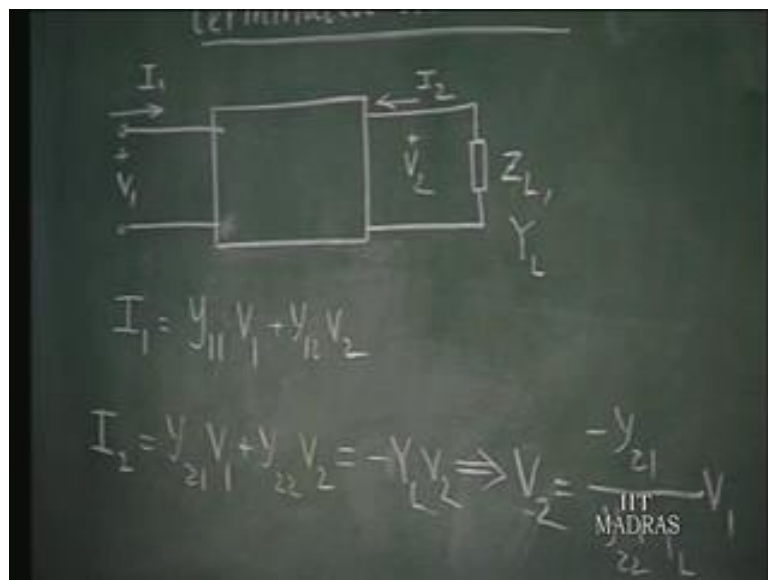


Networks and Systems
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Lecture - 33
Network Functions (4)
Terminated 2 - port networks
Parallel and Cascade Inter-connections
Exercise - 6

In this lecture, we will continue our discussion of the analysis of 2 port networks, using the various network functions that we have discuss defined so far.

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Often, we have a situation where a 2 port network is terminated in a load, which is let us say has an impedance function Z_L occurs and admittance function Y_L occurs. We would like to find out the ratio of the voltage V_2 to V_1 under these conditions. The input impedance measured under these conditions and so on and so forth. So, analysis of such networks can be done on the basis of the Y parameters or Z parameters or other sets of parameters. Let us, take the Y parameters to illustrate.

So, I_1 equals $y_{11} V_1$ plus $y_{12} V_2$, that is the standard equation. I_2 equals $y_{21} V_1$ plus $y_{22} V_2$ standard equation. But now, because of the load termination V_2 and I_2 are ((Refer Time: 02:06)) constrained by this load impedance. In particular, we have I_2 equals minus Y_L times V_2 . Because, Y_L times V_2 is the current in this direction. But,

since our reference direction for current is opposite. Therefore, I_2 be have to be written as minus Y_2 minus Y_L times V_2 , this is restriction.

Therefore, here we have written I_2 equals $y_{21} V_1$ plus $y_{22} V_2$ equals minus Y_L times V_2 . From this, we can eliminate I_2 . Therefore, you take this equation V_2 when transferred this on other side minus of y_{22} plus Y_L times V_2 equals $y_{21} V_1$. This gives this relation V_2 equals minus y_{21} divide by y_{22} plus Y_L V_1 . So, when you substitute this into this, you will get a relation between I_1 and V_1 . So, for V_2 we substitute this equation then you get a relation between I_1 and V_1 .

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The image shows a chalkboard with the following handwritten text and equation:

Input admittance

$$Y_{in} = \frac{I_1}{V_1} \Big|_{\text{Load}} = y_{11} - \frac{y_{12} y_{21}}{y_{22} + Y_L}$$

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Therefore, the input admittance at port 1, when port 2 is terminated in a load admittance Y_L can be obtained as I_1 over V_1 with load at port 2. This can be written as y_{11} minus $y_{12} y_{21}$ over y_{22} plus Y_L . It can be shown like this.

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Handwritten derivation on a chalkboard showing the input impedance Z_{in} in terms of various network parameters. The equations are:

$$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

$$= h_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L} = \frac{AZ_L + B}{CZ_L + D}$$

Below the equations, there is a small circuit diagram showing a voltage source V_1 and a load admittance Y_L connected to a two-port network. The IIT MADRAS logo is visible in the bottom right corner.

Similarly, the input admittance which is of course, the reciprocal of that. But, you can independently write this as, we can show this z_{11} minus $z_{12}z_{21}$ over z_{22} plus Z_L . Alternately, this can be written in terms of h parameters h_{11} minus $h_{12}h_{21}$ over h_{22} plus Y_L . In terms of the ABCD parameters write this as AZ_L plus B over CZ_L plus D . So, these are in terms of the various network parameters, you can find out the input impedance or admittance in this form. The derivation of this I will leave this as an exercise to you.

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Handwritten derivation on a chalkboard showing the input admittance Y_{in} in terms of various network parameters. The equations are:

$$Y_{in} = \frac{V_2}{V_1} \Big|_{\text{Load Termination}}$$

$$= \frac{z_{21}z_L}{\Delta_z + z_{11}z_L} = \frac{-y_{21}}{y_{12} + Y_L}$$

$$= \frac{-h_{21}}{\Delta_h + h_{11}Y_L} = \frac{Z_L}{AZ_L + B}$$

On the left side of the board, there is a partial equation $\frac{L+B}{Z_L+B}$. The IIT MADRAS logo is visible in the bottom right corner.

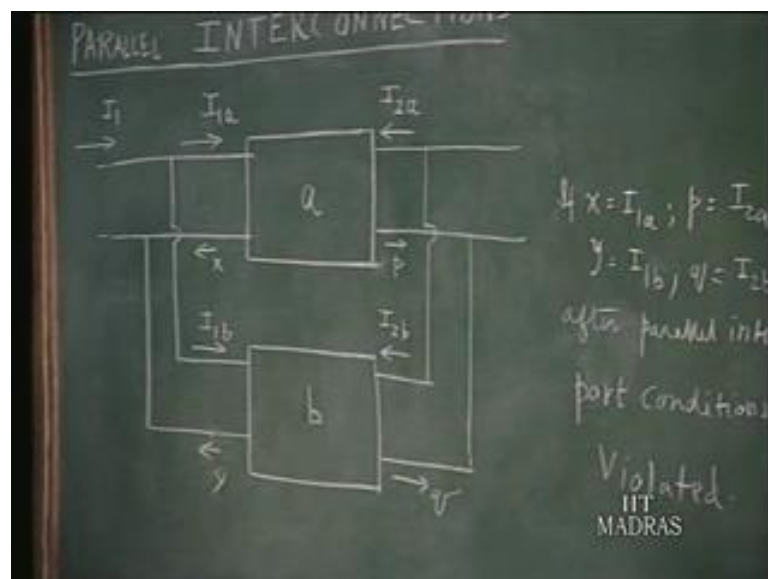
Similarly, the voltage ratio G_{21} of s which is V_2 of s over V_1 of s with a load termination is also important. Because, we will filter problems and so on. You terminate this 2 port in a load. And we would like to find out the ratio V_2 of s V_1 of s . Analyzing in the same fashion, you can show that this is equal to $z_{21} Z_L$ divided by Δz the determinant of the open circuit impedance matrix. And $z_{11} Z_L$ can also be shown to be $\frac{-y_{21}}{y_{22} + Y_L}$.

We can also show this to be equal to $\frac{-h_{21}}{\Delta h + h_{11} Y_L}$. These are all straight forward relations, which can be obtained by simple manipulation of this various equations. Z_L equals Z_L in terms of the ABCD parameters $A Z_L + B$. So, in terms of the various network functions you can calculate. The ((Refer Time: 05:58)) network functions that you are of interest was in a terminated condition.

Note that, we have used the capital G here to distinguish between small g , when under open circuit conditions the ratio of V_2 to V_1 we used small g . In general, load conditions load terminal conditions we use the capital G . Similarly, instead of small y_{11} we can write capital Y and Z in this case. Because, ((Refer time: 06:24)) they are not under normalized terminal conditions. But, under specific loads, which are defined by Z_L or Y_L as the case may be.

It is often convenient for us to analyze a complicated 2 port network as an interconnection of simpler 2 port networks.

