Networks and Systems
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Lecture – 30
Network Functions - 1
Driving-point and Transfer Functions
1-port Networks

While discussing the applications of Laplace transform technique. We introduced ourselves to the concept of system function. We said the system function is basically the ratio of the Laplace transforms of the response to the Laplace transform of the excitation, with zero initial conditions in the system. Now, a system function defined for a particular network is more commonly known as a network function.

So, network function is nothing but, a system function applied to a particular network. And the network function is defined for a particular pattern of a excitation and response. Now, while dealing with networks that we normally deal the variable of interest verse are the voltages and currents. Therefore, it turns out that the network function is the ratio of the Laplace transform of a current or voltage response to the Laplace transform of a current or voltage input.

So, in this and the next couple of lectures, will be discussing special types of network functions, applicable to the common accruing types of network, that we deal with. And you would like to steady, the special properties of the various network functions, that we are going to define and this was.

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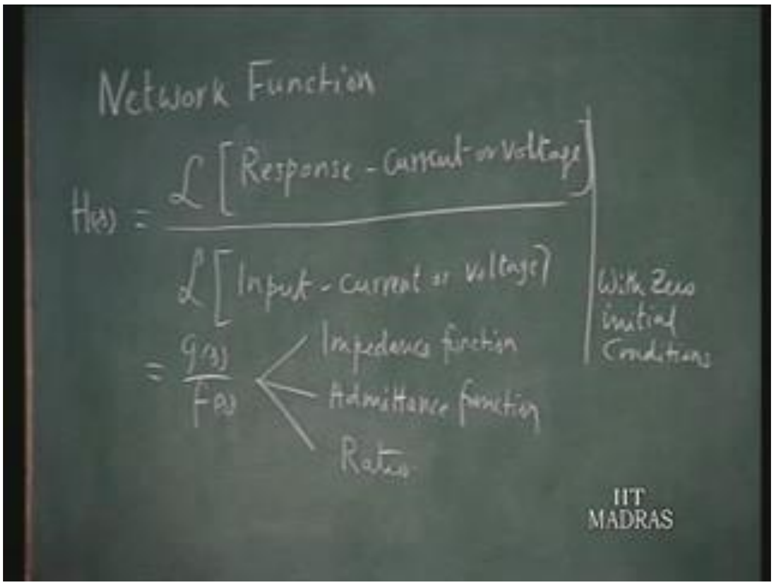

Let us see, so broadly speaking a network function is the Laplace transform of response, which is either a current or voltage. By the Laplace transform of the input excitation, which is again a current or a voltage. And this is done with zero initial conditions. So, if you call a network function, general name H of s. It is the ratio of the Laplace transform of a voltage or a current to the Laplace transform a current or a voltage.

So, depending up on the... So, this in the general a polynomial p s or q s or g s over f s the ratio two polynomials. And depending up on, whether the response is a current or voltage are the input as a current or voltage. The dimensions of H off s will be, either an impedance are in the admittance are a pure ratio. Therefore, as for as this is concerned g of s it could be an impedance. So, it is called, then in that cases called the impedance function, it could be an admittance, it is called the admittance function or it could be a ratio.

If the response is a current and the input is the voltage, then it would be an admittance function. If the response is the voltage and the input is the current, then it becomes an impedance function. If the both the response is current are input are both currents. Then, it is a ratio, if both the response is input or voltages also then it is a ratio. We call them, impedance function other than impedance, because this is function of s.

So, whatever suppose is z we can valid z of s. Similarly, y of s is the ratio of some functions of s, this are all functions of s. Therefore, there call impedance functions or the admittance functions. But, loosely speaking we refer to them as impedance this admittances as long as there is no misunderstanding. A second point observe is, that f of s equal 0, the roots of equation which are the poles of H of s represent in natural frequencies of the network a part of the network.

A property, that we already discussed in our earlier lectures. So, the natural frequencies of the network are given by the poles of H of s. Now, the natural frequencies of the given network are unique, no matter how you excite the network. And what type of response to measure. Therefore, for a given network configuration, you have the same natural frequencies, no matter what response to take and what input you in certain to the network.

Therefore, it is stands to reason, that the poles of network functions defined for a particular network will be more or less unique, with minor exceptions here. And there they are unique.

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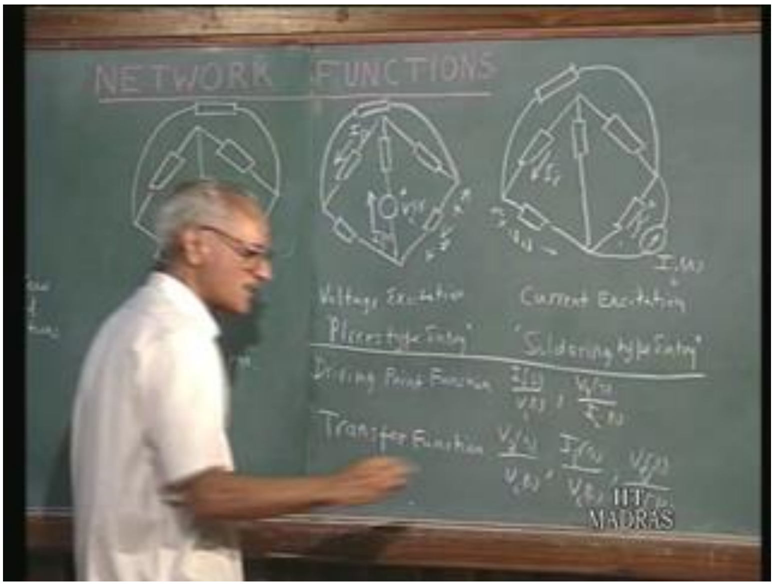

So, let us say that we having a network like this. Suppose, this is the basic form of the network. Now, when you introduce a voltage source as an excitation, he must introduced such a way. That, when the voltage is made equal to zero, the basic form of the network is obtained. Therefore, you should introduced voltage source. For example, by cutting in to this and put placing a voltage source here.

