Network and Systems
Prof. V. G. K. Murti
Department of Electrical Engineering
Indian Institution of Technology, Madras

Lecture 31
Network Functions (2)
2-port network parameters
Reciprocal 2-port networks

In the last lecture, we discussed the network functions associated with 1 port networks the driving point impedance functions the driving point admittance function. Now, let us check the configuration of 2 port network which has got 2 terminal pairs 1 and 1 prime are the terminals associated with port 1, 2 and 2 prime are the terminals associated with port 2.

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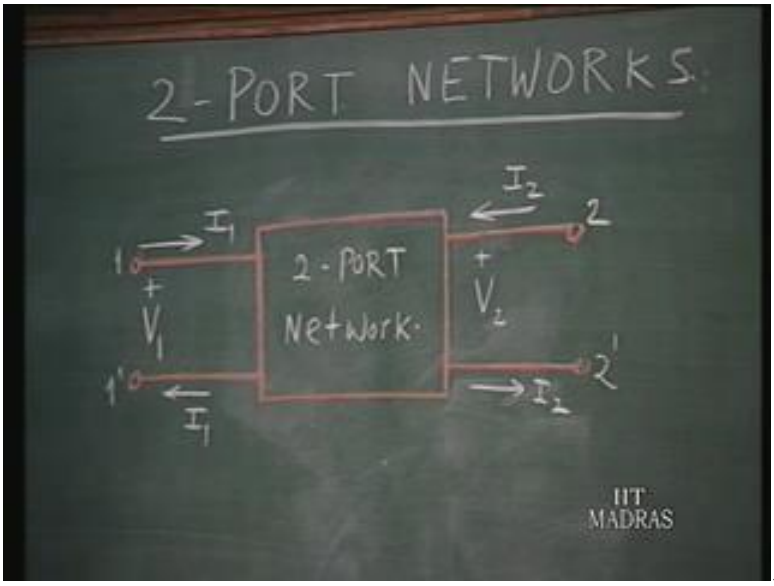

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The conventional reference signs for the port voltages and currents are given in this manner. Port current I 1 enters the network to the positive reference for port 1 V 1. And the port current I 2 enters the network with the positive terminal I mean the positive reference sign for the voltage at port 2. This is the conventional way of taking these polarities of the voltages and currents. And the port concept ensures that if the current I 1 enters the terminal 1, the same current must leave to the terminal 1 prime.

And like wise, current entering terminal 2 must leave the network through terminal 2 prime. Now, we have four variables to deal with V 1 V 2 I 1 and I 2. So, any two of these are linearly related to other two by virtue of the linear time invariant property of the network. So, depending upon the particular occasion contest we take two of them as the excitation functions and the other two as the response functions.

So, you have choice of taking any two as the excitation function and other as the response function and depending upon the pair that you take as the excitation function, you get different types of parameters which describe this 2 port network. We will take up to start with a particular set of parameters called short circuit admittance parameters. This is one way of choosing two of as the input quantity and two as the response quantity.

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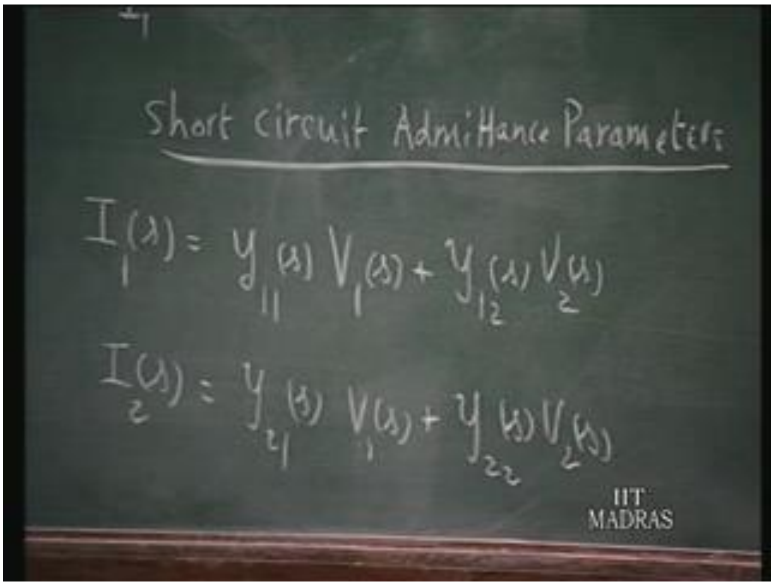

Here, we treat I 1 and I 2 as the response quantities and V 1 and V 2 as the input quantities. So, I 1 of s is linearly related to 2 the input quantities V 1 and V 2 and the proportionality constants for that are written as y 1 of y 1 1 of s times V 1 of s plus y 1 2 of s times V 2 of s. So, the response in this the response of current I 1 is really related to the input quantities V 1 and V 2 by the proportionality constants y 1 1 of s and y 1 2 of s.

Similarly, I 2 of s is y 2 1 of s times V 1 of s plus y 2 2 of s times V 2 of s. So, the network now is characterized by four parameters y 1 1 y 1 2 y 2 1 and y 2 2. They are called short circuited admittance parameters. Because dimensionally, they are admittances because the ratio of current to a voltage and these are the four parameters are called short circuited admittance parameters.

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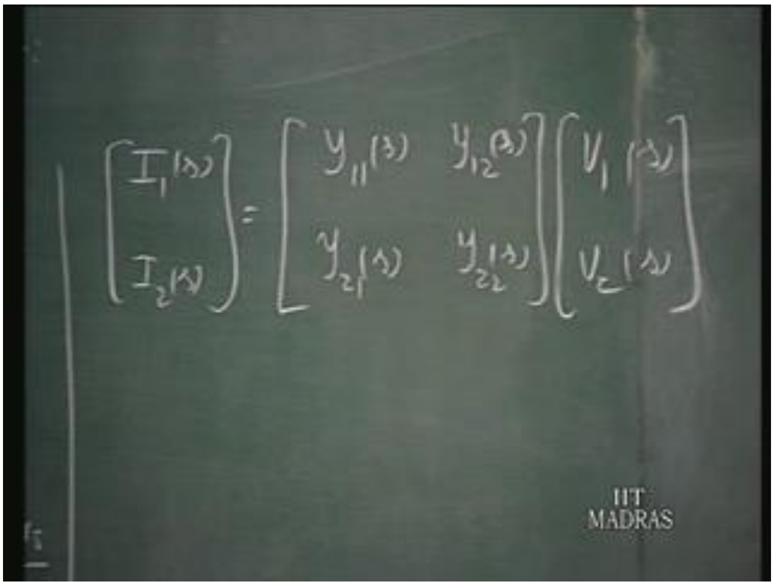

In matrix form the same set can be written in very convenient fashion I 1 of s I 2 of s is obtained as the y 1 1 of s y 1 2 of s y 2 1 of s y 2 2 of s multiply it V 1 of s V 2 of s. So that is the convenient way in writing down the parameters in matrix form relating the currents to voltages.

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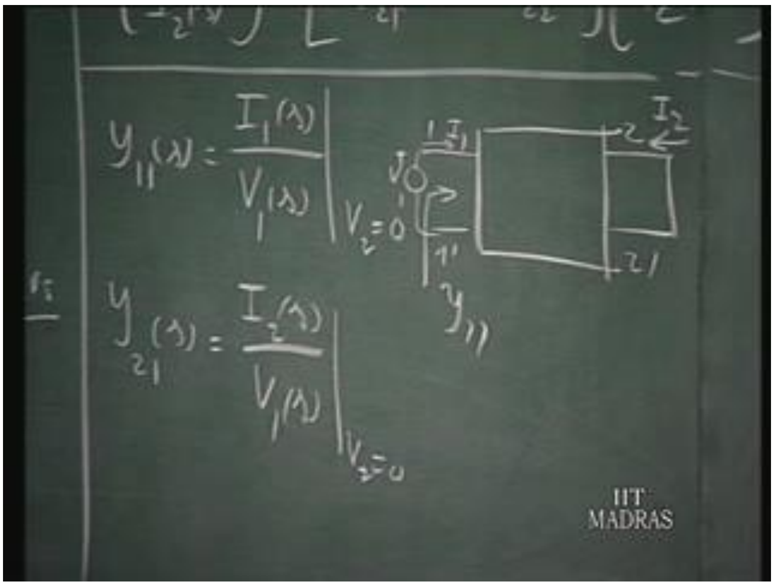

If you observe that if I want to define y 1 1 of 1 s, (Refer Slide Time 04:26) if I want to find out what y 1 1 of y 1 1 of s is I have to write y 1 1 of s is the ratio of I 1 of s to V 1 of s when V 2 is 0. Suppose I forced V 2 0 by short circuiting these two terminals and (Refer Slide Time 04:26) measure the current I 1 of s in response to excitation V 1 of s the ratio of I 1 of s to V 1 of s is y 1 1 of s.