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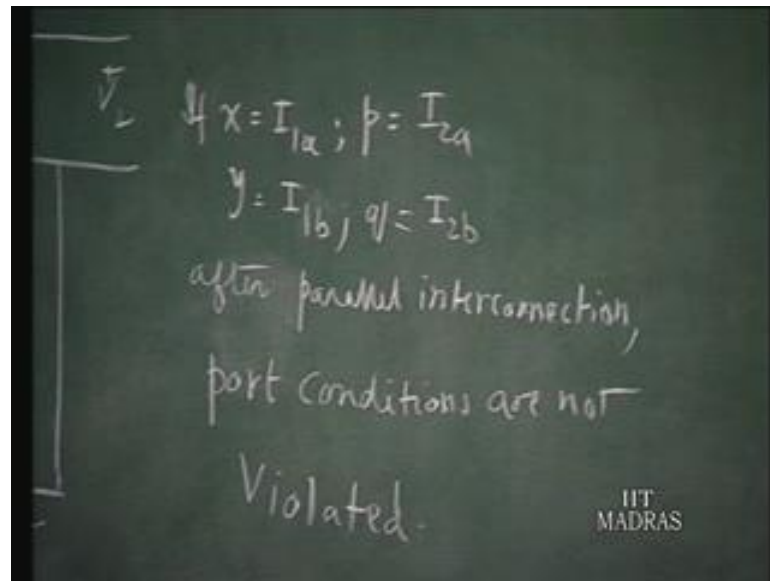
For example, I may have a complicated 2 port network, which can be thought of as the interconnection of two 2 port networks a and b. Or, put it other way let us say, there is a 2 port network a, which is connected in connected to a 2 port network b in this fashion. Now, this is called a parallel interconnection. Because, port 1 of the network a. And port 1 of network b are connect in parallel. The same voltage V_1 appears across port 1 of a and port 1 of b.

The same voltage V_2 appears across port 1 of a and port 1 of b. So, this is called a parallel, parallel interconnection. Both ports are connect in parallel. And now, it would be possible for us to find out the Y matrix of the overall configuration, in terms of the Y matrix of the, a network. And Y matrix of the b network provided some conditions are satisfied. Suppose, we are having only port a, network a in operation. It is not connect in parallel b with network b.

That means you disconnect this. Then, port requirement gives us the condition that $I_1 a$ must be equal to x . That means, the same current must flow through both the terminals. The port requirement is $I_1 a$ is equal to x . That would be true if network a, was operating in isolation. Similarly, $I_2 a$ is equal to px . On the other hand, if port b alone was operating $I_1 b$ equals y . And $I_2 b$ equals q . Now, when you interconnect them, when you put them in parallel.

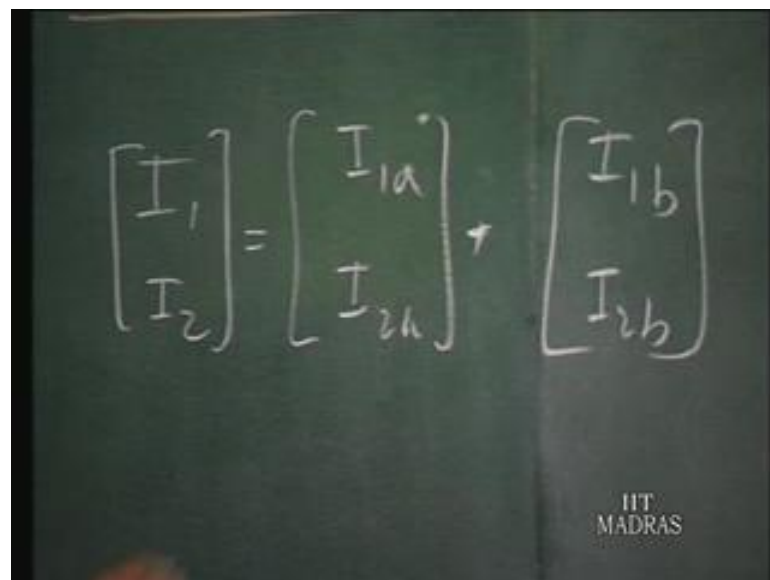
Then, if the same conditions are still valid that is x continues to be equal to $Y_1 a$, p is continues to $I_2 a$. y continues to be equal to $Y_1 b$ and q continues to be $Y_2 b$ after parallel interconnection. Then port conditions are not violated then it can be shown.

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That the Y matrices of these two will add up to form the overall Y matrix of this network. The proof will be something like this. If I the total current is I 1 and this current is I 2.

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Then we can say that I 1, I 2 these two currents are equal to I 1 a I 2 a plus I 1 b plus I 2 b. Because, as you can see, this current I 1 is I 1 a plus I 1 b. I 2 equals I 2 a plus I 2 b. This is what we give.

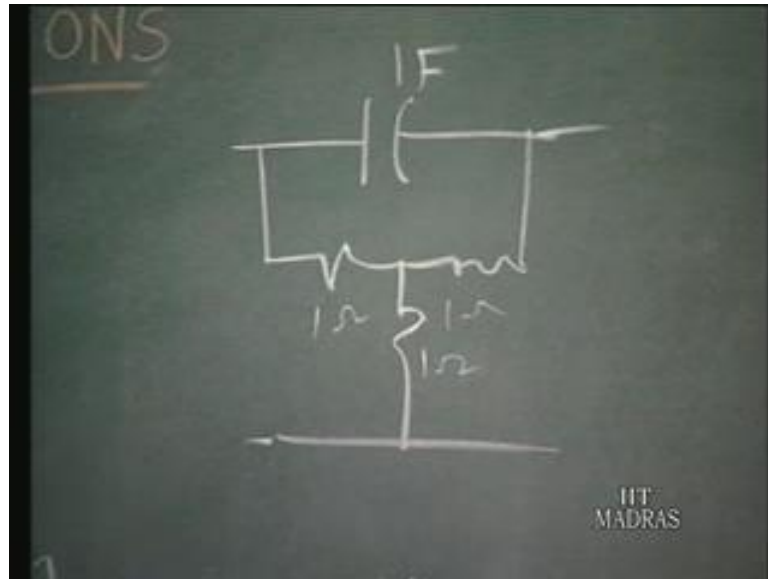
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$$I_{2a} = [Y_a] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + [Y_b] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
$$I_{2b} = [Y_a] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + [Y_b] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
$$Y = Y_a + Y_b$$

And now, this will be equal to the Y matrix of the a network, times V 1, V 2 plus the Y matrix of the b network times V 1 V 2. And you can now, have this is the common matrix therefore, you can write this as Y a plus Y b multiplying V 1 V 2. That means, the overall Y matrix of this 2 port network is Y a plus Y b, where each of these is the 2 port network. That means, when you have two 2 port networks in parallel. Y equals Y a plus Y b as long as the port condition is not violated. And you can take it for granted, that as long as we have 3 terminal 2 port networks.

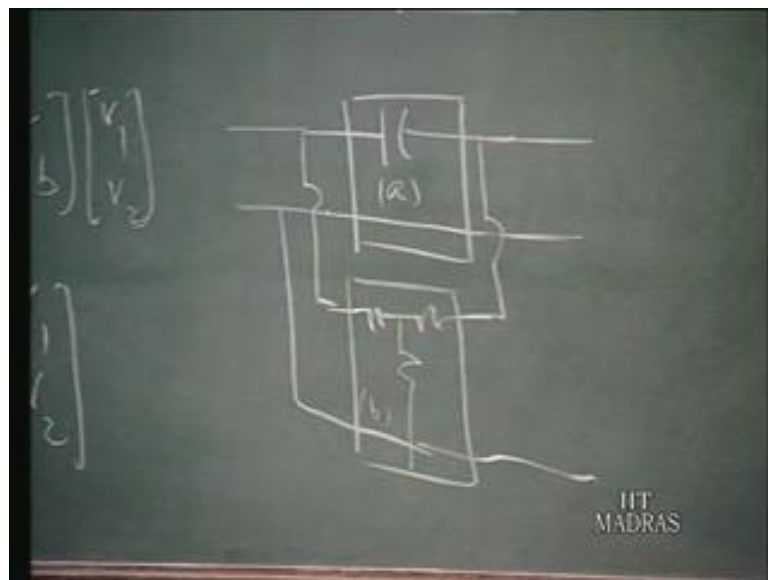
That means ((Refer Time: 10:24)) this is a common lead going through, when you have a 3 terminal 2 port networks. The port condition is never violated and this particular rule will be always valid. In other cases, we have to take special test we must make special test to see that the port conditions are not violated.

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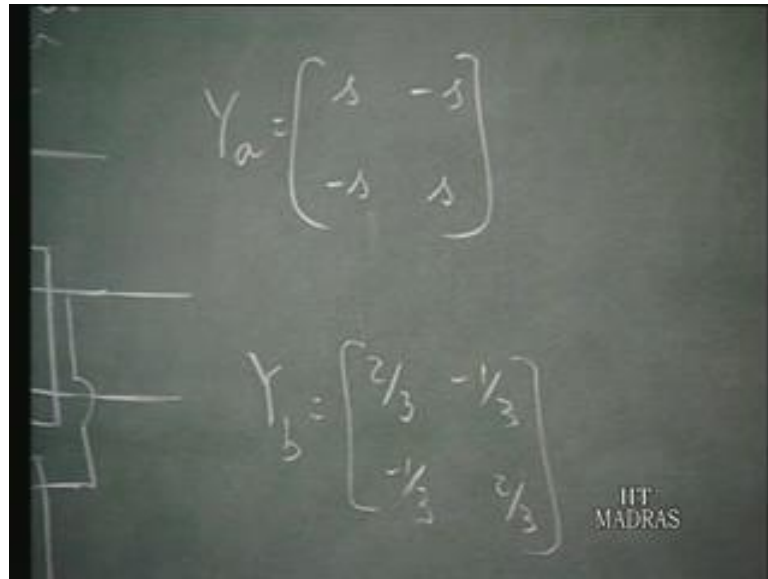
Let us an example, suppose we had this same example, which we consider ear earlier, where we had this 2 port network of 1 ohm, 1 ohm, 1 ohm and 1 farad.

