Networks and Systems
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Lecture - 32
Network Function (3)
2-port networks: Symmetry
Equivalent networks Examples

In the last lecture, we acquainted ourselves with the concept of a network function in general way and looked at the network functions pertaining to 1 port and 2 port networks. As far as 1 port network is concerned we have 2 network functions to deal with, the driving point impedance and the driving point admittance functions. These are functions of s, but I said we refer to them simply as impedances.

As far as 2 port networks are concerned there are 4 variables terminal variables V 1 V 2 I 1 and I 2 any two of them can be expressed in terms of the other two. And, we have several sets of parameters which characterize a 2 port network as far the terminal behavior is concerned. These are the z parameters, the open circuit impedance parameters. The short circuit admittance parameters or the y parameters, the h parameters, the hybrid parameters and the transmission parameters and the ABCD parameters.

We saw the definition of each one of these sets of parameters. And we also saw the consequences on the parameters, arising out the reciprocity theorem. That is if the network contains purely bilateral and reciprocal 2-terminal pair elements, what are the restrictions on this? We saw that z 1 2 equals z 2 1 y 12 is equal to y 2 1h 12 equals minus h 2 1 and the determinant of the ABCD matrix equals 1. Therefore, for a reciprocal 2 port only three of these four parameters in each set are independent. So once, we know 3, the fourth one can be obtained easily.

Now on the other hand, there are some 2 port networks, which provide identical behavior whether you view it from port 1 or port 2. Then, such 2 port networks are called symmetrical 2 port networks. That means the networks have a kind of symmetry with reference to ports 1 and 2.

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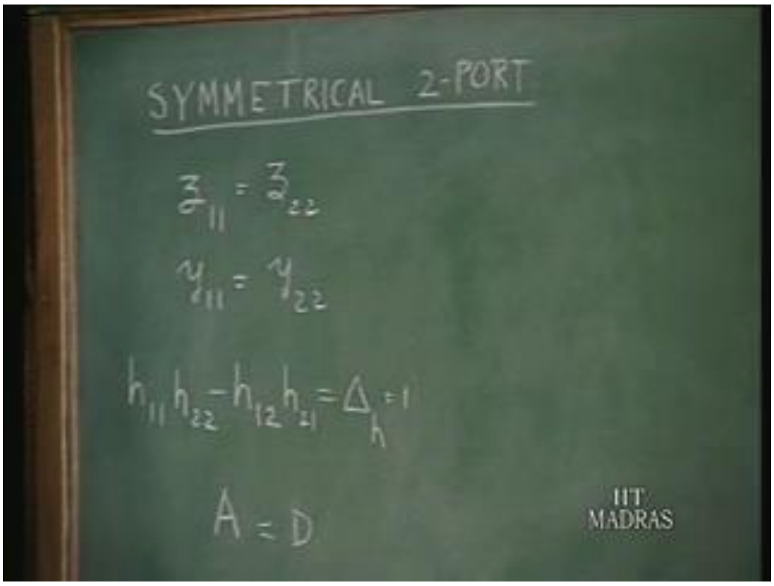

Now if you have a symmetrical 2 port network. The consequence is that the driving point impedance, whether you look at port 1 or port 2 is the same z 11equals z 2 2 y 11equals y 2 2. And likewise in terms of h parameters, the determinant of the h parameter matrix h 11 h 2 2 minus h 12 h 2 1 that is the determinant equals 1 and A equals D.

So, if a network is symmetrical and reciprocal then two restrictions on these parameters are valid. Therefore, for a symmetrical reciprocal network we have only two of these parameters in each set are independent the other two follow from that.

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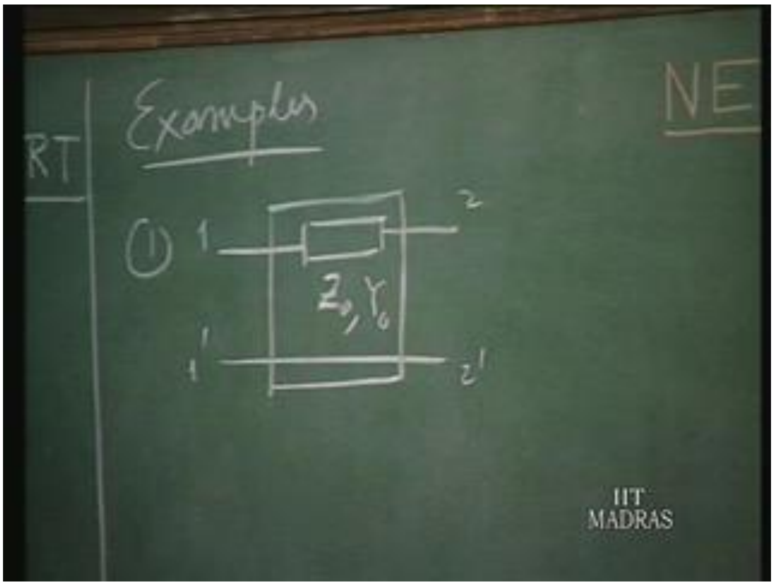

Now, based on these definitions of the various network parameters let us work out a series of example to illustrate the meaning of these various terms. Examples, suppose I have a very simple network 2 port network which consist of just one series element. Let us, say the impedance of this is Z not or the admittance of this is Y not. So, one is the reciprocal of the other a 1 port here with a impedance function Z not of s or it is reciprocal of s the admittance function 11 prime 2 2 prime.

Now, if you try to find out Z 11for this. You keep this open circuited and try to measure the input impedance that is infinity. Because, once you keep this open circuited then input impedance is infinity. Z 2 1also turns out to be infinity, because if you want to drive a current I 1 here. That I 1must go through infinite impedance. And therefore, consequent layer infinite voltage will be developed here at a tell a across port 2.

So, Z 11Z 2 1will be infinite. Similarly, it can be shown that Z 12 and Z 2 2 are infinity. So, all the four parameters are infinite themselves no useful purpose for us.

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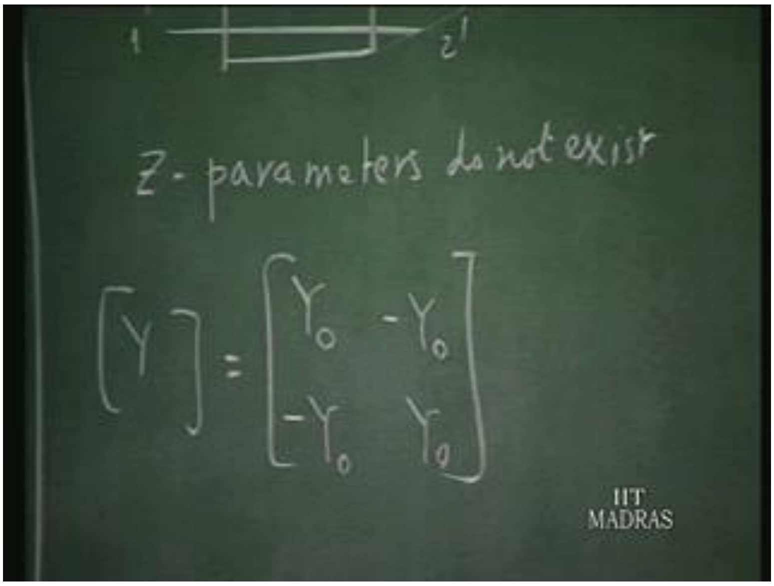

So, simply we say Z parameters do not exist do not exist, what about Y parameters, the Y matrix. (Refer Slide Time 04:01) Now, if you short circuit port 2 to port 2 and try to measure the input admittance here that is equal to Y not. Because, after all the total impedance in is that of the 1port network only. Therefore, admittance is the impedance is Z not the admittance is Y not, therefore Y 11is equal to Y not.

So, if you apply voltage V 1 here, the current that flows here is Y not times V 1 that is the current that flows here that is I 1. The same current flows here also; that means, you have depression I 2 is this current, this is I 1this is I 2. So, I 2 happens to be minus Y not times V 1. So, the depression of Y 2 1is I 2 divided by V 1, when 2 2 prime is short circuited or the port 2 is short circuited, therefore Y 2 1equals minus V not minus Y not.

And since, this is a symmetrical network; you find the same type of element arrangement whether use port 1or port 2. Y 2 2 is also the same, this is also a reciprocal element, because this is a bilateral type of impedance. Therefore, this is minus Y not. So, series element constituting a 2 port network we have a short circuit admittance matrix given in this fashion.

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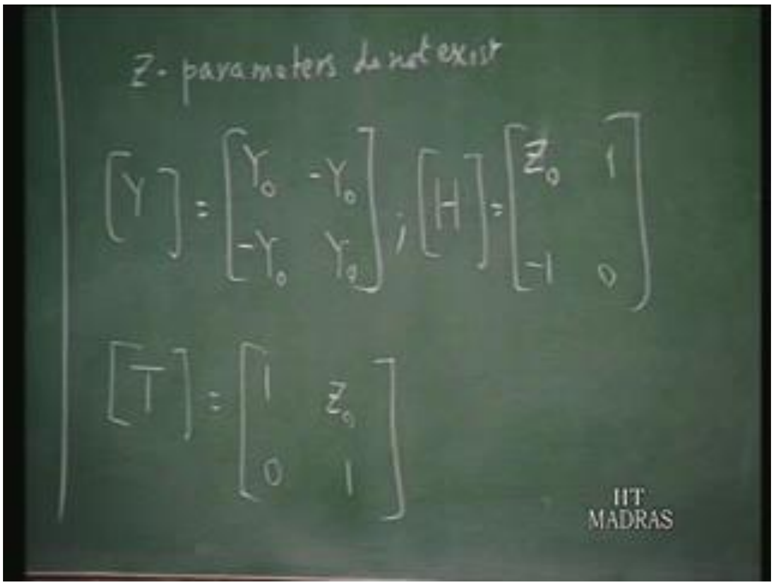

You can calculate the by similar reasoning the hybrid parameter matrix, the H matrix for this turns out to be 1over Y not or Z not 1minus 1 0. Again, this is a symmetrical network you have 1 minus 1 H 1 2 is equal to minus H 2 1. The determinant of this is Z not times 0 minus of product of one and minus 1. The determinant of h matrix is equal to 1 and that is a consequence of symmetry as we have already explained therefore, it all adds up.

