Networks and Systems
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Lecture - 03
Introductory Concepts

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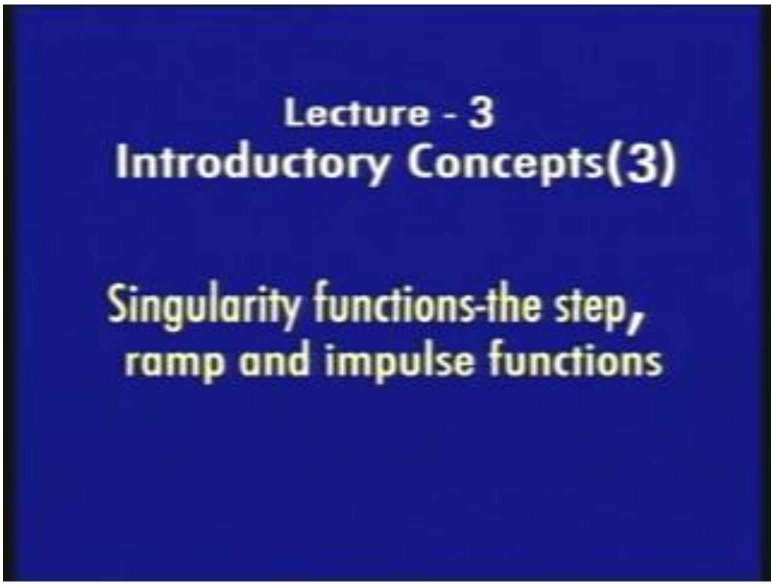

We were looking at, some of the standard signals which we come across in the study of networks and systems and their waveforms. We recall among these standard signal waveforms we mentioned the D.C Signal, then the sinusoid a cos omega t plus theta being its general expression. We also mentioned that the Exponential Signal A e to the power of s t where s is a complex number in general sigma plus j omega plays also an important role. We term this quantity s the complex frequency, simply for the reason that it has the dimensions of 1 over time something per second and something per second is called frequency and since this is general complex we call it complex frequency.

We should not read more meaning into that, we should not interpret this as indicating a repetitive phenomenon like a periodic phenomenon. We also mentioned that any signal like that in general will yield a complex value for real values of time. So, in order to for us to have a real function of time every such signal is accompanied A e to the power of s t is always accompanied by a conjugate e to the power of s conjugate t. In fact, every time every time is a physical situation where you have such signal this corresponding signal with conjugate coefficient and conjugate of the complex frequency will be present simultaneously. These 2 will yield a real signal 2A e to the power of sigma t cosine omega t plus phi where A is the magnitude of this complex coefficient a bar and phe is the angle associated with its complex number k.

So, this is the and we saw for different values of sigma and omega different natures of their values positive or negative as the case may be we saw how the waveforms look like. And so, associated with this exponential signal we have special cases e to the power of j omega t.

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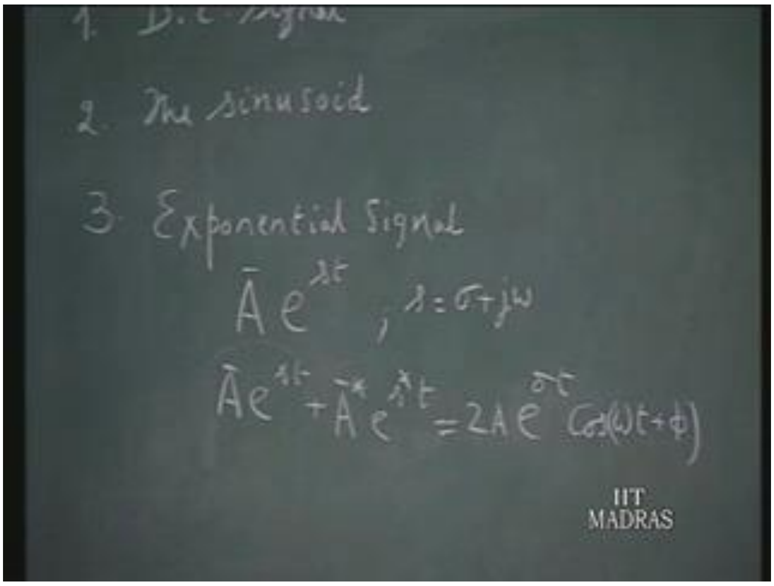

This is a very important signal in that; a sinusoid is closely related to this. This is a special case of e to the power of st. This is also an exponential signal, but the value of the complex frequency is purely imaginary. And if you have cos omega t this is always e to the power of j omega t plus e to the power of minus j omega t by 2 and similarly, sine omega t is e to the power of j omega t minus e to the power of minus j omega t divided by 2 j.

So, sinusoidal functions of time cosine omega t and sine omega t can always be expressed in terms of e to the power of j omega t and this is a special case of a exponential signal with complex frequency.

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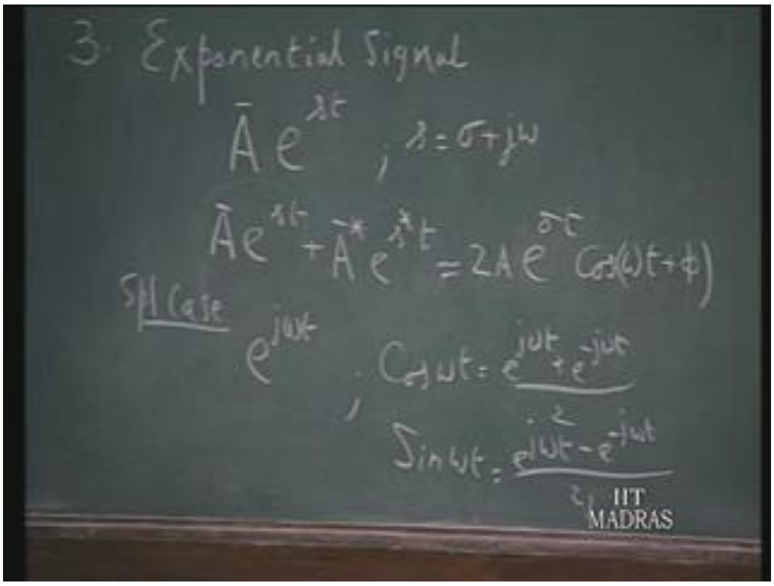

In fact, e to the power j omega t can be thought of as almost synonymous with a sinusoidal quantity. Because after all the real part of that yields cos omega t the imaginary part of this sine omega t and it is always convenient for whenever we have to deal with excitation functions of the type cos omega t or sine omega t we replace them immediately by e to the power of j omega t. Because, we recognize the cos omega t as the real part of e to the power of j omega t and work with e to the power of j omega t and finally, take the real part of the solution to get the resultant that you would get if indeed excitation had been cos omega t.