I will explain that what happens if you do not do that? Let us say, so for voltage excitation, we have the basic form would be suppose you will ((Refer Time: 07:34)) a voltage source here. And then, so this is suppose this is V i a voltage, all the functions of s. This is the Laplace transform of the voltage source that introduced here.

So, will call this voltage excitation. So, when you introduce a voltage excitation, you must make sure, that if the voltage made equal to zero. That means, this is short circuited, the original forming the network is obtained. If you are obtained, if you are put a voltage source here, then the voltage is reduced to zero, this is short circuited. Therefore, the original network is modified, it is not the same network as before.

So, if you introduced a voltage source. And would like to find out the natural frequencies in the network. It should be run such that, the voltage is introduced in to one branch in series. So, it is refer to as pliers type entry, that is you take a cutting pliers, cut this wire open and insert that voltage source.

So, this usually ((Refer Time: 09:03)) pliers type entry. On the other hand, if you had if you want to introduce a current a source. Then, we think to do would be to introduce a current source in parallel with a particular element. Let us say this the current source. So, I i's, so this all we are taking ((Refer Time: 09:36)) Laplace transform let us say I i of s. So, that when the current is made zero; that means, this opens circuited you get original basic form the network.

So, when you introduce a current excitation. Then, the entry supposed is set to be a soldering type of entry. That is you take the current source and solder the leads to this two nodes. So, this is the basic distinction between how introduce a voltage excitation, the current excitation to the basic network. The principle is that when you reduce the voltage to zero or replaced by short circuit or the current excitation to zero are replaced the current source by open circuit, the original basic form of the network is obtained.

So, when you do that, no matter how you introduce excitation into what branch whether current or voltage. And find out the response that any other point, then the poles of the whatever network function, we are taking about will be more or less the same. Because, the basic form of the network remains the same. So, this is important to keep in mind, that for a given network configuration, the natural frequencies have unique.

And therefore, no matter what type of network function you define the poles or likely to the same. I said more or less and also said with minor exceptions. Sometimes of minor exceptions are arise. Because, sometime the poles are can cancel out with zeros, leaving some of the poles may not be present in particular network functions. But, by and large before the cancellation takes place, all the poles will be the same.

Now, we also want to make a distinction here between, what is called the driving point function. The driving point function is obtained the ratio of response to current, when the both the response and current are associated with the same time null pair. So, if you are for example, this is I i and I want to find out the current response in this say I y of s.

So, the ratio of I i of s by V i of s, this is the current and the voltage are associated with the same terminal pairs. The same terminal, that is called a driving point function. In this case it happens to a driving point admittance. In this case suppose I have this V y. So, I measuring the voltage across this node, the Laplace transform of the voltage, that is what meant by V i V y and the ratio of that I i of s. So, V y of s over I i of s this is driving point function. In this case, the statutes to be a driving point impedance function.

On the other hand a transfer function is associated with the ratio of the response, which is measured and different pair of time ((Refer Time: 13:05)). So, not necessarily at the driving terminals. For example, if I take this is the voltage and I take the current here, this are voltage across the points.

So, the response is measured across this pair of terminals. The voltage is introduced here both of them are not associated with same pair of terminals. Then, v z by v of s by V i of s is a transfer function it is actually a transfer ratio, because the ratio of two voltages. Let me say, that if I measured the current here I p, then I p of s divided by V i of s is also a transfer function.

Because, the current is measured in different branch, than the branch in which the voltage is introduced. The terminal pair associated with voltage sources, this different from the terminal pair, which is associated with the current I p. So, this will be a transfer function. And in this case is a transfer admittance, in the case of the current excitation suppose I measure this voltage V q.

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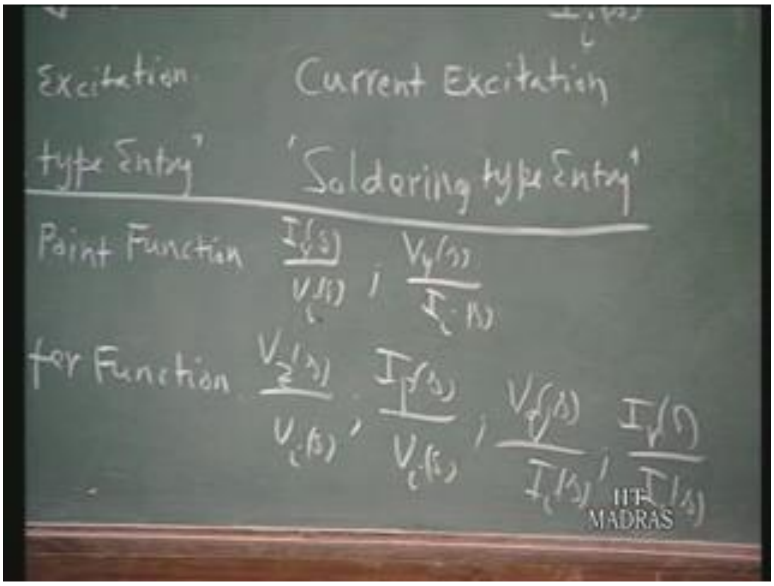

The, V q response V q divided by this current I i of s is also a transfer function. But in this case it happens to be a transfer impedance, ratio of voltage to a current. Let us say I am interested in finding out, this current of the response I r of s here. Then, the ratio of I r of s to the input current I i of s is again a transfer function, which is now a transfer current ratio.

So, in general we have different types of this network functions, some or driving point functions, some are transfer functions. The driving point functions are associated both the response and excitation or resources would be the same terminal pair. And therefore, this one has got to be the current the other has got to be voltage. Then, the driving point functions or the impedance functions, the admittance functions.