The transmission matrix the ABCD matrix for this can be computed from the definitions of the ABCD elements. This turns out to be 11 0 Z not. All these, Y not's and Z not's are function of s of course in a general case. So in this simple example, we have Z parameters do not exist Z matrix does not exist. But, the other type versus other parameters exist and these are the values.

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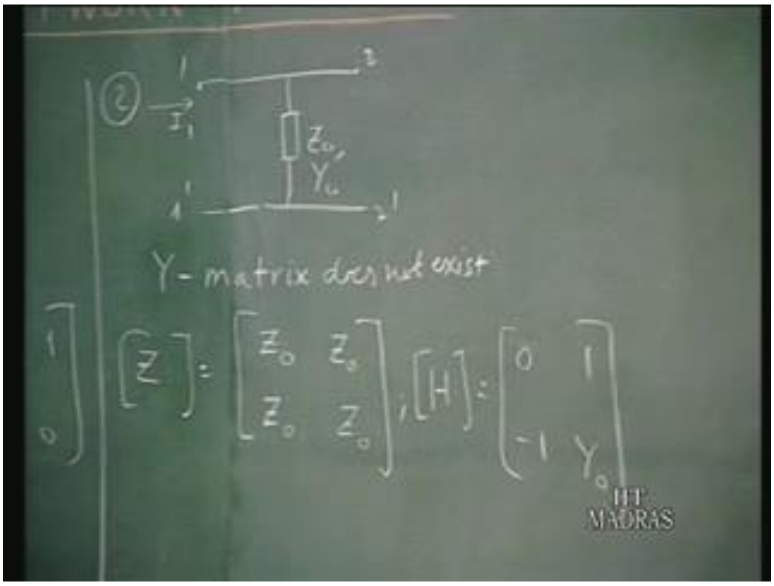

Now let us, take another simple case, where we have a shunt element 11prime 2 2 prime. Now, if you short circuit this and measure the input admittance that is going to be infinity and so it turns out that all the Y parameter elements Y parameters are going to be infinity. Therefore, we can say Y matrix do not exists, what about Z matrix? If you keep the port 2 open and measure the input admittance that is equal to the impedance of this element.

So, let us say this is Z not or it is admittance Y not. So, the input impedance when port 2 is open is Z not. Keeping port 2 open if you drive a current I 1 here, the voltage that is developed across port 2 is the same as the voltage that is developed across this impedance which is I 1times Z not. Therefore, V 2 divided by I 1when I 2 is 0 that is the value of Z 2 1 and it happens to be equal to Z not. Therefore, this is also Z not.

And again, this is a symmetrical reciprocal network. You if you calculate Z 2 2 and Z 1 2 they will also be the same. So, for a simple 2 port network constituted by 1 shunt impedance this is the Z matrix for this.

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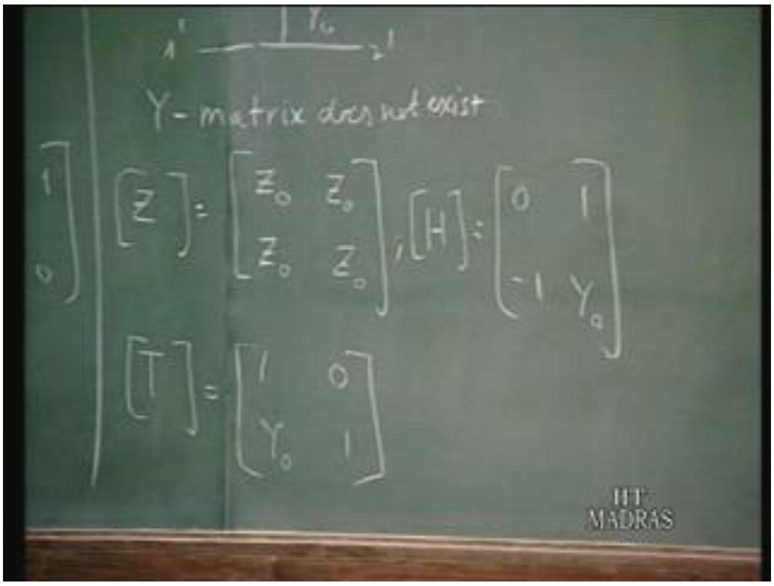

The H matrix for this still likewise can be computed 0 1 minus 1 Y not. Again, it is a symmetrical network it determinant is going to be 1 and H 1 2 is equal to minus of H 2 1 which is a consequence of reciprocity. The transmission matrix for this will be 1 0, we will also have a very simple form 1 0 Y not and 1. And again, you see the determinant of the transmission matrix A D minus B C is equal to 1 that comes as a result of the reciprocity of this network.

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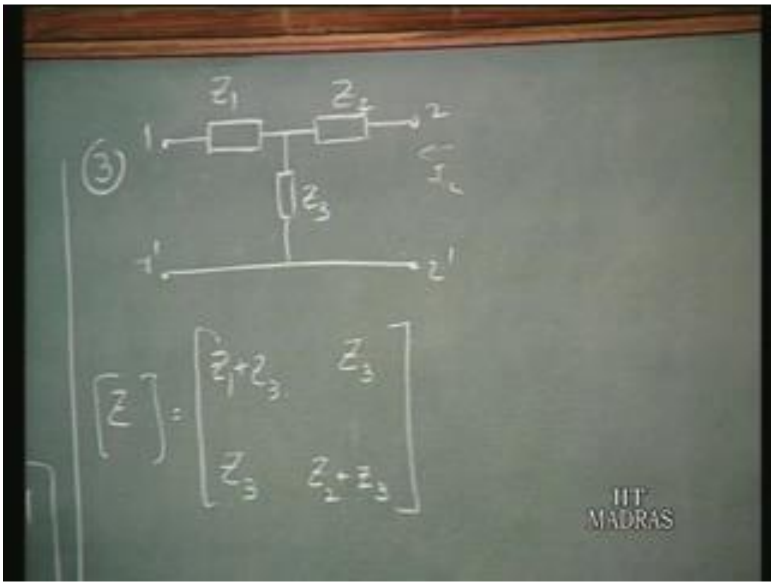

Let us, now take a third example here, we complicate this a little bit let us consider a T network consisting of 3impedances, impedance function that is Z 1 Z 2 Z 3. These are all functions of s. So, 31 port networks connected in a T formation, this is called a T network. Because, 3impedances have the configurations corresponding to a T letter T. Therefore, this is a T network and if Z 1Z 2 Z 3are functions of s.

If you want this particular configuration enables us to determine Z matrix of this 2 port network very conveniently. Z 11 is the impedance seen at port 1 keeping port 2 open. So, if you drive a current I 1 through this 1 port terminals that current passes through these two impedances. Because, port 2 is open and it develops a voltage Z 3 times I 1 across port 2 and across port 1it develops Z 1 plus Z 3 times I 1. Therefore, the Z matrix for this shall have Z 11 is Z 1 plus Z 3 Z 2 1 is simply Z 3.

Because, a current I 1 flowing through this will develop a voltage Z 3 times I 1 across this shunt network shunted branch which is also the voltage across port 2. So, Z 3 will be the Z 2 1 element and when you do the same operation connecting a current source here have I 2 and keeping this open the voltage that is developed across port 1is Z 3 times I 2. Therefore, Z 12 which is V 1divided by I 2 when I 1is 0 it happens to be Z 3. Therefore, this is Z 3.

And the looking in impedance here, when port 1 is open is this sum of this 2 which is Z 2 plus Z 3. So, you observe that in the case of a T connected configuration in a 2 port network the values of the Z parameters can be computed by surprising is very simple.

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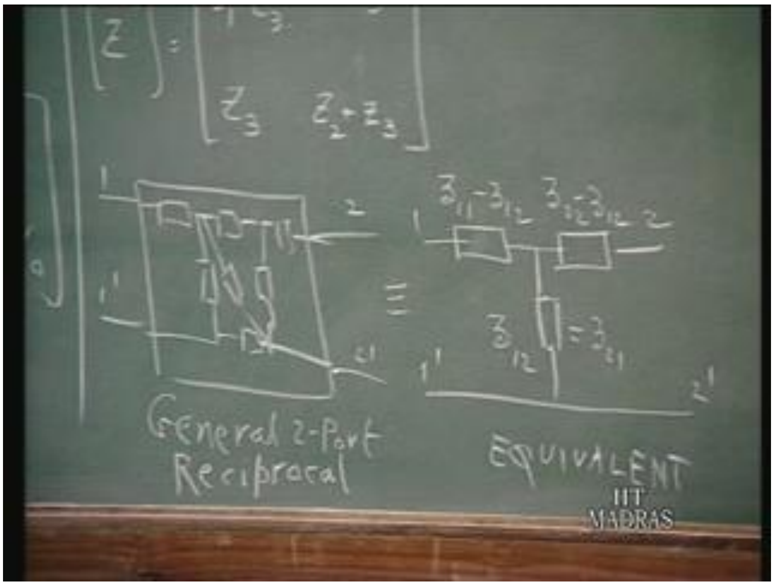

In a converse fashion, suppose we have a complicated 2 port network. Let us, say highly complicated 2 port network may be like this a general 2 port. Now, this general 2 port will have some Z parameters. You can compute them using various methods of network analysis that are available to you.

But, if that general 2 port network has got a Z matrix which is equal to Z 1 it is given by Z 11 Z 1 2 Z 2 Z 2 1and Z 2 2 in our general configuration. We can form an equivalent T network for this in this form. So, this is an equivalent network which has identical terminal behavior as the original network. So, what is meant by an equivalent network at that if you have 2 network equivalent then the terminal behavior of the two networks are identical? So in the case of, a 2 port network the relation between V 1V 2 I 1and I 2 will be identical in both cases.