More about this we will learn later, but we can always say that e to the power of j omega t we can always use that whenever we have to use a sinusoidal functions of time. Very often that turns out to be case. And In fact, manipulation of e to the power of j omega t is easier than the manipulation of trigonometric functions because you have only 1 function to deal with and differentiation and integration of e to the power of j omega t that " " function is convenient than differentiation and integration of trigonometric functions because you alternately between sign and sign with plus sign and minus sign and so, forth.

It is much more convenient to handle e to the power of j omega t. Now, the characteristic of all these functions are that you can their derivatives exist up to an infinite order. So, D. C Signal sinusoid you can go on taking the derivatives the successive derivatives.

They are all smooth functions and all the derivatives exist. Before, we proceed further let me take examples of signals and say what complex frequency terms are present in these signals? So, let us as an exercise let us take an example. So, i will write down a series of signals which are not the elementary kind that we have been talking about, but then they are composed of signals of this type. e to the power of st complex frequency signals. Then complex frequency is present in this signal. What are the components which have certain complex frequencies present in the signal?

So, we will take 1 by 1 cos 2t plus 30 degrees. Now, this is a pure sinusoidal signal, but as i said cos 2t can always be expressed as e to the power of j 2t and so on; therefore, the complex frequencies present here are plus or minus j2 e to the power of j2t e to the power of minus j2t together those terms will be present with appropriate coefficients a and a conjugate and these are the 2 complex frequencies present. In the complex frequency plane, this is the complex frequency plane with the x axis representing sigma and the y axis representing omega you have therefore, 2 frequencies at plus 2 and minus 2.

Second, suppose i have 2 cos t cos 3t plus 4 sine 3t again you have a sinusoidal. Naturally you can combine these 2 into cos some a cos 3 t plus theta. But, even if as they are you have the complex " " present here plus or minus j3 and so, are the complex frequencies here. So, plus or minus j3 or the complex frequencies present in this particular signal and once again you can portray them in the complex frequency plane in the same manner as for the first example. As a third example, suppose i say e to the power of minus 2t plus e to the power of minus 3t cos 4t plus t.

Here, as far as the first signal is concerned first part of this signal is concerned e to the power of minus 2t; therefore, there is a complex frequency there omega is 0 and sigma equals minus 2. So, that is the complex frequency associated with this component. The complex frequency associated with this component is minus 3 corresponding to sigma you recall, that if you have A e to the power of s t sigma plus j omega e to the power of sigma t plus cos omega t plus p. Therefore, minus 3 is the value of sigma and corresponding to the cosine term you have plus or minus j4. So, as far as this signal is concerned there are 3 frequencies complex frequencies present here minus 2 and minus 3 plus or minus j4.

So, and the complex plane here 1 frequency at this location and another conjugate frequencies at minus 3 this is minus 2 this is 4 this is minus 4 sigma omega. Suppose i have, e to the power of j 2t plus e to the power of minus 3t cos 4t plus theta again as before. Now, corresponding to this you have j2 and corresponding to this as before minus 3 plus or minus j4. Notice here, that this e to the power of j2 plus j2 there is no minus j 2. That means, this particular signal is not a real function of time.

So, very rarely you will come across this you should not come across this for any as long as the signals are real, real functions of time. Therefore, just to indicate that this is not a realistic signal that you are likely to come across I just gave an example. So, if you substitute a particular value of t you get an imaginary term. So, this is not a type of signal you are likely to encounter. Whenever you have e to the power of j 2t, you also should have e to the power of minus j 2t. So that, those 2 together will give rise to a real function of time. But mathematically if you are looking at this hypothetical signal these are the frequencies present there.

Lastly, suppose I have 4 plus 5 e to the power of 2t cos 5 t plus theta. Now this constant terms, D. C can be thought of as a special case of this where both sigma and omega are 0 when s equals 0 this is a constant therefore, a D. C corresponds to s equals 0. So, you have frequency corresponding to s equals 0 and corresponding to the second term you have plus 2 plus or minus j 5. So, only in the complex plane you have a D. C term corresponding to a point sitting right at the origin and now 2 points corresponding to 2 plus or minus j 5 and what it means is these 2 frequencies together will give rise to a term which is e to the power of 2t cos 5t plus theta or whatever it is.

Therefore it is, a growing oscillations a sinusoid which is growing in its amplitude and generally, those are signals which are you are likely to comes across as far as a physical situation is concerned because they can you can come across such signals for a limited period not they cannot go on indefinitely because they reach infinite proportions with respect to time. So, as long as you have a sustained signal which can last for a long time or forever then you need to have a negative real part because you have seen from the waveforms of the complex exponential signals for different values of sigma and omega that we plotted in the last lecture that if you have the real part of the complex frequency positive that represents a growing signal.

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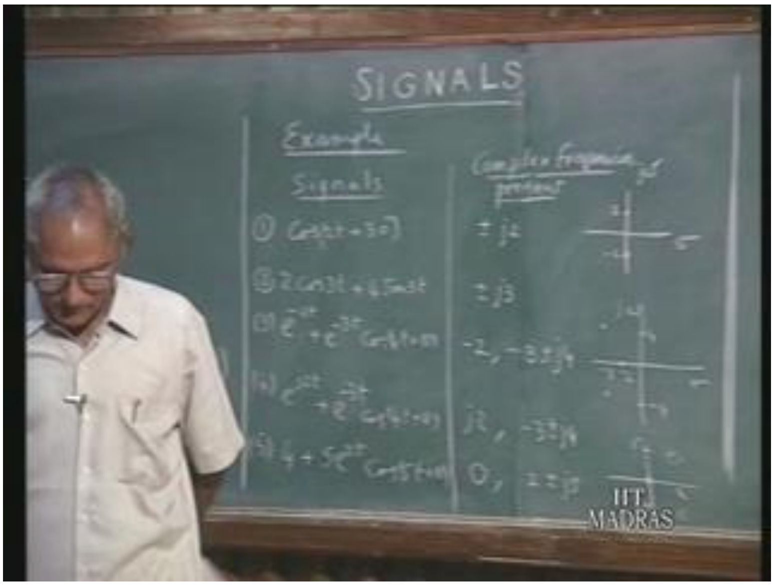

Now, all these signals as i mentioned are those which are smooth which are continuous by themselves and also have continuous derivatives. There is a need to have the compact notation for signals which are either discontinuous or have discontinuous derivatives.