Therefore, I 1 1 of s is written as I 1 s by V 1 of s and V 2 is 0; that means, you are short circuiting the output port 2 2 prime. This is 1 1 prime and we are measuring this admittance. You are connecting a voltage source V 1 and measuring the current here. Now let us see, what is the y 2 1 of s? y 2 1 of s (Refer Slide Time 04:26) is the ratio of I 2 to V 1 when V 2 is 0. Once again in this equation if you force V 2 to 0 y 2 1 of s is I 2 of s for V 1 of s. So, at the same excitation conditions this current I 2 to this V 1 is your y 2 1 of s.

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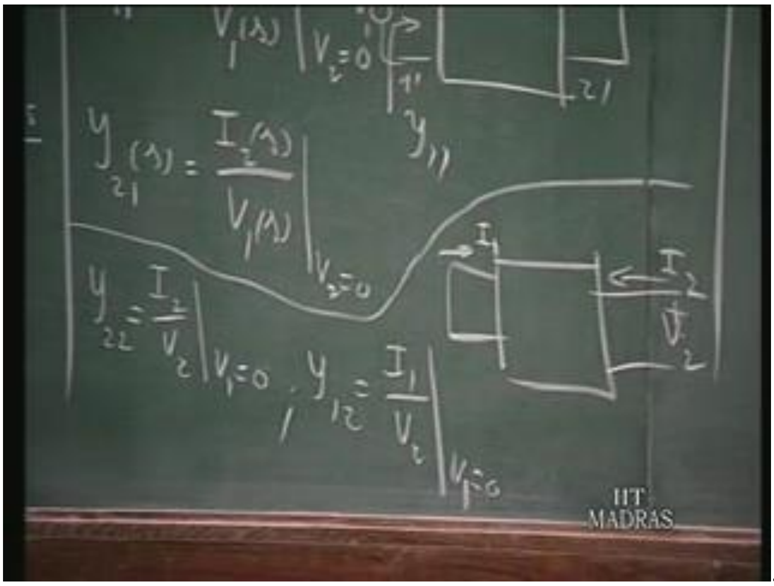

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Now, suppose I want to find out y 1 2 of s and y 2 2 of s. So, y 2 2 I will drop the functional notation s within brackets to simplify my writing y 2 2 is I 2 by V 2, when V 1 is 0. So, I 2 by V 2 at V 1 0; that means, the experiment that you suggested is in this 2 port network you short circuit this and you apply a certain voltage V 2 and measure I 2 the ratio of these two is Y 2 2 of s.

And in the same experiment, if you find out the ratio of I 1 to V 2 I 1 to V 2 when V 1 is 0 I 1 to V 2 is Y 2 Y 2 therefore, Y 1 2 is ratio of I 1 to v 2 by at V1 0. So, you have these four individual parameters can be calculated or can be defined as the ratio of excitation to the Laplace transform to excitation in Laplace transform response under these terminal conditions.

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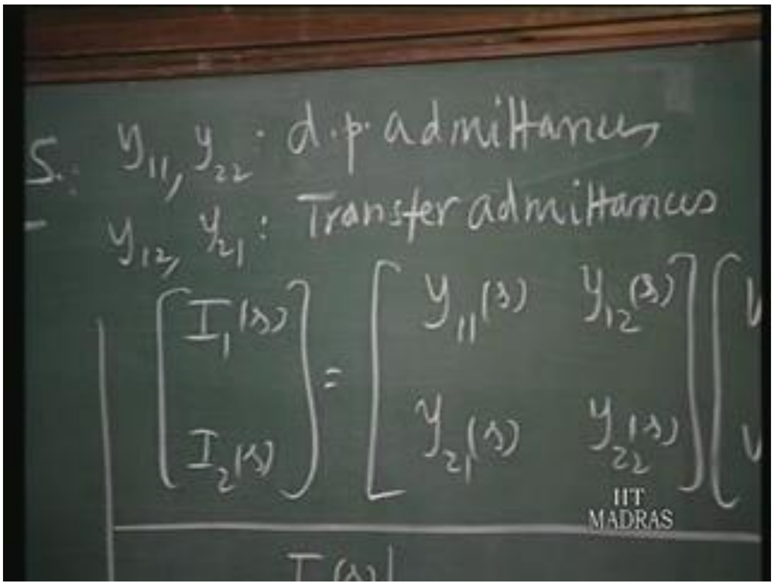

Now, y 1 1 and y 2 2 are called driving point in admittances y 1 1 and y 2 2 are the driving point admittances, because the response. And the input at the same location associate the same terminal pair either in the case of y 1 1 or in the case of y 2 2 I 2 or V 2.

On the other hand y 1 2 and y 2 1 are called transfer equations, because the response location is different from the excitation location. Strictly as I said these are called driving point admittance function transfer admittance function. But to simplify our referring to them, we simple refer to them as admittances driving point admittances and transfer admittances.

One thing you should notice is there is the particular order of the subscripts which is important (Refer Slide Time 05:33) when you are calculating y 2 1 it is I 2 by V 1, the first subscript refers to the response location. All our network functions are the response that side, Laplace transforms with response to Laplace transform in the excitation. The first subscript here refers to the response location, the second subscript to the excitation location y 2 1.

There are driving point function both the response and excitation at the same location. Therefore, you have the same subscript y 1 1. Similarly, y 2 2 (Refer Slide Time 07:14) look for y 1 2 the response location is I 1 port 1. The excitation is at port 2. So, I 1 by the V 2 and this is y 1 2. So, there is an order in writing out the subscripts the first subscript refers to the response location.

The second subscript to the excitation location and that is also in accordance with this matrix. Matrix notations the first subscript use the row address. The second subscript refers the column address and that is exactly the, what we have here also. So, there is some consistency in writing down these equations and that we should keep in mind when defining the other parameters.

Another point to keep in mind is we use the lower case letters for this y's not the capital letters, but the lower case letters small y and later all will be small z. Because, these are all admittances defined under certain normalized conditions. The general admittances with the other types of tabulations can be used capital letters. But, when you are using specifically for short circuit conditions they are called the lower case letters. Therefore to be must you must keep that in mind y 1 1 y 2 1 y 2 1 y 2 2 use small y's.

Now in all these definitions here, you observe that the natural form of the network is when both ports are short circuited. This we have introducing a voltage V 1 here. And when this V 1 is destroyed or deactivated it may reduce to 0, this is short circuited and this is also a short circuited. Similarly, here you are finding the response of I 1 and I 2 when a voltage is introduced here.

So, when the voltage is removed or made equal to 0, both the ports are short circuited. So, these are the parameters measured under the short circuit conditions of the network both ports are short circuited. Therefore, they are called the short circuit admittance parameters. That is the reason why we call them short circuit admittance parameters.

(Refer Slide Time 01:37) Because, we assume that the natural form of the network is like this. Are we introducing a voltage source here and measuring the response. Similarly, you introduce a voltage source V 2 and measuring responses. So, the natural form of the network is both ports are short circuited. And therefore, the poles of all these network functions y 1 1 y 1 2 y 2 1 y 2 2 turn out to be the natural frequencies of the 2 port network when both the ports are short circuited.

And with our with minor exceptions here and there which may come out as a result the cancellation of common factors with numerators and denominators except for that fact. All these functions will have the same denominator polynomial. Because, they represent the natural frequencies of the network under short circuit conditions, therefore when you calculate y 1 1 y 1 2 y 2 1 and y 2 2, you will find that the denominator polynomial all these function is one at the same. It is only the numerator which must be aware.

Now, how do we calculate this y 1 1 y 1 2 y 2 1 and y 2 2 for simple networks, you can use these definitions and find out y 1 1 y 1 2 y 2 1 using these definitions.