So, if I have this general 2 port described by a Z matrix, I can describe the terminal relations of the general 2 port by means of this equivalent network by writing this as Z 11minus Z 1 2. This as Z 12 and this as Z 2 2 minus Z 1 2, so; that means, when you look from this side the total impedance when keeping port 2 open is Z 11minus Z 1 2 by Z 12.

Therefore, Z 11 which is the driving point impedance of this network I use that value here I; that means, all this parameters are those which relate to the general network, if I calculate them and then incorporate three impedances having these values. This particular network will have the same terminal relations as this original network. All this is just on the basis that this is the reciprocal 2 port network in which Z 1 2 is the same as Z 2 2; that means, general 2 port network which is reciprocal this is very important.

If this is not reciprocal that equivalent will not be valid. So, we have a general reciprocal 2 port network which follows the reciprocity theorem which means Z 1 2 is equal to Z 2 1. And there in that case this is an equivalent configuration for this general 2 port networks. So, instead of dealing with this complicated 2 port network, we can always replace this by its equivalent and use this for in the analysis problem.

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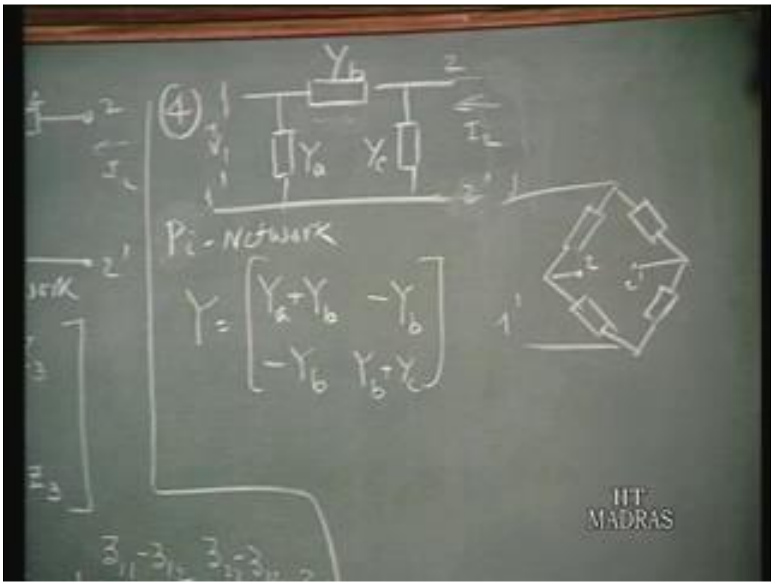

In a similar fashion, if I have a, what is called a pi network. Suppose, I have a configuration like this with three elements, let us say this is Y a this is Y b this is Y c. Then, for such a network calculation of the Y parameters turns out to be very convenient. So this is the port 1, 1 1 prime 2 2 prime the Y parameters for this is very convenient to calculate.

Because, when you short circuit this and measure the admittance at port 1 that is the parallel combination of Y a and Y b. Because, one position is shorted Ya and Y b come in parallel and that is the total admittance seen at port 1. Therefore, Y 11equals Y a plus Y b. Now, what is the current in port 2, when this is shorted what is I 2 in terms of V 1. When you have applied voltage V 1 and short circuit these terminals this entire V 1 appears across Y b.

So, the current in this direction is V 1 times Y b. But, our reference direction for current I 2 is in the opposite direction. Therefore, I 2 divided by V 1 turns out to be minus Y b. Therefore, this will be minus Y b. And similar arguments by applied when you short circuit port 1 and may apply a voltage across port 2 will lead to just to the result that Y 1 2 is equal to minus Y b and this is Y b plus Y c.

So, when you have a pi network this is called this; obviously, a part of the form of the pi. The letter pi symbol pi here and this is called a pi network. So, when you have a pi network usually referred to as pi network. This is called a tee network. So, when you have a pi network the calculation of the Y parameters turns out to be very convenient.

And we have a negative sign here that is important to note. Because, we have taken the reference direction in this fashion and therefore, when you apply a voltage V 1current actually flows in this direction. Suppose, you have few resistance the current will go in this direction. If this is a DC source and resistance the current will flow in this direction. But since, our reference direction is opposite sense it turns out as far as the transfer admittance are Y matrix is concerned.

You will have lot of negative coefficients. So that has to be taken kept in mind. That as far as the network of a 3-terminal configuration; that means, there is a common ground between port 1 and 2 it turns out that the coefficients in the transfer admittances will turn out to be negative. It may or may not be. So, when you have a four terminal type of 2 port network.

For example, if I have a 2 port network in this like in this fashion suppose this is port 11 1 prime and this is port 2 2 2 prime. There is no common connection between 1 prime and 2 prime in such cases the transfer admittances may have positive coefficients or negative coefficients or both. But on the other hand, if it is a 3-terminal 2 port you have negative coefficients like this. So, this is.

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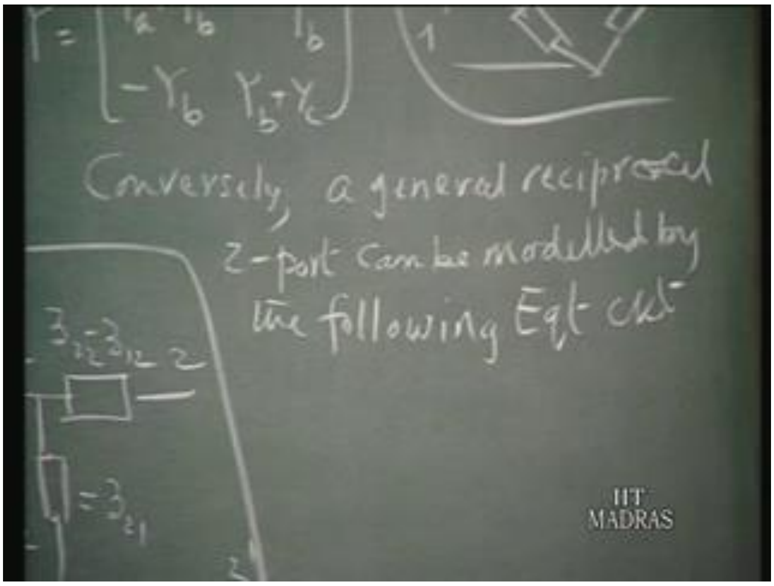

So, conversely a general reciprocal 2 port can be modeled by the following equivalent circuit. That is just as we have found out an equivalent circuit for a reciprocal general 2 port networks here. You found out that an equivalent circuit in this form similarly when you have a general reciprocal 2 port network.

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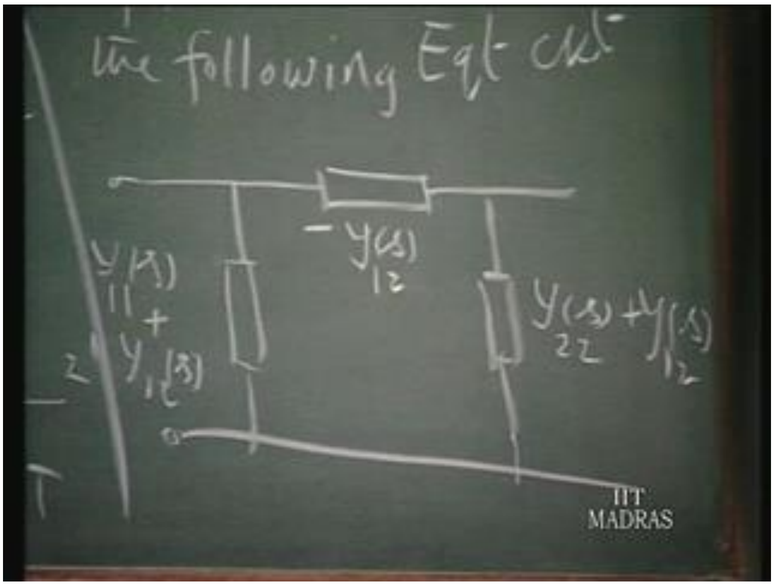

We can find out the equivalent circuit in terms of the Y parameters in this fashion. You have see (Refer Slide Time 15:43) you observe that this shunt element here is minus of I mean the series element series impedance is branch here as an admittance equal to negative of Y 1 2 or Y 2 1. Therefore, I can write this as minus Y 1 2 of s and when you short circuit this.

The total admittance seen from port 1 must be Y 11. Therefore, this must be whatever you are having here is equal to Y 11 Y 11of s plus Y 1 2 of s. So, this 1 port network must have total admittance Y 11 of s plus Y 1 2 of s. So, this plus this when you short circuit this will provide a total admittance at port 1 which is equal to Y 11 of s. Likewise this admittance will be Y 2 2 of s plus Y 1 2 of s. So that is the configuration.

So, we have seen form these two examples that there is a duality between them. If we have a t network you can find out the open circuit impedance parameters quite conveniently or in a general 2 port reciprocal network it can be modeled in terms of the Z parameters by an equivalent T network quite conveniently.

Similarly, if you have a pi network the Y parameters are easy to compute or conversely a general reciprocal 2 port network can be modeled by a convenient equivalent circuit in the form of a pi network using the Y parameters as shown here.

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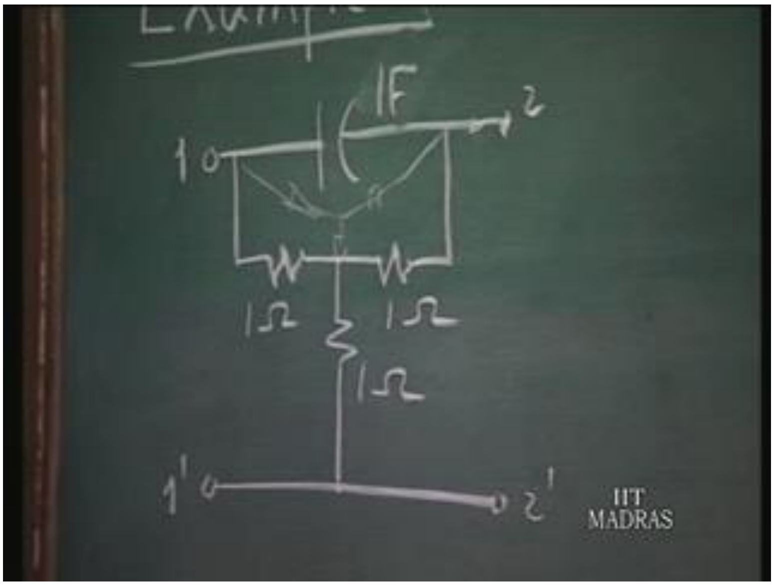

This is a slightly more complicated example. Let us say that we are interested in finding out the Z parameters of this 2 port network. We have seen earlier that if the network is in the form of a T network then the evaluation of the Z parameters is quite convenient. How are this configuration is not in the form of a T.