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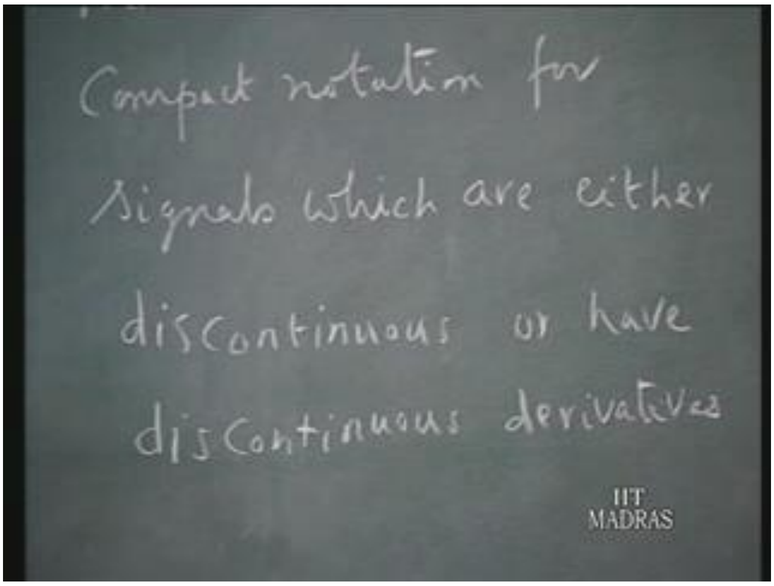

Take for example, this particular function of time or for that matter another 1 like this. Suppose these are the 2 functions of time that are given to you f2 and t. It is difficult to describe this elliptically because we have to say that function is 0 for negative values of time and from this point to this point it has got such you give an elliptical expression for this. From this point to this point you give another elliptical expression for this and beyond that once again you say it will " ". Similarly, if you want to describe this f of 2 elliptically we have to say it is 0 for negative values of time. It is a constant up to for t in this range another constant for t in this range, another constant for t in this range and it has got a certain straight line relation in this range and beyond that it is 0. That means, you have to describe give separate expressions for 5 different regions as far as this function f 2 is concerned.

So, it becomes little cumbersome, but 1 would like to see if we can express this by means of some notational expression, some notations just like we have said this is A e to the power of s t if it possible for us to arrive at some kind of expression for these signals f1 and f2 where we do not have to qualify further as saying this is valid for this interval of time and this expression is valid for a different interval of time. And this leads us to the topic of what is called singularity functions which are used to describe functions of this type which are discontinuous by themselves or have discontinuous derivatives. And it is the singularity functions that we will take up for discussion later and 1 of the most important singularity functions in the step functions which i will describe in a moment.

This is originally introduced into the literature by oliver heaviside a british engineer who made signal contributions to communication theory and operational methods of analysis of networks and systems. He was a controversial figure and most of his work was not founded on rigorous mathematics. So, he got into trouble with mathematicians of his day, but he introduced the step function among other things and the operational calculus methods and as a retort to the mathematicians who were not happy with his work he would say do i stop eating, because I do not exactly understand the process of digestion.

That was his retort to those people who were critical of his work. Because he introduced some methods when they were working and. So, he said why should you poke further into this as long as it produces results we should be happy with them. That is, just a brief historical note on oliver heaviside who introduced the unit step function which we will take up now. We shall now take a look at 3 important singularity functions. These are called singularity functions because, either they have discontinued this or the derivatives have discontinued this and therefore, in the regular classical sense of mathematics we do not have derivatives. If derivative fails to be continuous the classical mathematics you say it does not have a derivative.

So, it is singular in that point that is why these are called singularity functions. The most important perhaps is the unit step function. And this is what i said was introduced by oliver heaviside. This is indicated as u of t symbol symbolically it is represented as u of t and this is defined as 1 for t greater than 0 and 0 for t less than 0. So, the wave form for this would be it is 0 up to this point t this time axis t 0 and then from this point onwards has a value 1.

So, as a function of time it has got a waveform like this. 0 for negative values of t and a constant 1 for positive values of t at t equals 0 itself, at this point we don't really care what it is it can be left undefined you can take it as 1 if you wish 0 if you like or half whatever it is. Normally we are not really concerned in most of the work.

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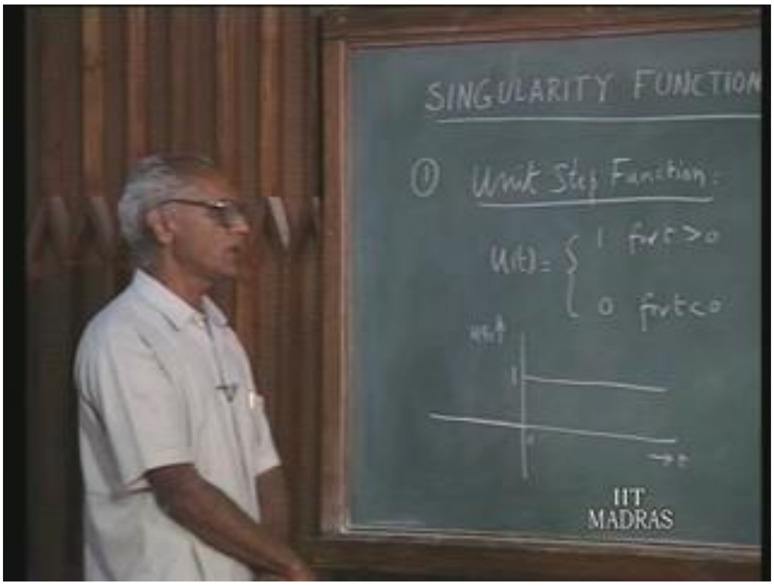

It does not matter how you take it. So, we will leave it undefined, but if really it becomes necessary you take it as half. Because, it is the average of the right and left extreme limits and in some limiting processes it will converse to that value half. So, you may take it as half, but in any case the important thing here is that it is a discontinuous function of time it is the left hand limit is 0 and then suddenly it jumps by a value 1 and stays constant from that point onwards.

So, it resembles a step here at this point a steep rise and that is why it is called a step function and since this height of the step is 1 it is called a unit step function u of t. Now, using this unit step function, w can describe a number of other functions of time which have discontinuities. So, let us take a series of examples to illustrate, how the unit step function can be used to describe different types of functions. Suppose, i have a function which has got unit 0 4 units are positive t and 0 for negative values of time then; obviously, this will be described as 4 times u t because it is a same unit function enlarged 4 times.

So, the amplitude is 4 4u t is this function. Suppose, I have a step function which starts at t equals 2 and 0 for negative values for values of t less than 2, then you would and the magnitude of this is suppose 1. Then, you would call that u of t minus 2 because as long as your argument t minus 2 is positive t greater than 2 it has 1. If t is less than 2 u of t minus 2 is a unit function unit step function of the argument negative values then therefore, it becomes 0. So, this describes this function is described by u t minus 2 you can also see if this is u of t. It is translated in time delayed by 2 seconds therefore, if this is f of t this must be f of t minus 2 therefore, from that argument also you can show you can see that this is u of t minus 2.