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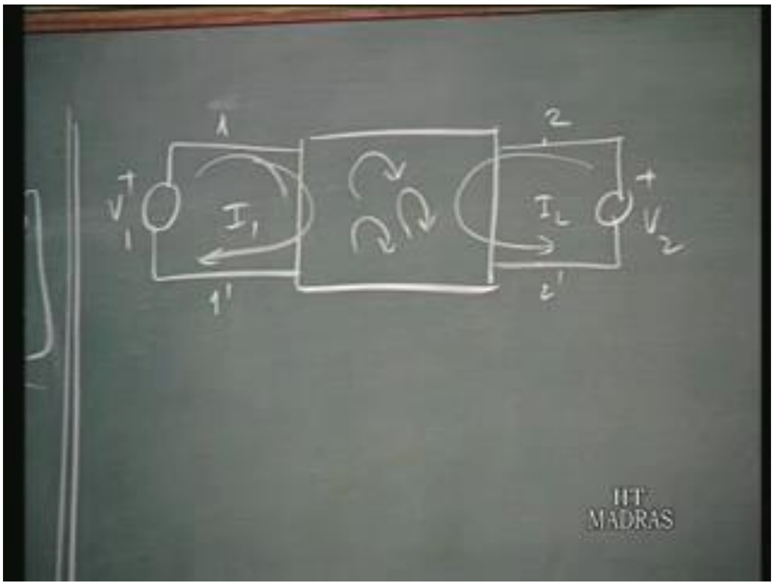

In a more general case suppose I have a 2 port network here. You connect a voltage source V 1 another voltage source V 2 here. And you perform the loop analysis. So through the network, you have this loop current I 1 through this you have loop current I 2. And therefore may be internal loops here depending upon the structure of the network.

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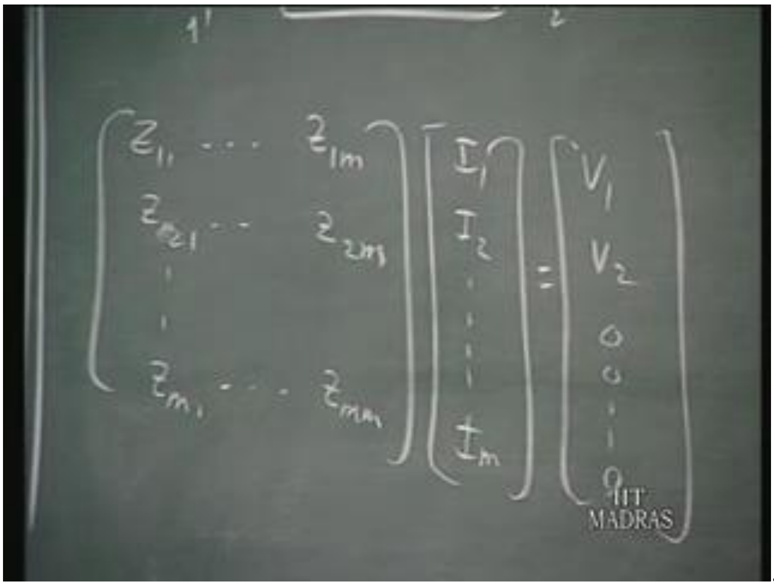

So, you can write the loop equations of this network, we have say Z 1 1 say there are m loops Z 2 1 Z 2 m Z m 1 Z m m. These are the loop equation I 1 I 2 I m as far as the forcing function is concerned, you have the sources only in this loop quadrants two. Because internally, it is suppose to be a passive network linear time invariant network no independent source are present. Therefore, we have V 1 of s and V 2 of s here for the second loop all the others are 0s.

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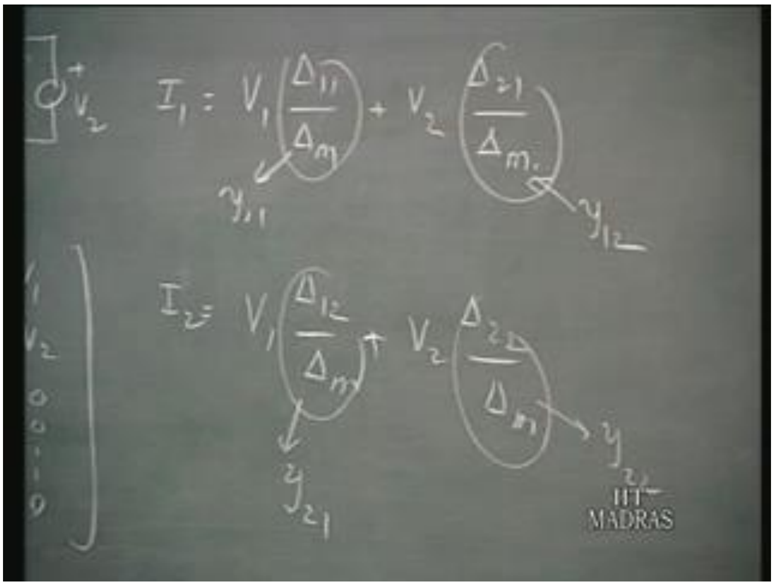

So, from this you can calculate I 1 and I 2 by taking the ratio of the appropriate determinants. And if you do that you will get what I 1 equals V 1 times delta 1 1 by delta m, where delta m is the mesh based a system determinant and delta 1 1 is the co factor plus V 2 times delta 2 1 by delta m. That is you replace the when you calculating I 1 you replace that in first column by this column V 1 V 2 and 0 and when you expand that you will get this.

And similarly, I 2 will be V 2 V 1 times delta 1 2 by delta m and V 2 times delta 2 by delta m. So, it is clear that this will be the y 1 1 and this will be y 1 2 this will be y 2 1 and this will become y 2 2. So, you can calculate can write down the loop equations of the network in general terms of generalized impedances and calculate the ratio of the appropriate co factor to the mesh based system determinant and that will give you y 1 1 y 1 2 y2 1 y 2 2 in a general case.

(Refer Slide Time 01:37) I mentioned earlier that out of this four quantities V 1 V 2 and I 1 and I 2. We can take any two as an excitation quantities and take the other two as responses. Let us, now consider that the currents are excitations and voltages as the responses.

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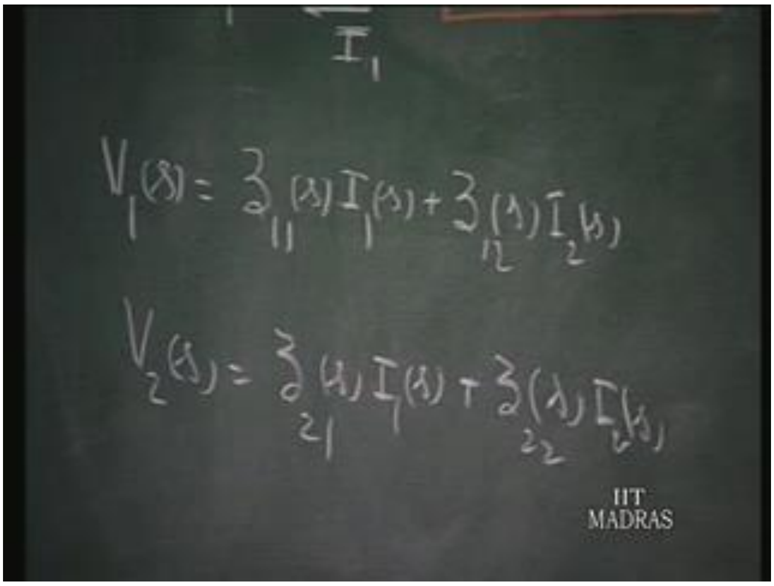

Therefore, I can write V 1 of s as an impedance z 1 1 of s multiplied by I 1 of s plus another impedance z 1 2 of s multiplied by I 2 of s. And similarly, the second voltage is z 2 1 of s times I 1 of s plus z 2 2 of s times I 2 of s.