On the other hand, transfer functions you have all possibilities. The response is measured a different pair of terminals, compared with the excitation. And depending up on the quantities that are involved transfer functions can be have the dimensions of impedances admittances or pure ratios. This is a transfer voltage ratios, this is transfer current ratio, this a transfer admittance functions, this a transfer impedance functions.

So, this is the general way in which we define network functions. But, you would like to talk about this network functions for particular categories of networks, which are important. But, before that we would like to talk about one other concept. Now, a network is said to have, suppose you bring out some terminals from the network. Then, we introduce the concept port. Port is defined as a pair of terminals in a network, through which a signal is either accessed to the network or from the network.

That means, whenever you introduce a signal into the network or take out the signal from the network, then it is set to be a port.

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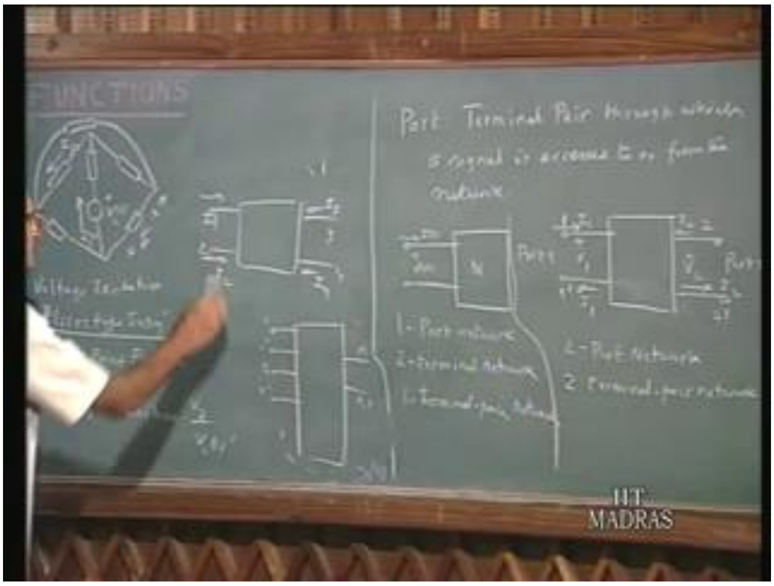

Now, port is a terminal pair through which a signal is accessed to or from the network. So, when you want to introduce a voltage source of the network, introduced terminal pair that is called a port. Now, suppose you have a network with two terminals. Then, this is called the port associated by this network n, this is called a one port network.

So, when you are dealing with one port networks, we are mainly interested in the voltage and the current relations of this network. So, in Laplace transform domain, this is Vs and I of s. So, the ratio of the voltage to current or current voltages, what you are interested in, we are not particularly interested what is happening inside.

So, we are interested in the behavior of the terminals. And this is called a one port network. That means, this network is connected to rational source to this two terminals. And the response is also measured in this terminals, this is also called a two terminal network, also called one terminal pair network all or one in this ((Refer Time: 18:14)). One terminal pair, we can extend this concept say for example, a two port network.

So, a two port network is represented in this fashion. You got two ports, normally labels are given 1 1 prime 2 2 prime, this is called port 1. And we call this V 1 and this as I 1. Now, here afterwards I will drop the functional notation function of s. Because, we understand that all this voltage and the current that we are going to talk about or Laplace transform variables. Therefore, it is needles to write specify that V 1 of s and I 1 of s it is simplifies ((Refer Time: 19:03)) V 1 and I 1. We understand that the Laplace transform variables. So, this is V 2 and this is I 2. So, this is port 1 and this is port 2.

So, we define the two ports separately at the two terminal pairs. This is called a two port network or two terminal pair network. We do not call it a four terminal network, you must be wondering while. See, a four terminal network, suppose I consider this as four 1, 2, 3, 4 terminals. Then, in a four terminal network we understand a four terminal network. That you can connect excitations to the terminals in arbitrary fashion and you can find measure the response arbitrary fashion.

But, as for two port network is concerned, if you want to introduced a excitation in the network, you have to connect this either to this terminal pair or to this terminal pair. We cannot connect an excitation for this two terminals or this two terminals. Not can be never responses across one two or one prime two prime. So, if we understand clearly, that the any method of inputting a signal into the network or measuring the response will be associated with specify terminal pairs.

Either this terminal pair or this terminal pair. But, no other terminal pair, then it is definitely a two port network. So, a four terminal network is four general type of network configuration. But, when you are talk about two terminal pairs, you always understand that the any input is given to the network, either this pair or this pair terminal. And the response also measured across this terminal pair or this terminal pair, not across any other terminal pair.

As a result of that, we can always the current is introduced here we can say the return current is always equal to I 1. Because, after all when you connected load or a source here, this current is tells like this. When, similarly this is I 2, this is going to be I 2. In a four terminal network, that condition need not be valid.

So, let me illustrate that suppose I have a four terminal network. This is 1, 2, 3, 4, I 1, I 2, I 3, I 4. They no guarantee I 1 equals minus I 2, all we can say is I 1 plus I 2 plus I 4 plus

I 4 is 0. So, it is possible that you can have a situation by connecting additional current source. So, that I 1 is difference from minus I 2. You can connect a current source between this two points, current source between this two terminal pairs and so on and so forth.

So, a four terminal network is different from a two terminal pair network or a two port network. We are interested for most part in this two port network in our discuss. Now, it also turns out, that it is not necessary that all the four terminals are distinct. It is possible in special cases, that there is these two are the same. In the electronic circuits applications, when you filter networks and so on.

Normally have a common ground to a two port network. Therefore, one prime, two prime may be identical. In which case is a two port network, with the special characteristics, that there is a common ground. That is one prime and two prime are the same, then it is called a three terminal two port network.