Suppose I have a function like this one for t less than 2 and 0 for t greater than 2. So, what will you describe this function as? Now this function has, a value 1 as long as t is less than 2 therefore, you write u of 2 minus t that describes this function because as long as t is less than 2, 2 minus t is a positive number and therefore, it must have a value 1 and as long as t exceeds 2, 2 minus t is a negative value; therefore, it becomes 0.That’s how it goes.

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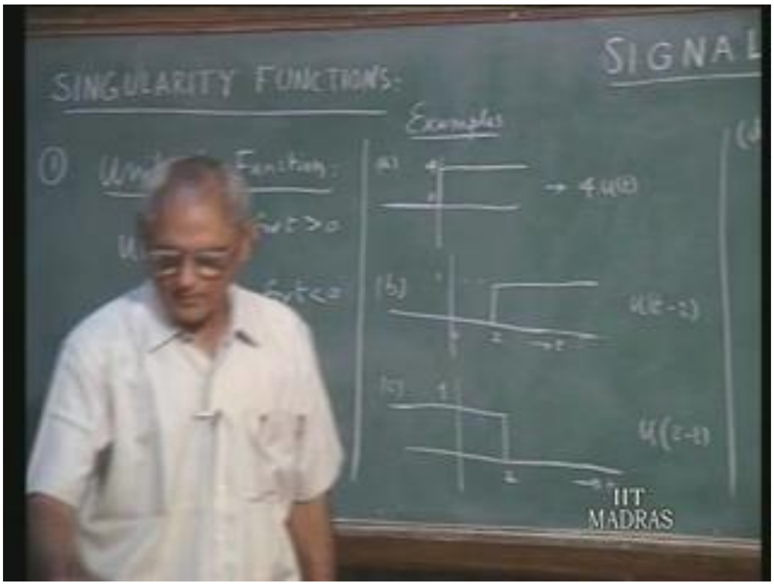

Let me now take a case, where you have a function whose value is minus 4 for t less than 2; therefore, this is u of t minus t, but instead of having plus 1 it has a value minus 4. Therefore, this will be minus 4 u of 2 minus t. You can substitute different values of t and see that it is in accordance with the definition of unit step function. The unit step function is also useful in describing functions like these. This is a kind of pulse sometimes called gate function because; it is like a gate between this intervals this signal is there afterwards this signal is not there at all.

Now, such functions can be easily shown to be the sum of 2 functions. For example, if i have a step function starting at 2 say f1 and another function f2 starting at 3. Notice, that if i subtract f2 from f1 from t equals 3 onwards this area is cancelled out by this area and what is left is only the value or the function between 2 and 3 and that is exactly what we are having. So, a gate function or a pulse function like this can be written as u of t minus 2, u of t minus 2 would be a step starting at 2 seconds and going on forever. You want to pull down that step to 0.

Therefore, at this point 3 you have to introduce a negative step of unit magnitude. So, that the original step that was going like this from that you are subtracting a negative step. So, that from point t equals 3 onwards it becomes 0. So, you need to subtract from this u of t minus 3. So, you can see that a square a pulse like this can be described in times of step functions and u of t minus 2 minus u of t minus 3. So, this is a very useful concept which we can use to describe discontinuous functions of this type. Another example, suppose you have this function which is symmetrically situated around the origin. Then you can think of this as a step function starting at this point and subtract from that a step function a negative step starting at this point.

The step function starting at minus 2 has 4 units. Therefore, 4 u of t plus 2 you must write because for t less than minus 2 then t plus 2 is negative therefore, this is 0. As long as t is greater than minus 2 our t plus 2 is positive you have a value 4. Therefore, 4 u of t plus 2 is a step which is going like this. But then you do not want this step to continue forever. At this point plus 2 you want to introduce a negative step of 4 units. So, that this portion is cancelled by this. So, from this you subtract minus 4 u of t minus 2. So, sum of these 2 is indeed this pulse that you are having a little more complicated situation.

So, suppose this is 1 this is 2 this is minus 1 time 2 3. So, this goes on like that forever beyond that t equals 3 it is continuous at minus 1. So, to describe this always what you should do is to start with the first step. And see; what are the other steps that have to be added to that, in order to describe the actual situation. So, this function is 0 up to this point and at that point it jumps by value 1. So, you can write this as u of t minus 1. That describes a unit step starting at t equals 1 and going on like that. And that explains satisfactorily the given function of time from t equals minus infinity on up to 2 this function describes the behavior at this point. At this point the function jumps up by another amount equal to 1.

So, to this step you have to add another step of unit amplitude. So, that will be another unit step starting at t equals 2. So, u of t minus 2 and now, what is the effect of these 2 steps. You have 0 up to this point 1 and then it continues like this. So, the effect of these 2 would lead to us would give us a curve which goes like this and then continues like this, but then what happens at t equals 3 you must bring this down to minus 1. So, you must introduce a negative step of 3 units in order to pull this value down to minus 1. So, minus of 3 u of t minus 3you do that; then this value is pulled down to minus 1 and then from that point onwards it continues at a constant value.

You can verify the result after all u of t equals 1 for t greater than 0. Therefore, suppose you take a large value of t say 6 or 7 then this is 1, this is 1 and this is minus 3. So, 1 plus 1 minus 3 is minus 1.

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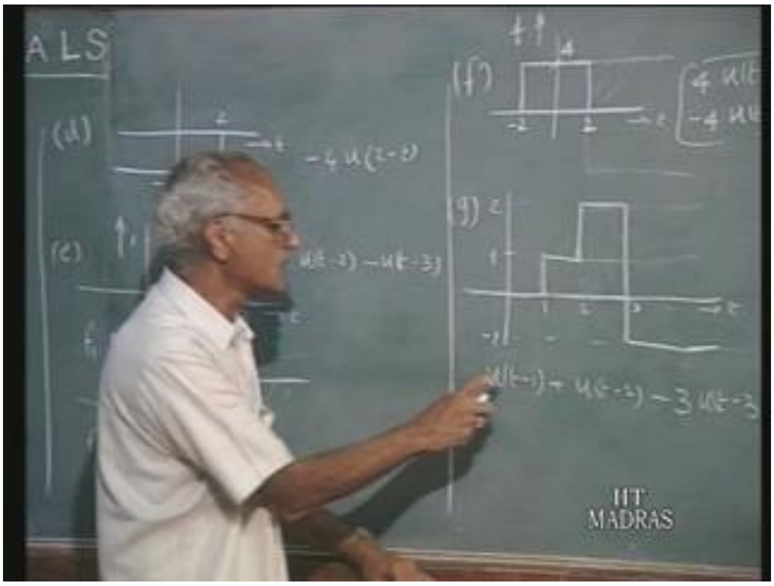

So, indeed that is minus 1. So, for any value of t greater than 3 this is going to be minus 1. So, these examples illustrate the usefulness of the step function in describing discontinuous functions and are piecewise constant functions and you must get the facility you must develop the facility of developing such functions. By means of appropriate steps with appropriate weights and appropriate shifts in time as we have indicated by means of these examples; the next singularity function that we would like to talk about is what is called a unit ramp function?

