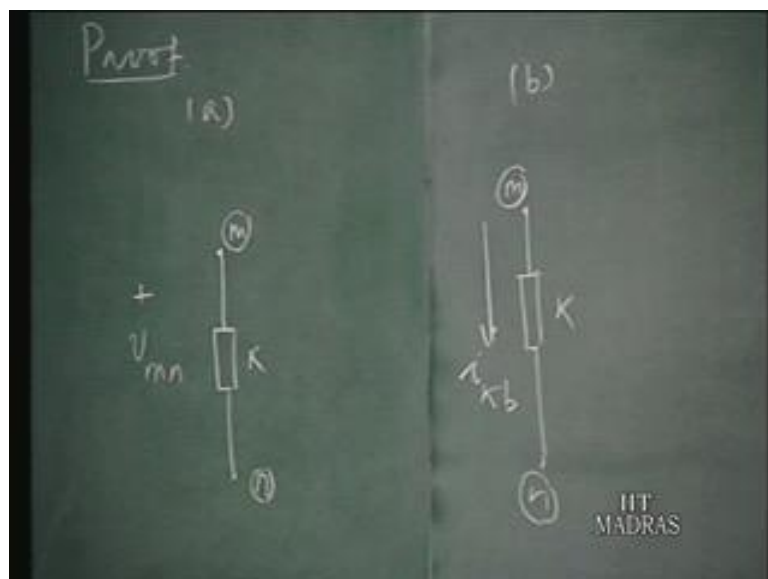
And let us take b network.

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By the way Tellegen theorem says us not only these two are equal, but they are also equal to the 0. You take the voltage of the k'th element multiply the current in the k'th element in the b branch sum them up it will be 0. Similarly, if you do the reverse operation; that means, both these expressions are equal to 0. So, we would like to show this product sum of this product is equal to 0, that is what we are going to establish.

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So, in the b network also we have a k'th element which is connected between the nodes m and n. And the current here is $i_{k,b}$ that is the current in $i_{k,b}$. Now, let me multiply this by $V_{m,n,a}$. Let me multiply this voltage by this current.

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$$\sum_{\text{all elements } k} (V_{m,n,a})_a i_{k,b} = \sum_{\text{all elements } k} [(V_m)_a - (V_n)_a] i_{k,b}$$

$$= \sum_{\text{all node pairs } m,n} [(V_m)_a i_{k,b} - (V_n)_a i_{k,b}]$$

$$= \sum_{\text{all nodes } r} N_r (\text{Algebraic sum of currents leaving node } r \text{ in network } b) = \sum (V_r)_a \times 0 = 0$$

So, $V_{m,n,a}$ multiplied by $i_{k,b}$ that is what we want to take the product. And then multiply do the sum them over all possible elements, so all elements k . So, we do that for all elements k we want to show that this is going to be equal to 0. Now, we can write this as after all the voltage between the nodes m and n can be written as $V_{m,a}$ minus $V_{n,a}$ in the a 'th network multiplied by $i_{k,b}$ summation all elements k . This node the voltage across the k 'th element $V_{m,n}$ the drop from m to n . Am writing the node voltage of m with respect minus the node voltage of n with reference to some datum node in this diagram.

Now, this can be further be written as all node pairs m,n , $V_{m,a}$ times $i_{k,b}$ minus $V_{n,a}$ times $i_{k,b}$. So, after all I have expand that and I have put this $V_{m,a}$ times $i_{k,b}$ minus $V_{n,a}$ times $i_{k,b}$. Now, if you look at this, what we are trying really doing is. We are taking the node voltage $V_{m,a}$ multiplied by the current that passes through $i_{k,b}$ leaving this node m in the network b . So, if you do this for all possible nodes, then what you will get is. Suppose you take r an r 'th node r .

So, we will say all possible all nodes are. So, let us take an r 'th node some node you have when you have an r 'th node here somewhere, then you have various elements

connected here. So, if you carry this summation over all possible node pairs, these r 'th node figures whenever there is a current involved in these three elements. And what we have seen is when the current i_{kb} leaves the node m we have given it a positive sign.

