Networks and Systems

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Lecture - 35
Network Theorems (2)
Thevenin's Theorem
Norton's Theorem
Millman's Theorem
Compensation Theorem

We discussed in the last lecture, the Superposition Theorem and the Substitution Theorem. And we also mentioned that, we had already discussed the reciprocity theorem while, we were dealing with the two port networks. We will now move ahead. And have a review of the Thevenin's theorem, using the framework of the Laplace transform domain.

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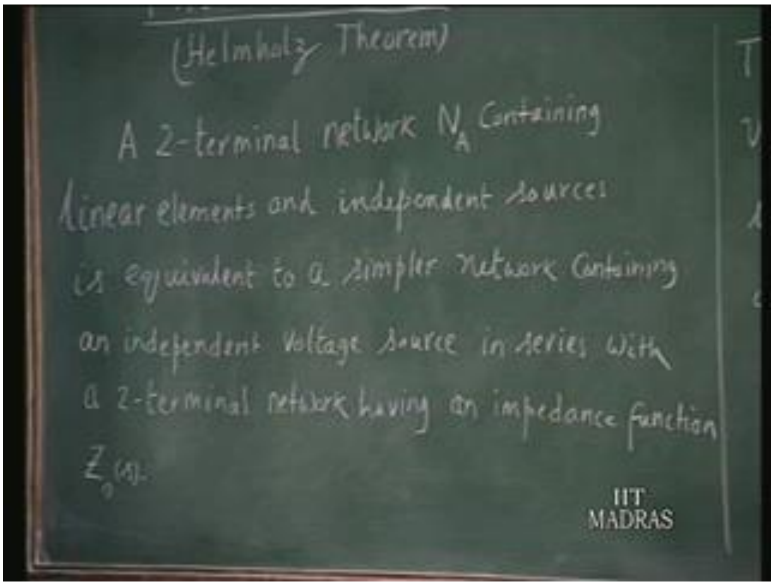

Thevenin's theorem is also known as Helmholz theorem, in the German literature. Because, there is some dispute about as who originally proposed this theorem. In the English literature is of course, is commonly known as Thevenin's theorem. In the, in German literature you will often find this referred to as the Helmholz theorem. The statement of the theorem is runs an analogous lines to, what you are familiar with in the context of DC circuits or AC circuits.

But, in the case of Laplace transform domain in the context of Laplace transform domain, we can put it in this fashion. A two terminal network, N A containing a linear elements and independent sources is equivalent to a simpler network, containing an independent voltage source in series with a two terminal network, having an impedance function Z naught of s.

So, a whole complex network containing several sources and a number of linear elements connected in a complicated arbitrary fashion; can be reduced, to a very simple series equation circuit, containing one independent source. And a two terminal network or a one port network in series with the source, which has a driving point impedance Z naught of s. Now, how do you calculate the source?

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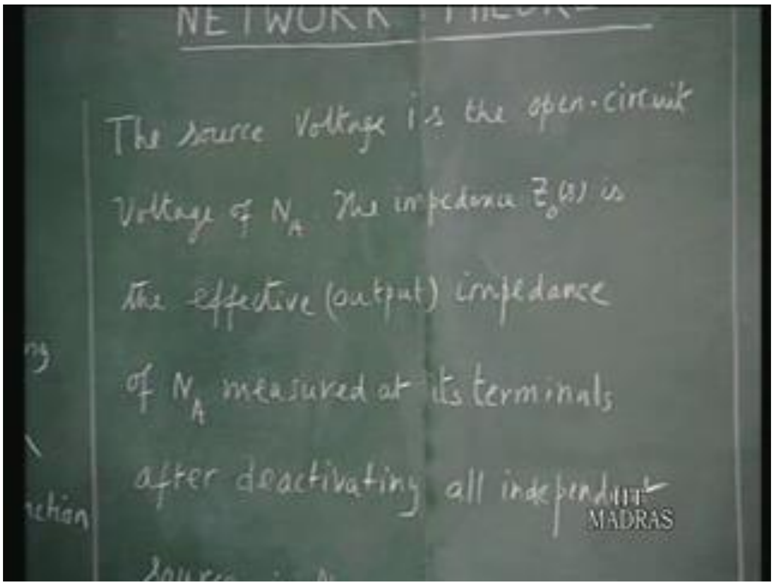
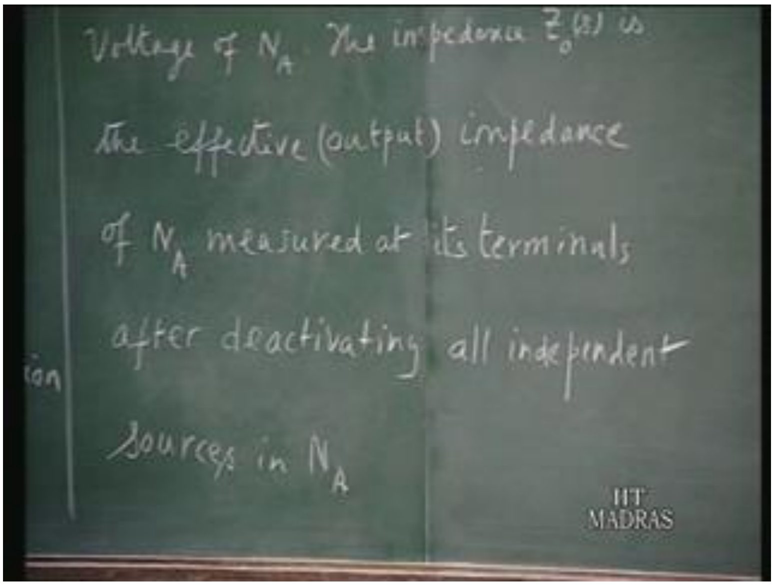


The source voltage is the open circuit voltage of N A. So, in time domain it is the open circuit voltage in time domain or you can use in Laplace transform domain. It is the Laplace transform of whatever is the open circuit voltage, obtained here. That is when the two terminals of N A are kept open. Whatever voltage you get it is Laplace transform will be then, the source voltage transform.



The impedance Z naught of s, which is to be connected in series with a source is the effective impedance, also called the output impedance of Na measured at its terminals after deactivating all independent sources in N A. When we say deactivate all independent sources, we mean that the voltage sources must be replaced by short circuits.

The current sources must be replaced by open circuits. Now, this statement of theorem is, I am sure familiar to all of you. Except that, you are now putting it in terms of transformed quantities. Now, how do we justify this theorem? Let us look at this chart.

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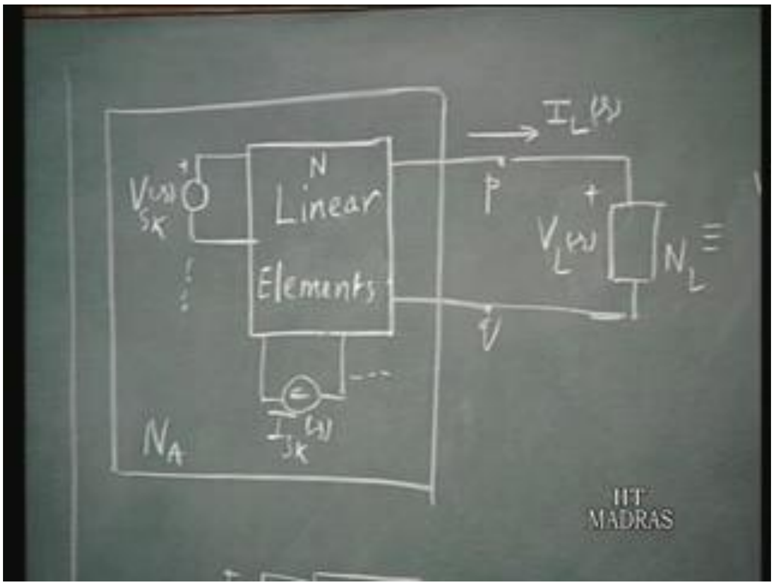

This is our N A, which contains linear elements acted upon by several sources. So, diagrammatically we can represent that we have a network N, which contains purely linear elements. And we have several voltage sources and current sources, acting in the network. So, we put all the linear elements inside this box and call that N. And represent the various current sources and voltage sources, as connected to this N externally.

So, this is a typical voltage source. I put V s k of s. That means, this is a k source. There may be several such sources. Similarly, there is a current source. One current source is represented. There may be several current sources, which are also acting at the same time. This is, the whole thing constitute a two terminal network N A with the terminals p and q. So, we would like to establish an equivalent for this entire network.

What do we mean by equivalent? That is, if this network has a terminal voltage V L of s and a terminal current I L of s, when it is connected to a external network N L. Then, if the equivalent is connected to N L, you should get the same quantities V L of s and I L of s. When we talk about an equivalent network, it goes without saying that the external conditions must remain invariant. We do not worry about, what happens inside this network as far as the terminals are concerned.

We get the same effect, when it is connected to any other arbitrary network. So, I would like to indicate this as N A, meaning that is an active network containing independent sources. N is the linear part of it. And what is connected externally is the, what we may refer to as load network. It could be any network, as long as it has two terminals. So, I refer to this as load network, for a simple for a simplicity.

And I represent this as a N L. So, when this two port network, when this one port network is connected to N L. Let us say there is a voltage V L and a current I L in the transform domain. Now, by substitution theorem there is a current I L passing through this.

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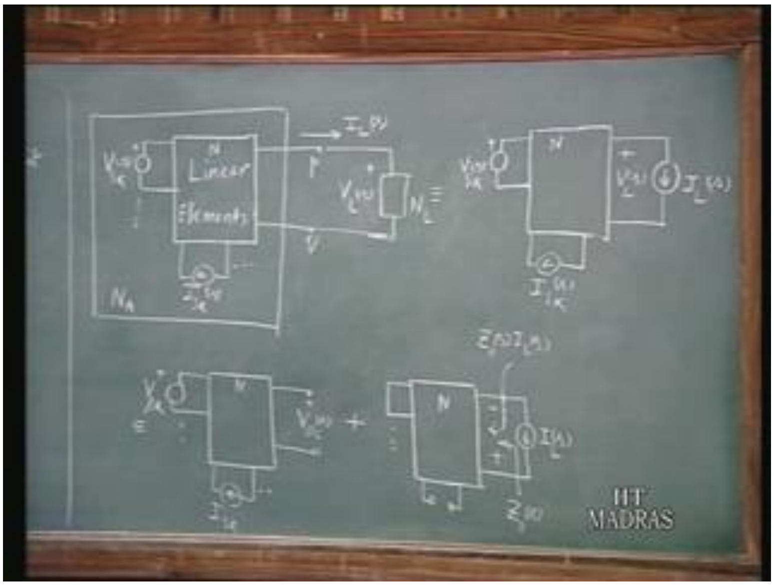

So, if I connect I L of s a current source of exactly as same magnitude as this, same value as this same function as this then, there should not be any disturbance created in the network. So, all elements will continue to have the same variables associated with them. So, if I connect a current source here, replace this N L by a current source as I should. I should still get V L of s here. And all elements inside, must have the same variable associated with them.

When we talked about substitution theorem, we said an element can be replaced by a current source or a voltage source. It need not be one element. We can have a whole, one port network can be replaced by I L of s, as long as the current in the one port is I L of s. So, this particular equivalent is obtained by using substitution theorem replacing N L by I L of s.