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You can consider this to be the parallel connection of 1 2 port network containing 1 farad capacitor. The other 2 port network containing 3 star connected resistances. So, you have the parallel combination of these two networks a and b. And if this is 1 ohm resistances this can be converted into 3 pi connected set of 3 ohms resistances as we have seen.

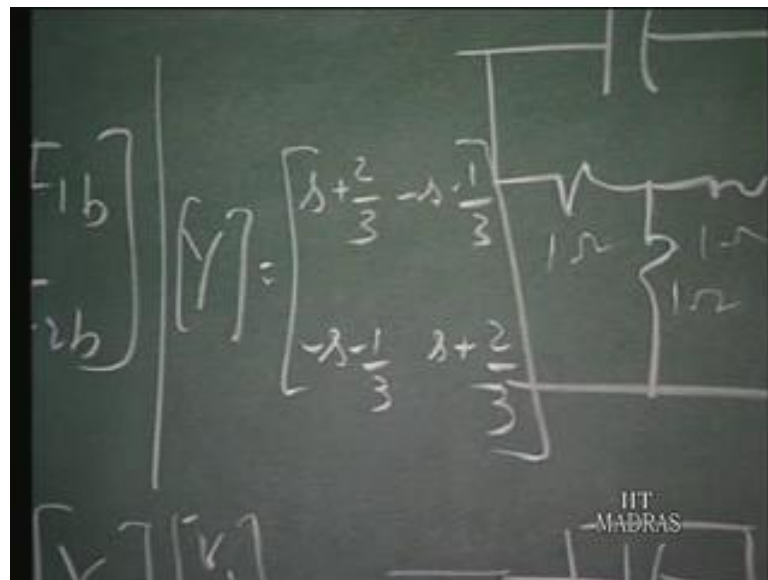
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The image shows a chalkboard with two handwritten equations for admittance matrices. The first equation is $Y_a = \begin{bmatrix} s & -s \\ -s & s \end{bmatrix}$. The second equation is $Y_b = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$. To the left of the equations, there are faint circuit diagrams. In the bottom right corner, the text 'IIT MADRAS' is visible.

And it turns out that the a matrix, the Y matrix for the a network for 1 farad capacitor turns out to be s, minus s, minus s, s. The Y matrix for the b network turns out to be 2 third, minus 1 third, minus 1 third and 2 thirds.

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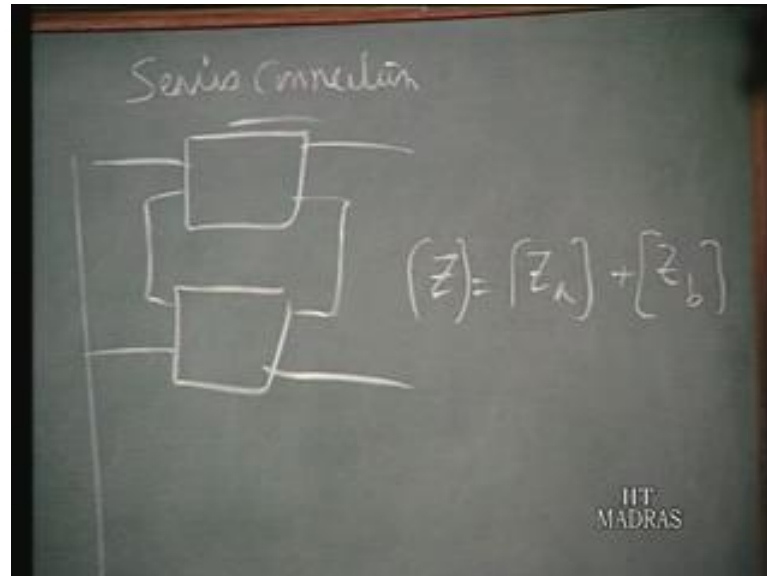


The image shows a chalkboard with a handwritten equation for the combined admittance matrix: $[Y] = \begin{bmatrix} s + \frac{2}{3} & -s - \frac{1}{3} \\ -s - \frac{1}{3} & s + \frac{2}{3} \end{bmatrix}$. To the left of the equation, there are labels $[I_b]$ and $[V_b]$. To the right, there is a circuit diagram showing a network of resistors and capacitors. In the bottom right corner, the text 'IIT MADRAS' is visible.

So, for this entire network the Y matrix is the sum of this two this will be the Y matrix for this will be s plus 2 thirds, minus s minus 1 third, minus s minus 1 third and s plus 2 thirds. This is the result, which we have already obtained. So, it turns out that when 2

networks are coming in parallel as long as the port conditions are not violated, the Y matrices add.

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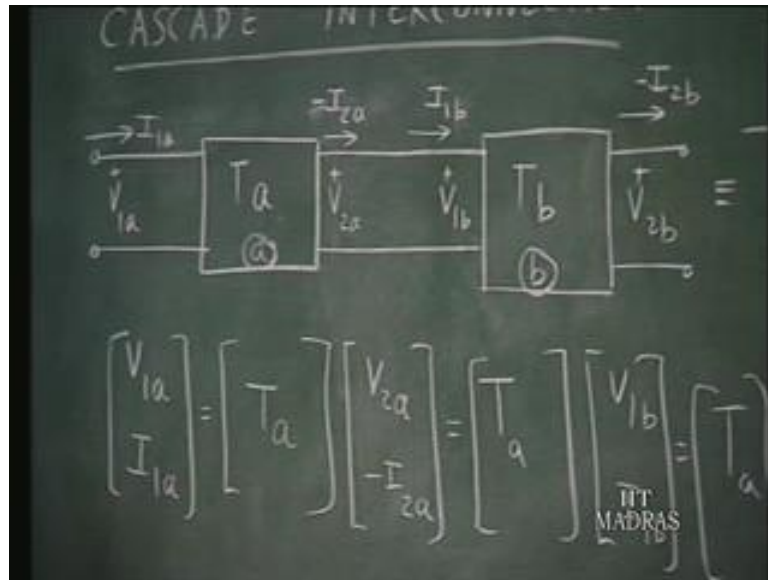
Similarly, it can be shown that when 2 networks are in series connection This is said to be series connection. Then again, when the port conditions are not violated that means, this current continues to be the same as this current continues to be same as this current and continues to be same as this current. The port voltage V_1 is $V_{1a} + V_{1b}$. The Z matrices add $z_a + z_b$. However, the port conditions are violated in this case for most of the practically useful networks.

And therefore, this particular interconnection is not very much used. It can be similarly shown, that in the case of h matrices it is you have one set of ports are in connected in series. And another set of ports connected in parallel. I will leave this as an exercise for you to figure out what type of interconnection is called for, if we have to have the sum of the two h matrices as the component of s to be added together to give the h matrix for the result at the network.

However, for the ABCD matrices the interconnection is what is called a cascade interconnection. That is what we will discuss in some in detail, because that is practically important. And there, we will have the overall port chain matrix or ABCD matrix is obtained as a simple product of the ABCD matrices of the individual networks. We will see that in a moment.

I will not discuss the series connection ((Refer Time: 14:13)) or the series parallel connection pertaining to Z and h matrices, because it is not ordinarily very important and useful in practical work.

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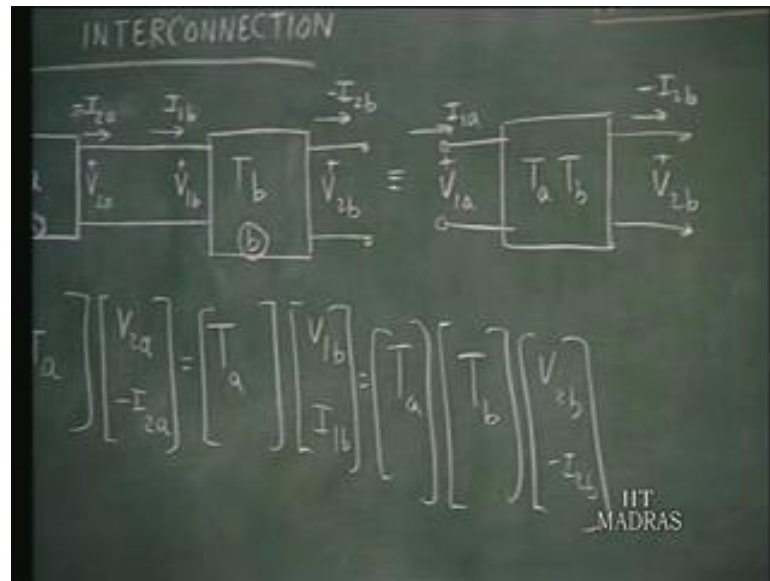


Two 2 ports networks are said to be connected in cascade, when they are connected in this fashion. That means, the input quantities 2 port, port 2 port network a will be there and then output quantities of network a, turns out to be the input quantities for network b. That means, if you uses a transmission line the side receiving end quantities of network a, will turn out to be sending in quantities for the network b and this will be the output of this.