But, what you are represented here, without connection between one prime and two prime. This the more general type, we simply assume that they are may or may not be common connection. So, we take it a general pair, that there is no common connections. So, it is called a four terminal port network and simple two port network. So, that is the distinction between a multi terminal network and a multi port network. And this can be this transept can be extended to n ports, four ports, three ports and so on.

So, if you have a n port network, then you have n terminal pairs port 1 1 prime 2 2 prime down the line to n n prime. So, port 1, port 2, port 3, port 4, n port. So, that will be little more complicated. But, in our discussion here we will confirm ourselves to the network functions as associated with the one port network and a two port network.

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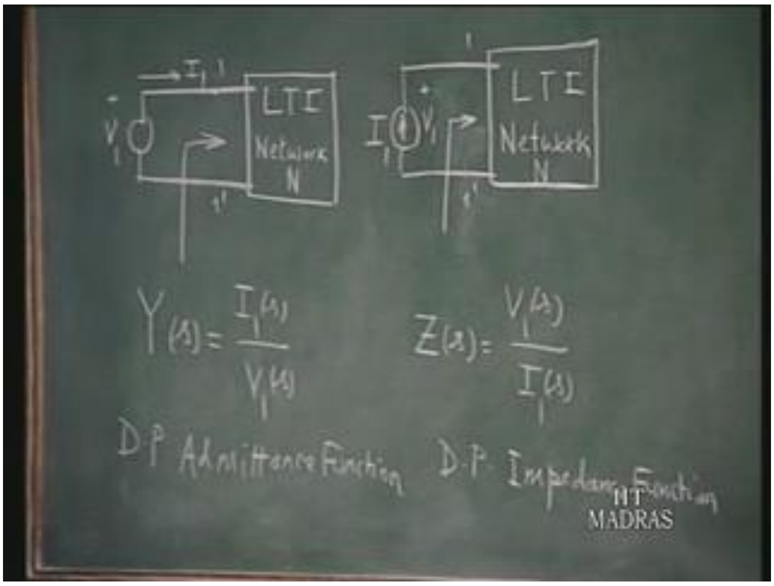

After having discussed the definition of a network function in general way. Let us now consider network functions applicable to one port network to start with. So, this one port network n is a linear timing variant network. Of course, it is under stud is also a continuous time to network. Usually abbreviate is LTI network, Linear Timing in variant network n. Because, it is only for such networks we have a proportional to relationship between the Laplace transforms of the voltage or current.

So, here if I introduce a voltage excitation to the network port 1, at this volume one port of course 1 1 prime on two terminals. Then, the response is the current, the ratio of the Laplace transform of the response to the voltage is called y of s. This is called driving point admittance function of the n port of the network n driving point admittance function.

It is a function of s, it depends up on the continuance of the network n various RLC elements there values and so on. So, a function of s that is why it is called driving point admittance function. But, loosely speaking we will simply refer to this admittance itself implying, that it is also a function of s we do not specifically mention that is y of s admittance function, we usually write y itself. But, strightly it is y of s.

Now, on the other hand if we introduce a current excitation to the network, to the current source. And measure the response voltage response. And find the ratio of the response to the excitation in transform domain. Then, you get z of s, v 1 of s, I 1 of s this is called driving point impedance function. The idea in defining this functions is that, if you know y of s. We can calculate the response for any voltage input, you know the current. Because, after all I 1 equals y 1 y times V 1 of s.

So, for any other excitations, you can immediately find out the current. Similarly, if you know the current excitation, you know the voltage for the corresponding current excitation no insert of us. So, the idea is you do not have to find out the internal construction of the network. Once, we know the y of s and z of s. We know the terminal properties, you for the given current you know what is the voltage, for a given voltage you know what is the current.

Now, it is also obvious, that once you have two terminals here. When, you measure the admittance of that, that must be the reciprocal of the impedance. So, after all the ratio V 1 to I 1 here must be the same as the V 1 to I 1. Because, whatever be the constitutes of the elements at the terminals, the impedance must be the same.

So, when you have got for example, in inductance here it is a pure inductance V 1 of s by I 1 of s is L s. Similarly, V 1 of s by I 1 of s must be a less. Therefore, it is stands to reason, that y off s z off s are related to reciprocal fashion.

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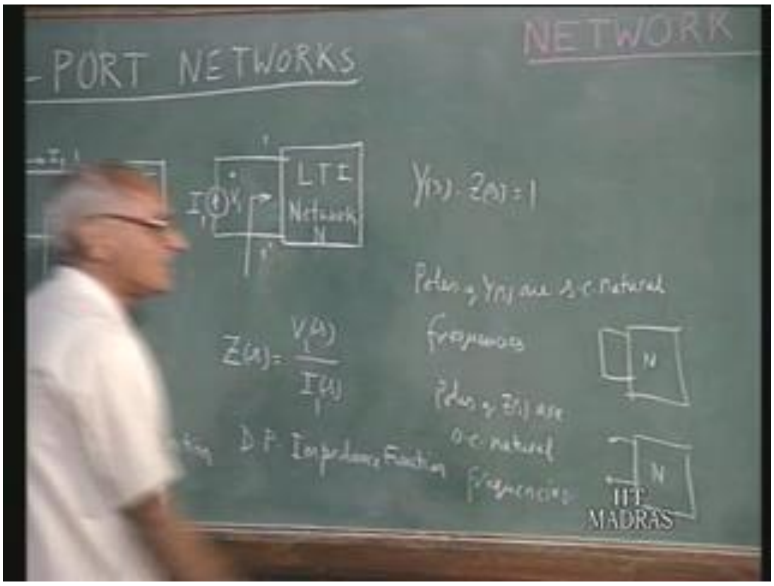

Y s times z of s is equal to 1. So, what is the reciprocal of the other, but the while which we defined as different. So, we also said any network function, the poles of the network functions of the natural frequencies the network.