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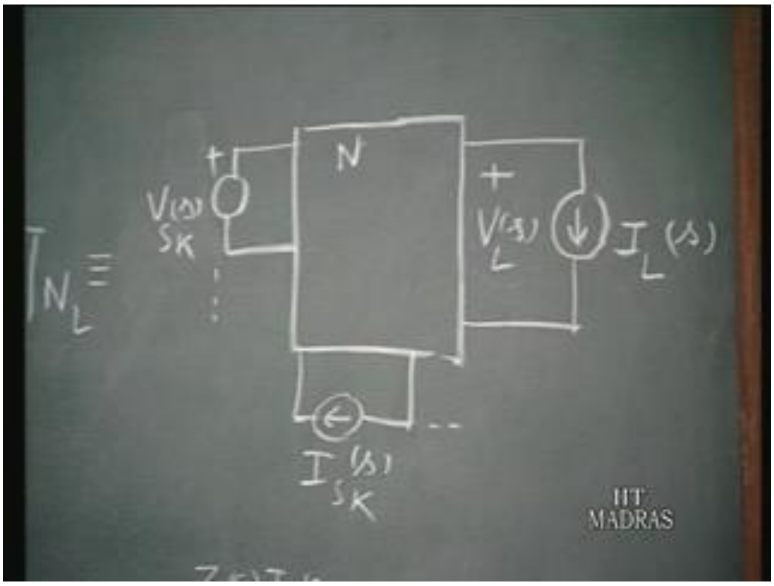

Now, I would like to calculate this V L of s in this network, by superposing the effects of because of, n is linear and we have several source acting on them. So, n is linear network. And this linear network is acted upon by several voltage sources, several current sources I s k of s L plus a another volt source, current source I L of s. So, in order to calculate V L of s, I take up the approach of superposition.

I consider all these voltage sources and current sources, one time. Find out its contribution to V L of s. Then, I deactivate these sources and consider only this source. And find out its effect. I superpose those two effects. I should get whatever V L of s would be. So, to do that I am saying this, the conditions in this network can be obtained by superposing the effects in these two networks.

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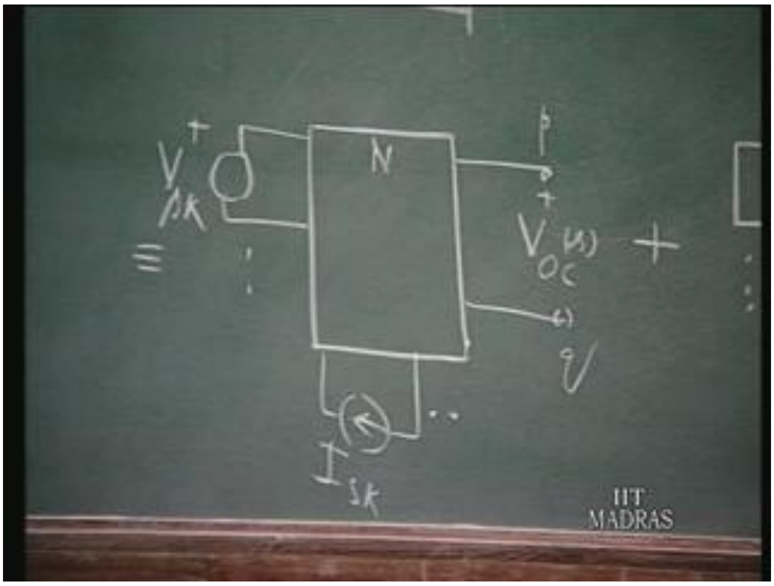

In the first network, I still keep this linear network. But, it is acted upon by the original voltage sources, that are present in N A. And the current sources that are present in N A. But, this I L of s is deactivated. So, that is kept open circuited. So, the two terminals are kept open circuited. And let as a result of this, we have a voltage V o c of s developed across p q. This is called the open circuit voltage.

So, in this original network N A, if you open circuit the terminals whatever voltage you get, it is Laplace transform is referred to as V o c of s. This is the open circuit connection. In time domain, it may be V o c of t. But, in transform domain it is V o c of s.

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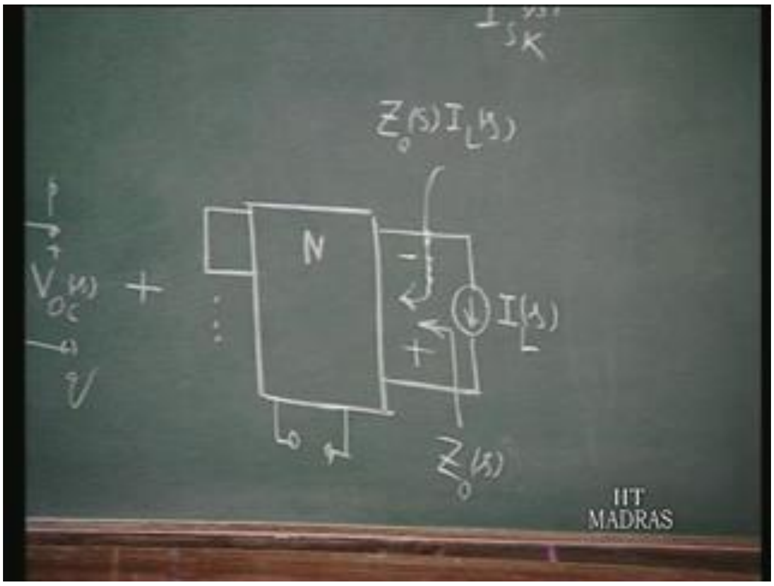

Along with this, if you take the current source acting alone and deactivate all the sources, independent sources in N A. These voltage sources are replaced by short circuits, because the V s k of s is made equal to zero. Its voltage is made equal to zero, that means its equivalent to short circuit. This current source strength is reduced to zero therefore, it is an open circuit. So, if all the independent sources inside are deactivated, you get a configuration like this, where this linear portion of the network N of s is acted upon by only one current source I L of s. All the other sources deactivated.

So, this is essentially therefore, a one port network linear one port network. A current source I L of s is driving that, current driving that one port network. And so, if looking the impedance of this is Z naught of s. That is the driving point impedance of this one port, after deactivating all the sources is Z naught of s. Then, naturally the voltage that is developed between these terminals p q is I L of s times Z naught of s, with q being positive with reference to p, as a reference direction.

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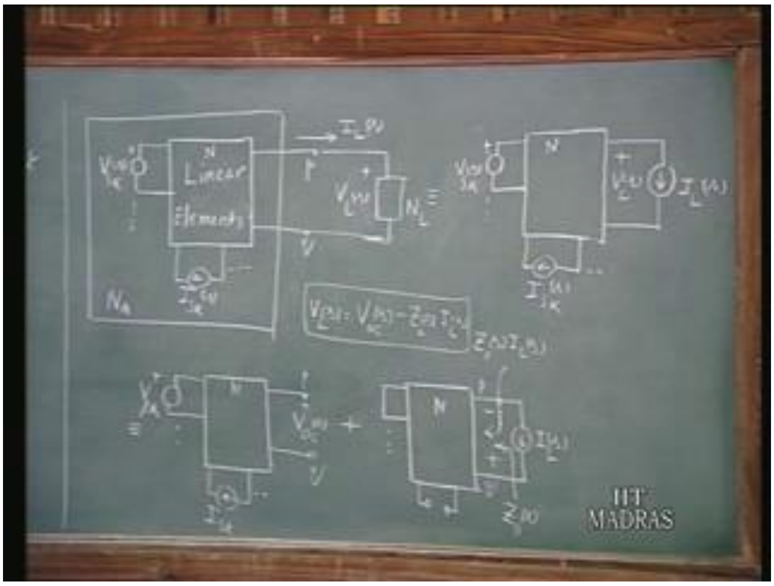

Therefore, the voltage that is developed across the terminals p q is minus Z naught of s times I L of s, if you take p positive. And Z naught of s times I L of s, if you take q as positive. Anyway, the overall V L of s therefore, after all the sum of these two must be equal to this one this voltage, which is equal to this voltage of course. So, as a result of this we get V L of s equals superposing these two effects.

V o c of s that is, the voltage here and the voltage between p and q here is minus Z naught of s times I L of s. So, we observe that in this network we have the voltage across the load network V L of s is V o c of s minus Z naught of s times I L of s, where I L of s is the current flowing in this. And V o c of s is the open circuit voltage. So, this particular equation is satisfying. No matter, what load you have by a simpler circuit like this.

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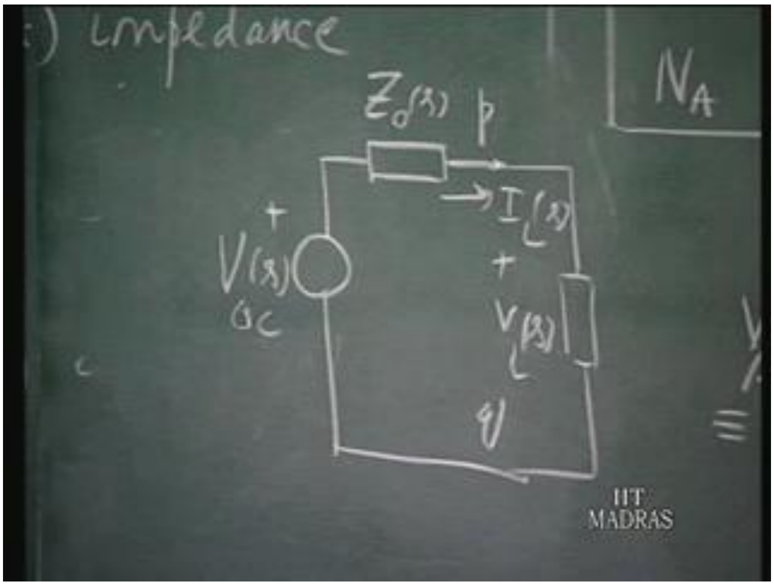

If I have a voltage source V o c of s, in series with a one port whose impedance function is Z naught of s. And I say, these are the terminals p q and whatever I connect here. If this is current is I L of s and the voltage is V L of s. So, V L of s equals V o c of s minus Z naught of s times I L of s.

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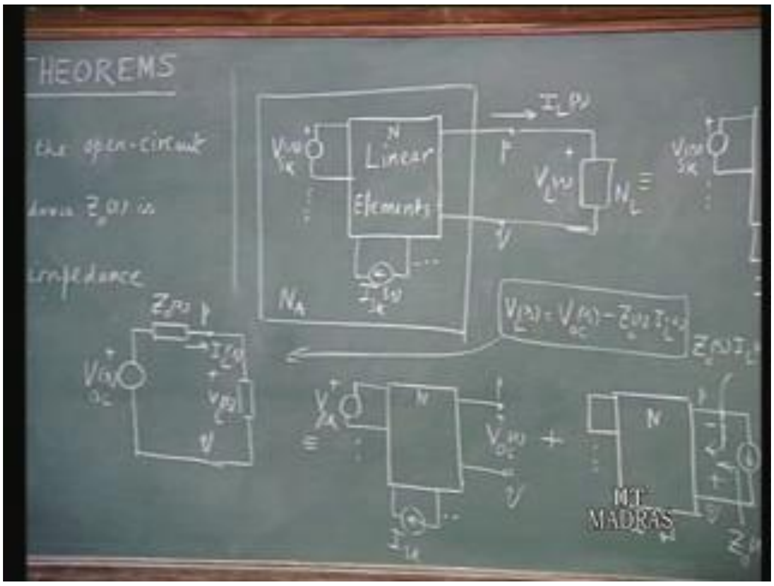

That is the exactly, what this equation states. Therefore, for any given I L of s, V L of s will be given by this equation. This will also be given by this equivalent circuit. And that is what constitutes the conditions in the original network. After all for a given I L of s, V

L of s is dependent on the conditions inside. And what that conditions are, V o c of s minus Z naught of s times I L of s. In this discussion, we have not taken note of any special properties of N L of s.