And when the current enters the node n we have got a negative sign here. So, if you consider state stock of such products summed up over all possible nodes r . Then we have V_r figures wherever there is a current. Wherever we take a branch which is incident at r is concerned.

And we are going to multiply V_r by a the current through that branch with a positive sign. When the current leaves that node r and with the negative sign, when the current enters the node r . So, if you look at this carefully we can write all nodes r times or a. Of course, a some algebraic sum of currents algebraic sum of currents leaving node r in network b . So, these are the currents in the network b .

So, we have take the node voltage r in the network a multiplied by the algebraic sum of currents leaving the node r in the network b . And by the Kirchhoff's current law this is going to be 0. Therefore this will be V_r a times 0 and so, the whole thing is 0. So, that is the principle of Tellegen's theorem. What we what it say is you take the voltage here multiply by the current in the corresponding branch, sum up over all branches that is going to be 0.

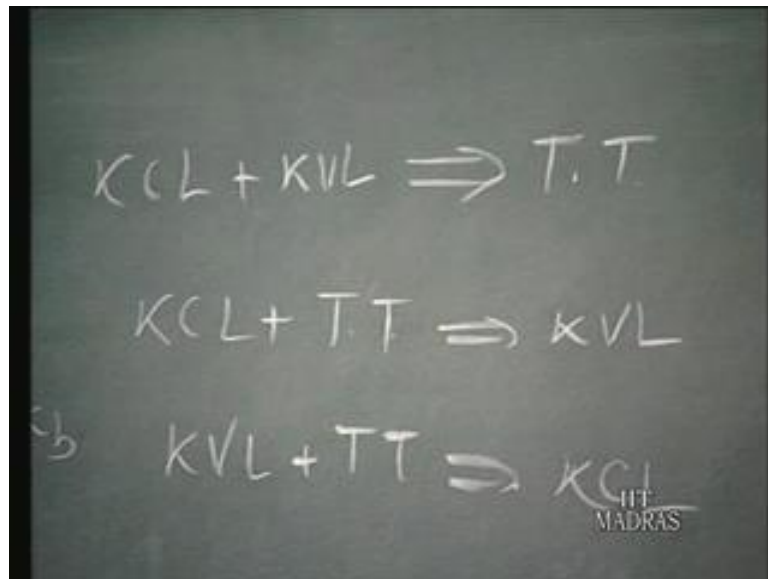
And like wise if you take the voltages here multiplied by the currents here and sum up over all branches that likewise is going to be 0. So, this is the principle of Tellegen's theorem. In this one thing we have should note is we have made use of Kirchhoff's current law and voltage law in deriving Tellegen's theorem. We have already made use of Kirchhoff's current law here.

You see unless this algebraic sum of currents leaving node r in the network b is 0 we can not get this result. We also made use of Kirchhoff's voltage law, whether we have recognized this or not. You observe that I said V_{mn} a the voltage between this I across this element k is V_{mn} a. We said is equal to node voltage of m with reference to datum minus the node voltage of n with reference to the datum.

Such an identification of node voltages is possible only if you have Kirchhoff's voltage law valid. Because, when you go round a loop between two nodes. If you get different voltages as you take two different paths, then Kirchhoff's voltage law is not satisfied.

So, we said we will have when we say a particular voltage node voltage is a definite value. Then we have assume that Kirchhoff's voltage law is valid. So, that is very important and that is what we have assumed.