So, the poles of y of s are the natural frequencies of the network, when network can as the excitation removed. So, v 1 is removed; that means, v 1 is replaced by a short circuit. That means, the poles of y of s of the natural frequencies network n when the two terminals are shorted. So, poles of y of s are so called short circuit natural frequencies.

That means, this network n, suppose you short circuit the terminals. And for example, it capacitor inside the network, you charge it. And let it go, there whatever oscillations, whatever complex which is represent in various response of the natural frequencies. So, the poles of y of s are the short circuit natural frequencies. On the other hand, when you come to z of s, this also a network function. The poles of z off s represent the natural frequencies.

But, what are the natural frequency, suppose the current source is removed. That means, this two terminals are open. Therefore, it is the natural frequencies under these conditions. So, the natural frequencies of that network are the poles of z of s. Therefore, say poles of z of s are open circuit natural frequencies.

So, that is the difference between y of s and z of s. But, we also know that y of s the reciprocal of z of s. That means, the poles of y of s are also zero of z of s. Because, after all y of s is 1 over z of s. So, poles of I of s which also are equal to zeros of z of s are the short circuit natural frequencies. The poles of z off s which are also equal to the zeros of y of s are the open circuit natural frequencies.

That means, the natural frequencies of the network with this termination. This are the poles of y of s and natural frequencies of the network with these terminals is important keep that in mind. That means as for as y of s z of s concerned both zeros and poles represent natural frequencies, the network under some termination are other. This may not be two in a general case.

But, as for as one port networks are concerned, both the poles and zeros at the driving point functions, represent natural frequencies under the appropriate terminal functions.

So, after having define this, now how do we go about calculating the driving point impedance or admittance are given one port network.

Now, all this definitions assume that there are zero initial conditions in the network. If there no initial conditions the network, every in the transform network every inductance is replaced by it is impedance function, which is L s generalized impedance function. Every capacitance as the impedance 1 over c s, every resistance having the impedance function r.

So, all the elements inside the network are replaced by the generalizing impedances are L s or 1 over c s as the case may be. And you try to find out the effective impedance from the looking in terminals from the terminals of the network of the effective admittance from the terminals of the network. Just as would calculate the effective resistance of d c in a resistive network, except that all feeling for all resistances, you replaced with elements having impedances R L s and 1 over c s.

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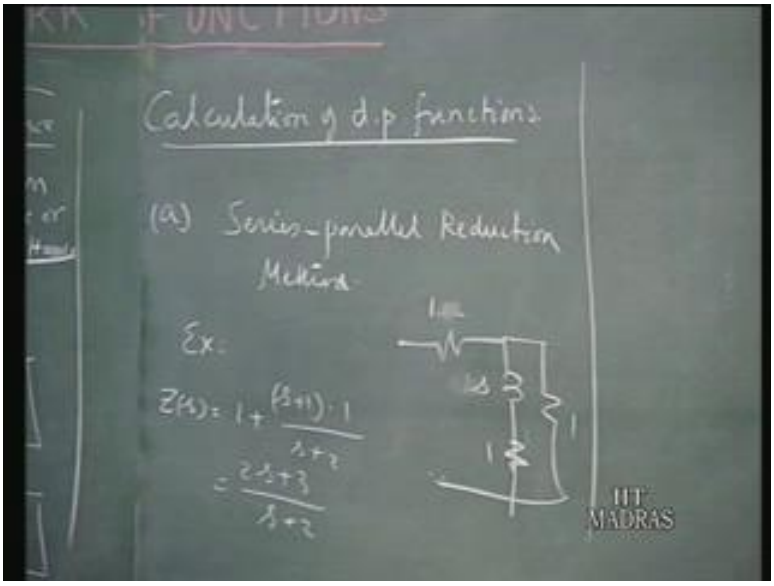

So, to calculation of driving point functions admittance the impedance the case may be, by the general name a impedance or admittance is called immittance ((Refer Time: 31:15)). So, some people use this immittance means, either an impedance or an admittance. So, a driving point function is a immittance an impedance or an admittance, how do we calculate this?

In simple cases, you can use series parallel reduction techniques. Example, suppose I have a one port network like this. Let us say this 1 ohm. So, one, so this is 1, this is L s suppose one Hendry inductance. Therefore, this is s, suppose this is also 1 ohm resister 1. And this is 1, suppose this are all the generalize impedances. Then, z of s is 1 ohm in series with a parallel combination of two branches, the respective impedances are got are s plus 1 by 1 over s plus 2.

So, this is impedance of the s plus 1, the impedances of this is 1. The parallel combination is impedances s plus 1 multiplied by 1 by s plus 1 plus 1 s plus 2. So, that will be s plus 2 plus s plus 1 2 s plus 3 divided by s plus 2. So, that is the impedance function ((Refer Time: 33:00)). So, we can use the same technique that we use in the case of finding out the effective resistance of the resistive network, same technique that we use here also.

To check the virility of the result, after all z of s is generalized impedance function of this network, one port impedance. So, under sinusoidal steady state z j omega represents the impedance at the particular angular frequency omega. So, let us see what happens when s equal 0, where s equal 0 this is 3 by 2. That means, the d c resistance of this must be 3 by 2 1 ohm in series with 2 1 ohms in parallel. Because, under d c a inductance is short circuit.

Therefore, one in parallel with one gives you half ohm resistance, half in series one with one and half ohms resistance. That is exactly what you get z of s when s equal 0 3 by 2. On the other hand, let us say for very large frequency when s goes to infinitive. That means, omega is going to infinitive j omega is going to infinitive. When, omega goes to infinity; that means, s goes to infinitive.