All we are saying is, whatever network which you connected I L of s is flowing, if certain voltage is developed. And therefore for a given I L, V L can be obtained from the properties of N A itself, without paying any regard to the nature of N L. And therefore, this is an equivalent circuit. So, this is what constitutes the Thevenin's theorem statement, in the Laplace transform domain. All we are talking about functions of s or the volts voltages and the impedance function.

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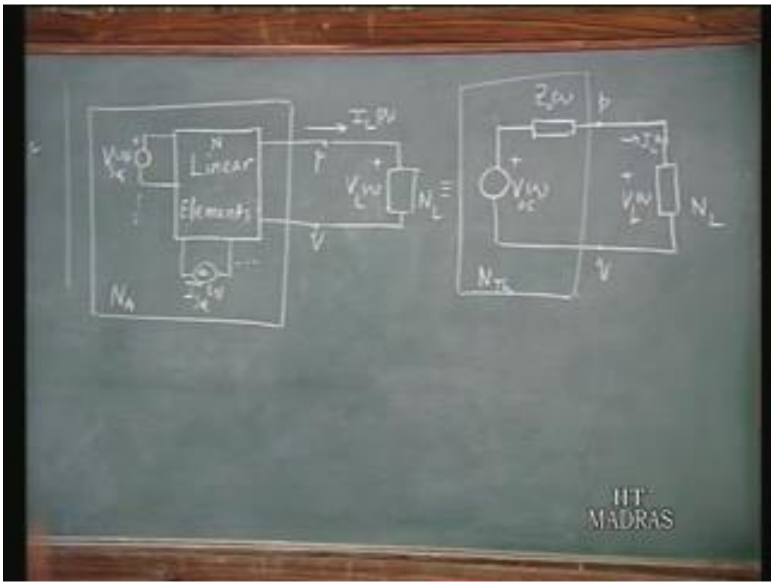

Our discussion so far, has told us that a network like this, with N containing linear elements plus acted upon by several sources. It can be replaced by an equivalent circuit of this type, where there is one source which has a voltage which is equal to the voltage across the open terminals of p q. When N L is disconnected, that is called the open circuit voltage. And in series with an impedance, which is the impedance measured.

At these two terminals after deactivating all these sources, which is called the output impedance also called Thevenin's impedance. So, this is an equivalent circuit. And this is often referred to as the Thevenin's equivalent of the N A. So, if you connect a particular one port network N L here and here, you get identical external conditions. When we say the two networks are equivalent, the external conditions are identical. We are not worried about, what is happening inside N A and N Thevenin's. Now, a few comments on this are in order.

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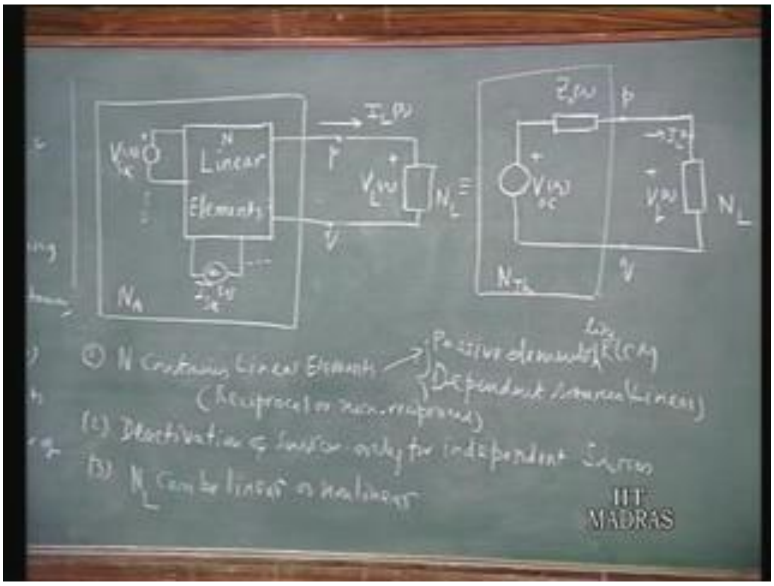

One N contains linear elements. This is important, because we used superposition principle so on and so for. And these linear elements can be passive elements, like RLCM. They can also have dependent sources, linear dependent sources. Linear dependent sources like voltage controlled, voltage source, current controlled voltage source and so on. So, N can contain any one of these passive elements like, I said like.

The elements can be reciprocal or non-reciprocal. For example, passive elements like this are reciprocal elements or bilateral elements, dependent sources or non-reciprocal elements. So, as far as N is concerned, all we will demand on them is and it is. That it should contain linear elements, the elements can be reciprocal or non-reciprocal. We do not really bother about this.

Two, when deactivating the sources we continue with our concept that, the voltage sources must be replaced by short circuits. And current sources must be replaced by open circuits. So, it is the same in the deactivation. But, we should do this only for independent sources. The dependent sources should be left intact, just as we used in the principle case of superposition application of superposition principle, only for independent sources.

So, when you are calculating the Thevenin's impedance or the output impedance, we should deactivate only for independent sources. The dependent sources if any, should be left intact. Three, N L. We have not said anything about N L. N L could be any network, can be linear or non-linear. There is no restrictions has on N L, as it could it should always demand is that, it should have two terminals only.

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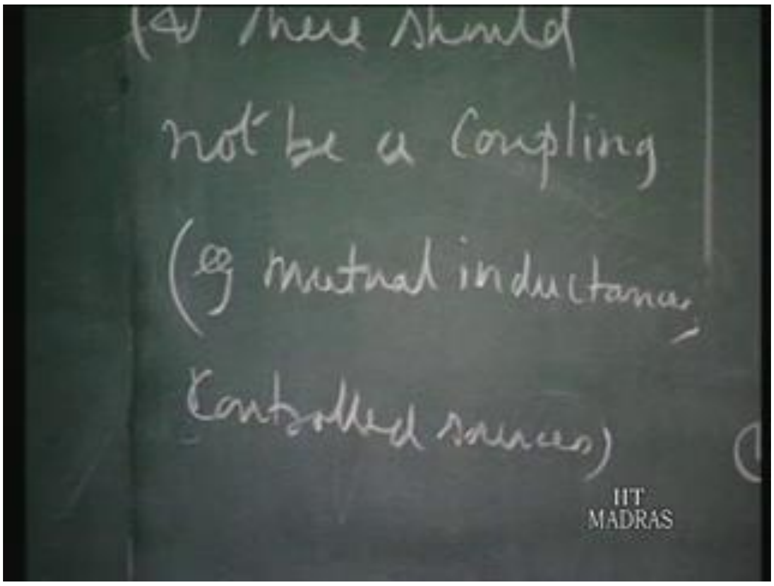

The fourth condition is, that there should not be any coupling between N L and this. There should not be any coupling between element here and element here. There should not be a coupling. How does a coupling between two elements arise? For example, mutual inductances. They can also arise from control sources. A current here may control a voltage here. A voltage here may control a current here. I said, they were due to mutual inductances control sources.

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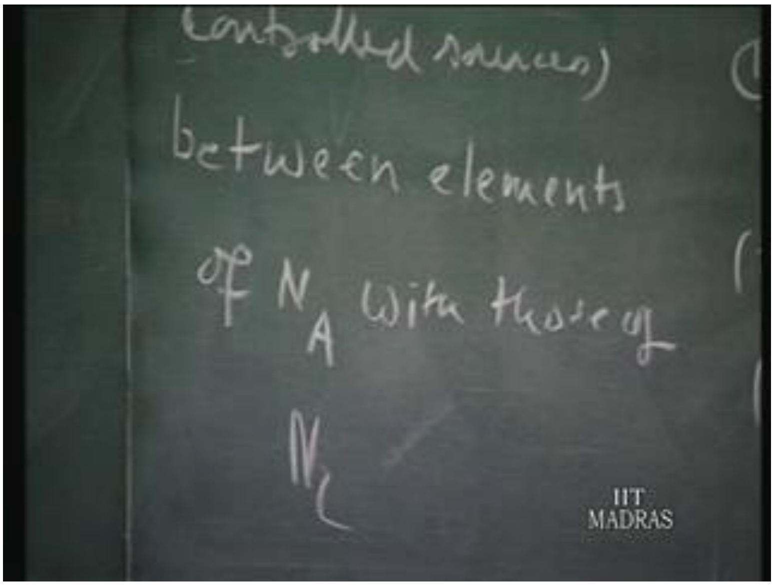

There should not be such a coupling between elements of N A with those of N L. That, so, all we demand is that the coupling between N A and N L must provide, come only through this two interconnections through terminals p and q. We cannot have a mutual inductance, primary here and secondary here. We cannot have a controlled source, whose controlling quantity is here and controlling quantity is here. Otherwise vice versa.

So, provided these conditions are met. Then, this Thevenin's theorem will work quite nicely. All we have to do is, you replace the entire network by an equivalent series connection of a voltage source and a current source. And we can find out, the currents in a external network. So, if suppose you want to connect ten different networks here and you want to find out the currents. If you take the original network, it becomes difficult because, you have to do the analysis ten times.

So, if you are going to have repeated analysis at this network, at these two terminals with different types of loads. It would be quite useful of course, to replace this entire thing by a simple equivalent circuit here. And then, carry out this analysis with this simpler circuit for the ten different loads. So, naturally this process becomes simpler. The overhead that you pay for establishing the Thevenin's theorem, will pay for itself. Because, you are going to do repeated additions and repeated calculations.

And why we demand that, there is no coupling between these two. An element here and element here is but, once you replace this entire N A by its equivalent Thevenin's network then, the identity of some of those elements inside are lost. Therefore, the original coupling turn between the coupling affects between the term here and element here, will no longer be available here. Therefore, this condition is vital. It is also important. So, with this let me workout an example.