So, we see for as far as the network a is concerned. If T a is the transmission matrix or the ABCD matrix. V 1 a I 1 a as far as a is concerned is related to V 2 a and minus I 2 a with this difference direction. By this chain matrix as the transmission matrix T a. So, V 1 a I 1 a is the ABCD of matrix pertaining to network a, which I have indicated as T a multiplying V 2 a minus I 2 a. But, we observe that V 2 a is same as V 1 b.

The port voltage of port second port voltage of a, turns out to be the first port voltage of b. So, V 2 a is the same as V 1 b. And minus I 2 a is the same as I 1 b, the input current to port b. The port 1 of network b. So, minus I 2 a equals I 1 b. Now, what is the usefulness of this representation. You observe that V 1 b and I 1 b are related to V 2 b to minus I 2 b by the chain matrix of network b.

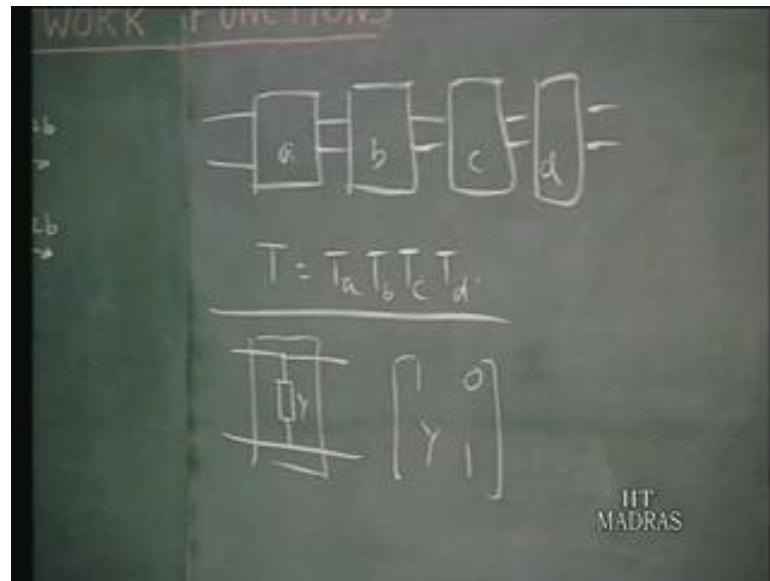
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So, V_1b I_1b is obtained as T_b times V_2b minus I_2b . So, if you substitute for V_1b I_1b , T_b times V_2b minus I_2b it will be T_a times T_b times V_1a minus I_1a . That means, the input quantities here will be related to the output quantities ((Refer Time: 16:17)) in the overall 2 port networks. Suppose, you consider this as an overall the cascade as an overall 2 port. Then, that the relation between the ((Refer Time: 16:27)) input quantities and the output quantity is the product of 2 chain matrices T_a and T_b .

So, you have V_1a I_1a as the chain matrix of this turns out to be $T_a T_b$ multiplied by this. So, in the cascade connection, if two 2 port networks are connect in cascade. The chain matrix of the cascaded network is the product of the chain matrix as the individual links in that chain. That is why this is called a chain matrix.

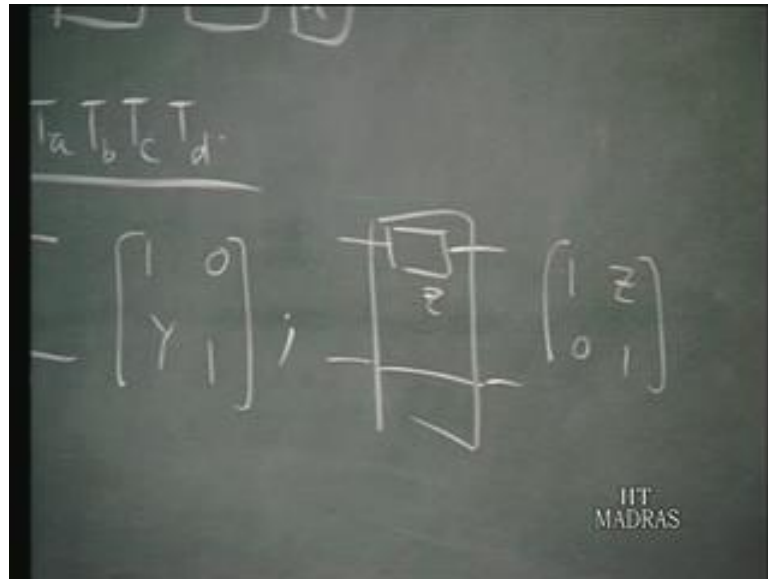
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So, you can see that if I have a lot of this networks in cascade. So, suppose ABCD in cascade. See, all the four 2 port network put in cascade. The overall chain matrix of this is T_a , times T_b , times T_c , times T_d where these are all individual matrices. The product of all those matrices will give us the chain matrix for the overall configuration. It is a very convenient arrangement. Now, in doing this in calculating the chain matrix of the overall configuration it is important to see, that for simple networks like this.

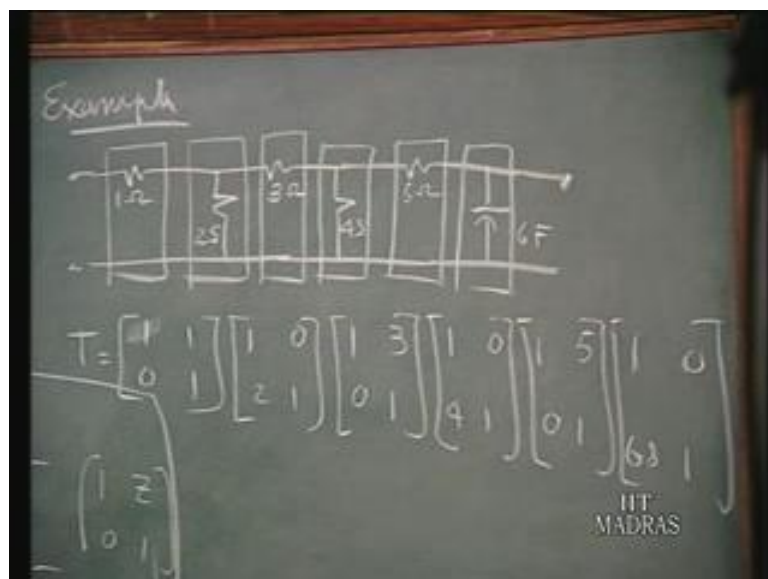
Suppose, I have this as the 2 port and this as Y . This chain matrix as we have seen in an example, which worked out earlier. You can go back and see, the chain matrix for this is equal to this, very simple. Suppose, your shunt element the chain matrix of that is $\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$.

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On the other hand, if I have a series impedance Z like this, the chain matrix for this is $1, Z, 0, 1$. So, if you remember these two relations chain matrices were very simple networks. When you have a cascade connection ((Refer Time: 18:17)) of a whole lot of these elements it is very easy to figure out, what the chain matrix of that would be. In an example, we will illustrate this.

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Suppose, I have a network like this, this is what is called a ladder network. This can be thought of as the cascade connection of this 2 port network, another 2 port network, third

2 port network, a fourth 2 port network, fifth one and a sixth one. So, we have six 2 port networks very ((Refer Time: 19:09)) simple configurations like this. All put in cascade giving us the overall configuration. Let us, take some numerical values 1 ohm, 2 semen, 3 ohms, 4 semen's that is the conductance, 5 ohms and 6 farads.

So, to obtain the chain matrix of the overall configuration. I need to multiply the chain matrices of the individual units. Remember in the rule, the chain matrix for the first 2 port network is this that is the 1. For the second 2 port network it is 1, 0, 2, 1 using this loop ((Refer Time: 20:00)), because this is a shunt element. Similarly, for the third 2 port network it is 1, 3, 0, 1 for the fourth it is 1, 0, 4, 1, because it is a shunt element.

Fifth again is a series element 1, 5, 0, 1. And the last is again a shunt element 1, 1 the admittance of this element is 6 s, in terms of the generalized admittance. So, that is your overall chain matrix. So, you can write this term by mere inspection. The chain matrix the ABCD matrix and the overall configuration.