On the other hand if you convert this delta configuration of a 1 farad capacitor and 2 1ohm resistors by means of convert this into a star like this. Then, the whole network falls out into the form of a T network. So, let us try to do that convert this delta into an equivalent star.

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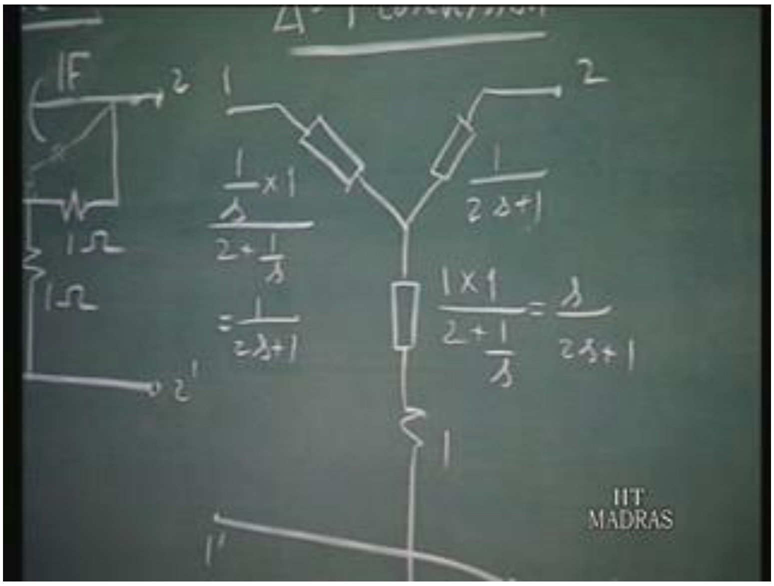

So, delta to star conversion now the rules for this convention follow exactly the same lines as we have in the case of resistive networks or in the case of impedances under AC fed circuit analysis. So, however we have to use the generalized impedances in this case. So, this delta if it has to be converted into a star, we have three generalized impedances like this.

And of course, we have this 1ohm resistance here, so this will be 11prime 2 2 prime. So, this delta will be converted into this equivalent star and we are can find out the open circuit impedance parameters of this altered configuration the terminal conditions will remain the same, because whatever changes have been brought about are internal to the 2 port.

The impedance of this is obtained by the product of these two impedances divided by the sum of three impedances. Therefore, the impedance of this will be one upon s that is generalized impedance of this times 1 divided by the sum of the three impedances which is 2 plus 1over s which will be of course, 1 upon 2 s plus 1.

By symmetry this will also be 1upon 2 s plus 1 that is the impedance of this box. The impedance of this box; however, is the product of these two impedances divided by the sum of impedances. So, this 1 times 1 divided by 2 plus 1over s this will be s upon 2 s plus 1. And of course, this 1ohm resistance will remain. So now, the entire network now is in the form of a T network and we can find out the open circuit impedance matrix of this port 2 with without further I do.

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So, Z o c of s of this would be the driving point impedance at port 1 is the sum of this impedance plus this impedance plus this impedance. This is 1over 2 s plus 1 this is s over 2 s plus 1. Therefore, s plus 1over 2 s plus 1 plus 1 of course, which will be of course, 3 s plus 2 over 2 s plus 1that is Z 11. By symmetry Z 2 2 is also 3 s plus 2 over 2 s plus 1 Z 2 1and Z 1 2 given by this impedance of this shunt branch s over 2 s plus 1 plus 1 that will be 3 s plus 1over 2 s plus 1 that one will be the open circuit impedance matrix of this 2 port network.

On the other hand let us imagine that we are interest in finding out the Y parameters of that. To find out the Y parameters its most convenient if the network is in the form of a pi. So, how can you convert this into an equivalent pi? The answer is quite straightforward, we have three resistors connected in star if you convert them into an equivalent delta we have one branch coming in parallel with 1 farad capacitor.

Another branch coming in parallel with first port, a third branch coming in parallel with second port the whole configuration will be in the form of a pi.

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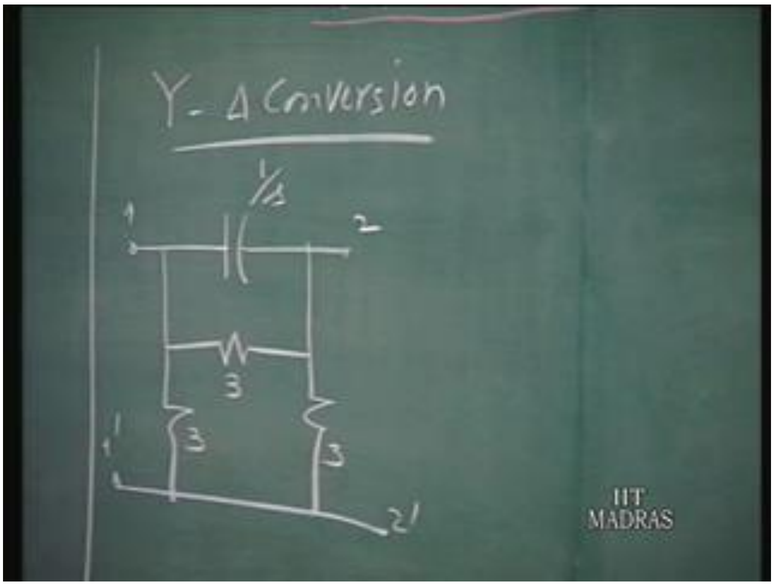

So, let us do that. So, star to delta conversion if you do that. We retain the identity of the 1 farad capacitance, which has got generalized impedance 1over s. In addition to these three resistors are connected into an equivalent delta. So that will be the altered configuration. The impedance of this 31 ohms and star as equivalent to 33 ohms in delta, so these are all the generalized impedances are 3 ohms each. So that will be the configuration that we have for the pi network that is obtained by converting the star in to an equivalent delta.

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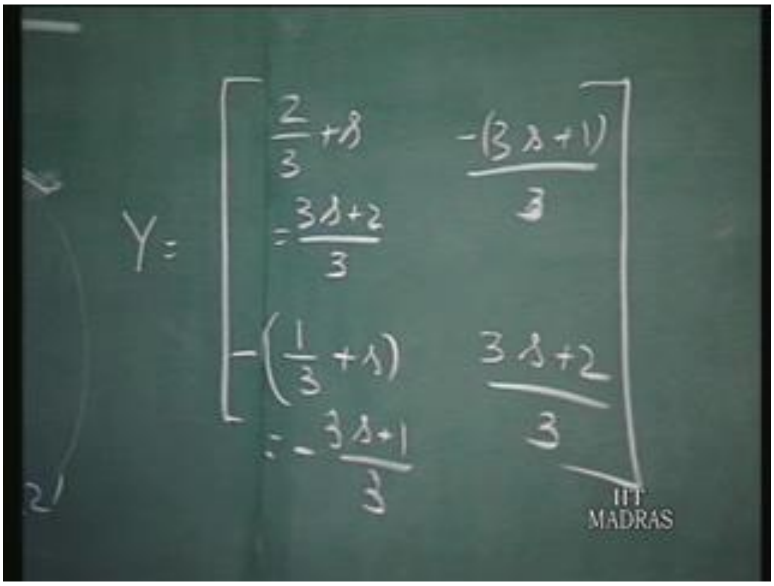

So, what are the Y parameters for this? In the short circuit admittance matrix from this now, (Refer Slide Time 26:49) the driving point impedance at port 1 is obtained by short circuiting second port and looking at the impedance as seen from port 1, which will be this is 3 ohm resistor it is admittance is one-third. The admittance of this is one-third and that is in parallel with the capacitor of admittance s.

So, all these three elements come in parallel because 2 2 prime is a effectively short circuited. So, port 1 is a admittance which is the sum of this impedance plus this admit this admittance this admittance plus this admittance plus this admittance that will be 2 upon 3 plus s which is 3 s plus 2 upon 3. And by symmetry the impedance seen from port admittance seen from port 2 as well be 3 s upon plus 2 upon 3.

Now, what about the transfer admittance? Suppose you short circuit this drive apply voltage here. The current that flows in 2 is limited by the admittance of this parallel branch. Because, this is ineffective because you have short circuited and the current here we do not care what it is we are interested in only the current flowing in this. So, the current flowing in this will be the applied voltage times the admittance of this parallel combination. That will be 1by 1 by 3is the admittance of this and s is the admittance of this.

So, one-third plus s times V 1 will be the current flowing in this direction. But, our port difference is in this direction the current which is measured always with this reference side. Therefore, we must put a negative sign out in front. So, Y 2 1 will be minus of one-third plus s. That will be minus of 3 s plus 1over 3 and Y 1 2 likewise will be minus 3 s plus 1upon 3. That will be the admittance transferred short circuit transfer admittance of Y 1 2.

(Refer Slide Time 25:00) Now we observe here, that Z 1 2 Z 2 1 Y 1 2 Y 2 1 or all of them having the same 0, s equals minus one-third makes this 0 this also zero. And so, does it make the transfer impedances Z 2 1and Z 1 2 0.

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Zeros of transfer function are called are called zeros of transmission. Essentially, they are the values of the complex frequency for which if you apply the signal as an input with that complex frequency the output turns out to be 0. Therefore, they are called zeros of transmission.