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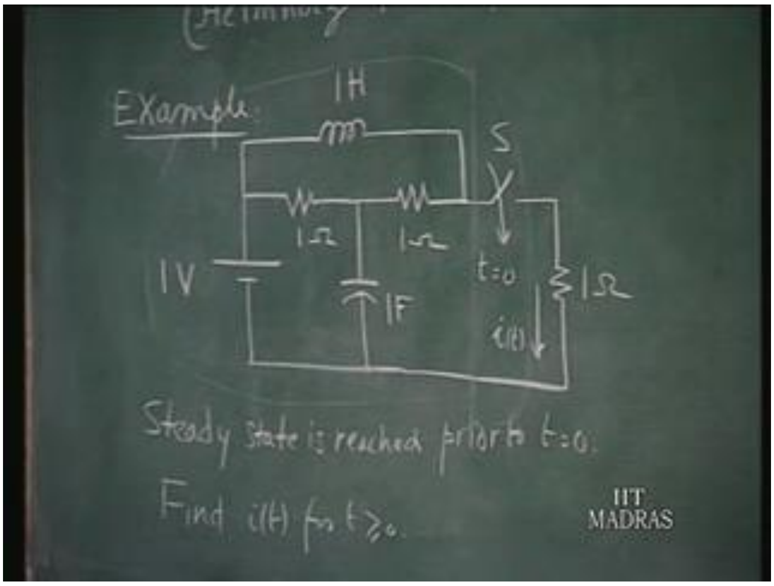

Let us now consider this example, as an illustration of the application of the Thevenin's theorem. We have in this circuit, two resistances one capacitor and one inductor. All of unit value, with a 1 volt source. The switch is kept open for a long time, still steady state is established. And it is then closed, at t equals 0. We are interest in finding out the current i t. So, to solve this problem there are several approaches, of course.

We can set up this transform domain equivalent circuit. And then, try to find out the current by the loop method or the node voltage method. We can also use the Thevenin's theorem, treating this entire network here. This portion as N A with two terminals or may be up to this two terminals. And then, find out the effect of this load resistance of 1 ohm on this two terminal equivalent. So, let us establish the two terminal equivalent of this network, taking these two as the terminals.

So, to do that, we want to work out in transform domain. Therefore, let us write the transform, transformed circuit of this configuration. So, we have first of all, the initial conditions. Before, the switch was closed the steady state was reached. That means, the 1 volt charges the source charges, the capacitor to 1 volt. And once, this is 1 volt there is no current here. And current in this, is also zero, because these two are the two nodes of the same potential.

Therefore, there is no potential difference between these two. The current here is zero and the current here is also zero. So, the inductor carries no current. The capacitor carries a voltage of 1 volt. The charge, the capacitor is charged a voltage of 1 volt.

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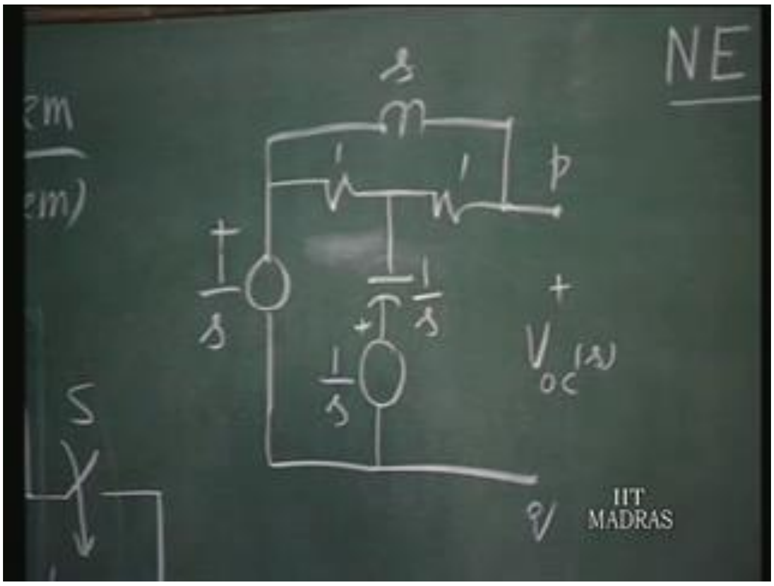

Therefore, the equivalent circuit of this will be 1 by s. That is the source voltage. 1 ohm capacitor has an impedance function s. We will put it simply 1. This is the generalized impedance of the 1 ohm resistance. The capacitor has an impedance 1 over s. And the replacement of initial condition, across the capacitor is a source of equal to 1 by s because, it has charged up to 1 volt. And now, in this we are closing the switch at t equal zero, connect this to 1 volt. That means, we want to find out the open circuit voltage. Therefore, if you call this terminals p q, whatever you measure here is V o c s.

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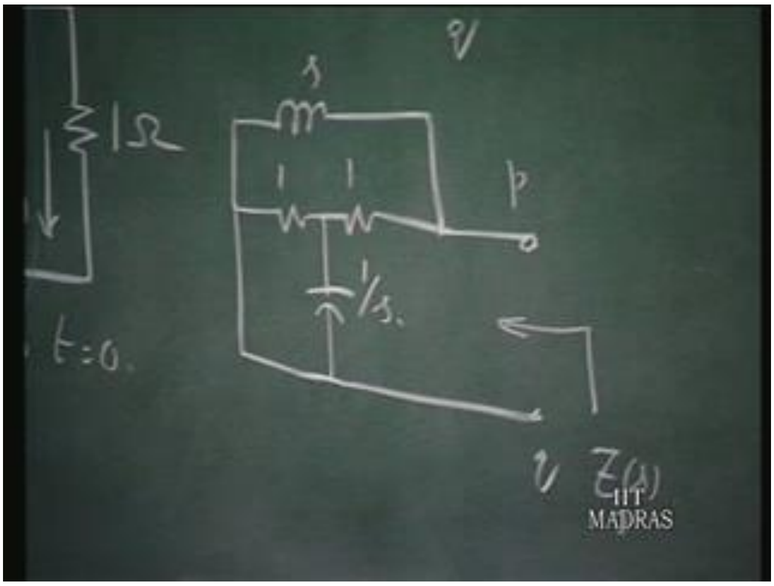

To find out the Thevenin's impedance, what you do is. You deactivate all the sources, including those sources which come about to replace the non-zero initial conditions. These are the two terminals, p q after deactivating all independent sources. And whatever you measure here is, Z naught of s 1 1 s 1 by s. Now, let us find Z naught of s to start with. Now, after all this impedance just is, it is corrected p and q.

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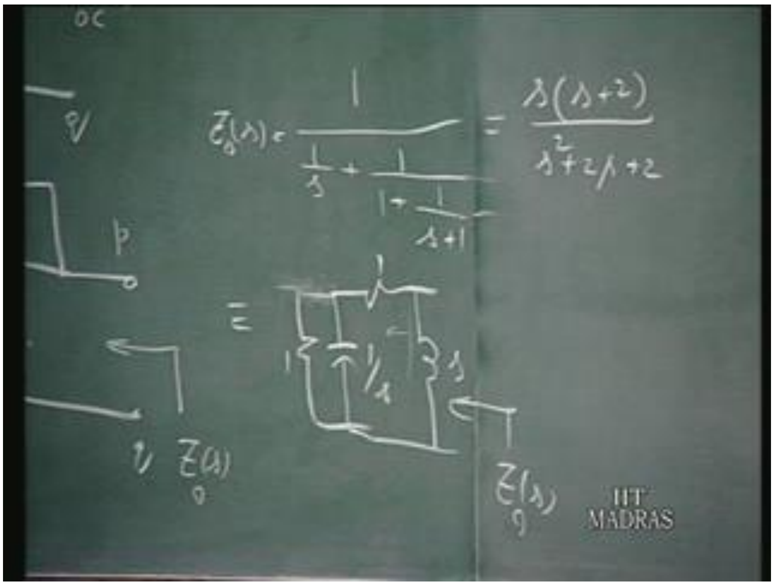

Therefore, you may as well put this here. That means, in other words this is you have. This s is coming here and you have the capacitance 1 over s 1. And this 1 ohm now is in parallel with 1 by s. So, that is what we are having. So, to calculate Z naught of s here, is quite straight forward. So, I can write the Z naught of s equals 1 over because, this element is in parallel with something else. Therefore, the impedance the admittance of this will be 1 over the impedance. The Z naught of s is the impedance.

So, I would like to calculate its admittance of the one port network. So, I must invert this. The admittance of this is 1 by s plus admittance of whatever follows. The admittance of this portion of network is what I should add there, but one element in the series. So, I would like to consider its impedance. So, the impedance of this combination is 1 plus the impedance of this. The impedance of this, I can consider on the admittance basis.

So, once I have the admittance basis, the admittance of this is s the admittance of this is 1. So, I can interpret Z naught of s by means of an expression like this, which is a continued fraction expansion as it is called. We can simplify this and it turns out. That this is equal to s times s plus 2 over s squared plus 2 s plus 2. That is what Z naught of s would be. Now, if you to find out V o c of s, you can use the load method of analysis or whatever it is.

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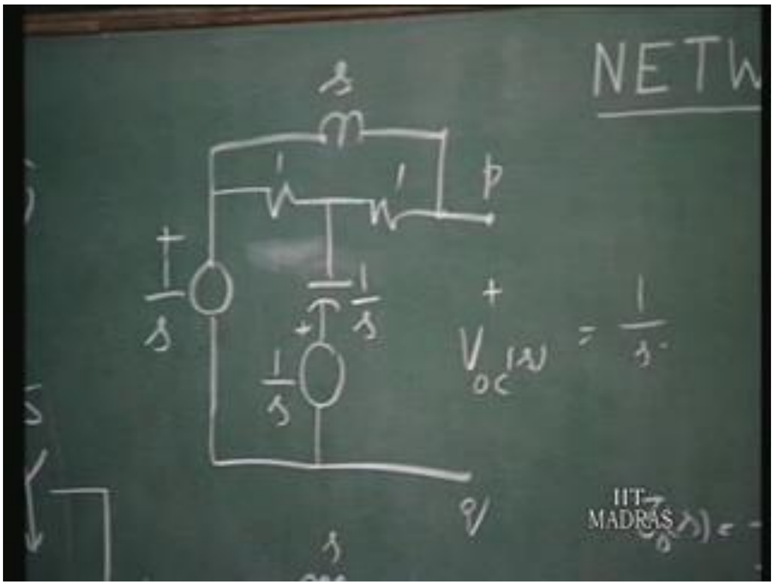

But, it will be very soon evident to you after the calculation. That this will indeed be, because just like you had 1 volt here and 1 volt here and no currents flow in this. Similarly, this there will be no currents here. And therefore, this will also be equal to V o c of s will also turn out to be 1 by s. You can analyze this and show. But a much simpler way of analyzing this, would be like this. Imagine, what is happening under open circuit conditions in this.

Suppose this switch was closed. But, still you are open, after all you finding the open circuit here. The switch is closed. No doubt, does not matter. Let it be closed. But, you are opening this out. Therefore, what is the open circuit voltage in this circuit? In this circuit, the open circuit voltage is exactly 1 volt DC because, there is no currents flowing through this. When the switch was open, what we are measuring is the open circuit voltage.

Therefore, the open circuit voltage in this network is 1 volt DC, pure 1 volt DC, because there cannot be any current here. And the switch is open capacitor, is already charged to 1 volt. There cannot be a current here. There cannot be a current here. Therefore, this 1 volt will appear here at the terminals. Therefore, the open circuit voltage is 1 volt DC. Consequently, V o c of s is also 1 over s. In other words, when you want to find out the open circuit voltage, you do not always have to go to transform domain. You can find out the open circuit voltage in time domain and take its Laplace transform. That will be the open circuit voltage.