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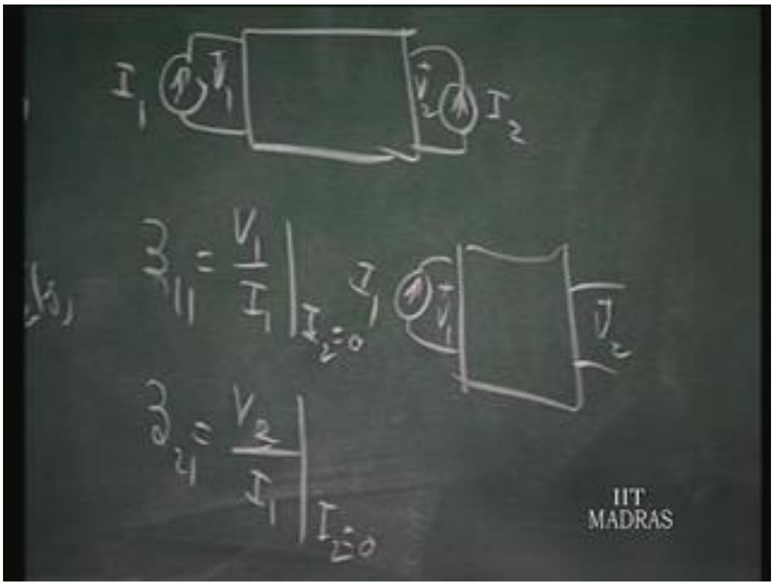

Now, we know have the voltage s as the response quantities and currents and the inputs; that means essentially, what we are thinking of is a network with few current excitations I 2 I 1 and V 1 and V 2 are the response quantities. So, these are called open circuit impedance parameters, for example if I want to measure z 1 1 all I have to do is z 1 1 is the ratio of V 1 to I 1 when I 2 is 0. Similarly, z 2 1 is V 1 V 2 by I 1, when I 2 equal to 0; that means, you are now thinking of a situation where a current I 1 is led in and you are measuring out the responses V 1 and V 2.

The ratio of V 2 to I 1 is z 2 1 the ratio V 1 to I 1 is z 1 1. (Refer Slide Time 04:26) So naturally, you get a similar set of defining relations for z 1 2 and z2 1 for z 1 z 1 2 2. We have to find out you have to make I 1 0.

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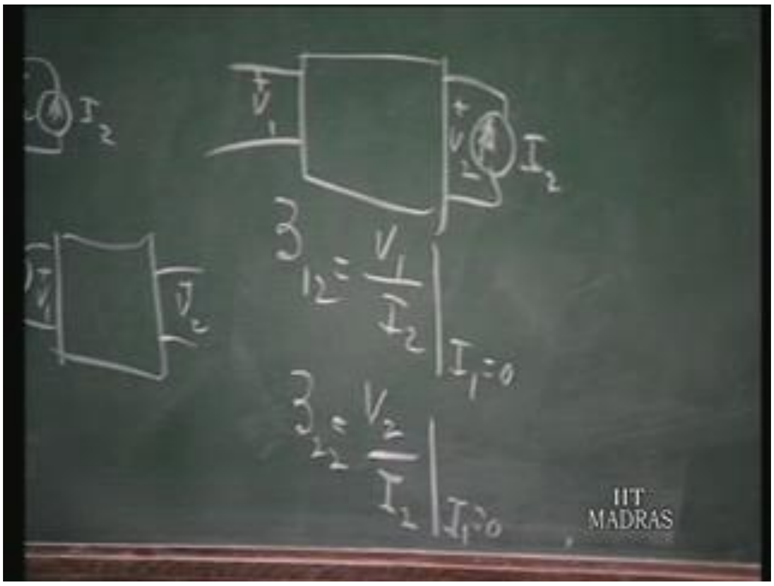

Therefore, you should make I 1 0 and introduce a current I 2 here. And this is the response V 2 and this is response V 1 z 1 2 is V 1 divided by I 2. When I 1 is 0 and z 2 2 is V 2 divided by I 2 when I 1 is 0. So, these definitions are completely in parallel with it defining relationship for the y parameters except now the currents are the excitations sources.

And therefore, we also observe that if this current I 1 is made equal to 0 both the terminals are open. Similarly, when I 2 is made equal to 0 then both of them; that means, these are network functions whose poles represent the natural frequencies of the network when the both the ports are open circuited. So, the raw form of the network is that this is both terminals are open. And you have connect a current source the soldering type of entry for the two currents and finding out the responses.

Whereas, in the short circuit parameters the raw or the natural form is the short circuited and we are introducing a voltage source in that the pliers of the entry at the voltage source. So, we have these are called open circuit impedance parameters. And naturally z 1 1 and z 2 2 are the driving point impedances because V 1 I 1 V 2 I 2 are individually measured in the same pair of terminals. So, Z 1 1 and Z 2 2 are the open circuit driving point impedance functions z 1 2 and z 2 1 are the open circuit transfer impedance functions.

So, to summarize the z parameters represent the network functions defined under open circuit conditions. By introducing current sources, when the current sources are open re removed then the 2 ports are open circuited. There the poles of all this network function represent the natural frequencies. The network under open circuit conditions Z 1 1 and Z 2 2 are the driving point impedance functions, z 2 1 and z 1 2 are the transfer impedance functions.

And we can define them in this fashion are individually and need not to find out these parameters for a general network. We can do the loop current in the node voltage method of analysis. You can introduce a current source here make this a lapper node point of the response for V 1 V 2. And then, you can perform the node analysis and get the values of z 1 1 z to 1 z 1 2 and z 2 2 in more or less similar fashion as what we are demonstrated for the case of admittance functions.

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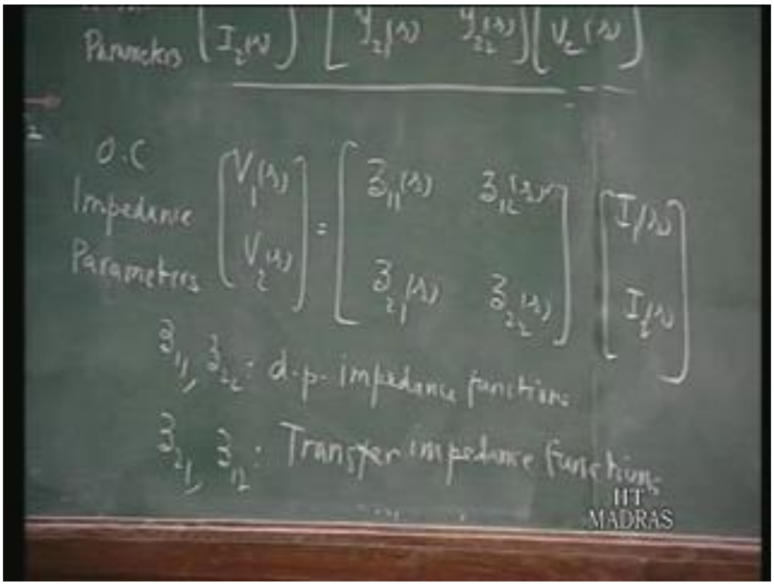

So, let me write this table here in this table, we have already discussed given the matrix relation short circuit admittance short circuit admittance parameters are called y parameters. In the similar fashion we can write V 1 of s and V 2 of s as z 1 1 of s z 1 2 of s z 2 1 of s z 2 2 of s and I 1 of s I 2 of s. These are called open circuit impedance parameters. Again z 1 1 and z 2 2 are the driving point impedance functions open circuit driving point impedance functions, where as z 2 1 and z 1 2 are the transfer impedance functions.

Now looking at these two, after all you have a matrix relation the currents are given in terms of voltages. (Refer Slide Time 08:31) So, voltages suppose from this you want express voltage in terms of currents you have to invert this matrix and write V 1 V 2 is this matrix inverse times I 1 I 2. So, one can expect that this entire matrix here is the inverse of this matrix. So, the inverse of this matrix is the inverse of the matrix is this. So, z and y matrices are inverses of each other that does not mean that y 1 1 is the inverse of y 1 1.

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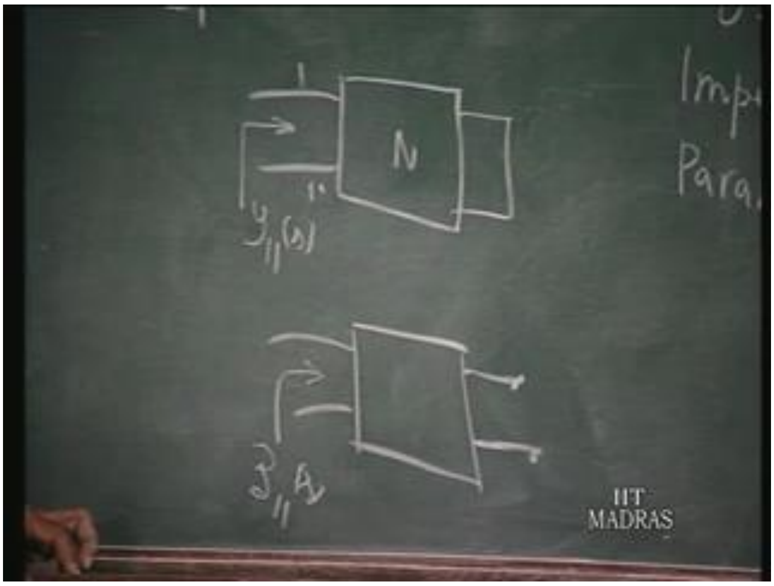

Look at the definition of Y 1 1, how do you define Y 1 1 is the ratio of I 1 to V 1 when V 2 is 0. So, you short circuit this and measure this admittance and that is y 1 1 of s, how do you define z 1 1 it is the ratio of V 1 to I 1 when I 2 is 0, so the you are keeping this open circuit at. And measuring these impedance that is z 1 1 of s. So, the network in which this admittance is measured and the networks in which this impedance are measured are different.