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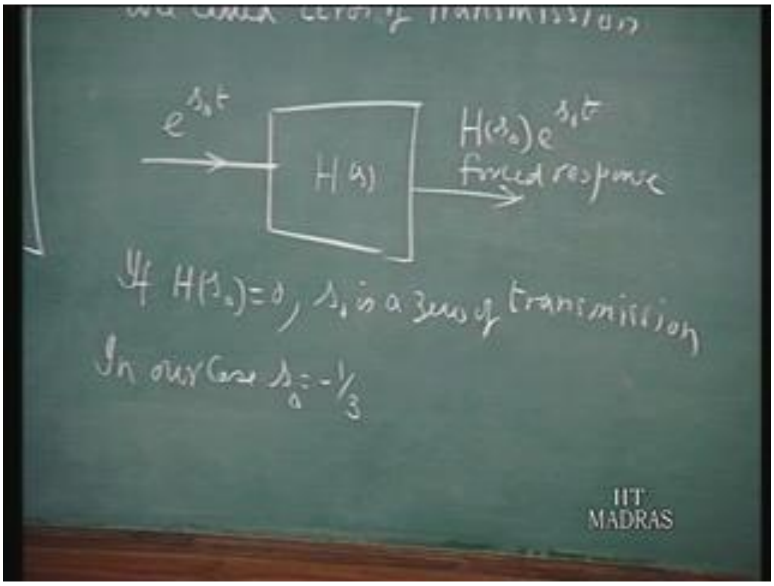

And the zeros of transmission are inherent for a particular network. So, if you are having a system function H of s a generalized system function general transfer function. If you apply a exponential input signal e power s e power s not T and the forced response is of course, H s not times e to the power s not t is the forced response. Then, if H s not is 0, then s not is said to 0 of transmission.

And this zero of transmission is inherent for a particular system. So, with minor exceptions here and there if there is a particular zero transmission associated with transfer admittance that will also be the zero of transmission with the transfer impedance transfer voltage ratio transfer current ratio and so on and so forth. So, H of s can be any type of transfer function for a signal flow of input from port 1to port 2.

In this case in our case there is only 1 zero of transmission that s not happens to be minus one-third. So, if you drive this 2 port network at port 1 with an exponential signal e power s not T where s not is minus one-third. Then, the forced response at the output is zero, a word about this zero of transmission or zeros of transfer function.

We know that the poles and zeros of driving point functions have certain restrictions as far as their natural response is concerned. The g from the point of your stability of a passive network we already observed that the poles and zeros of passive transfer driving point functions of passive networks must be in the left of plane. And if they are in the imaginary axis they should be simple.

So, from the point of view of stability of passive networks the poles and zeros of driving point functions have got certain restrictions. Same restrictions also are valid for the poles of transfer functions. For example, this transfer function has a pole at s equals minus half. So, poles and zeros of transfer functions the poles of transfer functions must have the same restrictions as poles and zeros of driving point functions as far as passive networks are concerned.

But as far as zeros of transfer functions are concerned, there is no such restriction. Even for passive networks you may find the zeros of transmission anywhere in the complex s plane. They need not be restricted to left of plane. They can be in the imaginary axis they can be in the right side of plane and so on and so forth. So, absolutely no restrictions and zeros of location of zeros of transmission are the zeros of transfer function.

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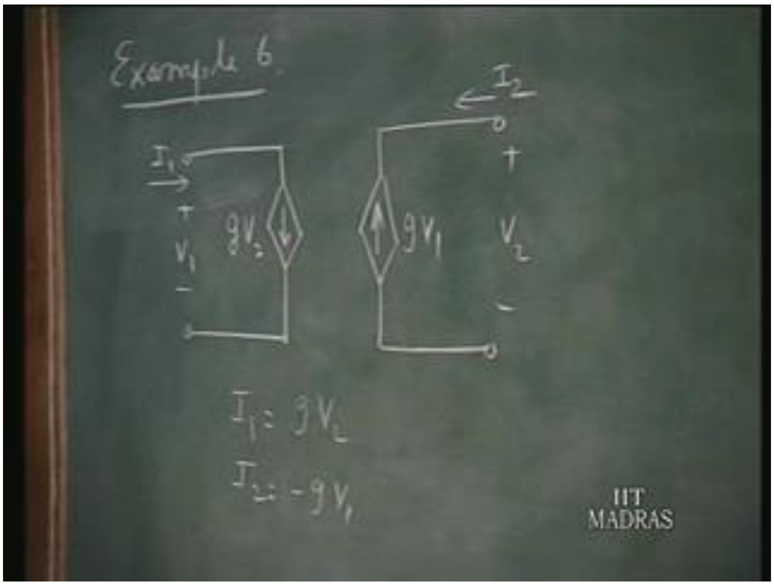

The poles of transfer function; however, are restricted in the same way as the poles and zeros of driving point functions. So far, we have been considering only reciprocal networks. So, let us consider a 2 port network constituted by dependent sources. We have two dependent current sources, the current here depends upon this voltage the current here depends upon this voltage.

So, we have for this network I 1 equals g times V 2, I 1of s is g times V 2 of s and I 2 which is a current in this direction is minus of g 1g V 1. So, minus g V 1, so these are the two conditions on the terminal variables.

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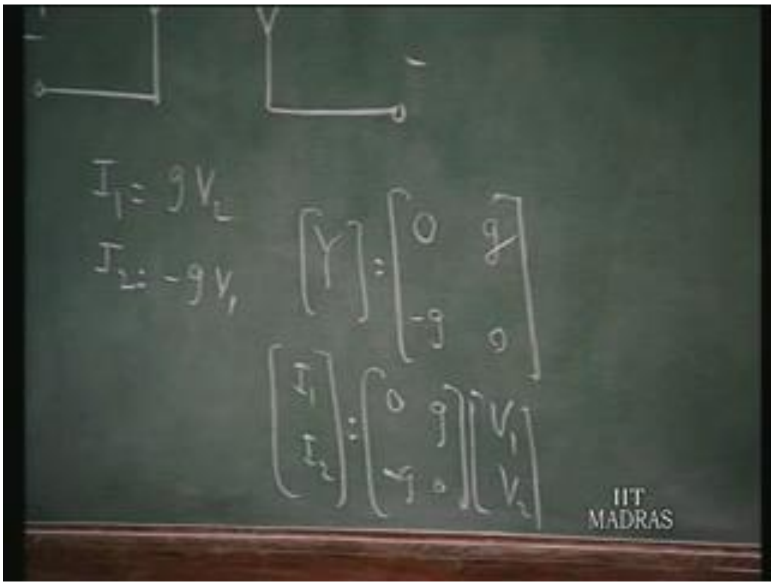

Therefore, the Y matrix for this will be I 1 equals g times V 2 and I 2 equals minus g times V 1so; that means, I 1 I 2 are given by this matrix 0 g minus g 0 multiplying V 1 and V 2 . So, you observe that this Y matrix is not the of diagonal elements or not in the same; that means, this is not reciprocal. So, V 1we have dependent sources in the 2 port network the reciprocity relationship may not be valid.

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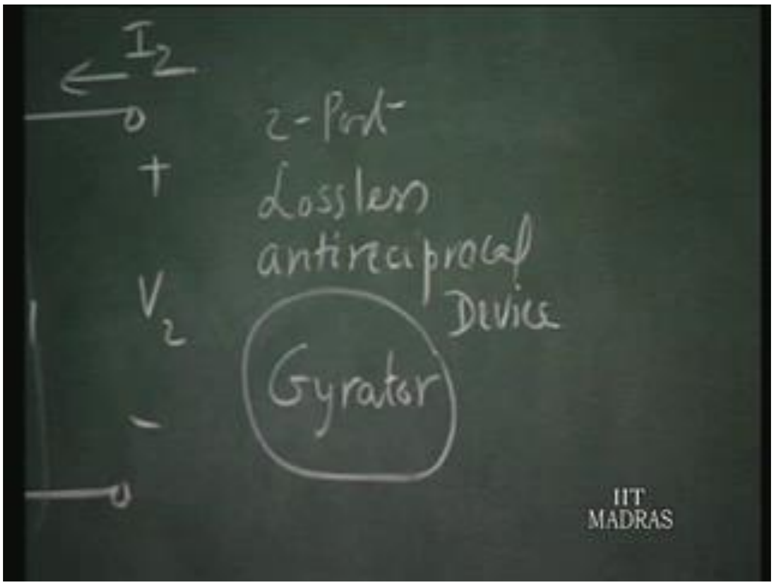

In fact, this is what is called anti reciprocal if this if the network is a reciprocal we expect these 2 to be equal is exactly the negative of this. So, this is called anti reciprocal such a

2 port network constitutes, what is called a gyrator? This is a 2 terminal pair 2 port lossless anti reciprocal device which can be constructed with active elements this is a one element this is used in active networks and it is a design of active filters. But, we will just note the terminal relations and leave it at that.

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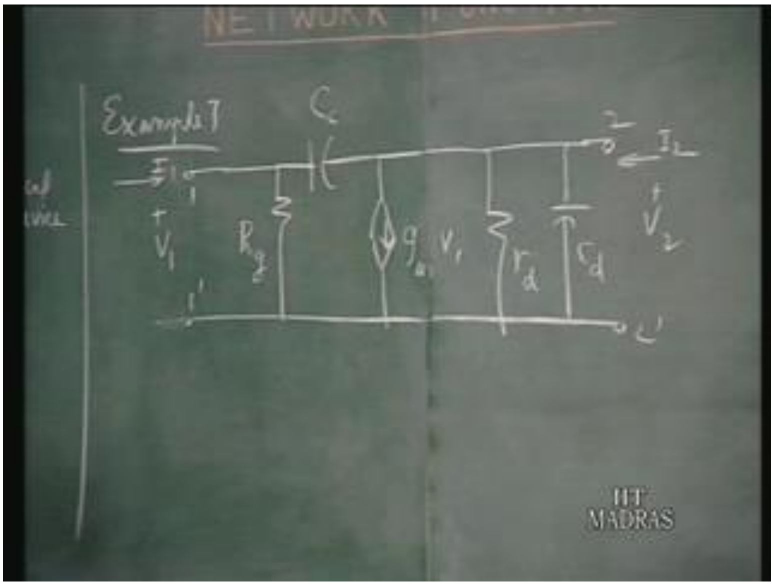

Let us, take another example slightly more complicated. Suppose we have, this is a model of some active circuits, we have gm times V 1. We have resistance r d and finally, you kept capacitance c d. So, this is 11prime 2 2 prime V 1 I 1 V 2 and I 2. We can calculate the Y parameters for this quite conveniently. So, let us do that Y 11 you short circuit V 2 and measure the driving point admittance. Once you short circuit V 2 there will be low currents in r d and c d.