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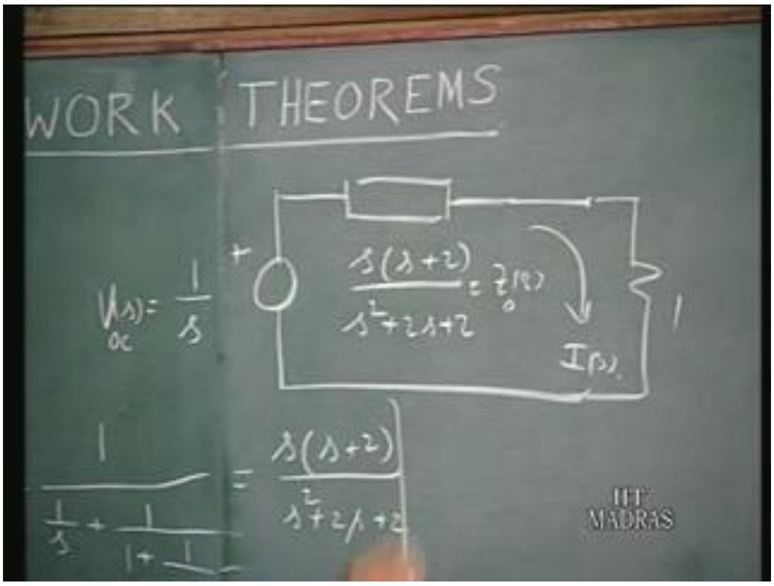

So, the up short of all this analysis therefore, is Thevenin's equivalent as 1 over s as the open circuit voltage. And a one port network, whose driving point impedance is s times s plus 2 over s square plus 2 s plus 2. This is equal to Z naught of s. This is the Thevenin's equivalent of our original configuration. Now, you have connected a 1 ohm resistance here. And you are asked to find out this current I.

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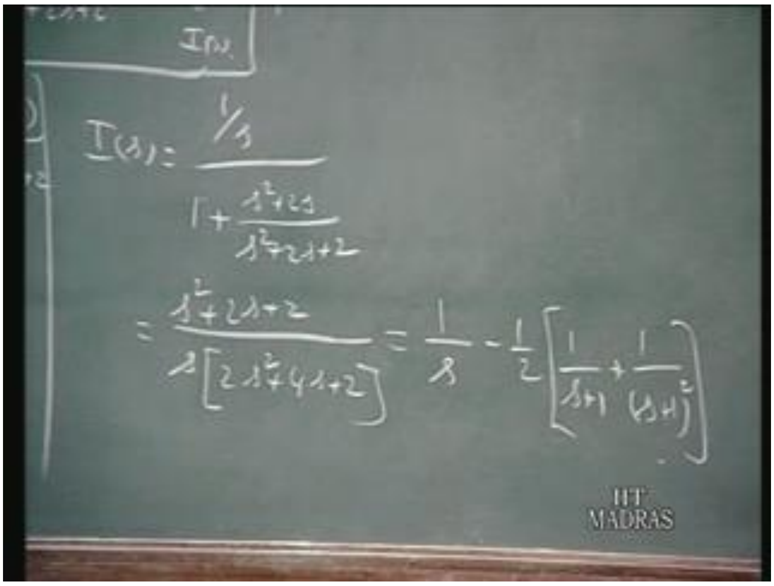

So, we analyze this, I of s will turn out to be 1 by s divided by 1 plus s square plus 2 s divided by s square plus 2 s plus 2. That will be s squared plus 2 s plus 2 divided by, s times 2 s square plus 4 s plus 2. And you make the partial fraction expansion of this, s square plus 2 s plus 2 times s square plus 4 s plus 2. You can make the partial fraction expansion of this. And you can show that, this is equal to 1 by s minus half 1 over s plus 1 plus 1 over s plus 1 whole square, which means that the current i t equals 1 by s.

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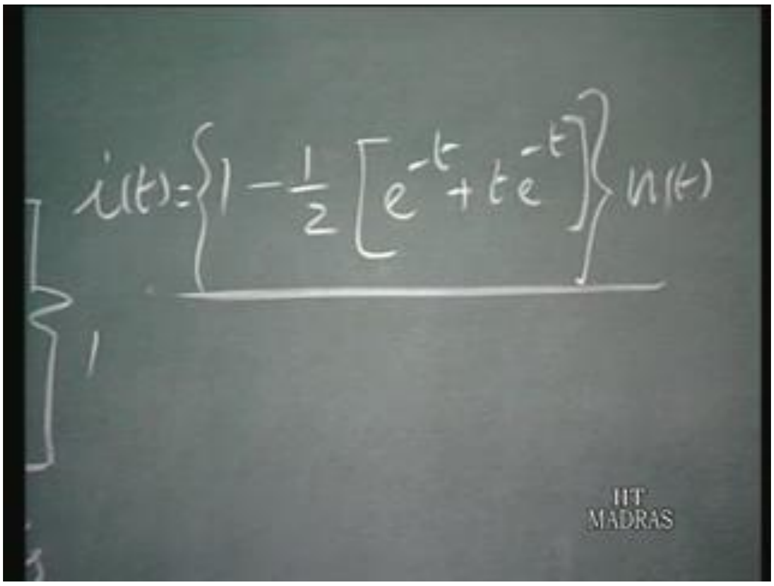

That is 1 minus half e to the power of minus t. From this, you get t times e to the power of minus t the whole thing of n of t. That is the current. 1 minus half e to the power of minus t minus half t into e to power of minus t. That is the solution for this. Now, I would also like to demonstrate the use of substitution theorem, to get the same solution. Now, let us look at this.

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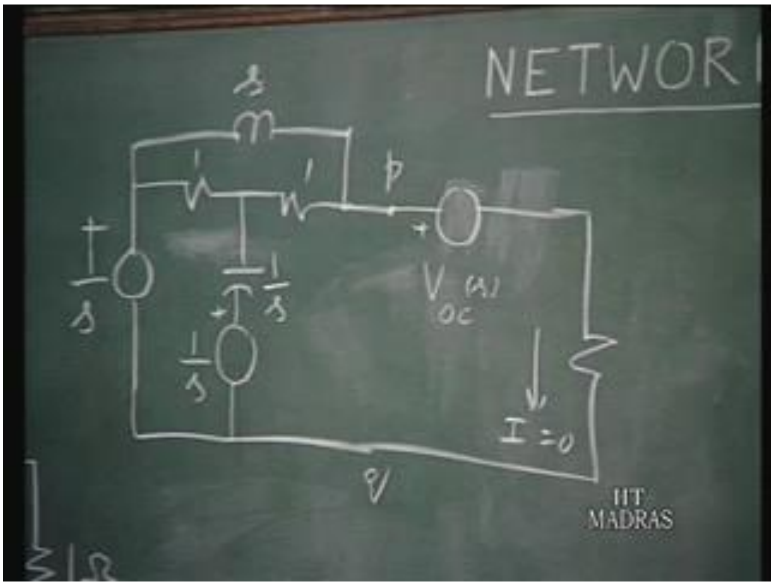

I would like to see what happens, when the switch is closed. This is what we are having. When switch is closed, you want to find out the current here, I of s. Now, let me introduce here 2 voltage sources. Let me say, when switch is open. Suppose, when the switch is open the voltage across this, which is V o c t. After all, that is the open circuit voltage. And let its Laplace transform be V o c of s.

So, if you replace the open switch by a voltage source V o c of s, which is exactly the Laplace transform of the voltage that is appearing across the open switch. Then, the conditions in the network are not altered. This is continues to be zero, I will be 0. So, if you replace this open switch by a voltage source, whose value at every instant of time is exactly the voltage that is appearing across the open switch. Then, the conditions in the network are not altered. That is the substitution. So, this is the, what we have.

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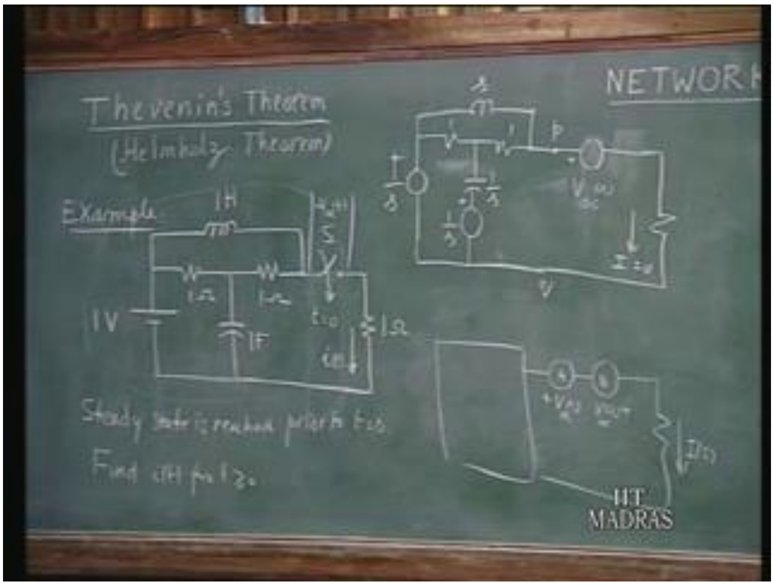

Now, what I can do is this entire thing whatever we are having here, let it be replaced like this. Suppose, I have two sources like this, which are equal and opposite, say source A source B. Together, they add up to zero. That means, the closer are the switch. When you close the switch, its equivalent to have zero voltage across the switch terminals, which is equivalent to connecting two voltage sources of equal and opposite values in series.

So, the close switch corresponds to this. So, when you have a close switch, you have a voltage across the terminals to be equal to zero. So, I choose to replace the close switch by two voltage sources, which are equal and opposite. But, I choose those voltage sources the magnitude of each source should be the same as open circuit voltage, that you have here. So, the current here would be I L of s. This is what I of s, which is what we are after this is the current we are interested in.

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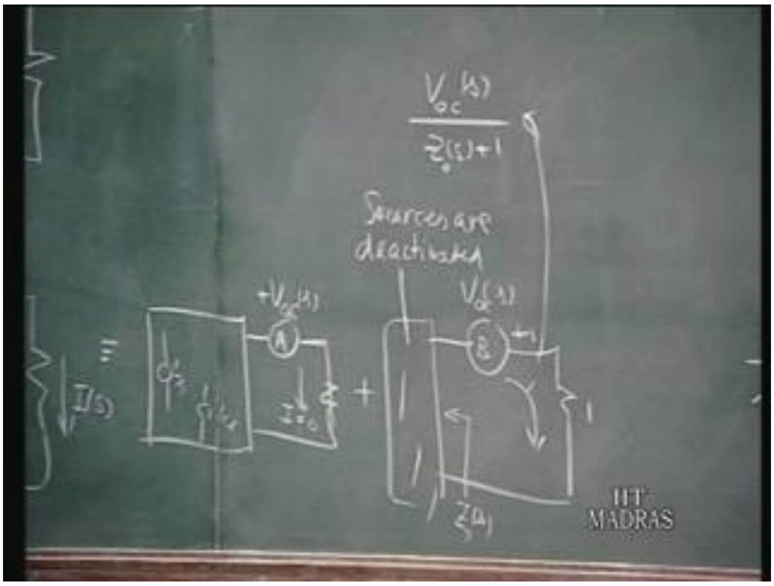

Now, I can use this principle of superposition. And say this is equivalent to one in which you have V o c of s, acting with all the internal sources 1 by s 1 by s etcetera plus another, where the second voltage source is acting with a 1 ohm resistance s. And here the sources are deactivated. The internal sources are deactivated. That means, when the switch is closed here we are replacing the close switch by two voltage sources. Each voltage sources, we has been so chosen to be equal to the open circuit voltage here.