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$$= \begin{bmatrix} 1350s + 43 & 225 \\ 942s + 30 & 157 \end{bmatrix}$$

And if, you multiply this out and simplify this turns out to be 1350 s plus 43 that is the a parameter, b parameter is 225, c parameter is 942 s plus 30 and 157 is the d parameter. So, that is the ABCD matrix of this overall configuration. We have seen, that you can verify that this follows the reciprocity condition A D minus B C equals 1. And you can see, that the chain parameter or the ABCD parameter of a cascade of a overall network

can be thought of particularly, in this form which is called a ladder network, can be thought of as a cascade connection of simple networks.

We change parameters of each of this is written down by mere inspection. And it is a question of algebra to find out the overall chain matrix. Let me, now summarize our discussions on the topic network functions. We started with a general definition of a network function. As the ratio of Laplace transform of the response, which is current or voltage to an excitation, which is again a current or voltage. So, this is a general network function.

And we said, for a passive network the poles of the network function represent the natural frequencies under the appropriate conditions. And the restrictions on the poles of the network function will be, such that the all the poles must be in left of plane. They can be marginally in the imaginary axis provided they are simple. This is from the point of view of stability, which is a requirement for a passive linear network. And when you talk about the network function it is a ratio of Laplace transforms.

It is not the ratio of time function this is what we should keep in mind. Then, we went to study the network functions are appropriate to 1 port and 2 port networks. For 1 port networks the ratio of the excitation, which is a voltage to the current or vice versa of the 2 network functions that one 1 can define. This is the driving point impedance function or the driving point admittance function, which are reciprocals of each other. We observed, that the reciprocal of a driving point function is also a network function.

And therefore, both the poles and zeros of driving point functions have got to satisfy the requirements of their locations, restricted locations from the point of view of stability. Then we discussed the various sets of parameters characterizing a 2 port network. We define the Z parameters, the Y parameters, the hybrid h parameters, the inverse h parameters are called the g parameters, the ABCD parameters. Of these the Z parameters represent the, or pertain to essentially to a network with the 2 ports open circuited.

Because, you introduce current excitation, when the current is removed the particular port is open circuited. That why, they are called open circuit impedance parameters. Similarly, the Y parameters are appropriate to the network with the 2 port short circuited they are called short circuit admittance parameters. The h parameters refer to networks

when both 1 port is open other port is shorted. Therefore, they are called hybrid parameters.

As far the ABCD parameters are concerned, there is no particular hesitation or open circuit or short circuit associated with that. In fact, we saw that the ABCD functions as a rest are not respectable network functions. Because, they are not in the ratio of Laplace transform response to excitation, as a matter of fact the inverse of ABCD the reciprocals of these are network functions, which we have seen. Therefore, the poles of ABCD do not represent the natural frequencies.

It is the reciprocals, which represent the natural frequency in the network under appropriate conditions. We worked out various examples to calculate the network parameters or ABCD or the various parameters of a 2 port networks. Then, we also saw how a load at 2 port network. That is 1 network terminated in a particular load impedance can be analyzed, using any one of the sets of parameters. We in particular we found the input impedance of a 2 port network terminate a load impedance z_n .

We also saw, the voltage ratio V_2 to V_1 . How it can be calculated in terms of the various parameters Z Y h or ABCD. We observed that a reciprocal 2 port network has got certain definite restrictions on the parameters. So, out of the four parameters in each set only three are independent reciprocity requires, that Z_{21} equals Z_{12} , Y_{12} equals Y_{21} , h_{12} is equal to minus h_{21} . And $A D$ minus $B C$ is equal to 1. Similarly, the requirements of symmetry were also discussed.

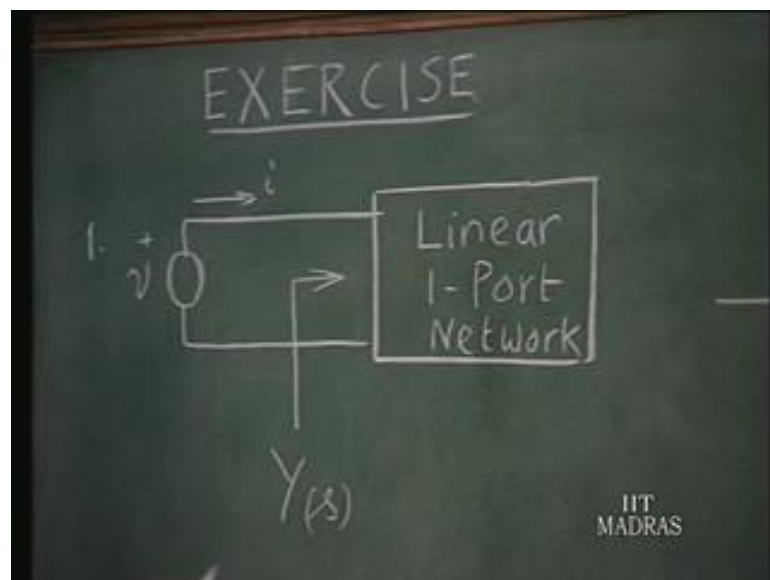
And then, we also finally, saw that how interconnect a complicated 2 port network can be viewed as interconnection of sub networks of 2 port networks. And of these various interconnections that are possible. The parallel interconnection is appropriate to Y parameters. As long as port condition is not violated, the Y matrix of the overall 2 port is the sum of the Y parameters of the individual 2 ports. Similarly, in a cascaded system of 2 ports.

The chain parameters the chain parameters the chain matrix of the overall 2 port is the product of the chain matrices of the individual 2 ports. But, you must keep that order in fact for example ((Refer Time: 26:30)), if you have got here T . T equals T_a , T_b , T_c . T_d you must maintain the order in the cascade chain. So, that is the rule with reference to the chain parameters.

Similarly, as far as Z parameters is concerned. The series connection will mean that the overall Z matrix the sum of the individual Z matrices provided the port conditions are not violated. Similarly, the h matrices will add, when 1 port is parallel the other set the other pair of ports is in the series. And the interconnections related to h parameters. This is series parallel interconnection or the series interconnection or not very common not useful not very useful practice, so we did not discuss them at length.

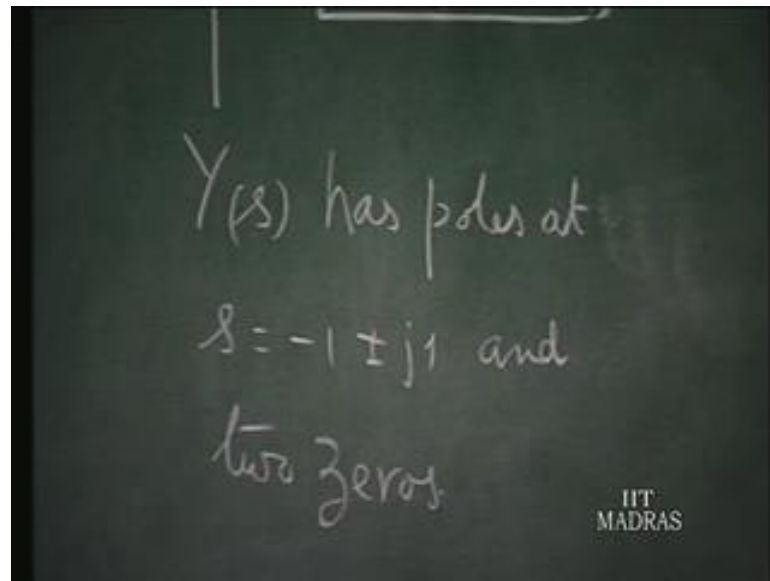
So, at this stage I will give you a set of examples for you as an exercise to work to illustrate the ideas that we have discussed under the heading network functions.

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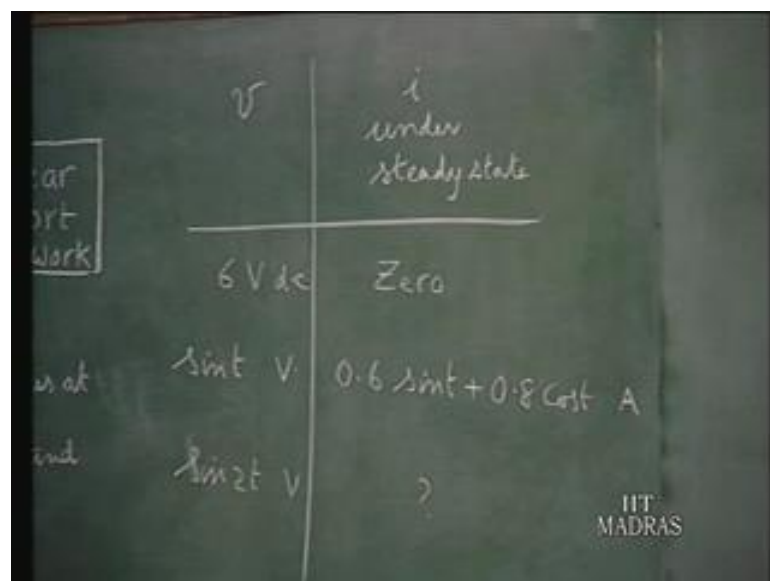
First problem in this exercise, deals with the 1 port network. This is a linear 1 port network with an admittance function Y of s , the driving point admittance is Y of s .