This N have other terminals as short circuited here the two terminals are kept open circuited. Because of this reason, z 1 1 is not necessarily equal to 1 over y 1. So, the short circuit driving point admittance at port 1 is not the reciprocal or the open circuit driving point impedance at port 1.

These two are not reciprocally related, but this entire matrix is the inverse of this matrix. So, you can show that z 1 1 is related to all the y parameters in some fashion you can invert the matrix and find out. But individually, y 1 1 is not equal to 1 over z 1 1, because the two networks are different.

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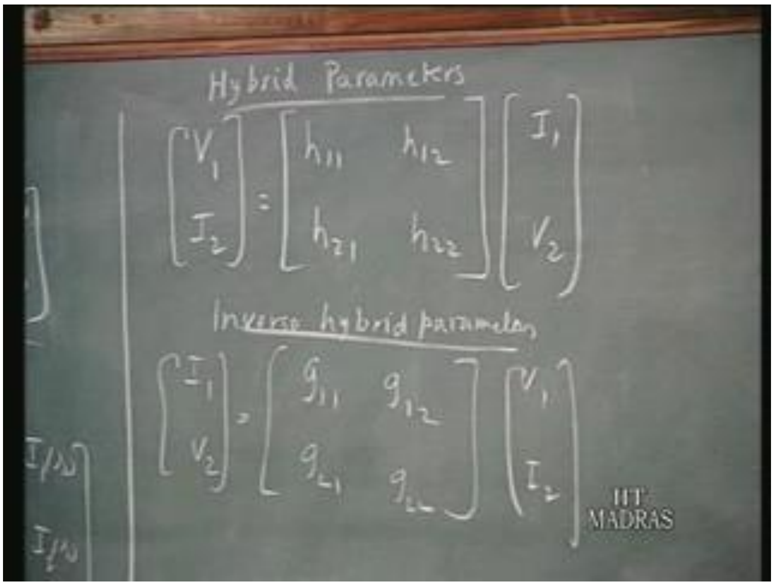

Now, apart from these two basic matrices, you have various other types of matrices which I would like to talk about now. You can express V 1 and I 2 in terms of I 1 and V 2. So, it will be h 1 1 h 1 2 h 2 1 h 2 2 times I 1 and V 2. Again I am dropping the functional notation for the convenience. You can also express I 1 and V 2 in terms of V 1 and I 2. Normally, these are referred as g 1 1 g 1 2 g 2 1 g 2 2. These are called hybrid parameters h parameters are hybrid parameters commonly used in transistor work.

The reason, why they are called hybrid they are mixed variable? Because, you are not talking about both the input quantities as the voltage are currents 1 is a current other is a voltage response also one is voltage other is current. So, therefore, they are mixed variables they are called hybrid parameters. These are called inverse hybrid parameters h parameters or inverse h parameters.

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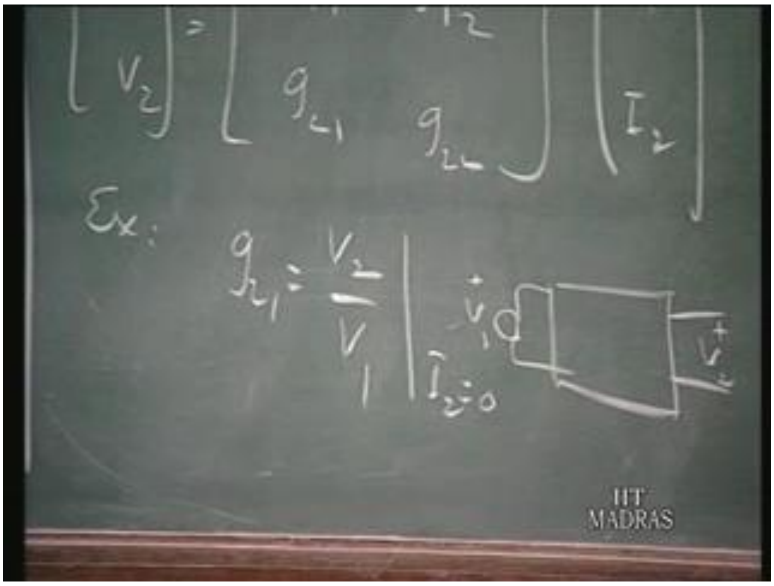

And once again, you can define example if I want to find out what g 2 1 is is V 2 over V 1 when I 2 is 0. Because, V 2 is g 2 V 1 plus g 2 2 I 2, therefore when make I 2 equal to 0, V 2 equal g 2 1 times V 2. So like this, you should be able to define each one of these parameters with the appropriate terminal conditions.

Now, you observe that this is the ratio of the response when I 2 0 and you are measuring the V 2 over with a particular excitation. So in the in defining, this g type parameters this the x pattern of excitation and response that you are having. So, the natural form of the network is when V 1 is 0, this port is short circuited and the other port is open. So, in this hybrid parameters it turns out that 1 port is open circuit the other port is short circuited.

Therefore, the poles here represent the natural frequencies when 1 port is open circuit the other is short circuited, whereas here both the ports are short circuited. This is the case where both the ports are open circuited. If the case hybrid parameters 1 of the ports will be open circuited, the other is short circuited. And the natural frequencies under those conditions will be the poles at the appropriate networks functions here.

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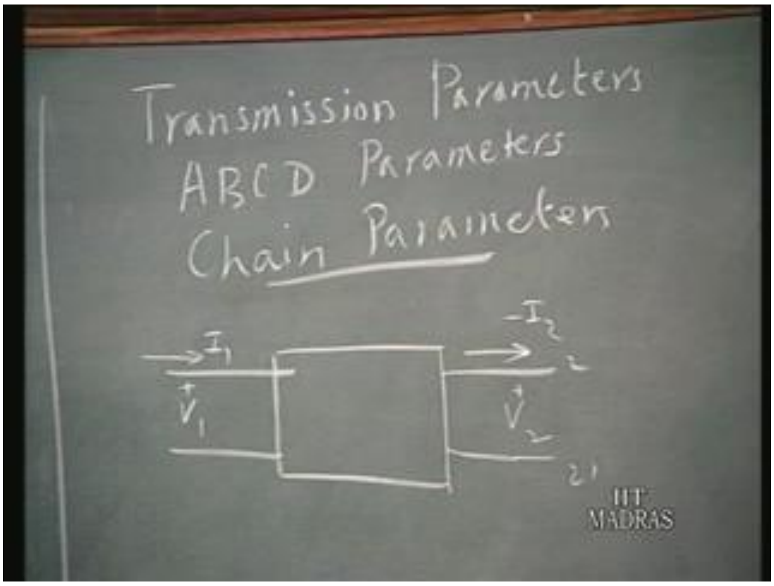

In addition you also have another important category of parameters which go by the name transmission parameters also called ABCD parameters sometimes called chain parameters. Here, assume that this is the transmission line and you are talking about the output of the transmission line. This current is minus I 2, because we are we are defining the port I 2 as leading inwards. Therefore, 2 2 prime this the voltage and current that is being fed from the at the output end.

And the receiving end here, at the transmission line let us say V 1 and I 1 are the voltage and current. So, we can think of V 1 and I 1 are the quantities at the sending end and V 2 and I 2 are quantities at receiving end like a transmission line. The current is going out of transmission line this is terminal voltage at the receiving end. The terminal voltage and current will sending in.