So, inside here you have two sources, these two sources. Now, I am to find out these current what I am saying is. I use the principle of superposition in the first place. I use the internal sources plus source A. And the second place, I deactivate all the other sources and keep only source B. Now, you observe that this is exactly the situation here. When the switch is open, you have open circuit voltage V o c of s all the other sources.

Therefore, this current is 0. Now, the current here that whatever current that comes here is, what V o c after all. We have deactivated all the sources here. So, if the looking in impedance is Z naught of s here. The current that flows here will be V o c of s divided by Z naught of s plus 1. That is the current here. So, original current was zero, the new current here is V o c of s over Z naught of s plus 1.

And that must be the current here. Therefore, this current I of s can be obtained by solving this network alone. And how do we solve that network? You must find out V o c of s. Find out the looking of impedance. And that is exactly, what we did in the case of Thevenin's theorem. So, we have interpreted the results, that we have got here as an application of this substitution theorem, instead of the Thevenin's theorem. Ultimately, the work will be the same.

You have to find V o c of s, whatever means you have. That V o c of s can be interpreted, as the open circuit voltage across the switch which is of course, 1 volt. Therefore, its Laplace transform is 1 by s. So, if it is 1 by s. You have got to find the Thevenin's impedance of that. And calculate that will give you I of s. The incremental changes produced as a result of this source, is this but the original current was zero. So, this plus, this is the total current. So, this is another way of looking at it.

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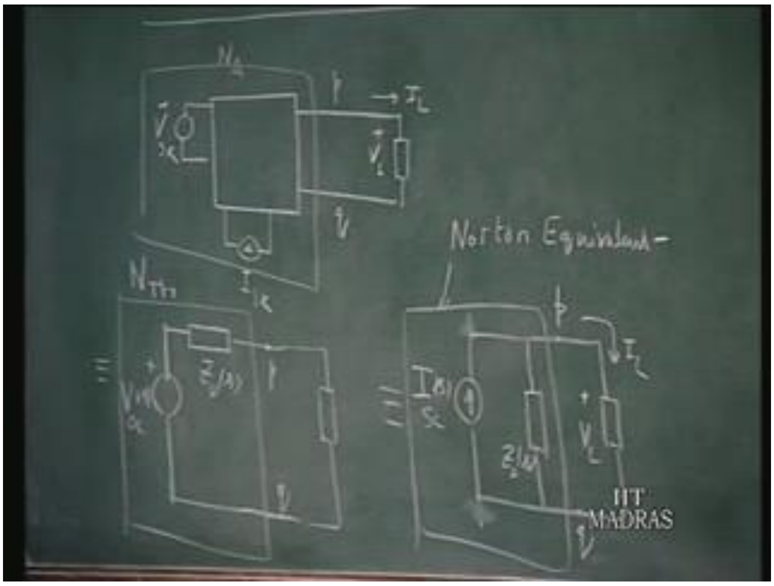

After having considered Thevenin's theorem, let us now see another theorem which is closely related to it, which goes by the name Norton’s theorem. This is a kind of dual, the Thevenin's equivalent. You recall that in the case of Thevenin's theorem, when we said that there is an active network containing linear elements and sources called N A, can be replaced. And these two terminals can be replaced by a voltage source V o c of s in series with a one port network, having an impedance Z naught of s.

So, for all practical purpose just, this gives identical results as the original two terminal networks. In so, far as it affects when external conditions are concerned. We have an alternate equivalent circuit in which, instead of the open circuit voltage here we have a current source, which is called the short circuit current source I s c of s. In parallel with a one port network whose impedance continues to be the same as that before.

So, the output impedance of this network is now put in parallel and you have a current source. So, this Norton equivalent is in the form of a current source in parallel with the output impedance, which is the same as the Thevenin's impedance. This is called a short circuit current source because, if this original network is short circuited to the terminals p q. So, if you have the original network N A and short circuit the terminals p q, whatever current that flows through that, is the short circuit current.

And its Laplace transform, if you call that I s c of s that is the short circuit current, that you have to incorporate here. In other words in the equivalent circuit, if you short circuit p and q the current that flows is I s c of s. This current this show, this source must allow this current to flow through. So, if you short the terminals externally, the current in the short circuit will be exactly the same as source current.

So, this is an alternative way of doing this. Sometimes, it may be easier for us to calculate the short circuit current. In which case, we can use this equivalent. Sometimes, it may be easier to calculate the open circuit voltage to make it easier to may be easier to adapt the Thevenin's equivalent. But, you observe that after all when you open circuit this, you get V o c of s. When you open circuit this network, we get I s c times Z naught of s.

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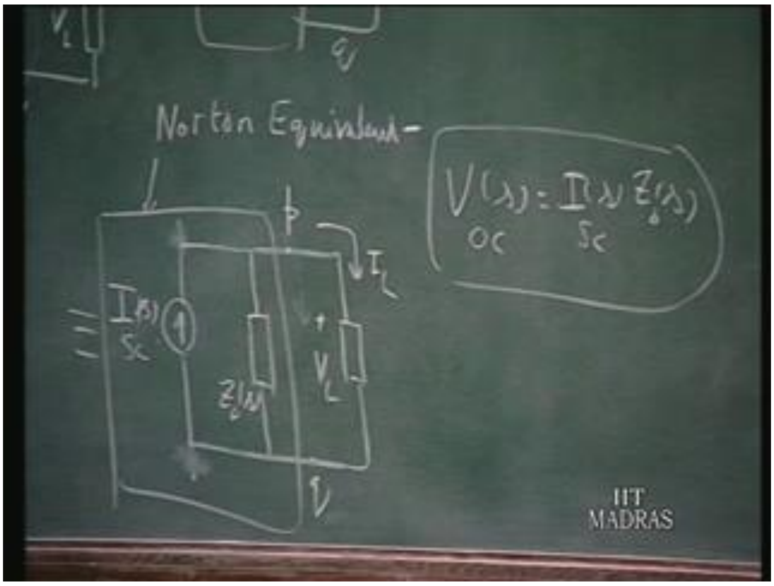

And after all these two are equivalent therefore, we have V o c of s equals a short circuit current times Z naught of s. So, in other words of the three quantities that you have got, V o c of s I s c of s and Z naught of s only two are independent. Any two can be used to find out this third. So, in certain problems it might be easier for us to find out V o c of s and I s c of s. Then, you do not have to calculate Z naught of s. If you calculate these two, you can find Z naught of s. And use either Thevenin's theorem or Norton’s equivalent.

So, it is not necessarily for us to calculate Z naught of s every time. If it turns out then, it is easier to calculate V o c of s and I s c of s. Use that information to calculate Z naught of s. And use that either in this equivalent or in this equivalent. I will not discuss this further because, the concept is straight forward. And some of you, am sure will know have worked with Norton equivalent circuits in the DC domain or AC domain. So, we will not assume this further.

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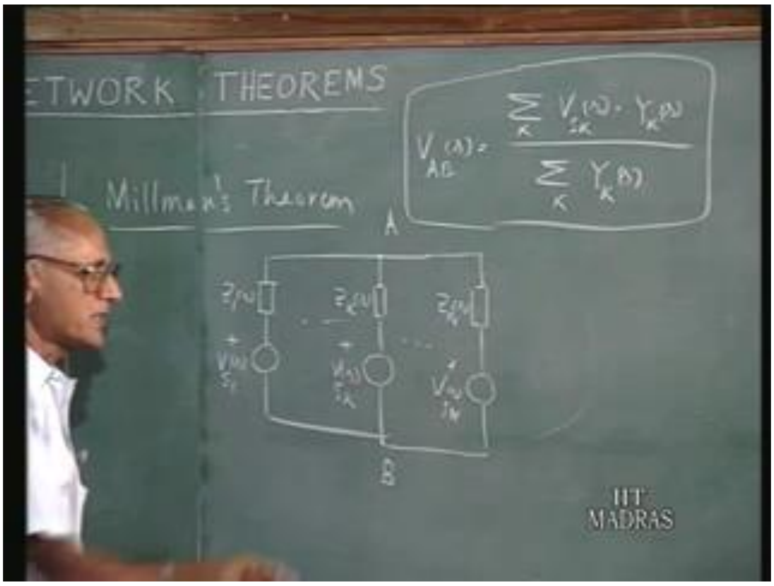

I would also like to mention in this context an extension of these results, which go by the name Millmans's theorem. These are all the straight forward extensions of this, same similar concepts. Millman's theorem is applicable to a parallel connection of several sources. So, V s s 1 of s Z 1 of s. Suppose, you have whole series of such branches and I will say, this is the kth branch V s k of s Z k of s. And like that, let us say that a last branch is Z n of s and V s n of s.

So, if I have a whole series of such parallel branches, each branch containing a voltage sources in series with an impedance. Then, the voltage between the two terminals between, which all this parallel branches are connected is given by V a b of s equals summed against k V s k of s times Y k of s, where Y k of s is the reciprocal of Z k of s the driving point admittance of this one port network divided by.

So, this is the result is known as the Millman's theorem result. It can be obtained quite straight forward manner, by application of Norton's equivalent. I will just outline the proof without going through that. Suppose, you short circuit this then the current that flows through the short circuit is V s 1 times Y 1 V s 2 times Y 2 V s k times Y k s V s n times Y n of s. That means, this is a short circuit current. So, that is the short circuit current.

Now, we looking in impedance will be the parallel combination of all these. That is the looking in admittance is equal to this. So, if you want to find out the open circuit voltage here, because after all nothing is connected. These two terminals are open circuited. So, if you want to find out the open circuit voltage of this, the I s c of s times Z naught of s or I s c of s divided by Y not of s. This will therefore, be this is the short circuit current I s c of s this is Y not of s and that is how it is.

You can also use node analysis. All this will fetch the same result. So, we will not discuss this further. Now, we will move on to the next theorem, which is important which is known as the compensation theorem. Now, before discuss this when you use the Thevenin's theorem or Norton's theorem what you can do is.

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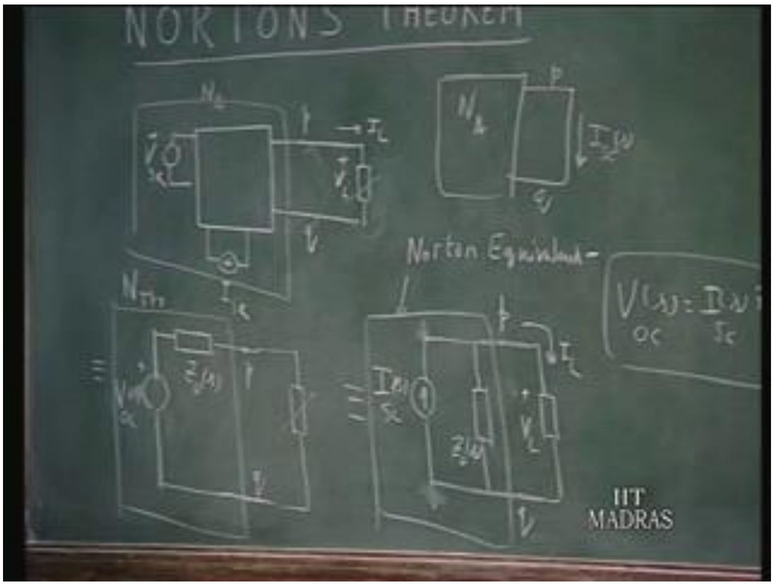

Suppose, you change this load impedance Z l, you can find out the new current here. That means, whenever the impedance here is changed. If you want to find out a new current here then, a Thevenin's or Norton equivalent are quite convenient. Because, you set up a similar equivalent and for different values of Z l, you can find out the new currents. But, suppose I change this impedance here and want to find out its effect on the current somewhere here in the circuit.