For large frequencies, this is the open circuit. Because, the impedance offered by inductor at very large frequencies is infinitely large. Therefore, practically this branch goes out to commotions it does not participate no current close to that. So, all we are left with 1 ohm in series with another ohm 2 ohms.

So, when z of s when s goes to infinitive is 2. Because, 2 s very large compare with 3 a s will large compared with 2. Therefore, this ratio to leading coefficients is 2. That means, the effective impedance is 2 ohms, that is what you got here. So, sometimes it will useful for us to check of the validity of our answer by looking at the limiting behavior as just goes to 0, which corresponds to be d c and s goes to infinity, which is the corresponding to very large frequencies.

Now, this is of course, a method which suggest itself are simple cases. But, in general you will have to use for a general case, loop current method of analysis are node voltage method of analysis. Are you would like to illustrate this, by taking one or two numerical examples, rather than discussing them in a general way. So, let me say will take an example, where I will write it here loop current method of analysis.

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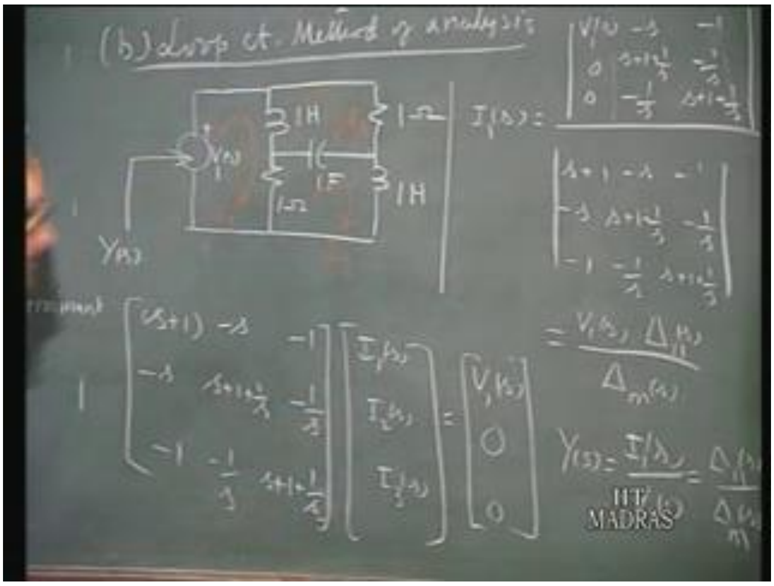

So, let me take a specific example. So, this is the one port network with two terminals seeking out. This 1 Hendry, 1 Hendry 1 ohm, 1 ohm and 1 ferrets. I would like to find out, the admittance of this y of s to do that, what we do is? We have a voltage source, say V 1 of s and you want to measure the current here, to do that let me assume that there are three loop currents here at I 1, I 2 and I 3.

So, you write the loop equations of this three loop network. In terms of the Laplace transform impedances and admittances. And so, first loop the total impedance in that loop is the impedance of this is s, the impedance of this is 1 Therefore, I have s plus 1 I write this in matrix from I 1 of s I 2 of s and I 3 of s convenient for this we to write this in matrix form.

So, s plus 1 times I 1 of s and the coupling between loops one and two is to this 1 ohm 1 Hendry inductance. And in that both the currents I 1 and I 2 are oppositely headed. Therefore, the coupling impedances is minus s at the coupling between loops one and three is in the 1 ohm resistances. And the two parents are oppositely directed therefore, by the should be minus 1.

This self loop impedance of loop two of this s plus 1 plus 1 over s. And the coupling of loop two with reference to loop one once again to this 1 Hendry inductance. So, that is impedance of that is minus s, this are all impedances. And the coupling between loops two and three is to this one ferrets capacitor at both the loops are currents are oppositely directed in this. Therefore, minus 1 over s.

And of course, the other side you have the forcing function, for the first loop, the total driving voltage is v 1 of s. For the second loop, there is load driving voltage therefore, that is 0 and in the third loop also it is 0. Therefore, you can finish this and the third loop impedances is self impedances is s plus 1 over s plus. Therefore, s 1 plus 1 over s and the coupling between loops two and three is minus 1 by s and the coupling between loops three and one is ((Refer Time: 38:51)) minus 1. So, you have this.

So, if you after all ((Refer Time: 38:54)) interest in finding out I 1 for a given excitation v 1. Therefore, we are interest in finding out the ratio of I 1 to v 1. So, if this is the loop basis, this loop impedance matrix. You write I 1 of s haves the ratio of two determinates. So, the entries here are the same as this s plus 1 minus s minus 1 minus s s plus 1 plus 1 over s, this is minus 1 over s, this minus 1 minus 1 over s s plus 1 plus 1 over s.

And as for the numerator is concerned, you replace the first column by the coursing function matter v 1 of s 0 0 minus s s plus 1 plus 1 over s minus 1 over s s plus 1 plus 1 over s minus 1 over s minus 1. So, you observe that as for the numerator is concerned this is v 1 of s times this quantity, which can be return as delta 1 1of s the whole factor of the first element 1 1. And the denominator is the mesh basis system determinant, we can write this delta m of s.

So, if that is so that is the calculate delta m of s and delta 1 1 s for this particular values. And we get y of s as I 1 of s over v 1 of s which is obtained as delta 1 1 of s over delta m s.