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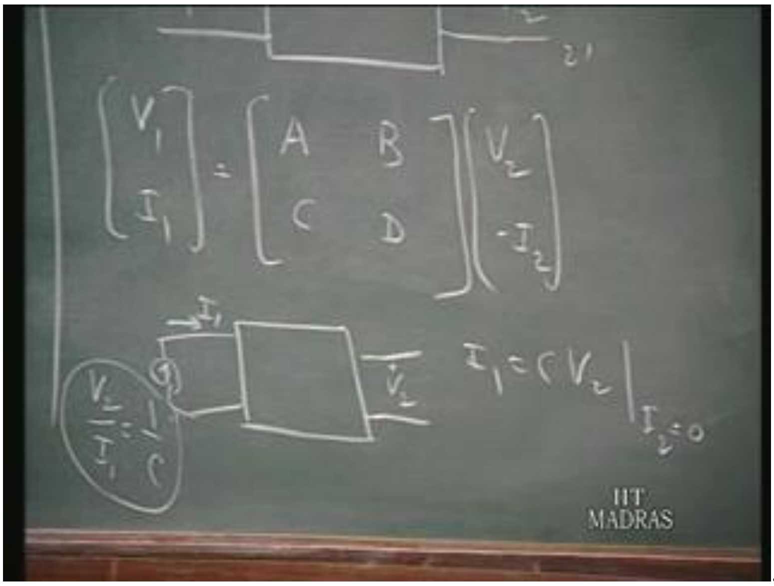

So, if you express V 1 and I 1 in terms of the receiving end quantities V 2 and minus I 2. Then, we these are related like parameters ABCD again ABCD a function surface at general case and this is called ABCD parameters. So that is how these are defined. Now let us see, how do you find out C? C is the ratio of I 1 to V 2 when I 2 is 0. So, I 2 is 0, this is kept open circuited and is the ratio of V V 2 2 I 1 to V 2.

So, the ratio V 2 I 1 is equals to C V 2 where I 2 is 0, I 1 equals C V 2 when I 2 is 0. So normally, when you define any network function you write C equals I 1 by V 2. You can think of V 2 as an excitation, but unfortunately here I 2 is forced to be 0. Therefore, you cannot think of a connecting a voltage source and again make sure that I 2 is 0. Therefore, you have to think of a current source here, I 1 that is the input and your measuring the V 2.

Therefore V 2 by I 1 the response to the input V 2 by I 1 turns out to be therefore, 1 over c. So, in the case of this chain parameters, this ABCD values do not turn out to be the network function in our standard format in the ratio of the Laplace transform of the response to the Laplace transform of the input. In turns out that in this particular case of the chain parameters, the reciprocals of these quantities ABCD 1 over C 1 over A 1 over B 1 over D turn out to be the network functions.

In our conventional sense at the Laplace transform of the response to Laplace transform of the input. Because, C normally would assumed to be I 1 by V 2 and V 2 being the response, but V 2 is the input. If V 2 is the input we cannot force the current also to be 0, because we cannot connect a voltage source here and then demand that current I 2 is 0.

So, it turns out that as far as the ABCD parameters are concerned it is the reciprocal of the ABCD's which are the conventional network functions. But, we use this ABCD also in network functions in this relationship. But they deviate from our standard definition of a network function. Now in all these cases in all these; that means, we have a essentially five important sets of parameters. Actually, we can have a 6th set also we can expect V 2 and I 2 in terms of V 1 and I 1 that is the inverse of this matrix that becomes 6.

(Refer Slide Time 01:37) After all we have four quantities here V 1 V 2 and I 1 I 2, you can select a pair of two quantities from four in six different ways. So, you have 1 2 3 4 5 and the, if you take inverse of this you get a sixth set of parameters. But, we normally that is not done that is not very important.

These are the five sets of parameters which are important the short circuit admittance matrices, the open circuit impedance matrices, the hybrid parameters, the inverse hybrid parameters and the chain parameters. The importance of the chain parameters are why you call them chain parameters it will be brought out later. And in the both y matrix z matrix h matrix and g matrix, we use the lower case letters, because they are under standardized terminating conditions It is conventional to use the lower case letters.

As far as the ABCD parameters are concerned, it is a conventional to write them by capital letters A B C and D. It is also important for us to recognize the dimensions of the each one of these quantities. As far as y and z are concerned (Refer Slide Time 25:21) their impedance and admittance is no problem at all h 1 1 as the dimensions of resistance or impedance because V 1 h 1 1 is V 1 divided by I 1 when V 2 is 0. Therefore this has the dimension of impedance h 1 2 V 1 divided by V 2 where I 1 is 0 that has a dimension.

It is dimensionless it is a ratio likewise this is a ratio this has the dimensions of an admittance. Similarly, (Refer Slide Time 25:21) g 1 1 has the dimensions of an admittance, g 1 2 has the dimensions as the ratio no dimension at all g 2 1 is dimension less g 2 2 has the dimension of a impedance. Likewise, you can associate the dimensions with each one of these.

So, important thing is for us you must have the familiarity with these, but you must be able to define each one of these parameters in any one of these sets like in this fashion. You must know the dimensions of the quantities that are involved. And further more by manipulating this equations, (Refer Slide Time 08:31) if somebody gives us the admittance parameters of a 1 port 2 port network from these y parameters should be able to derive any other set of parameters.

So, after all from this you have to write down two equation I for I 1 and I 2 in terms of V 1 and V 2. And using those two equations you can manipulate this equations to express I 1 V 2 in terms of V 1 I 2 by manipulating these equations. (Refer Slide Time 25:21) Therefore, in other words given y parameters we should be able to get g parameters in 2 port parallel network given y parameters you should be able to get h parameters and so on and so forth.

So, you should be able to manipulate these equations and find out one set of parameters from any given set. And that is straight forward it is not very difficult I mean tables are set up there is available in textbooks. But, we do not have to read always use these tables we should be able to deduce one set of parameters from the other by manipulating these equations. one or two examples you work out little later.

So, at this point of time, let us now look at these, let us take one or two examples to illustrate these ideas which we have discussed just now. That is to derive one set of parameters from the other set. And then, we will try to work out some other examples using these various parameters.

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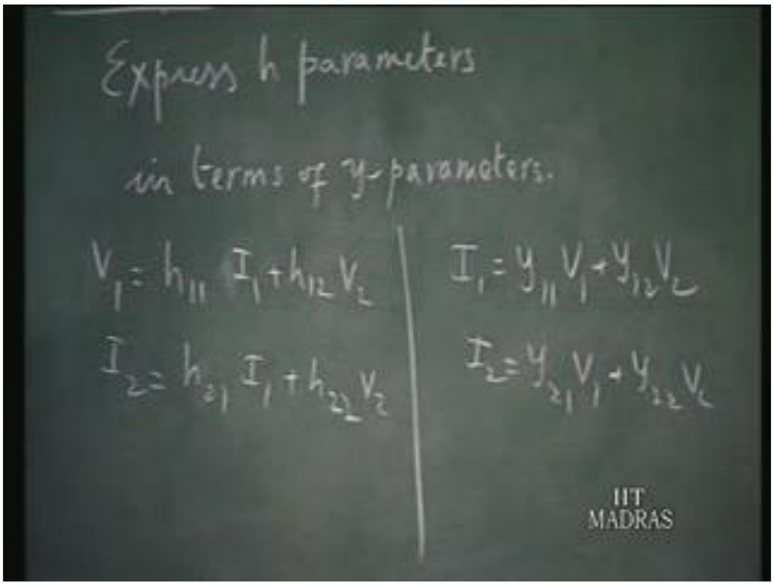

Let us, now take an example express h parameters in terms of y parameters. Suppose that is the question that is asked. You recall that h parameters have this form h 1 1 I 1 plus h 1 2 V 2. And I 2 equals h 2 1 I 1 plus h 2 2 V 2. But, y parameters have the form I 1 equals y 1 1 V 1 plus y 1 2 V 2 and I 2 have the form y 2 1 V 1 plus y 2 2 V 2. So, now we have to express V 1 and I 1 in terms of I 1 and V 2.