If I change this impedance say, let us say from 2 ohms to 3 ohms, what happens to the current inside? Then, neither Norton's equivalent nor Thevenin's equivalent will give us a clue to this. Therefore, we should like to discuss this in with a more powerful theorem, which enables us to find out the effect of change in an impedance on the rest of the circuit. And this is, what is called the compensation theorem. Let me mention the statement of the theorem. Then, we will discuss that further.

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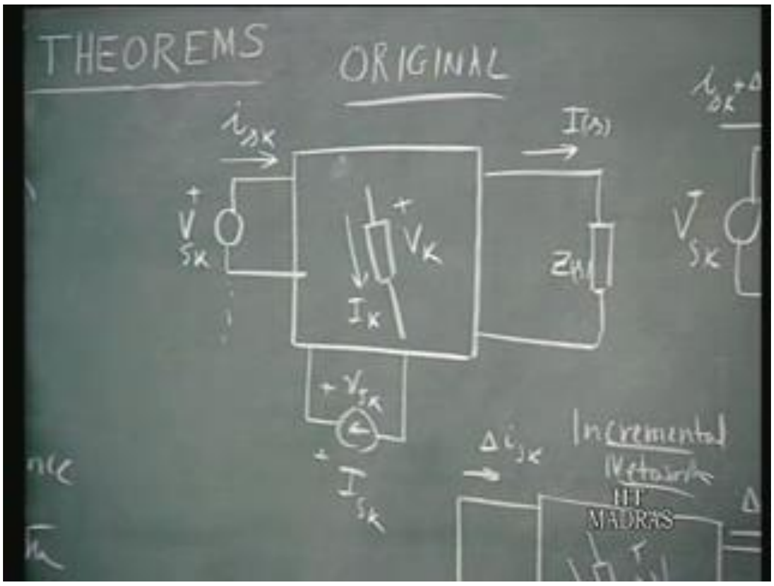

To introduce the compensation theorem, let us consider this network. This box contains linear elements. It is acted upon by several voltage sources, I represented one of them called V s k. Several current sources, the kth current source is represented here carrying a current I s k. The current produced by the voltage source, I call I s k. Just to distinguish between this and this, I use the lower case letters here.

Similarly, the voltage across the current source, I use small v s k just as to distinguish from here. They are functions of s. The several elements inside carrying voltage V k and I k, this is a typical element. And we have an impedance in the network Z of s carrying a current I s. The question which you would like to ask is, if this impedance changes by a certain amount what are the consequent results changes that are produced in the entire network. To do that, let us consider this altered network.

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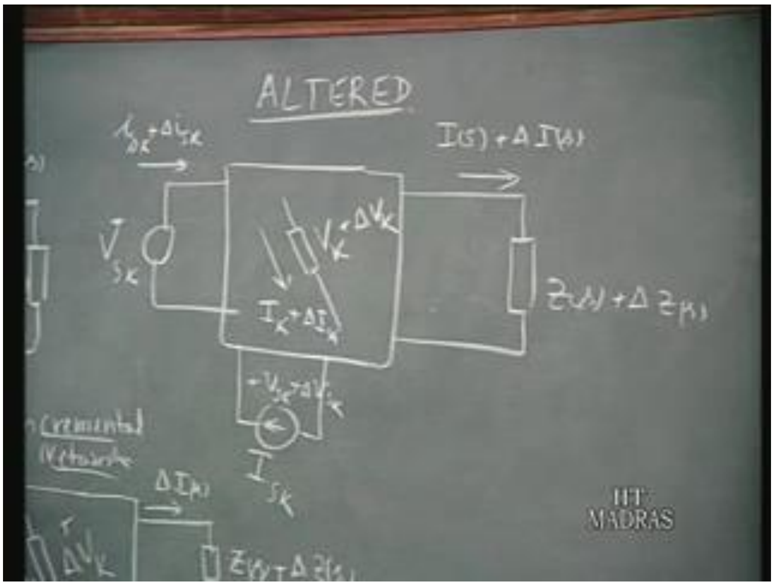

In this altered network, we have the same voltage sources and current sources. Same elements inside but, one particular impedance Z s is changed to Z s plus delta Z s. Consequently, all the values except the strength of the independent sources will be re-altered. The current in this originally was I s, may altered as I s plus delta I s. The voltage here may be altered from V k plus delta V k, I k current I k plus delta I k.

The current in the voltage source, originally I s k may have changed to I k I s k plus delta I s k. The voltage across the current source is from V s k changes to delta V s k. That means, all the incremental quantities all the changes are indicated by delta times the original values. So, this is the altered network. In order to get the, calculate all the variation or the alteration or the changes in this.

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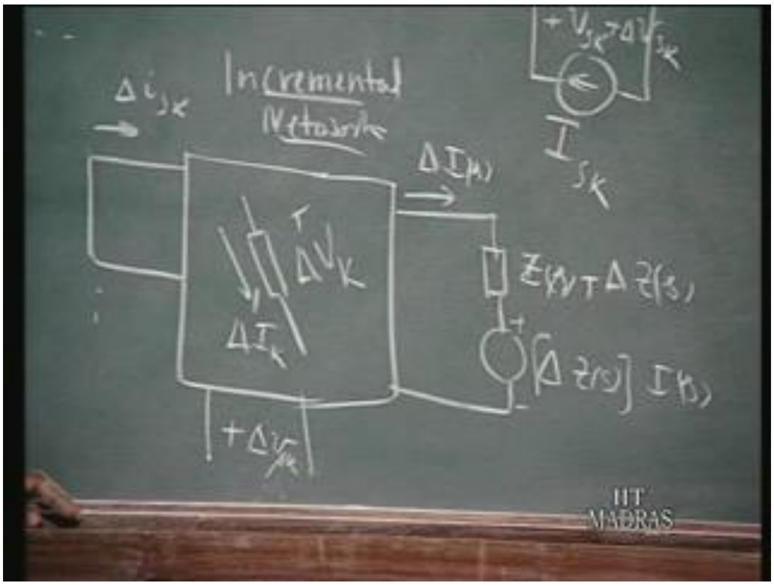

The compensation theorem tells us, you consider this network in which all the independent sources are deactivated. So, the voltage sources are also replaced by short circuit. The current sources are open circuited. In this are internal elements. So, the original current impedance Z l is carrying a current I s. So, you have a voltage source whose strength is delta Z s, the change in the impedance times the original current.

And you put the value as the new impedance that, you are having here. So, this voltage source in series with Z s plus delta Z s acting in. The original network with all this sources deactivated, whatever you are having. Is the solution of this network will give you, exactly the increments or change that are produced, as the result of change of impedance from Z s to delta Z s?

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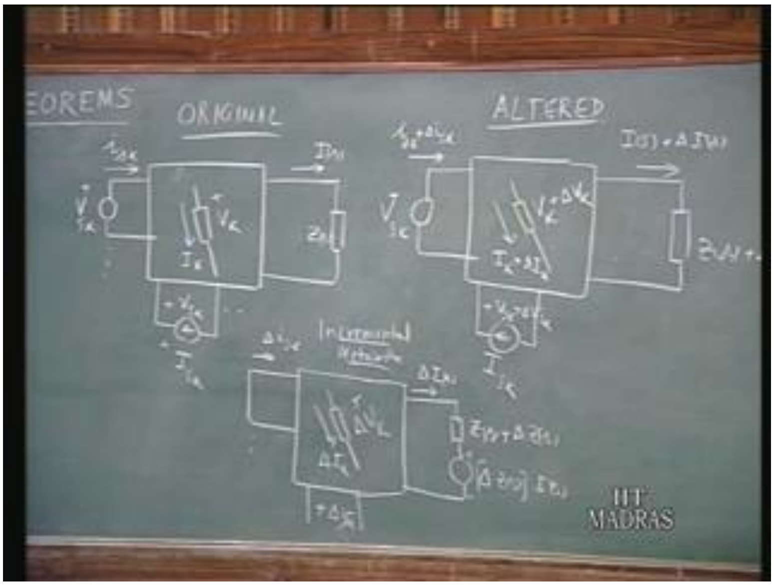

In particular, the current associated with this is delta I k. The voltage associated with this is delta V k. The change in the voltage source current is delta I s k, this is delta V s k. And the current here is I s, was changed to I s plus delta Is. That means, to calculate the changes that are produced in the original network as a result of the change in impedance from Z s to delta, Z s plus delta Z s. You do not have to re-do the calculation with all the sources present. You can do the calculation with only one source present. And that solution will give you the incremental value, the changes that are produced all over the network. That is the statement of the compensation theorem.

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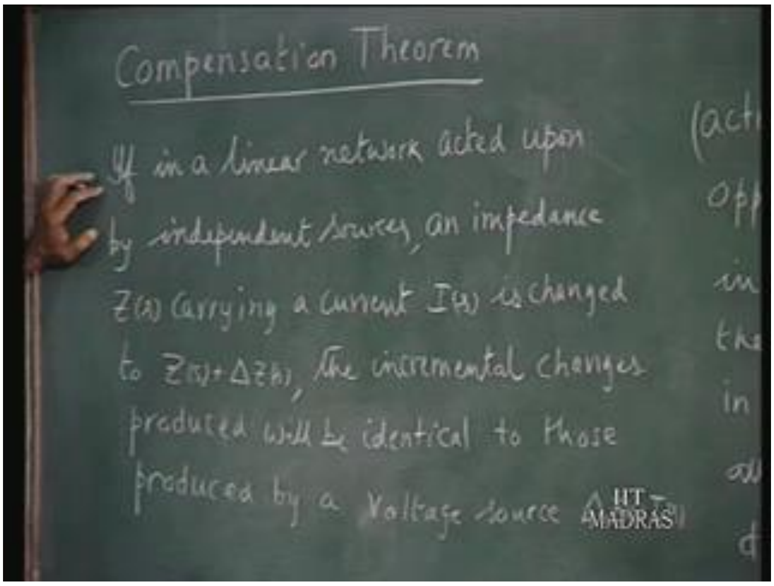

So, the statement of the compensation theorem will be something like this. If in a linear network acted upon by independent sources, an impedance Z of s carrying a current I s is changed to Z s plus delta Z s. The incremental changes produced that is inside the network, will be identical to those produced by a voltage source of strength delta Z s times I of s.