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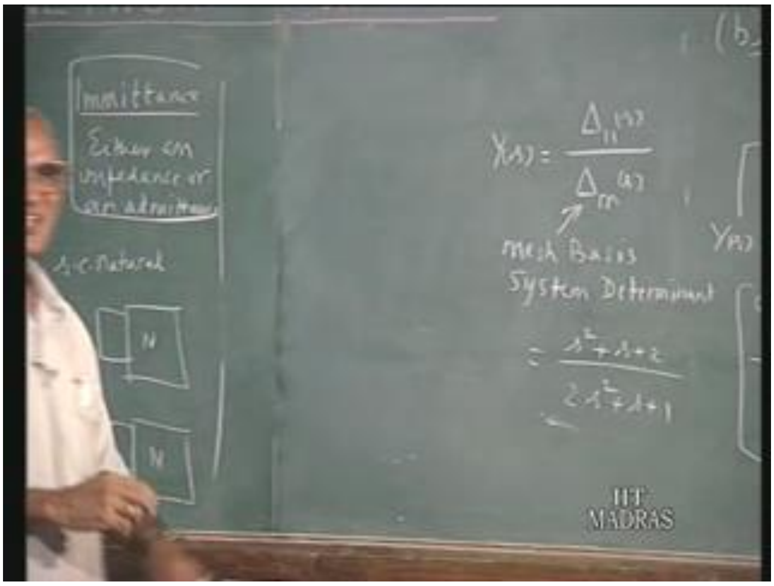

So, ultimately done we have y of s is the denominator is delta m s. This is the mesh basis system determinate, this is called mess basis or loop basis system determinant. That means, the determinate of the loop impedance matrix s. And the numerator it is the cofactor of the first term in this loop impedance matrix delta 1 1 s.

So, you can calculate this for the particular we have got all the values here, you can calculate this delta 1 1 here at this and you show that this is equal to s square plus s plus 2 divided by 2 s square plus s plus 2. Again you would like to check, the validity of this some verify, that this is in accordance with what we except for small frequencies or large frequencies. When, s suppose you take the d c case, that is y j omega and omega equal 0 will be obtained by substituting s equal 0 it must be...

That means, ((Refer Time: 42:18)) must be the admittance of this network, under d c if you see, under d c the capacitance is open circuit, 1 Hendry is a short, 1 Hendry is a short. So, 1 ohm in parallel with 1 ohm provides you, half ohm resistance or two semen’s admittance, that exactly what we are have the s equal 0. For large values of s this provides you half semen’s admittance. That means, 2 ohms must be the resistance of the network for very large frequencies.

So, very large frequencies capacitor is a short circuit, the two inductors are open circuits. Therefore, this two are not participating. Therefore, 1 ohm this is the part terminals are one, this is the short circuit another one. Therefore, this two ohms impedance is presented for the network of large frequencies, that is exactly what we are done here. Now, you can also carry out this analysis by note admittance.

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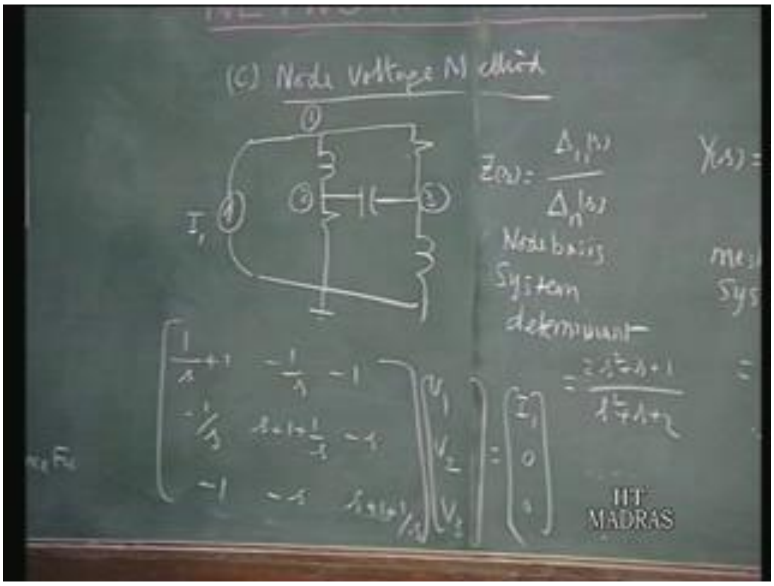

So, you can also have node voltage method for evaluation of the network function. So, what we have done is, when you want to make the node analysis, the current excitation is more appropriate. Therefore, you can write here I 1 and then, the same elements that you are having here. Suppose, you take this at the z term node, this is node 1, this is node 2, this is node 3. And you can set up the node basis determinant as follows I will just give the final result, you can complete the work.

Because, now you are talking terms of the admittances. The total admittances connected to node 1, the admittance of this place the, the admittance of this is 1 over s the admittance of this is one. So, 1 by s plus 1, so by s minus 1 minus 1 by s s plus 1 plus 1 by s minus s minus 1 minus 1 s plus 1 plus 1 by s v 1, v 2, v 3 there are Laplace transform variables. The current is entering node one only, now that is place therefore this I 1, 0, 0.

So, on this basis proceeding in a similar fashion, as we had in the other case z of s can be written as, the node basis system determinant. That means, the determinant of this matrix. And on the numerator, you have the delta 1 1 of s where this is the co factor first co factor of this. So, you can calculated in this fashion and you would end up with the result, which is 2 s square plus s plus 1 divided by s squared plus s plus 2 which is of course, the reciprocal of y of s.

So, you have alternate ways of doing this, either series parallel reduction, start delta conversion. Whatever the whole gamete of analysis taking the available to you. The d c networks case can be employed here as well. And you can find out the effective impedance at the terminals of the port network, in exactly in identical fashion.