This is also a causal signal like the u of t in this it is 0 for negative values of time. It is indicated as r t and it is defined as t for t greater than or equal to 0 and 0 for t less than 0. So, it is clear that its wave form would be like this goes on like this and it has a value of 1 at time t equals 1. So, it is for positive t it is described as t and for negative values it is 0. So, this is called a ramp function because this is like a ramp a slope.

So, it is called a ramp function and since this slope is equal to 1 it is called a unit ramp function and suppose i indicate this slope by brackets this slope equal to 1 it rises by unit amount the unit time this is called unit ramp function. Now, unit ramp function once again let me illustrate its applications through examples. See how we can describe several interesting wave shapes through the unit ramp function.

Suppose I have a function like this which is 0 for t less than 1 and increases at the rate of 1 unit per unit distance travelled along the x axis. Then this would be called r of t minus 1 because it is the same unit ramp function shifted in time delayed in time by 1 second and therefore, this will be r t minus 1.

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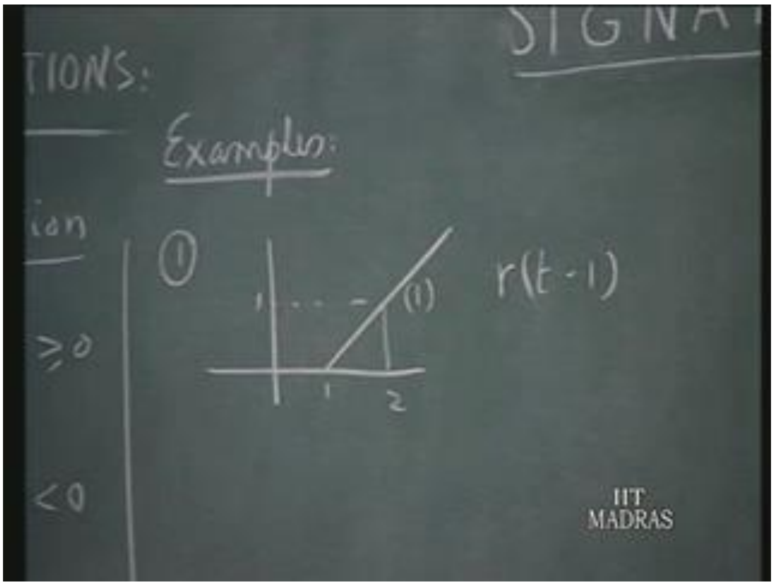

We should be able to recognize whenever a function is delayed by an amount tou, then f of t becomes f of t minus tou. Suppose it starts at 1 and there is a negative going ramp. At 2 it is equal to minus 1 then it will be minus of r of t minus 1.

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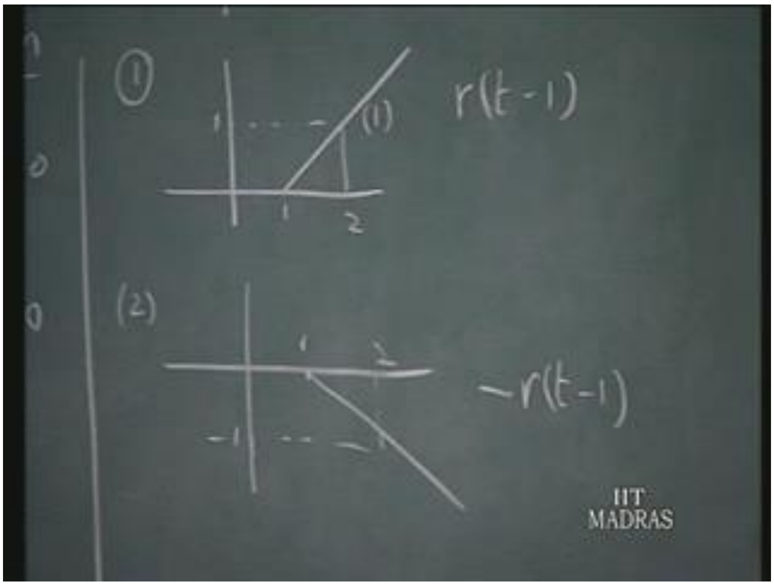

It is a ramp function with a coefficient of minus 1 and it starts at t equals 1. That's how you describe it. Suppose I have a function which is 0 for values of 3 beyond 3 in the positive direction and then it increases the negative direction of t with a slope equal to 3 or slope in the as t increases it is minus 3. Then first of all, it leads the ramp function has values for t less than 3. Therefore, you must have basically something like r 3 minus t because if t exceeds 3 this becomes 0.

When t exceeds 3, 3 minus t is a negative number and therefore, this is 0. So, basically you have r of 3 minus t. Now, you must fix the coefficient to suit this now when t equals 0 for example, then this is equal to 9. This slope is 3; therefore, if you put here 3 r t minus 3 this becomes answers this description. Because all you have to do is you must ensure that this is negative for t greater than 3 and therefore, the base point comes at t equal 3 and beyond that if you fix 1 point on the straight line the straight line gets fixed up.

Therefore when t equal 0 this is r of 3, r of t is equal to 3, r of 3 is equal to 3. Therefore, 3 times 3 is 9 and that answers this particular description. So, that is how 1 can fix up this you can do it in different ways, but at least this is 1 argument to show that f of t is equal 3 r 3 minus t.

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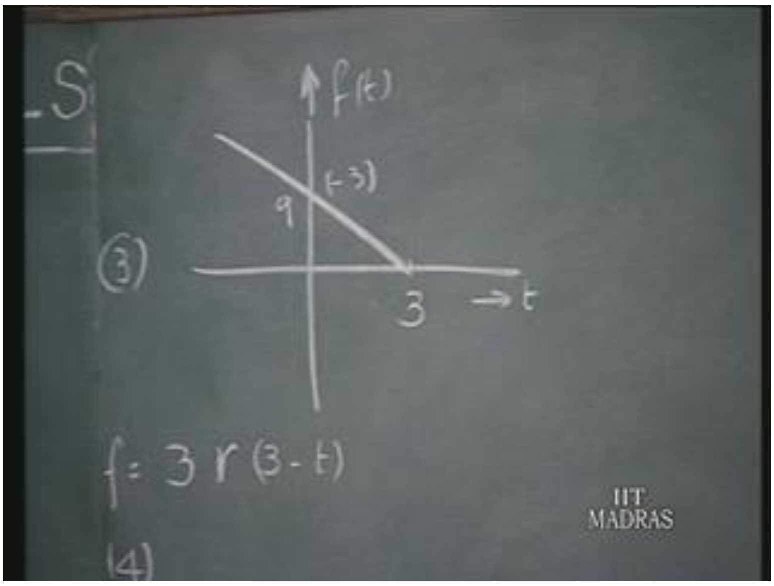

The 4th example: suppose I have, a triangular wave shape 1 minus 1 4. This is a function which you can decompose as first of all; at t equal minus 1 there is a ramp function going like this at t equals minus 1. So, if a ramp starts at minus 1 and increases with a slope of 4, then you would describe this as 4 r t plus 1 because this is delayed this is advanced by 1 unit; therefore, r of t plus 1. So, such a line that is a line starting from here and then going on like this could be called 4 r t plus 4 all right.