You can forget about that and the impedance the admittance as seen from port 1 is the admittance of this plus the admittance of this. Because, these two are now coming in parallel and that what constitutes the current here and whatever current is generated in this dependent source will flow through a short circuit gm V 1, because that is the path with the least impedance.

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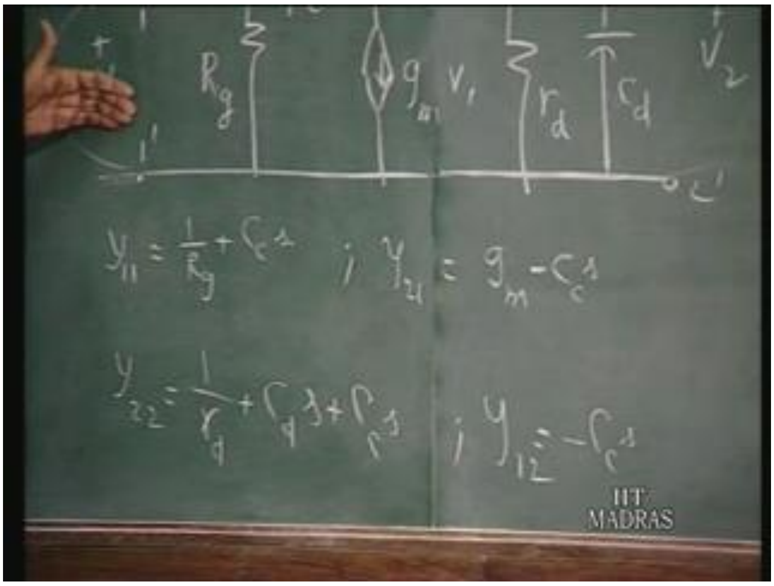

So, Y 11 will be the admittance of this plus the admittance of this. So, 1over R g plus C c s that is Y 1 1, Y 2 1the current in this I 2 short circuited current in the short circuited terminals when applied voltage V 1is there. V 1 drives a current through C c like this therefore, there is a current C c s times V 1. In addition we have a current through this dependent source gm V 1 going like this.

And since the direction positive reference direction for current I 2 is in this direction. So, you have that current divided by V 1 will turn out to be gm minus C c of s that is Y 2 1 g m minus C c of s that is Y 2 1. Y 2 2, if you short circuit the port 1 apply voltage V 2 and measure the current in the driving terminals that current will be constituted by the current flowing in r d C d.

Since, V 1is shorted the current source strength is 0. Therefore, this is open circuited you have a current passing through the cap coupling capacitor C c through these terminals. Therefore, the parallel combination of this this and this constitute the total admittance. So, you have 1 over R d plus C d s plus Cc s that is your Y 2 2. Y 1 2 is the ratio of current flowing through the shorted terminals in this direction divided by V 2. So, V 2 drives a current through C c in this direction which is C c s times V 2.

So, the current in the direction reference direction for I 1 is minus C c s times V 2 therefore, Y 1 2 is minus C c s. Notice here, once again Y 1 2 is not the same as the Y 2 1. This is because this is not a reciprocal 2 port network, because of the presence of the control source gm V 1. So, when you have control sources present can reciprocity relations will not be valid this is an active 2 port and reciprocal this reciprocity is not valid.

We have seen in these examples and some of the earlier examples that for a 2 port network one set of parameters make exist another set may not exist. We have seen that in the case of a pure series branch Z parameters do not exist, but other parameters may exist. In general it may be that 1set of parameters may exist others may not.

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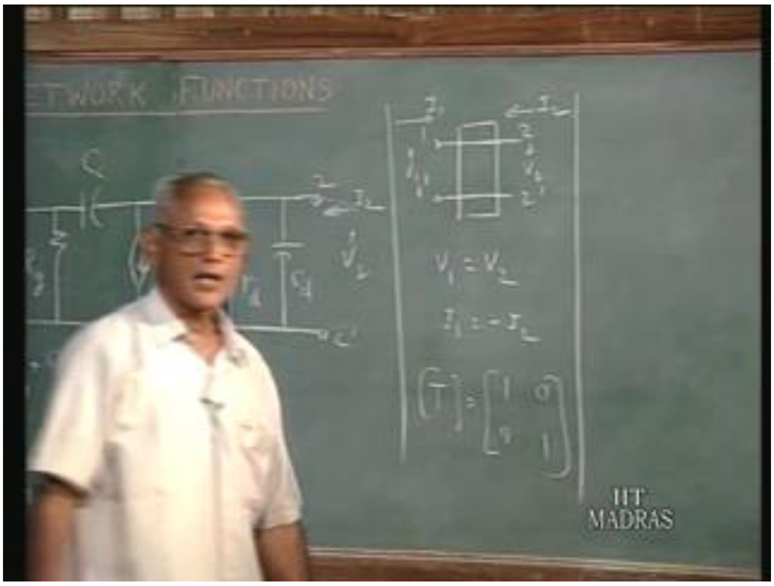

So for a very simple case, suppose I have 2 port constituted by two line like this these are two lines going right through from port 1 to port 2. You will find that all parameters except the ABCD parameters exist. So in this case, we have V 1 equals V 2 and I 1 equals minus I 2. Therefore, the chain parameters are ABCD parameters are exist which is equal to 11 0 0. Other parameters do not exist. You would not have x you would not have Z parameters you will not have Y parameters you do not have H parameters for this 2 port network.

Now, we have constructed an equivalent circuit of a 2 port network in terms of Z parameters and Y parameters. And we said those 2 equivalent circuits in terms of T T l T configuration and pi configuration are valid for a reciprocal 2 port. Now, what happens if you have a general 2 port of this with a lot of active elements present? And if you want to find out the equivalent circuit for such sit situations, then we will have to model the equivalent circuits in terms of active elements.

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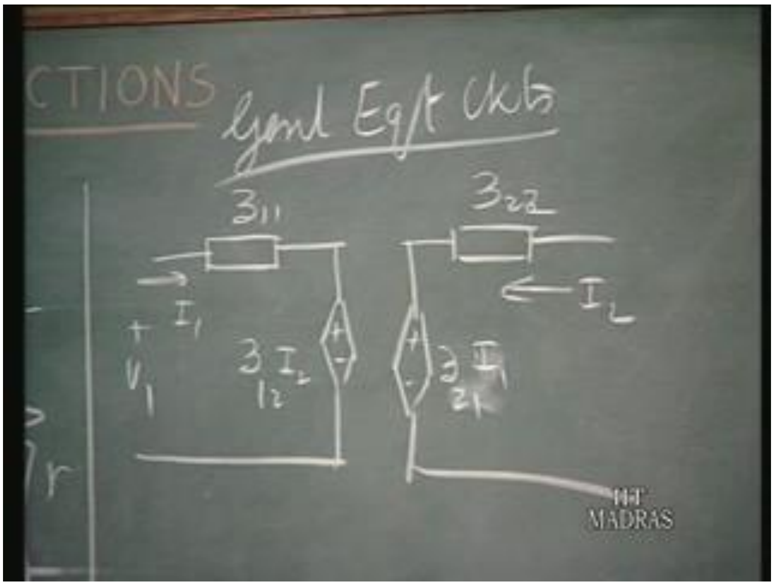

So, the general equivalent 2 port networks general equivalent circuits in terms of active elements can now be given. We can say for Z parameters we can write Z 11 plus a voltage source which is equal to Z 1 2 times I 2 V 1. And likewise, you have Z 2 2 I 2 plus any equivalent a source here which is a dependent voltage source which is Z 1 2 times am sorry Z 2 1 times I 1. That is an equivalent circuit in terms of Z parameters for a general not only the reciprocal network.

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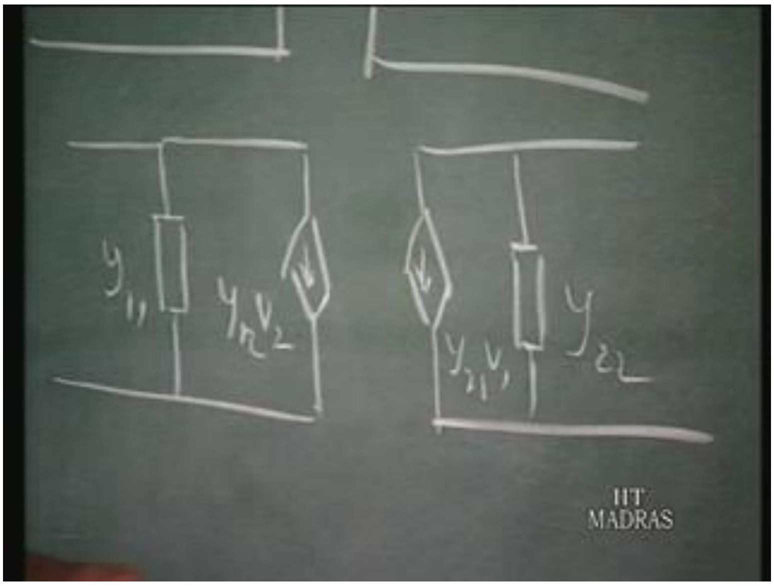

In terms of Y you have Y 11 V 1 plus a current source which is equal to Y 1 2 V 2 and this is the port. And for port 2, we have Y 2 2 V 2 that is the current here and you have Y 2 1 V 1. That is the a port network which will be an equivalent circuit in terms of Y parameters for a general 2 port network.