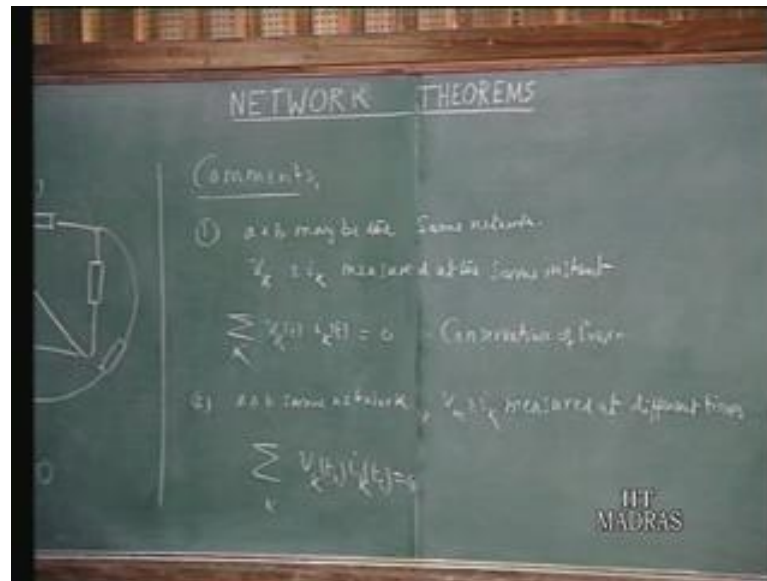
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So, in a sense Kirchhoff's current law plus Kirchhoff's voltage law have been used to deduce Tellegen's theorem. T T means Tellegen's Theorem. In fact, it can be shown that any two will lead to the third. We can also show I do not do it here, but Kirchhoff's current law plus Tellegen's theorem will imply Kirchhoff's voltage law.

Kirchhoff's voltage law plus Tellegen's theorem implies Kirchhoff's current law. So, these three are intertwined in such a way that any two will lead to the third one a few comments before we take up some example.

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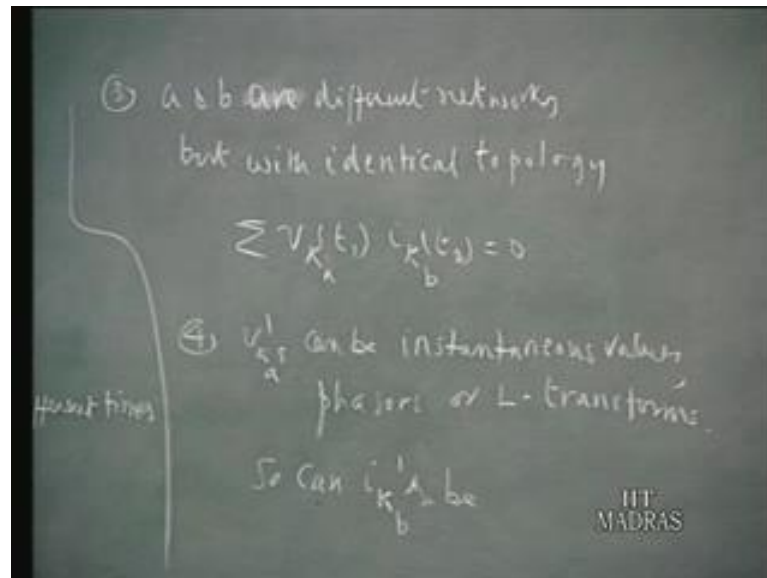


Few comments on Tellegen's theorem comments or amplifications of this. It is possible that we can make identify a and b, a b may be same network. That means, we are applying this Tellegen's theorem in the same network. Currents in the one network and the currents in the same network. And v_k and i_k measured at the same instant. So, if you take the currents and voltages to be at the same instant Tellegen's theorem says $v_k t$ times $i_k t$ summed up over all k after there is no point in now saying a and b, because it is the same network $v_k t$ times $i_k t$ is 0.

This is a statement of conservation of power in the network. If you take instantaneous power associated with each element add them up over all element that is going to be 0. This is something which we already know. This is a consequence of Tellegen's theorem this is a conservation of power. It is a statement of conservation of power which of course, is not very surprising.