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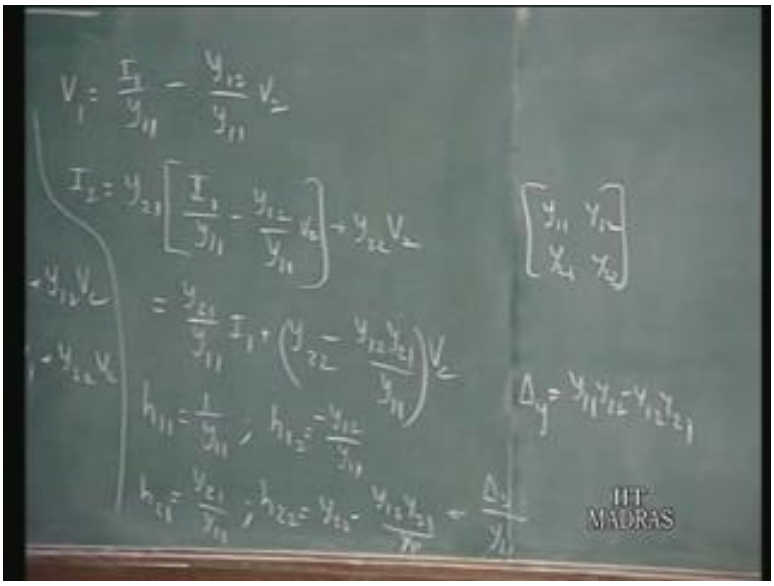

Therefore, we have to find out from this V 1 equals from this I 1 by y 1 1 V 1 equals I 1 by y 1 1 minus y 1 2 by y 1 1 V 2. That is what we are having? I 1 by Y 1 1 minus Y 1 2 by Y 1 1 V 2 so; that means, that particular equation is in accordance with this form. So, so we already have h 1 1 h 1 2. As far as I 2 is concerned, we express I 2 in terms of I 1 and V 2. But, we have I 2 expressed in terms of V 1 and V 2.

But, V 1 expression we already know, so substitute this here. So, you get I 2 equals y 2 1 times for V 1 we substitute this expression I I 1 divided by y 1 1 minus Y 1 2 by Y 1 1 times V 2 plus y 2 2 V 2. So, we have now expressed I 2 in terms of I 1 and V 2; that means, this will be y 2 1 divided by y 1 1 times I 1 plus y 2 2 this is starting from this y1 2 y 2 1 divided by y 1 1 times V 2.

So, we have expressed V 1 and I 2 in terms of I 1 and V 2. So, from this we can readily see that h 1 is 1 over y 1 1 h 1 2 here minus y 1 2 over y 1 1, h 2 1 equals y 2 1 over y 1 1 and h 2 2 is this quantity y 2 2 minus y 1 2 y 2 1 divided by y 1 1. That is what we have added. So, this you can write this if you have a common denominator this is delta y divided by y 1 1 plus delta y is the determinant of the y matrix y 1 1 Y 2 2 minus y 1 2 Y 2 1.

That is you have the y matrix is this y 1 1 y 1 2, y 2 1 y 2 2. So, if you take the determinant of this matrix this will be y 1 1 y 2 2 minus y 1 2 y 2 1 and that is what we are having here delta y by y 1 1. So, in this way you should be able to convert one set of parameter into ((Refer time: 39:56)).

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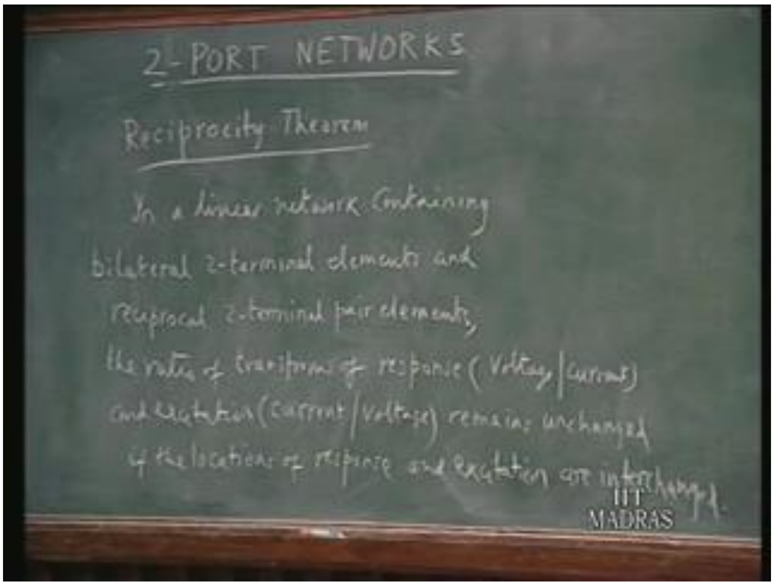

We shall now consider some restrictions imposed on this parameters by a network which is called a reciprocal network. Reciprocal network is a network which satisfies reciprocity theorem, what is reciprocity theorem? Reciprocity theorem states is in a linear network containing bilateral 2-terminal elements and reciprocal 2-terminal pair elements.

The ratio of transforms of response, response means either voltage or current and ratio of transforms of response and excitation there is excitation may be current or voltage. The ratio of transforms remains unchanged or remains the same, if the location of response and excitation are interchanged, if the recursions of response and excitation are interchanged.

First of all let us see, what this means? We are talking about a linear network containing 2 terminal elements bilateral 2 terminal elements; that means, the ratio of voltage to current the form of the relationship remains the same, whether the current is the one direction or the other.

All over or else the elements are bilateral 2-terminal elements reciprocal, 2- terminal pair elements; that means, you have 2-terminal pairs your mutual inductance m 1 2 is same as m 2 1. So, there such elements are called reciprocal 2 terminal elements. So if you have RLCM elements, the ratio of transforms to response to excitation will remain the same if the location are interchanged; that means, let us take an example.

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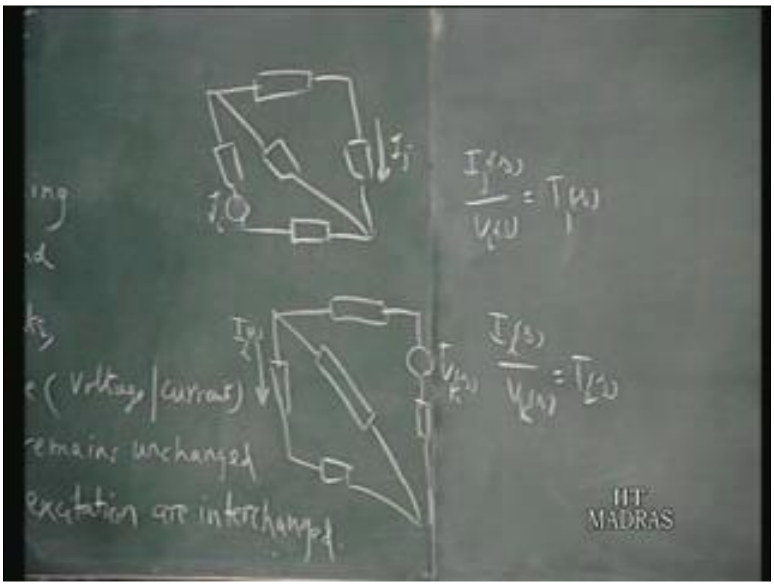

Suppose, I have a network like this suppose I inter I put a voltage source Vi here and measure the current here I j. So, I am finding out the current here in response to an excitation introduced here. So, I get I j of s over V i of s. So, let us say this a one particular transfer ratio, transfer function T 1 of s. Now, if I interchange that; that means, I am measuring the current here by introducing a voltage source here. Suppose I reverse this, I introduce a voltage here say V k of s.

And then, I measure the response here I l of s. So now, where I have originally the excitation point I need to finding out the response and where I was earlier measuring the response I am introducing the voltage here. So, I l of s divided by V k of s let us is T 2 of s. So, this another transfer function the reciprocity theorem tells us that these two ratios are the same, because the response ratio the ratio of response to the excitation remains the same, when their locations are interchanged.

In viewing this, we must be careful that this current is entering that point as far the source is concerned this point is also entering that point. As far as the this source is concerned. So, you must have some consistency in the symmetry in the reference sides that is important.

And secondly, I purposely write response voltage current here and reverse the order here. So, when you are measuring the ratio response to excitation. You must take the response if its voltage the excitation is the current if the response is current voltage is response excitation is voltage. It does not work out when both the responses are voltage. But, response and excitation are voltages both the response and the excitation are currents.

We will illustrate that little later why it is so. So, one must be a current, the other must be a voltage that is important. And any network which satisfies this is called a reciprocal network. So, a reciprocity theorem is applicable to the types of networks which contain these elements. So, we should be also be familiar, what types of elements will not have reciprocity theorem satisfied. If you have linear dependent sources, control sources and so on. It does not satisfy reciprocity theorem.

So, the types of networks for a another element which some of you may be familiar with what is called a generator? It is an anti reciprocal 2-terminal pair element that also it does not is not will not satisfy reciprocity theorem. So, reciprocity theorem is valid as essentially for RLC networks with mutual inductances also present.

