

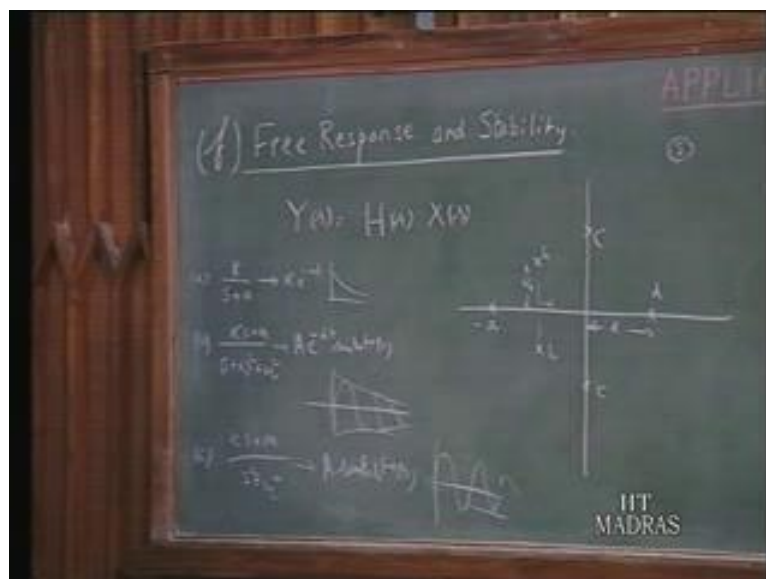
Networks and Systems
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Lecture – 29
Application of laplace transforms (4)
natural response and stability.
forced response to exponential and sinusoidal inputs.
Exercise 5

In the last lecture, we were considering, the various characteristics and implications of the properties of the system function $h(s)$ which is defined as the ratio of laplace transform of the output to the laplace transform of the input with 0 initial conditions and the system. When we say 0 initial conditions and the system, we do not really end of the any loss of generality because any initial conditions this system can be replaced by the equivalent source as we have thought of in the case of the electrical networks.

Similarly, in the case of systems as additional inputs we can think of nonzero initial conditions additional inputs and therefore, each non zero initial condition can be thought of as a separate excitation and 1 can use the super position principle therefore, when we say 0 initial conditions this really knows loss of generality, we considered various aspects of the characteristics of $h(s)$.

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Now let us start, with consider once again The free response and the system as given by the poles of $h(s)$ we said in the last lecture, for that as for the output is concerned it consist of 2 parts 1, which corresponds to the poles of $x(s)$ and 1. Which consist of poles of $h(s)$ the poles of $x(s)$ give raise to the forced response part of the output at the poles of $h(s)$ will give the parts corresponding to the free response of the system on the natural response of the system and the poles of $h(s)$ and indeed of $1/s$ of $f(s)$ which is the denominator of the polynomial in $h(s)$.

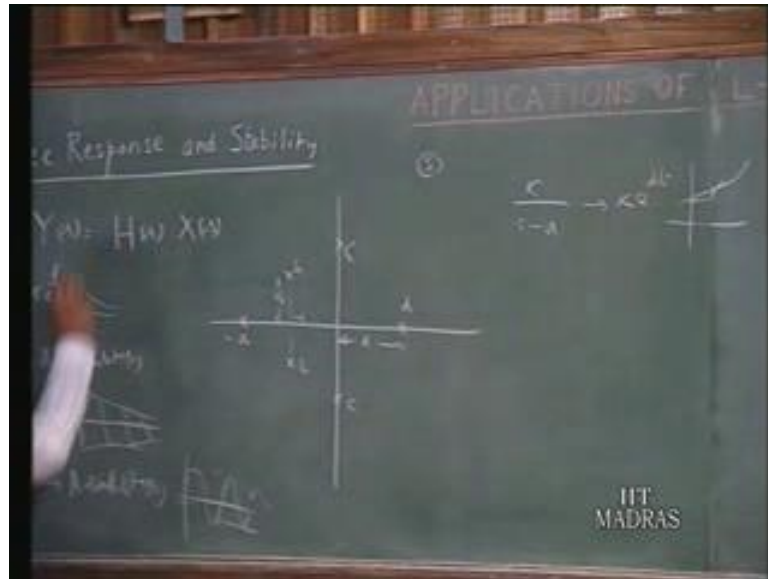
Now, depending on the location of the poles of $h(s)$ we have different types of behavior. So, in the complex plane, if this is the complex plane, s plane suppose, I have a pole here its location a than that corresponds to in the partial fraction expansion k over s plus this is minus. Let us say, k over $s + a$ and this will give rise to a response $k e^{-at}$ to the power of minus a .