Suppose you have a and b same network v_k and i_k measured at different instants measured at different times. So that means, if you take v_k at t_1 and multiply by i_k at t_2 then also it is 0 on something. This is not something which is immediately evident it is not a manifestation or conservation of power. You can take currents in this voltages in a network at a time t_1 multiply the currents at another time t_2 still their product is 0, this is surprising.

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Third thing a and b of different networks of different networks, but with identical topology then only this theorem will work. Then v_k^a multiplied by i_k^b in general $\sum v_k^a$ that is also $\sum i_k^b$ this is also surprising. In fact, we can even generalize this v_k^a 's can be instantaneous values, phasors or Laplace transform. So, can v_k^a 's, so can i_k^b 's.

So; that means, you can have all this all the v_k^a 's can be either instantaneous values can be phasors can be laplace transforms. So, can i_k^b and it is not necessary, that if you choose a set of v_k^a 's as a Laplace transforms i_k^b 's should also be Laplace transforms. The surprising thing is I can multiply the Laplace transform of voltages multiplied by the instantaneous values of currents. In the second network they have no physical meaning the product, but still this is 0.

It means as long as set of voltages that you take satisfy Kirchhoff's voltage law as long as the set of currents you take satisfy Kirchhoff's current law. And you know phasors satisfy Kirchhoff's current law, the Laplace transform satisfy Kirchhoff's current law, instantaneous values satisfies Kirchhoff's current law.

So, as long as the whole set of v_k^a 's satisfies Kirchhoff's voltage law. The whole set of currents satisfy Kirchhoff's current law. Then you can use this theorem and you can know there is no compulsion for that for us.

That if v_k 's are Laplace transforms i_k 's should likewise be i_k 's could be instantaneous values. Here as a matter of fact any set of variables, which satisfy voltage a constraint Kirchhoff's voltage law constraints. Current law constraints can be used to for this in this in this particular application of Tellegen's theorem.

And another important feature is the network elements are arbitrary, they can be linear or non-linear. Active or passive reciprocal or non reciprocal. Whatever is this um absolutely no restriction on the network elements. The only requirement is Kirchhoff's current law and Kirchhoff's voltage law must be valid and the two networks must have identical job atleast. So, that you can put one element in one network in one to one correspondence with the other element.

So, this is very powerful theorem and as I said, when it was proposed by Tellegen around 1950 did not receive the attention, that was due it was only later on that people found out the generality of this. And there is a nice book written by Penfield's Spence and Duinker around 1970, which is titled the Tellegen's theorem. And it is applications it shows how Tellegen's theorem can be employed to prove several other network theorems.

And so, that is a very general kind of network theorem and it is a very important one. So, in this lecture we had first of all started with the proof of the compensation theorem, how an impedance in a network changes from Z to ΔZ Z plus ΔZ . How the increments brought in the network have to be computed by analyzing the network with a single source which is equal to ΔZ times I . So, if the ΔZ carries initial currents and initial energies.

Therefore you have to view another source also. And but, all the other independent source inside the network are deactivated. We worked out one or two examples to see the advantage of application of compensation theorem in solution of networks. Then we went to discussion of Tellegen's theorem. We mentioned Tellegen's theorem applies in its generality to two networks which have identical geometry or topology as they are called; that means, the manner of interconnection of elements is identical.

So, that one element could be placed into one to one correspondence with another element in the second network. Tellegen theorem says that if you take the voltage of a element in one network multiplied by the current in the other network. And do this for all networks and add them up will turn out to be zero. no matter what the network elements

are, no matter what type of voltages what whether it is instantaneous values or Laplace transform variables that you are talking about.

No matter what the current variable are, the only requirement is individually the voltage variables must satisfy Kirchhoff's voltage law. In every loop in network in one network. And the current variables must satisfy the Kirchhoff's current law in network in which they relate. We will in the next lecture illustrate the Tellegen's theorem by means of an example. And then we will also use it to prove another kind, another network theorem named as reciprocity theorem applicable to two port networks; that will be in the next lecture.