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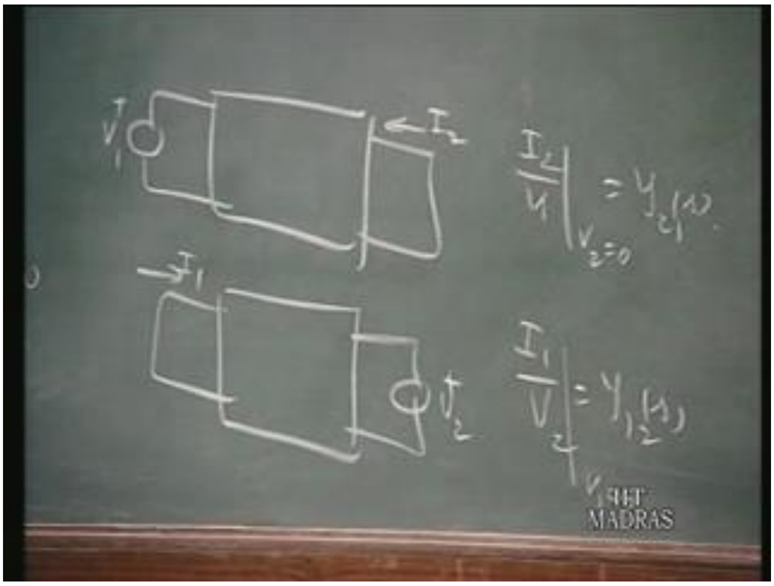

As an obvious of this, what we have here is? Suppose I have 2 port network and let us say this is the raw form of the network. This is a basic network, if I introduce a voltage source here, V 1 and measure the current here I 2. So, this is I I 2 by V 1 when V 2 is 0, this we call this as y 2 1 of s we know that.

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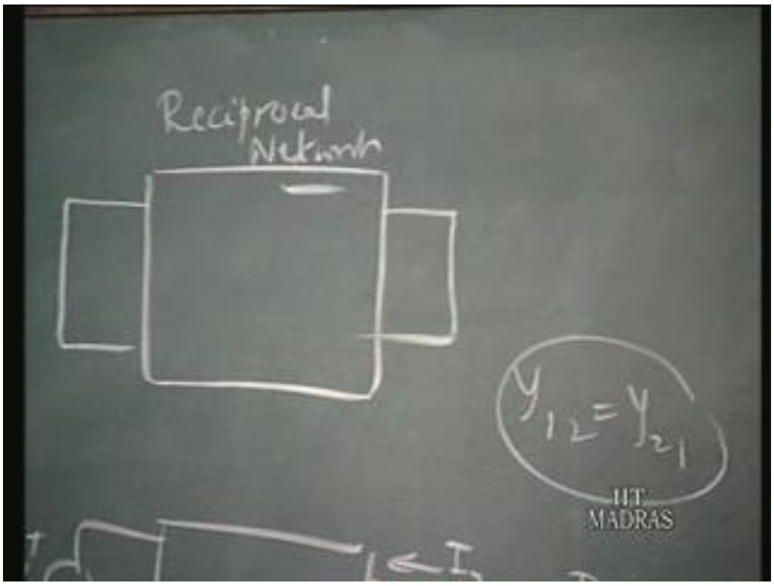

Now in this raw form of the network, suppose I introduce a voltage here V 2 and measure the current response here I 1. Then, I 1 by V 2 is y 1 2 of s, this is what we say when V 1 is 0, this 1 how we define. So, if this is reciprocal network y 1 2 must be the same as y 2 1. So, if this is the reciprocal network it satisfies the reciprocity theorem y 1 2 is equal to y 2 1.

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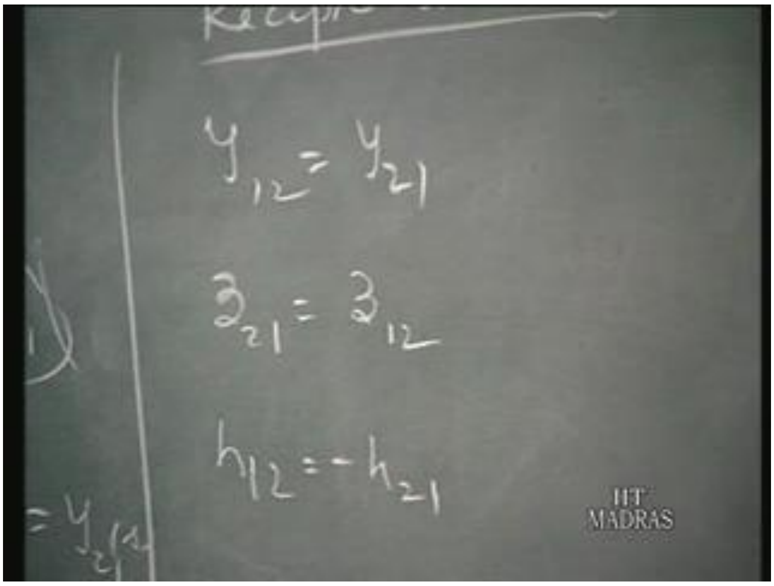

Similarly, it can be shown that for a reciprocal network; that means, the 2 port is a reciprocal of course. We know that y 1 2 is equal to y 2 1. Similarly, it can be shown that z 2 1 is equal to z 1 2 and it can be shown that h 1 2 equals minus h 2 1. Again h 1 2 and h 2 1 can expressed in terms of z parameters and this can be shown.

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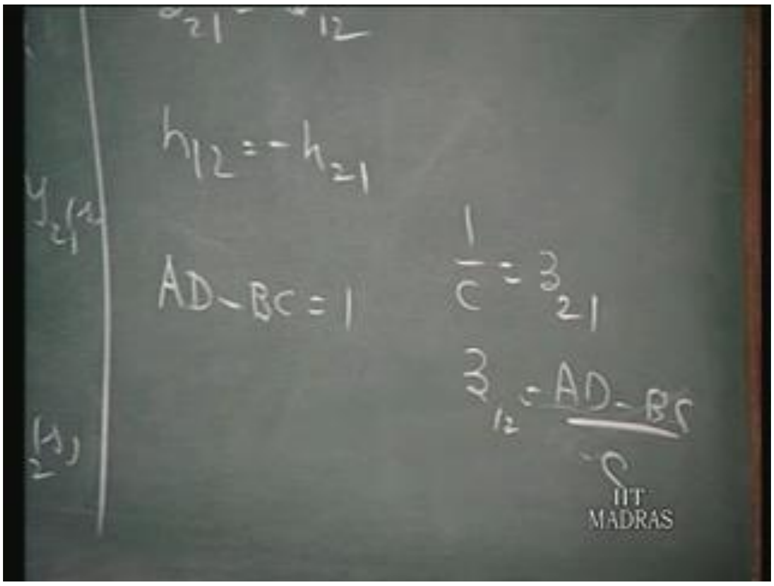

And similarly, AD minus BC for a reciprocal network is equal to 1, this comes from the fact that 1 over C happens to be z 2 1 and z 1 2 happens to be AD minus AD by C minus B. So; that means, AD minus BC by C this is AD minus BC by C. So, for a reciprocal network these two must be equal. Therefore, AD minus BC is equal to 1. So, this is the consequence of reciprocity as far the parameters are concerned.

So in this lecture, we have introduced ourselves to the various sets of parameters characterizing a 2 port network. These are the z parameters, the y parameters the hybrid parameters h, the inverse hybrid parameters g parameters and the transmission parameters ABCD parameters. We saw that they these two those are independent specifications one set of specifications can be obtained from the other.

We should be able to convert one set of parameters into any one of the other required sets. And we also saw an example, where this such a transformations can be done. Next, we consider the case of reciprocal 2 port network which basically consist of RLCM elements. In such cases, we have additional constraints on the parameters in any individual set.

Basically, the z parameter that two the transfer impedance parameters must be the same, in the case of short circuit admittance parameters the two transfer impedances must be the same. In the hybrid parameters h 1 2 must be equal to minus h 2 1 and in the case of ABCD parameters AD minus BC equals to 1; that means, the determinant of the ABCD matrix must be equal to 1.

These are all equivalent conditions; we will study some additional properties of these parameters. And then, the how for particular configurations one set of parameters is easier to calculate than the others. All these questions will be taken up in the next lecture.