Now, if this is the line that is going then this describes a wave form which is going like this, but now at this point at t equals 0 you must introduce, you must pull this down by. So, that it will have a slope of minus 4 from that point onwards. Therefore, in order to pull this down to introduce a resultant slope of minus 4 this has a slop of plus 4 at this point you must introduce a negative going ramp to pull down this slope to minus 4.

So, at this point you must introduce here another ramp with a slope of minus 8 units. This has a slope of plus 4 units, this type of slope of minus 8 units and this ramp is introduced at t equals 0 therefore, this will be called minus 8 r of t. That will be that particular characteristic which starts at t equals 0 and has a negative slope. Now, what is the result of these 2, if you have these 2 together then you load then no doubt this increases. At this point you pull this down and then it is going all like this. Now, at t equals 1 we must arrest this downward slope we must put this back to 0.

That means; this slope resultant slope of minus 4 must be stopped and then it must be restored to a D.C 0 slope. Therefore, at t equals 1 you must introduce another ramp with a positive slope of 4 units. So that, that particular ramp and this what you had earlier picked up will lead to a 0 identically for values of t beyond 1 and therefore, this would be called 4 r. This ramp is introduced at t equals 1 therefore, 4 r t minus 1.

So, ultimately this particular and if you do that then this resultant negative slope and then this positive slope will be cancelled out will cancel out and yield identically a 0 value from t equals 1 onwards. Therefore, this particular triangle triangular wave form can be described as 4 r t plus 1 minus 8 r t plus 4 r t minus 1, the sum of these 3 that is how this ramp function can be employed. Notice that, r t is not equal to t they are not the same r t is a function like this whereas, t is function like this; this is r t, this is t.

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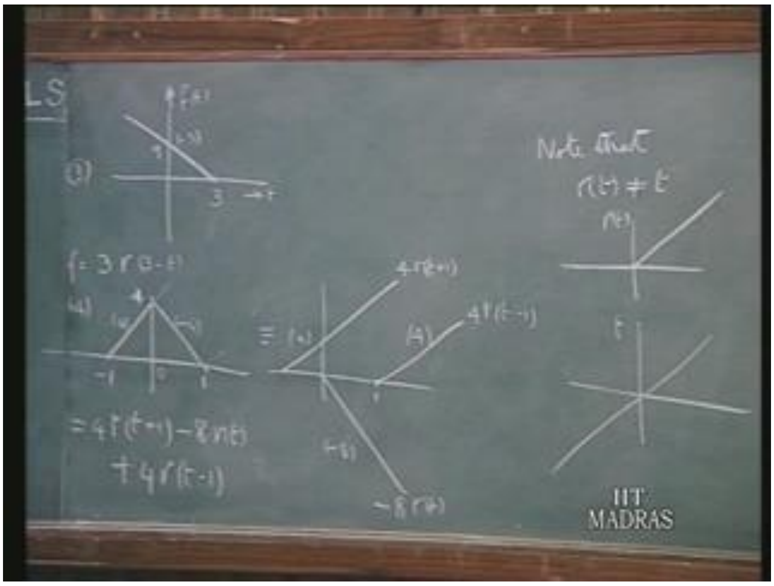

T has a value for negative values of time as well whereas, r t is 0 for negative values of time you can; however, say r t equals t times u t. You can say that because if you take this t and multiply by unit step function the negative values will be multiplied by 0 it gets cancelled out and for positive t that t multiplied by 1 t u t will provide r t. So, r t can be written alternatively if you wish as t u t, but it cannot be equal to t by itself.

The last singularity function which we will talk about is the unit delta function or unit impulse function. It is also called unit delta function also referred as dirac delta function because it was introduced by dirac. Now, 1 can introduce this in this fashion suppose i have, a symmetrically situated pulse symmetrically situated around the situation of width d and height 1 over d. So, this pulse has a width d and a height 1 over d. Now let us imagine, what happens when d is reduced by a factor 2 then the pulse width diminishes, but the height increases. So, that the area under the curve is 1. So, the area is equal to 1.

So, with maintaining that area as you make d smaller and smaller then the pulse width becomes smaller, but the height grows. So, that the area under the curve is maintained equal to 1 and this is d by 2 this is minus d by 2 and it is still centered around the origin. So, imagine what happens; when d goes to 0, when d goes to 0 you should have identically ideally a pulse which has got 0 width, but a height which is infinitely large. So, that is what is called the impulse function or delta function. Which is indicated because, we cannot mark the point at infinity in a black board or in your note book.

We will indicate an arrow like this and say this is 1 indicating that the area under this curve is equal to 1 and this is called delta t this function is called delta t. So, delta t is we can define it in this manner limit as d goes to 0 of this symmetrically situated pulse width d and height 1 over d that is 1 way of defining this delta function.

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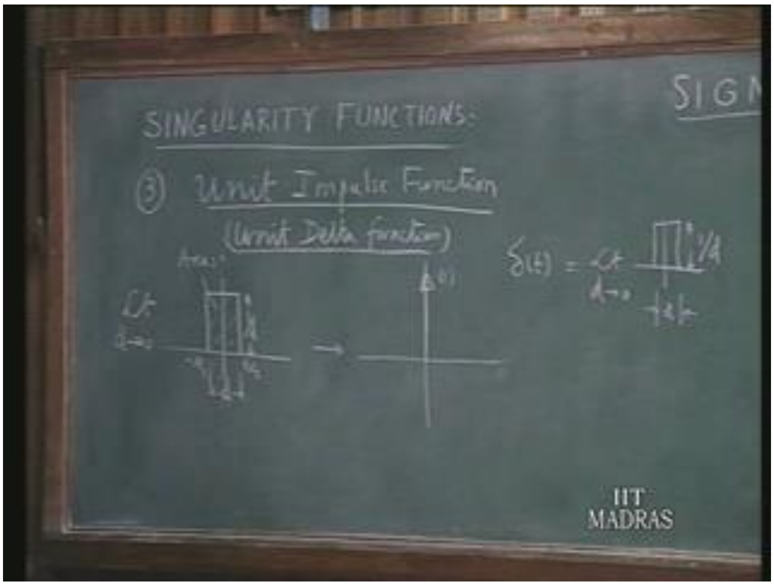

You can also think of sort with other wave forms like triangle wave form and take the limit and it goes 1 that is also possible, but we will confine our discussion to this. A second way of defining delta t mathematically would be delta t equals 0 for t not equal to 0. So, this is 0 for every value of t except t equals 0. For t equals 0 it has infinite type. We do not know how to describe this. So, all we can say is if you integrate delta t from an interval a b, such that 0 is in between a and b; that means, the range of integration includes that delta. This is equal to 1.