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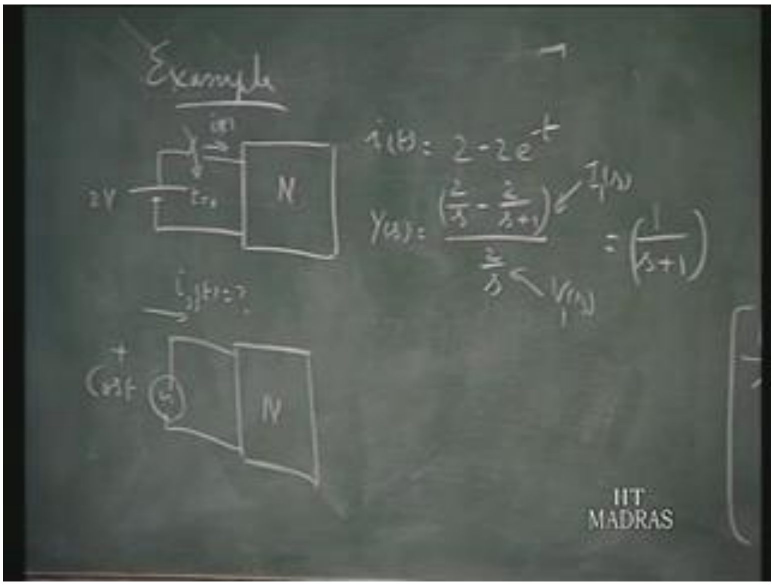

One last example, repeat example. Suppose, I have one port network, this is again assume to be time in variant, linear network. On closing the switch t equals 0, I find that I of t in this case happens to be 2 minus 2 e to the power of minus t. This is node, this network when I close this switch with 2 volts with 0 initial energy stored in n as given me this.

What I would like to know is? In this network n is connected to a sinusoidal source of cos t volts I would like to find out, the current and steady state how much is the current? I do not know, anything about inside the constitution of the network. So, form the terminal conditions I should be able to do it. Will how this is the question that we asked. Now, we can find out from the given data the y of s, the admittance function of this network.

The current is I t therefore, the response is I t therefore, the Laplace transform of that is 2 by s minus 2 by s plus 1 that is I of s divided by v of s. What is the input? Input is 2 volts V c therefore, it is Laplace transform is 2 by s. So, this is the V of s v 1 of s, this is I 1 of s. If you simplify this, it turns out to be 1 over s plus 1. So, 1 over s plus 1 is the admittance function of this network.

Now, we want to find out the steady state current with sinusoidal excitation. Therefore, this the general Laplace transform, the ratio of Laplace transform. So, we want to find out the frequency response corresponding to this. So, y j omega omega happens to one for cos t. So, we are interested only, not in the total response. We can find out the total response by multiplying the Laplace transform of this with y of s.

But, we are interest only the steady state response. Therefore, I am taking only the frequency response function corresponding to this, this 1 over j omega plus 1. Therefore, this is 1 by root omega happens to be 1. Therefore, this be 1 by root 2 at an angle of minus 45 degrees, which means if you have a cos t, the facer for the voltage and the current are related by this admittance.

Therefore, I steady state has got the amplitude of this is one. Therefore, the amplitude of the current is 1 by root 2. And there is a face of minus 45 degrees therefore, this is cos t minus 45 degree. So, that is the steady state current, you can also use this information to get the total response. When, costly ((Refer Time: 49:07)) equals 0 by finding out the Laplace transform of cos t multiply this y of s and find out the inverse Laplace transform become the current.

So, you can see how the concept of the admittance function of the impedance function, one port network can be used for finding out the response of the other type of excitations. You can find out, suppose cos t switch the t equals 0. And you have find the total current, all you have to do is multiply the Laplace transform of cos t, u t with the y of s. Find out the corresponding I of s, find out the inverse Laplace transform. Then, that will give the total response of which certainly this will be the steady state part.

So, in this lecture we have accounted ourselves, with the general nature of the network function. We said it is network function is nothing but, a system function, apply to networks. This is the ratio of the Laplace transform of the response to the Laplace transform to the input, with zero initial conditions of the system. And such that, function are defined only for linear time in variant system networks with zero initial conditions of course.

And then, we also introduce ourselves to the concept by the port. Essentially, a port is the terminal pair to which excitation is introduced in the system or response is which measure. And the distension between the ports, n port network and n terminal network as been wrong. And the essential concept of port is, in a terminal pair one current is going through one terminal. The same current must written to the other terminal of associate with the same pair, it can be otherwise.

And in a multi terminal network, that particular property is not necessarily valid. Then, we went to discussion of the distinction between one port network and two port networks. And then, we took for special consideration one port network. And we sinusoidal one port network, the network function that, we can think of the ratio voltage to current at the current voltage.

So, depending up on the excitation, we define the driving point impedance and driving point admittance of the function, both are reciprocals of each other. We mention, that the poles of a driving point impedance function of the natural frequencies of the network, under open circuit conditions when the terminals are open. The poles are the driving point admittance are the natural frequencies is the network, when the two terminals are shorted.

That means, if you had a capacitance inside the network ((Refer Time: 51:37)) let at system go on, it soon the capacitor discharge. And then, various voltage and currents exhibits some natural frequencies. And those natural frequencies will turn out to be, the poles of y of s. And things y of s is related to z of s in a reciprocal fashion, the poles of y of s are also zeros of z of s wise versa.

Then, we went adjust we looked up on the various ways, in which we can calculate this impedance functions said the entire techniques, that are available to you for calculate the effective resistance of the rectitude network. In the case of d c can be used here, except that we deal with generalized impedance, rather than pure resistance.

So, functions of s are involved to it associate with each term. We can use the series parallel detection technique, the star delta conversion technique are more generally. The loop current method of analysis are load voltage method voltage of analysis, worked out one or two examples to illustrate the application of this technique.

We also mention that, once we have the admittance function or impedance function known to us. We can find out the response to any other excitation, through taken the Laplace transforms of the input, multiply by the network function, finding out the Laplace transform of the response and taking the inverse Laplace transform.

We can also use this to get the steady state response, under sinusoidal conditions by taking ((Refer Time: 52:59)) frequency response function from the given network function by circuiting j omega for s and taking the appropriate value of omega. In the next lecture, will talk about to continue this discussion in relation to two port networks.