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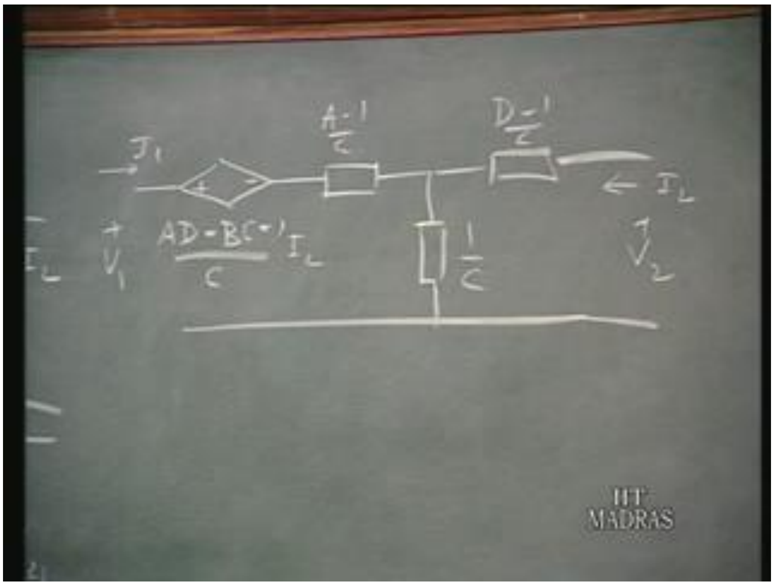

Current in terms of the ABCD matrices, you have you can write this in the form have A minus 1 upon C this is an impedance of this. This is D minus 1 upon C that is also impedance 1 over C is an impedance. And in addition, you can have a dependent source which is AD minus BC minus 1 divide by C times I 2.

So, if you analyze this network you will find out that it will also determine relations of a 2 port in terms of ABCD term parameters V 1 equals A times V 2 minus B times I 2 etcetera. And for a reciprocal network this becomes 0, because AD minus BC equals 1therefore, this dependent source will vanish.

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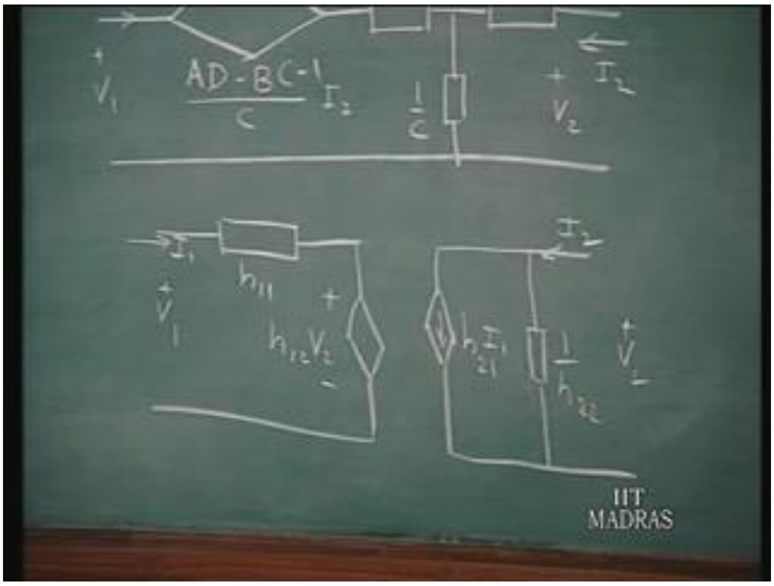

In the case of h parameters the equivalent circuit would be like this we have V 1 equals I 1 times h 11 that is a impedance plus h 1 2 times V 2. So, it is a voltage source dependent voltage source which depends on the value V 2 and h 1 2 is a dimensionless factor. On the other hand, the current I 2 will be h 2 1 times I 1. So, this is a current source of value h 2 1 times I 1 in parallel with an element whose impedance is 1over h 2 2.So, if this is V 2 the current here is V h 2 2 times V 2.

Therefore, the total current is h 2 h 2 2 times V 2 plus h 2 1times I one. So, this is a current source whose current ratio is h 2 1. And this is a voltage controlled voltage source whose value is h 1 2 V 2. So that is the configuration which is gives an equivalent circuit of the 2 port network in terms of h parameters. So, these are the general equivalent circuits which can be made use of for analyzing the 2 port network, instead either you can use the equations or this equivalent circuit.

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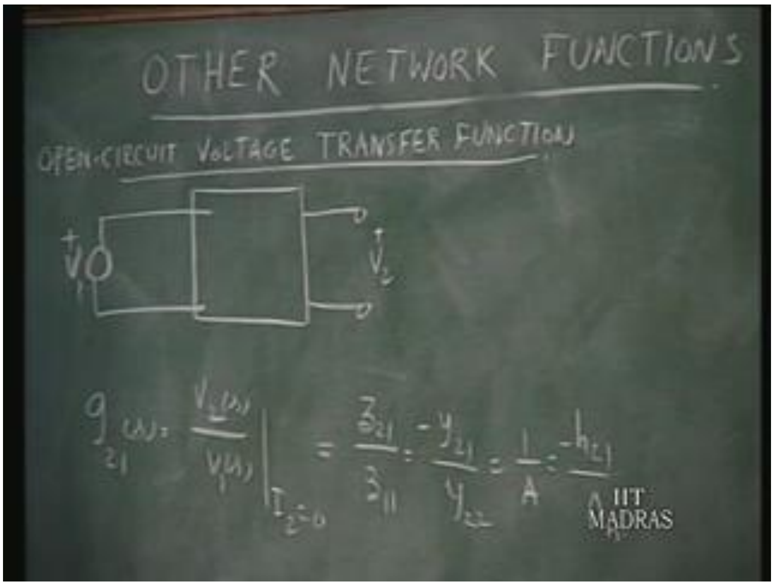

When dealing with 2 port network, we often use network functions other than those which have already we discussed in this particular example. We have port 2 open, we apply an input voltage V 1. The ratio of V 2 of s to V 1of s under open circuit conditions is called the open circuit voltage transfer function in the forward direction that is if the signal is going from port 1 to port 2.

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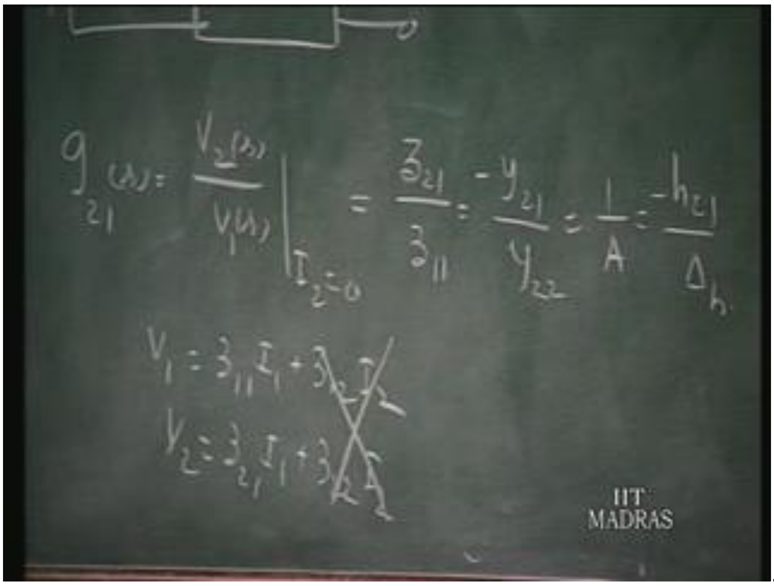

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And it can be shown that this Z 2 1 by Z 11 for example, you have V 1equals Z 11 I 1plus Z 1 2 I 2 and V 2 equals Z 2 1 I 1plus Z 2 2 I 2 and when I 2 is 0. These two terms get cancelled out. The ratio of V 2 to V 1is Z 2 1over Z 11 likewise you can show that this is also equal to minus Y 2 1over Y 2 2, this can also shown to be 1over A or minus h 2 1 by delta h.

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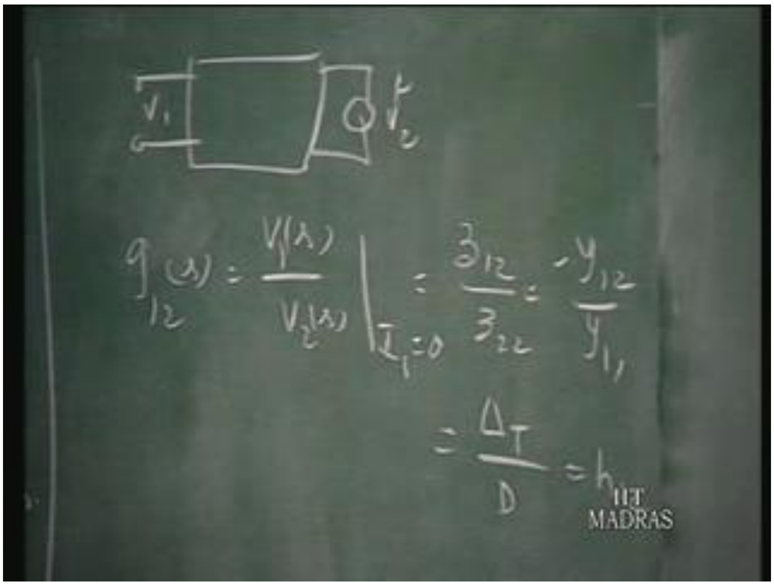

Similarly, if we have kept port 1open and apply a voltage to the port 2. So, V 1 this is g 1 2 of s V 1of s over V 2 of s when I 1 is 0 that can be written as Z 1 2 over Z 2 2 minus Y 1 2 over Y 11is the delta T over D, where this determinant this is the determinant of the transmission matrix is also equal to h 12. So, this can be we can work this out yes.