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And it is given that Y of s as poles, at s equals minus 1 plus or minus j 1. Poles at s equals minus 1 plus or minus j 1 and two zeros. The location of the zeros are not known to us. So, this is a network function with a quadratic in the numerator and quadratic in the denominator with a particular scaling factor. The zeros are not known, the poles are known to be at minus 1 plus or minus j 1, the take complex conjugate poles. Further, data given to this network is ((Refer Time: 28:23)) the current under steady state conditions for different types of excitation voltages V .

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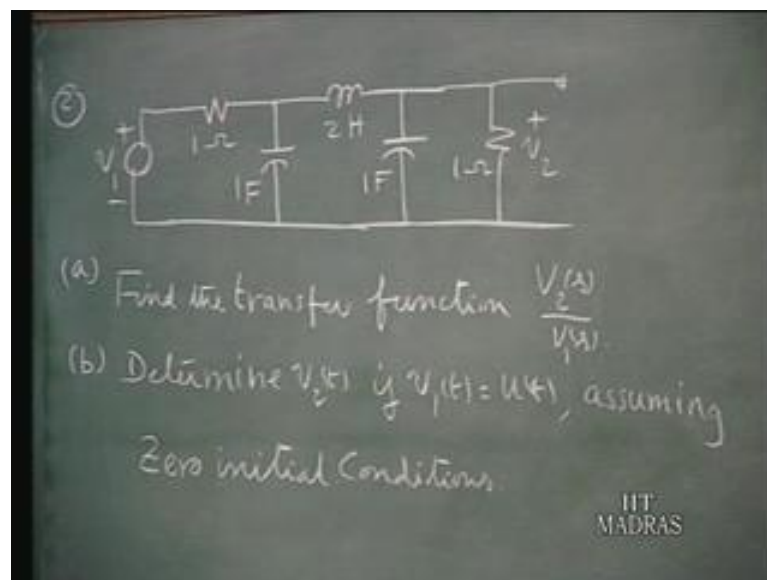


So, if V equals volts DC, if the applied voltage is 6 volts DC. Then the steady state current into the network is 0. It is known that this is 0. Now, the second excitation is when the input voltage is $\sin t$ volts. If you drive this network 1 port network a sinusoidal voltage source $\sin t$, this steady state current is $0.6 \sin t$ plus $0.8 \cos t$ amperes So, when you drive this 1 port network with a sinusoidal input voltage $\sin t$, this happens to be the current.

You are asked to find out the current when the input is $\sin 2 t$ volts. If the input is $\sin 2 t$ volts. What is the steady state current for this plane? So, using these two items of data we should be able to calculate the locations of the two zeros as well as the scaling function that you are having. Because, after all the network function of this will be of the form h times $Z 1$, it is s minus $Z 1$, times s minus $Z 2$ divided by s minus $p 1$, times s minus $p 2$.

But, $p 1$ and $p 2$ are known. So, three constants are to be evaluated using these two items of data using that you must be able to find out the steady state current, when the input is $\sin 2 t$ volts.

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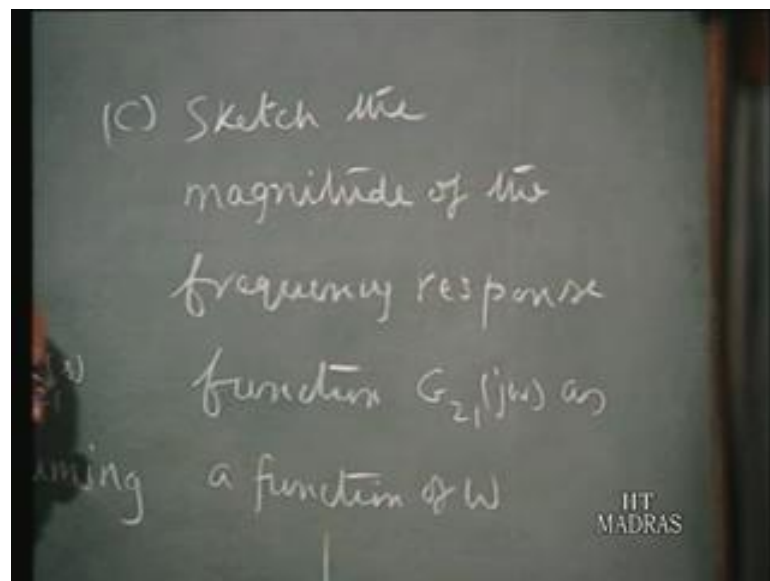


Second example, you have a 2 port network of this configuration 1 ohm, 1 farad, 1 farad, 2 henries, 1 ohm. You have the input voltage $V 1$. So, first question is find the transfer function $V 2$ of s over $V 1$ of s , for this network So, the Laplace transform of the output voltage $V 2$ of s $V 2 t$ to the Laplace transform of the input voltage $V 1$. Then b,

determine V_2 of t if the input voltage V_1 of t is a unit step function $u(t)$ assuming zero initial conditions.

So, we assume that all capacitors are initially uncharged. And the inductance does not carry any current using this transfer function given this particular input excitation find out the output in time domain.

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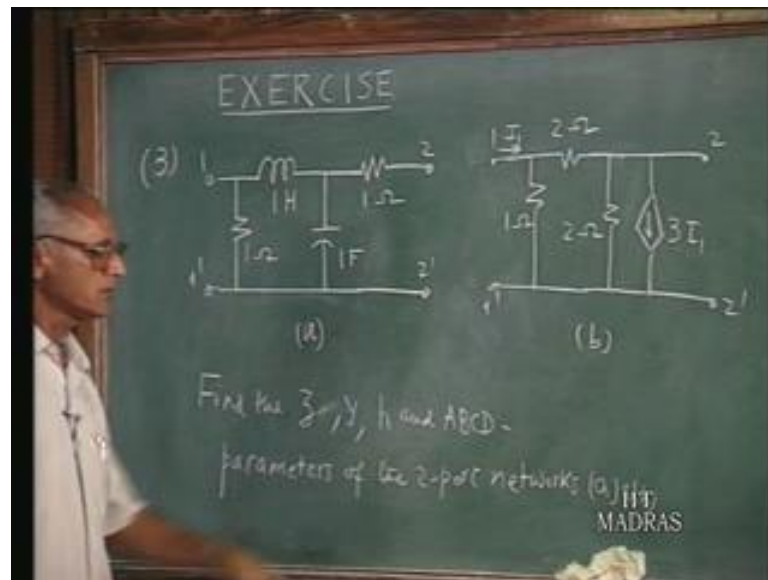
C sketch the magnitude of the frequency response function.

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This V_2 of s by V_1 of s I will call that as G_{21} of s . So, response function G_{21} $j\omega$ as a function of ω . Magnitude or the frequency response function as a function of ω . That is in another words you have to plot G_{21} $j\omega$ magnitude as a function of whatever curve you get.

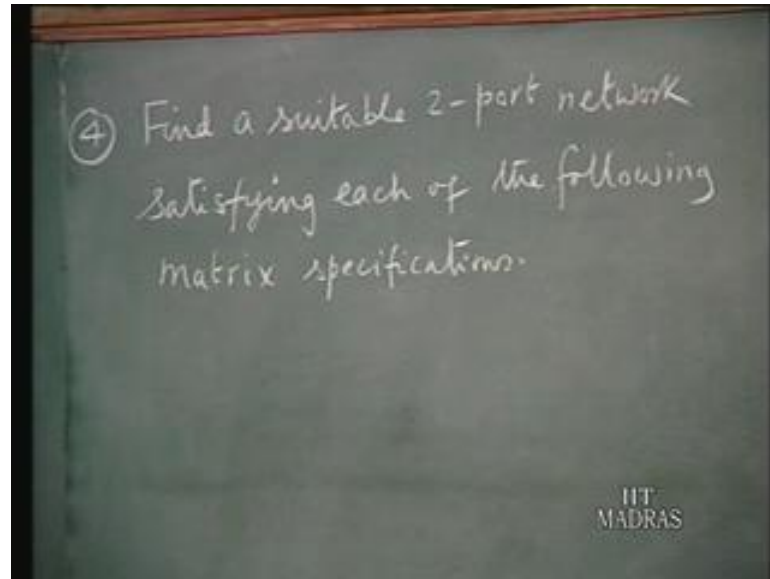
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Problem number 3, you have given a 2 port network here. Find the Z , Y , h and $ABCD$ parameter of the 2 port networks a and b . The two 2 port networks for each one of these, you find the Z , Y , h and $ABCD$ parameters. First network consist of a 1 ohm resistance, 1 Henry inductance, 1 farad capacitor, 1 ohm resistance a ladder network Second 2 port network 1 ohm resistance, 2 ohm resistance, 2 ohm resistance and a dependent current source here, whose value depends upon the input current I_1 .