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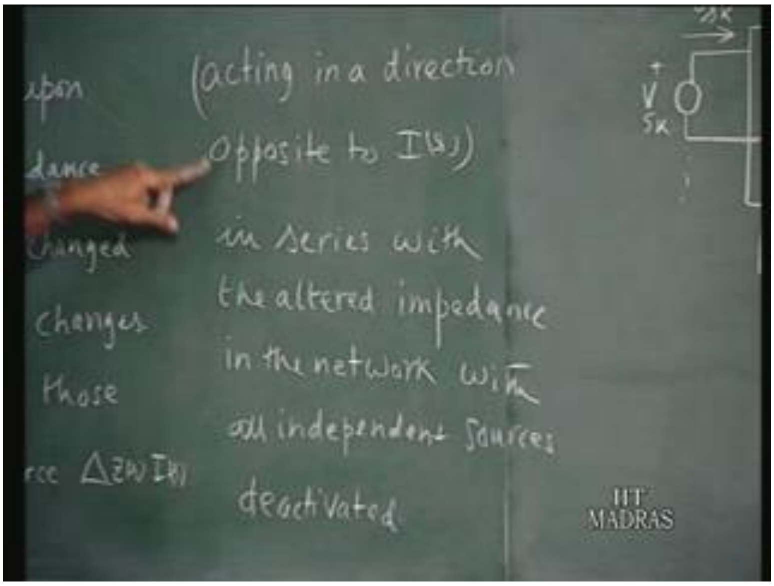

In series with the altered impedance in the network, with all independent sources deactivated and the direction or the polarity of the voltage source should be, it should act in a direction opposite to I s. The meaning of this is clear here. You are having a current I s here. And your voltage source is such as to drive a current opposite to I s. That is the, what is meant by acting in a direction opposite to I s. So, this is the statement of compensation theorem. Let me give a quick example to illustrate this. We will prove this in the next lecture.

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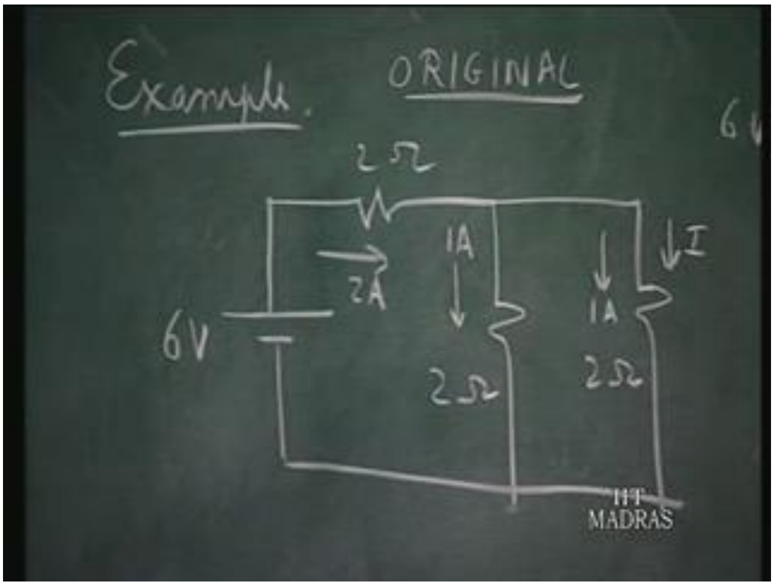

To illustrate this theorem, let us consider a simple DC circuit, a 6 volt source acting in a circuit consisting of 3 and 2 ohm resistors. If you analyze the circuit, you have 1 ampere here 1 ampere here, because after all 2 parallel 2 is 1 ohm. So, 3 ohm is the effective resistance. 2 ampere is the current here. 1 ampere current here, 1 ampere current is here. Now, the question you would like to ask is, if these 2 ohms is changed to 3 ohms what is the resultant changes that are produced in a circuit?

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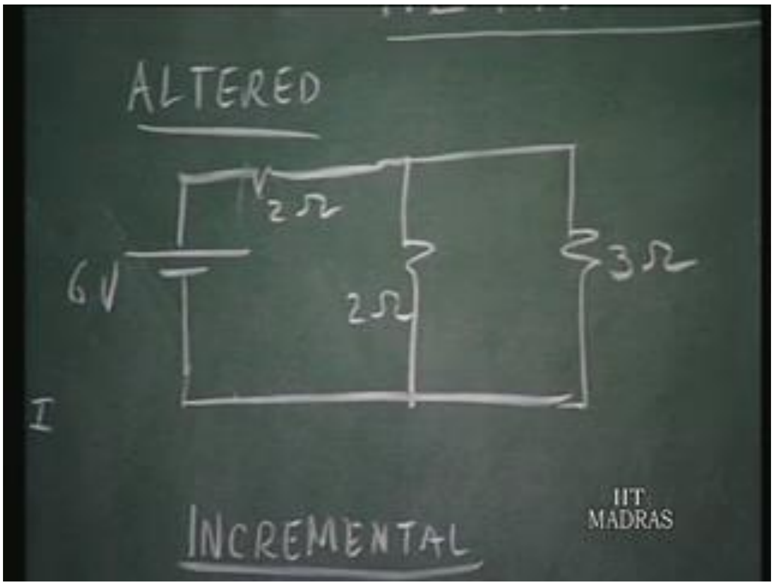

So, the altered network here has got 6 volts again. Original source voltage, 2 ohms 2 ohms and this is changed from 2 ohms to 3 ohms. In order to find out the currents here, the changes that are brought about. We consider this incremental circuit, in which we incorporate the source. After all this, 2 ohms is changed to 3 ohms. So, delta R is equal to 1 ohm. The original current in the network is 1 ampere.

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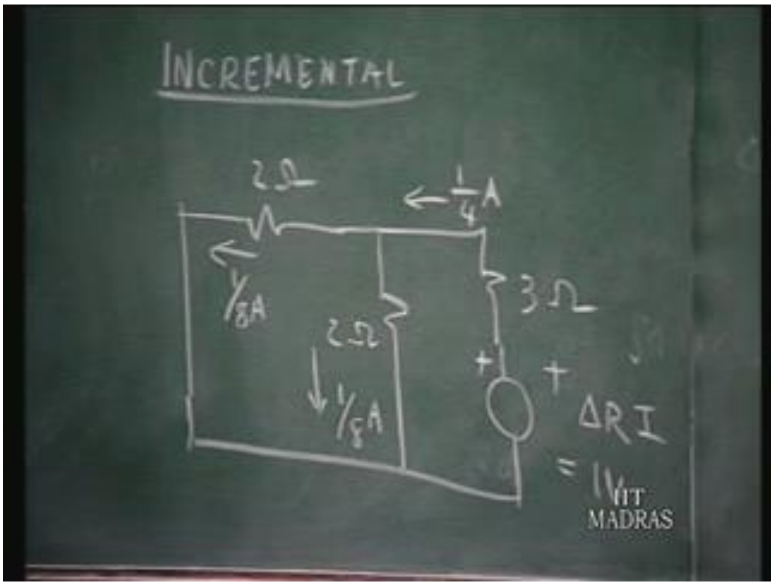

So, you have delta times original current 1 volt. Change is 1 ohm. Current is 1 ampere. Therefore, this is a source of 1 volt. A DC source acting in a circuit, in which the original sources are deactivated. So, these 6 volts is deactivated. It is replaced by a short circuit. So, if you analyze the circuit which consists of 3 ohms and 2 2 ohms in parallel. The currents will be one fourth ampere one eighth ampere and one eighth ampere.

So, in this new circuit you have these are the changes, these are the increments. So, these are the changes that are brought about in the original circuit. Originally, it was 2 amperes. Now, there is an additional current of one eighth ampere here.

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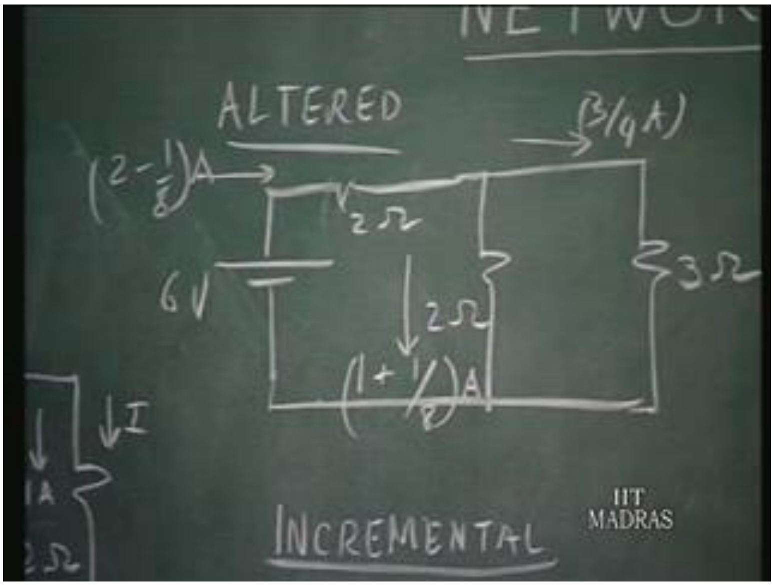

Therefore, the net current here is 2 minus 1 by 8 amperes. The current here is 1 ampere originally. The change that is brought about, that is one eighth. Therefore, the current here is 1 plus 1 eighth of ampere. The current here is originally 1 ampere. This is in opposite current of one forth of ampere. Therefore, this current is three fourth amperes. So, this is how it is done. An advantage is, if you have several sources here and you want to carry out this analysis for second time, it will be little more complicated.

But in this approach, you are dealing with only one source. Therefore, the changes that are brought about can be computed by analyzing this network with just one source. That is the advantage of the compensation theorem. We will prove in the next lecture. Then, justify the statement of the compensation theorem. And then, workout further examples. So, in this lecture we have discussed the Thevenin's theorem in the framework of transformed diagonal in the Laplace transform.

We worked out an example and showed its utility by establishing in a simple equivalent circuit for a complicated linear network, with several sources present. And we mention that this Thevenin's theorem approach is valid even, if the network replaced. Network contains reciprocal or non reciprocal elements as long as the elements are linear. And the load network can be either linear or non-linear, the equivalent circuit is to be valid.

We also saw close relatives of the Thevenin's theorem, the Norton's theorem and the Millman's theorem which can be derived quite easily in a straight forward fashion. And finally, we discuss the compensation theorem which enables us to calculate the changes that are produced in an entire network, consisting of linear elements and independent sources, if one of the impedances is changed by certain amount.

And in the discussion of the compensation theorem, even though we said Z has changed from Z plus delta Z, it does not mean that delta Z is going to be very small quantity. It can be, the impedance can be doubled or tripled. Delta Z by no means, should be a small quantity. We would not have to put that restriction. So, all we say is if Z is changed from Z plus Z plus delta Z.

The changes that are brought about in the entire network can be calculated by solving a very simple network, in which all the original independent sources are deactivated. But in this new network, you have just one source whose strength is equal to a voltage source, whose strength is equal to the change in the impedance delta Z multiplied by the current, that was originally existing in Z in the original network which I of s.

So, delta Z times I of s acting in a network in series with the altered impedance Z plus delta Z but, in the network all the original voltage sources as current sources are deactivated. The solution of this, will give you the altered quantities in the network. We will continue our discussion of the compensation theorem, in the next lecture.