That means, the under this curve this infinitely large amplitude 0 width curve is said to be equal to 1 and therefore, as you integrate through the origin integral from a to b of delta t d t equal to 1. So, that is 1 way of defining this.

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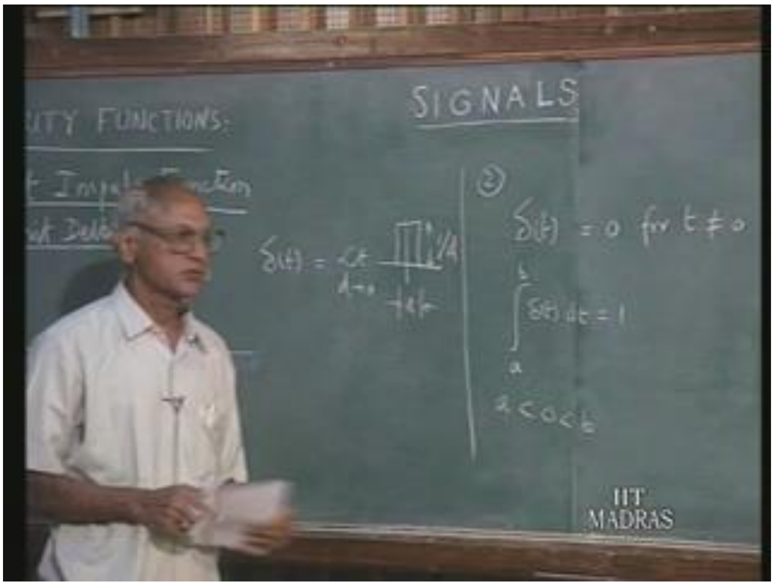

A third way of looking at this delta function is this. Delta t equals 0 for t not equal to 0 as before, but suppose you take a function f of t multiply by delta t and integrate from a to b. Then, what do you have; you have this delta function multiplying a function f of t. So, you have a continuous function. Let us say, f of t is a continuous function and then you are having this delta function. When you multiply these it is 0 everywhere except at this point. At this point f of t is multiplied by delta t.

So, f0 this value f0 is this value is multiplying this delta t. And the integral delta t d t is equal to 1 all we have now is instead of delta t dt you have f0 delta t dt. Therefore, this is equal to f 0. So, this is another way of looking at a delta function any continuous function f of t if you multiply by delta t and integrate it samples that value of f of t at the point where the delta is situated delta is situated t equals 0. So, you want to find out the value of the function f of t where t equals 0 you multiply f of t delta t dt integrate to get f of 0. So, these are all equivalent ways of looking at this delta function.

We can adopt this as the definition sometimes we can even use this as a definition or you can take this as a definition all are equivalent ways of looking at this.

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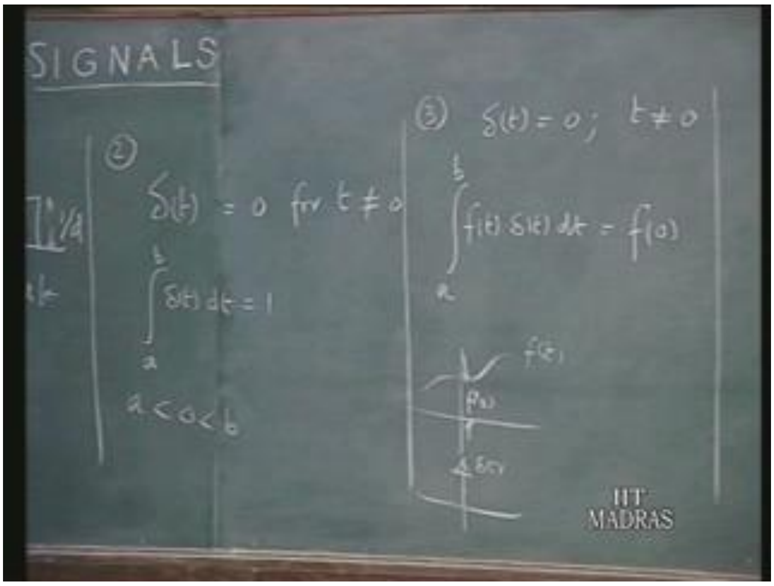

Important thing is that this delta function is something again as i mentioned which may not be which is not coming under the fold of the classical mathematics. So, originally when it was introduced it was introduced by applied mathematicians and engineers. And later on, mathematicians established a theory called theory of distributions which makes the manipulations of delta functions puts it on a more sound basis of rigorous mathematics and therefore, we do not go into the theory of distributions, but there is a branch of mathematics called theory of distributions which gives a theoretical basis to the use on manipulation of delta functions.

So, let us now let me now; plot some examples of, how different types of delta functions can be conceived. Suppose I have 4 delta t, it is an impulse function starting at t equals 0 and its magnitude is 4. I mean, we indicate its magnitude 4 in the sense that it is no doubt infinity, but the area under the curve is 4 units. So, the magnitude of a delta is indicated within brackets in this fashion. Suppose, I have 3 delta t minus 2 it means; the magnitude of the delta function is 3, but it is now sitting at t equals 2 because t wherever the argument of the delta is 0 then only delta exists otherwise it is 0; that means, this will be t equals 2 and its magnitude is 3 that is how 3 delta t minus 2 looks like.

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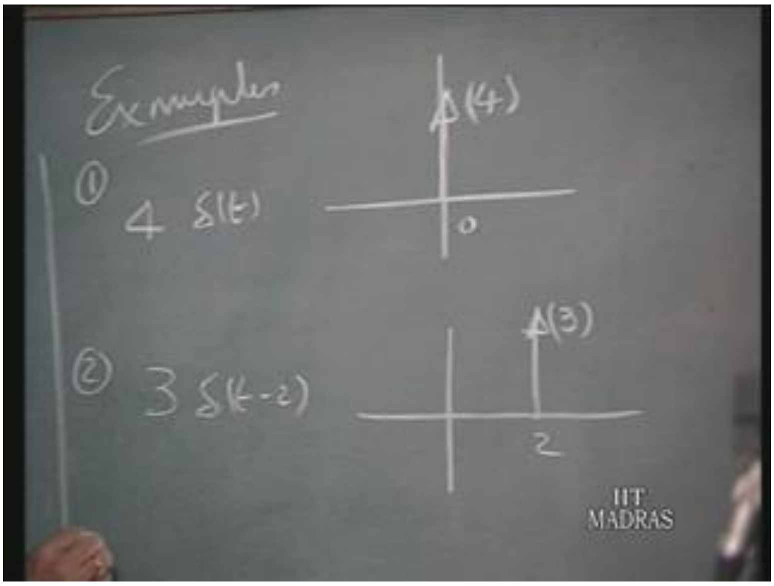