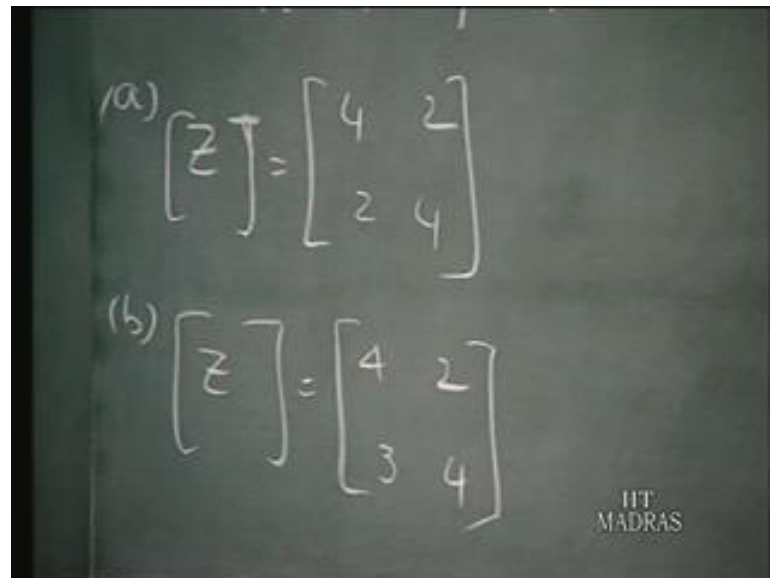
So, whatever current I_1 you have here. The current here is 3 times I_1 . So, this is a dependent current controlled current source. And therefore, you will find that this network is not going to be reciprocal. If this is a reciprocal network, this is not a reciprocal network. So, you will expect that Z_{12} is not equal to Z_{21} , Y_{12} is not equal to Y_{21} , h_{12} is not equal to minus of h_{21} . And the determinant of $ABCD$ parameter matrix is not going to be equal to 1 for this network b .

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Fourth problem, find a suitable 2 port network satisfying each of the following matrix specifications. So, this is the reverse of analysis, you are given the matrix specifications you have to find a network which satisfy this. You can use the concept of equivalent networks to do this.

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A, the Z matrix of this 2 port network has to satisfy this relations, this is the Z matrix of this. B, the Z matrix of another network 4, 2, 3, 4. So, immediately you see, that this is a

reciprocal network this is not a reciprocal network. So, you need to incorporate some dependent sources in order to find a network, which has got this Z matrix.

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ifications.

$$(c) [Y] = \begin{bmatrix} 5 & -3 \\ -3 & 6 \end{bmatrix}$$

$$(d) [Y] = \begin{bmatrix} 5 & 3 \\ -3 & 6 \end{bmatrix}$$

HT
MADRAS

C the Y matrix for this is 5, minus 3, minus 3, six. D, Y matrix for this is 5, minus 3, let us make it plus 3, this is minus 3, again 6 again this violates the condition for reciprocity therefore, the network that you get here should cannot be modeled using purely bilateral reciprocal elements.

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swing

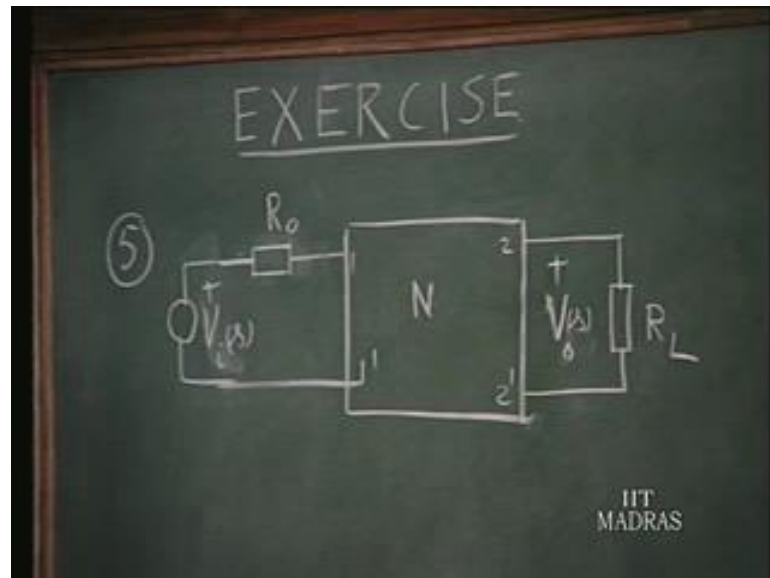
$$\begin{array}{l} -3 \\ 6 \end{array} \left| \begin{array}{l} (e) [T] = \begin{bmatrix} 26 & 5 \\ 5 & 1 \end{bmatrix} \\ (f) [H] = \begin{bmatrix} 4 & 2 \\ 1 & 4 \end{bmatrix} \end{array} \right.$$

$$\begin{array}{l} 3 \\ 6 \end{array}$$

HT
MADRAS

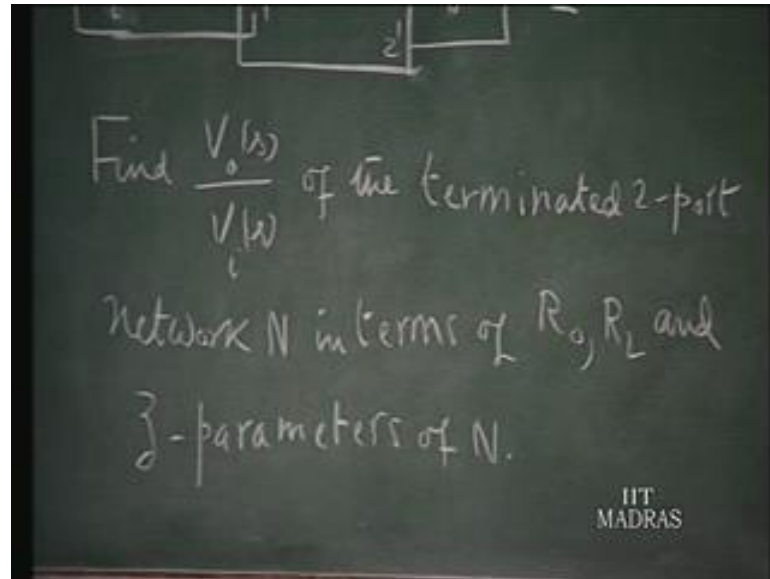
E, the T matrix the ABCD matrix for this is $26, 5, 5, 1$. You observe that the determinant of this T matrix is equal to $1, 26$ minus 25 equals 1 . So, it should be possible for you to find out a reciprocal network satisfying this. F, the H matrix for this $4, 2, 1, 4$. So, you find out for each one of these specifications, a simple 2 port networks satisfying the, these equations.

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Problem 5 concerns what is called a doubly terminated 2 port network. This network N, 2 port network $1, 1'$, $2, 2'$ is terminated a load resistance R_L at one side. You have resistance R not in the other side. So, in that you introduce a voltage source V_i of s . So, you can view this as a termination of N or you can view this as internal resistance of this source whatever it is. So, we have a resistance at one side and a resistance at the other side.

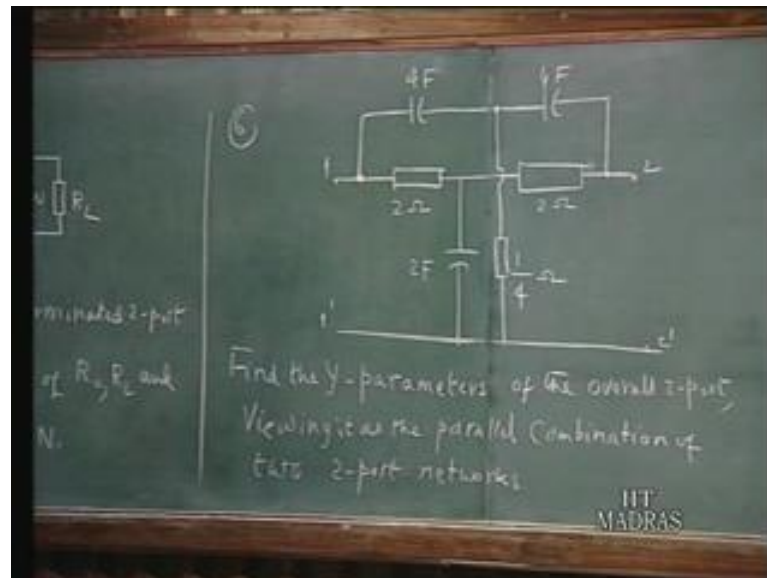
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You are asked to find out, find the transfer function $V_o(s)$ over $V_i(s)$ of the terminated 2 port network N in terms of naturally, you must have R_o and R_L entering the picture. R_o and R_L and the z parameters of N . We have done a similar example in the lecture, where there is only R_L , R_o is 0. Now, we extend that to find out the ratio of $V_o(s)$ to $V_i(s)$ in terms of the z parameters ((Refer Time: 38:56)) and R_o and R_L .