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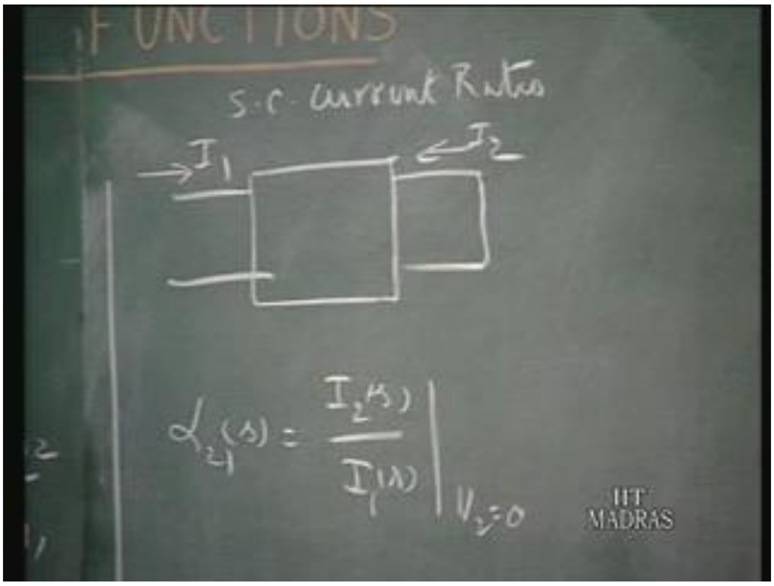

Often we may also be interested in finding out the short circuit current ratio; that means, you apply a certain current I 1 here and measure the current I 2 under a short circuit conditions. So, you write this alpha 2 1 of s is I 2 of s over I 1of s when V 2 is 0 that is this is called the short circuit current ratio.

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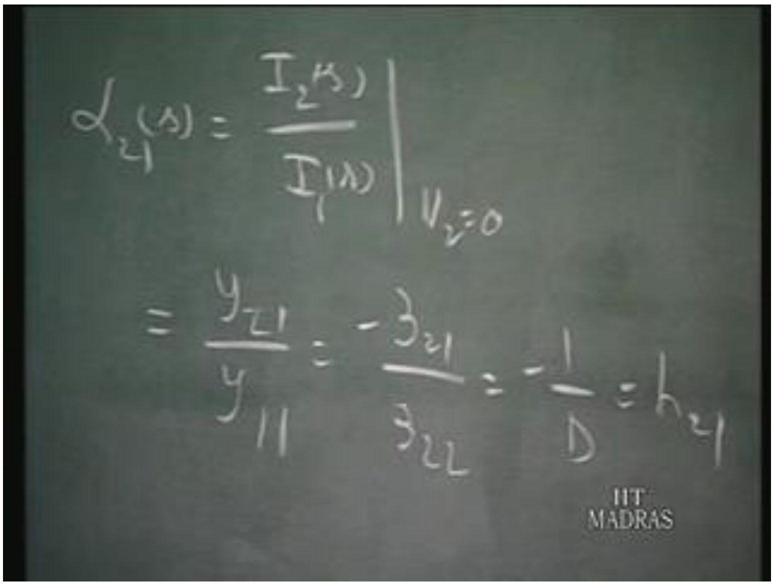

And you can show that this is equal to Y 2 1over Y 11 equals minus Z 2 1over Z 2 2 equals minus 1over D equals h 2 1.

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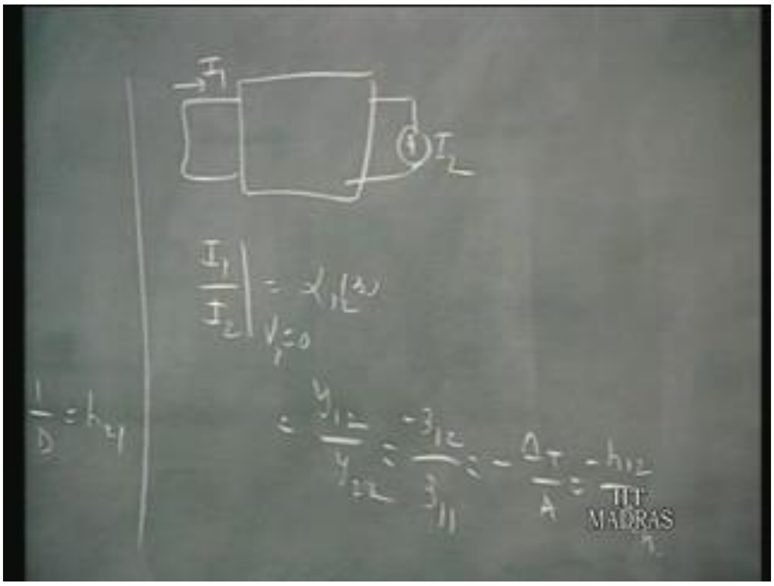

And to complete this picture you might have a situation where you want to find out the ratio of I 1 to I 2 I 1 to I 2 when V 1is 0 this is alpha 1 2 of s. And this can be shown to be Y 1 2 over Y 2 2 minus Z 2 1minus Z 12 by Z 11 equals minus delta T over a equals minus h 1 2 over delta h.

The important thing to note in this one important thing that you can note is that the transfer voltage ratio in the forward direction which is Z 2 1over Z 11 will turn out to be the negative of the short circuit current ratio in the opposite direction minus Z 1 2 over Z 11, because Z 12 is same as Z 2 1for a reciprocal network.

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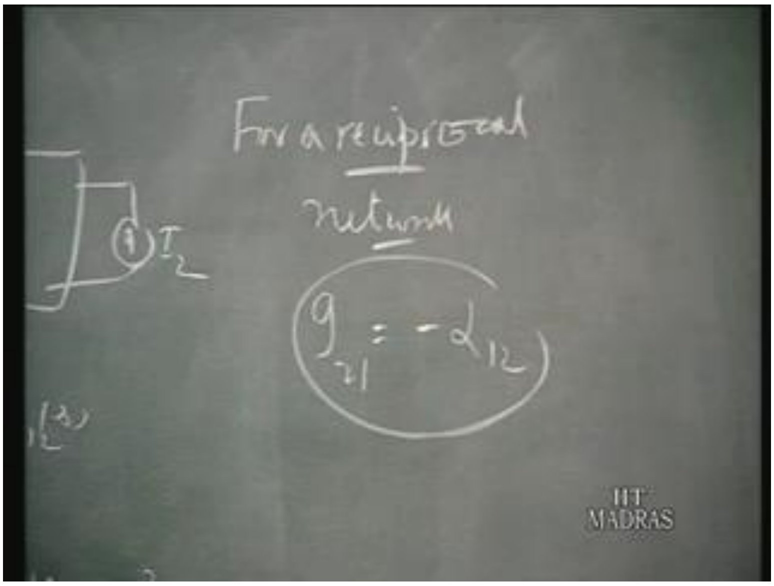

So for a reciprocal network, we can say for a reciprocal network g 2 1 equals minus alpha 1 2. The open circuit voltage ratio in the forward direction is the negative of the short circuit current ratio in the reverse direction.

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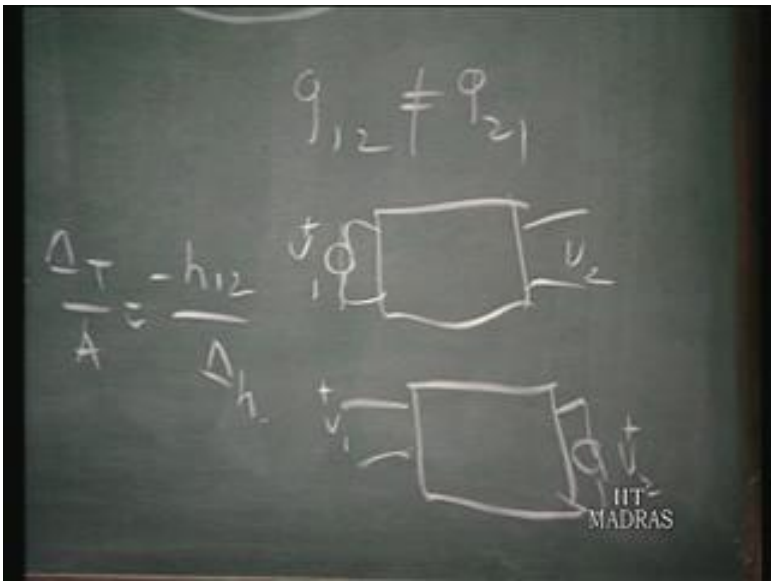

We cannot say that g 1 2 is not equal to g 2 1, because when you are measuring g 2 1 your keeping this open and you are applying a voltage source. This is V 2 over V 1to find out g 2 1. On the other hand when you want to measure g 12 you are keeping this open and you are applying a voltage here. So, this is V 1divided by V 2.

You observe that in the natural configuration natural state of the system when you short circuit this port 1 is shorted port 2 is opened in this port 2 is shorted port 1is opened. So, the natural state of these two networks before the excitation is introduced is different. Therefore, even though we may have reciprocal elements here it does not follow the reciprocity theorem.

Because, the natural states of the 2 networks are different one is short circuited here kept open? The other is short circuited here and this is kept open therefore, g 2 1is not equal g 12 is not equal to g 2 1 even in the case of a reciprocal network. On the other hand for a reciprocal network g 2 1 equals minus alpha 1 2.

So, in this lecture, we have expanded our knowledge of the 2 port network parameters. We considered various examples saw the consequences of the reciprocity and symmetry on the values of the network parameters. We saw that when we have dependent sources inside the network reciprocity property is violated. And at particularly appear occurs when we have active elements inside.

And then, we also saw that in the case of a open circuit of a um port 1. We define what is meant by an open circuit voltage transfer ratio g 1 2 which can be expressed in terms of Z parameters and Y parameters. Similarly, the short circuit current ratio can be expressed in terms of these parameters. And we saw reciprocal network the open circuit voltage ratio in one direction is the negative of the short circuit current ratio in the opposite direction. We will continue our discussion in the next lecture.