Suppose you have minus 3 delta 3 plus 4 it means; that at equals minus 4 that is the position where delta is fixed and it is a value equal to minus 3 therefore, it is a negative going impulse with magnitude of minus 3 that is how it goes a few further properties;

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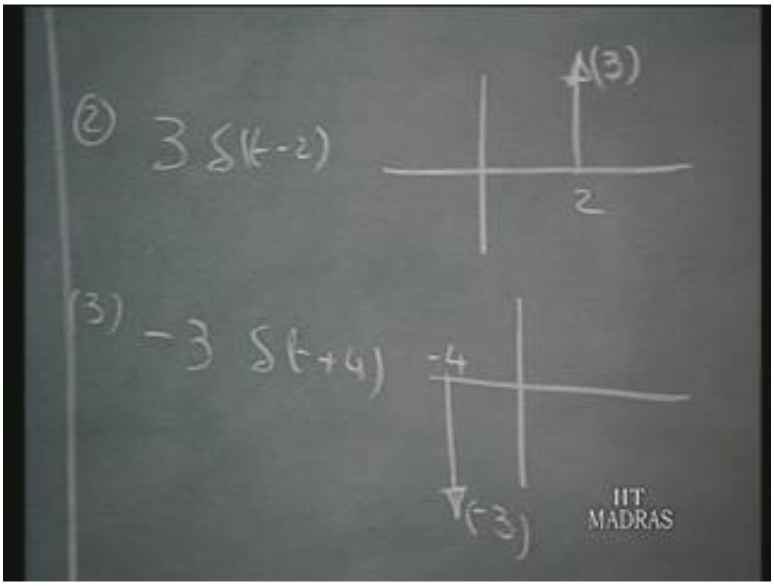

Delta t equals delta of minus t, it is an even function of time. So, delta exists at t equals 0. Delta minus t also exists at t equals 0 and nothing else. Therefore, whether it is plus 0 or minus 0 it makes no difference therefore, this is an even function of time. Secondly, delta t multiplied by f of t equals f 0 multiplied by delta t because f of t multiplied by delta t is 0 for all values of t except t equals 0.

So, the value if at all exists it exists only at t equals 0 at the time f 0 time delta t is the value that we have. Thirdly if i integrate delta t minus t nought times f of t dt over a range which includes t nought over a range which includes t nought then; obviously, this will be everywhere this integrand is 0 except when t equals t naught because at that time delta t minus t nought exists. So, at that time f t nought, which is a constant delta t minus t nought is delta function is being integrated therefore, this will be f t naught. So, this will yield f t nought that is how it goes in other words if you multiply f of t delta t nought this is after all f of t nought delta t minus t nought and you integrate f of t nought delta t minus t nought dt.

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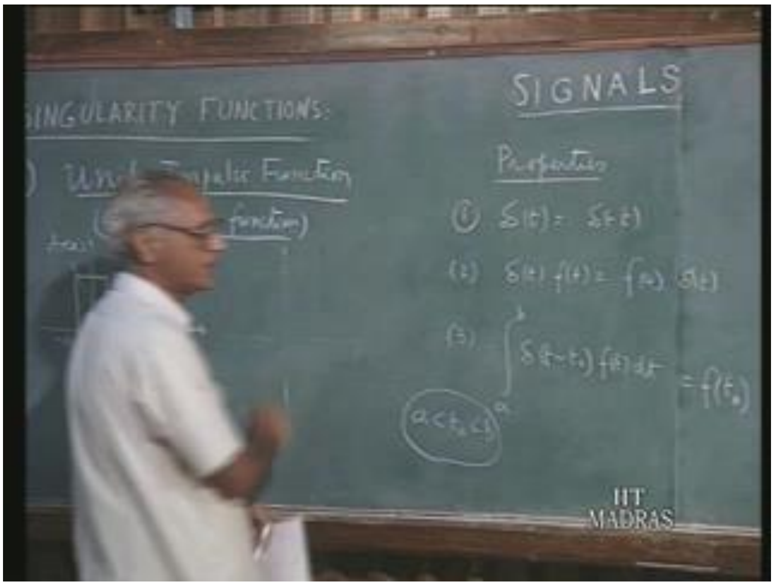

Whenever you integrate through a delta the value increases by 1 unit and that 1 unit multiplied by f of t nought leads to this. So, this can be put in this fashion. Whenever you integrate through a delta at a particular point whenever whatever, suppose you have some kind of curve like this which includes a delta function. If you integrate that at this point because of delta it jumps “ “, because of the delta function.

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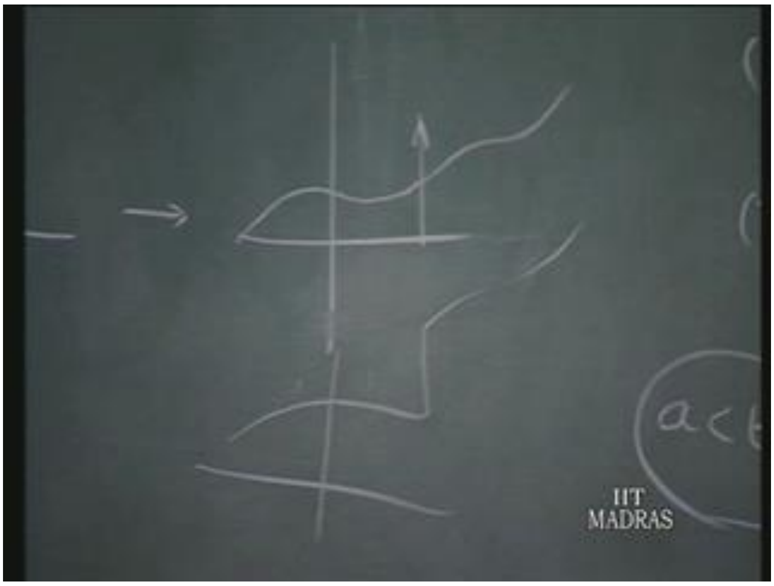

To summarize, we had in this course of this lecture reviewed the concept of complex frequency signal and identified the complex frequency components present in some example signals. Which we have taken up and then later on we introduced ourselves to a 3 singularity functions which play a very important role in describing functions which are either continuous or discontinuous which are either discontinuous by themselves or are discontinuous derivatives.

The 3 functions that we have familiarized ourselves with are the unit step function, the unit ramp function and the unit delta function. All these play a very important role in describing either response variables or excitation variables which are piecewise continuous, but have discontinuities at some points either in themselves or in the derivatives we will take up some examples involving the use of these functions in our next lecture.