This is a very practically important situation, where N could be a filter network, where you have a source with an internal resistance R_o . And the filter is terminated in a load resistance R_L , this is very practical situation. And you try to find out $V_o(s)$, $V_o(s)$ over $V_i(s)$ in terms of R_o and R_L and z parameters of the network N .

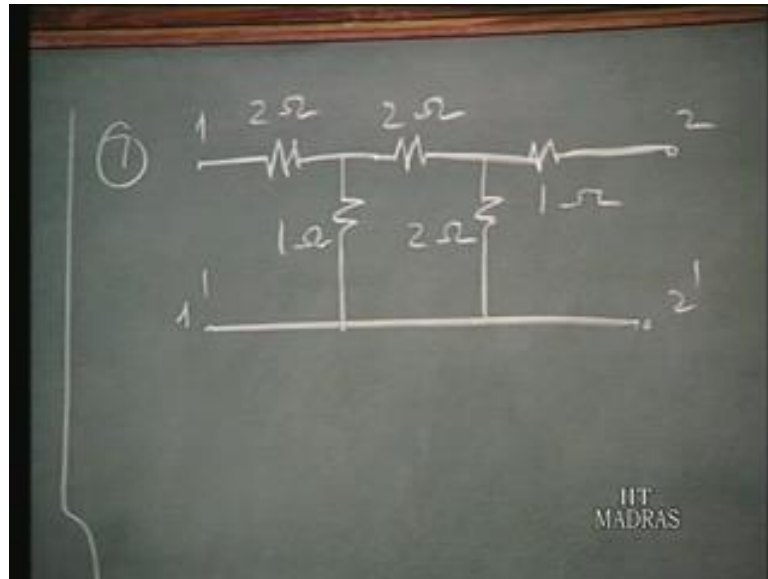
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6, next problem is slightly complicated network. You have these boxes represent resistances. So, you have 2 T networks these three elements constitute a T, these three elements constitute a T and 2 T networks are put in parallel. This is 2 ohms, this is also 2 ohms. This has a 2 farad capacitor, 1 fourth ohm resistor, 4 farad capacitor and 4 farad capacitor. So, this is the overall network. So, the question is find the y parameters of the overall 2 port viewing it as the parallel combination of two 2 port networks.

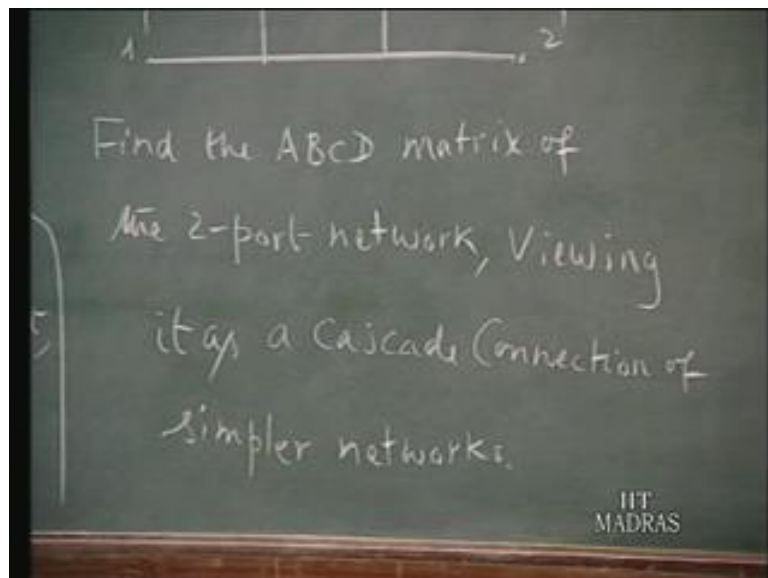
So, it means yes, that you have to consider the T network constituted by these three elements as 1 2 port network. The T network constituted by these three elements as an another 2 port network. Both of them are now in parallel. Find out the y parameters of network a, find out the y parameters of network b. Add them up that will be the y parameters of the overall network.

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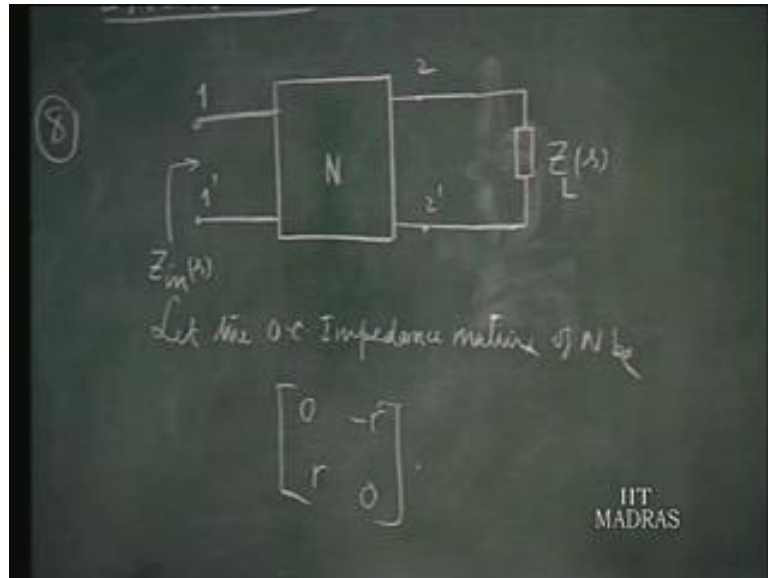
7, so this is a 2 port network, which is built entirely of resistors, this has the ladder configuration.

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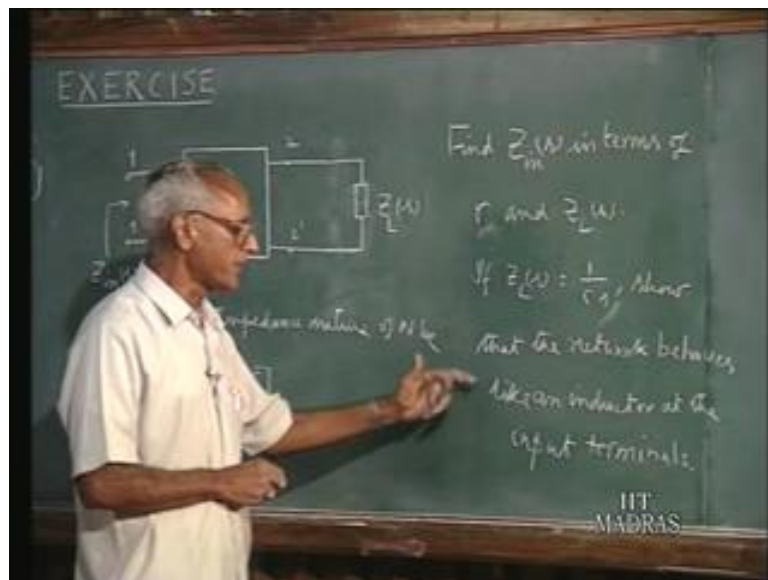
Find the ABCD matrix of the 2 port network, viewing it as a cascade connection of simpler networks. Think of this, as cascade connection of several simple 2 port networks. Find out the ABCD matrix of each one of these simple networks multiply them out. And get the overall ABCD matrix for the entire configuration.

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Last example, we have a 2 port network N terminated in a load impedance Z_L of s. let, the open circuit impedance matrix that is the Z matrix of N be 0, minus r, r, 0. That means, the Z matrix of this, if z_{11} z_{22} is 0, z_{12} is minus r, z_{21} is equal to r. This is the situation for this. Now, if you measure the input impedance here Z_{in} of s with this load termination.

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Find Z_{input} of s, that is the input seen that the port 1 when the port 2 is terminated in Z_L of s. In terms of, this r which forms a component in this matrix and Z_L of s. If Z_L of s is the impedance. Suppose, Z_L of s is taken to be one capacitor suppose that means, 1 over c s, show that the network behaves like an inductor at the input terminal. That is, if

this input terminating impedance is a capacitor. As you see, ((Refer Slide Time: 48:56)) from the input terminals 1 1 prime, it behaves like an induct.

So, this is the property of a network, which has got this kind of 2 port open circuit impedance matrix. And as I mentioned earlier, this is characteristic of what a device called gyrator, which I mentioned in parsing, while discussing the theory. And So, a gyrator terminated in a capacitor is equivalent to an inductor at the input terminals. So, you work this out, find out Z input in terms of a general Z_L of s . And if, Z_L of s happens to be $1/c s$. Find out, the corresponding input impedance in terms of R and c .