So, that is something which is case with time on the other hand. If I have the pair of complex conjugate poles here say at locations b say this is a location b this will be $k s + j m$ divided by $s^2 + \alpha^2 + \omega^2$ that will be the type of expansion partial fraction expansion that; to get plus 1 into the locations in the b positions this is minus α and this is ω^2 . And this gives to response, which is sum say $a e^{-\alpha t} \sin \omega t$ s beta.

Which means; that we are having a declined oscillations with the amplitude declined exponentially. On the other hand, if you are having 2 pair of poles and the imaginary axis locations c that corresponds to something like: $k s + m$ once again over $s^2 + \omega^2$ and this gives rise to a response, which is $\sin \omega t$ plus beta which is sustained oscillations.

Therefore, the poles of $h(s)$ take the natural response of the system on the other hand, if the pole in the right of plane suppose, the pole here that corresponds to suppose, this is $d s - d$ then this corresponds to $k e^{dt}$ and that is: represent an increasing exponential which d indicates in stability. So I can say that, if the system is stable. If you have a bounded input given to $x(s)$ which is: finite then in natural response must decay with time are atleast style with time.

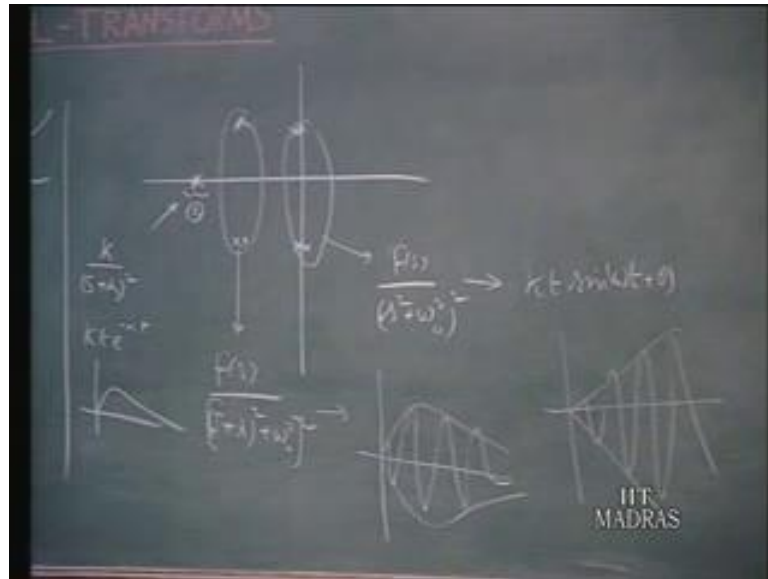
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We should not take ω_n to go indefinitely to large quantities therefore, first stable system, we require all the poles of $H(s)$ to be in the left half plane and in the limit as a border line they can be on the imaginary axis because that is: the limiting behavior however, what happens if the poles are repeated.

Repeated poles really need that: in the denominator polynomial $H(s)$ are factors like $s^2 + 2\alpha s + \alpha^2 + \omega_n^2$ are $s^2 + 2\alpha s + \alpha^2$ plus ω_n^2 plus α^2 plus ω_n^2 whole square, if the repetition is other 2. So let us consider, what happens.

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If you in the complex plane you have a pole of order 2 here that means: multiple this 2 repeated pole it can be repeated 2 times 3 times on. But let us, illustrate it for the case of 2 that means; if the partial fraction expansion here k over s plus α whole square and the time domain response corresponding to this will be of the form $kt e$ to the power of minus αt .

So, that means; you have it exponential decays but, starts with 0 amplitude on the other hand, if you are having 2 complex conjugate poles repeated like this: that will be of the form some f off s over s plus α whole square plus ω not square whole square that means; this 2 complex conjugate poles are repeated twice at this gives as to response which will be once again this envelope like this.

So, even though the poles are repeated in the there in the left of plane it gives as a stable response will response ultimately decays, what happens, if the repeated poles in imaginary axis suppose they are the 2. So this corresponds to a term like: f off s over square plus ω not square whole square. And this gives as a time response which is some $kt \sin \omega t$ plus θ that means the time response corresponding to that would be come to like this,

So, this certainly should be excluded, if you want a stable system. If you have a reasonably well behaved bounded type of input the output cannot go indefinitely large for a stable system. So, that means; we cannot have repeated poles on the imaginary axis

certainly we can not have poles in the right of plane whether, it is repeated simple and in the imaginary axis can not have repeated poles even repetition of by rather duplication of poles that is multiple roles rather 2 themselves or not permitted certainly higher order poles can not be permitted.

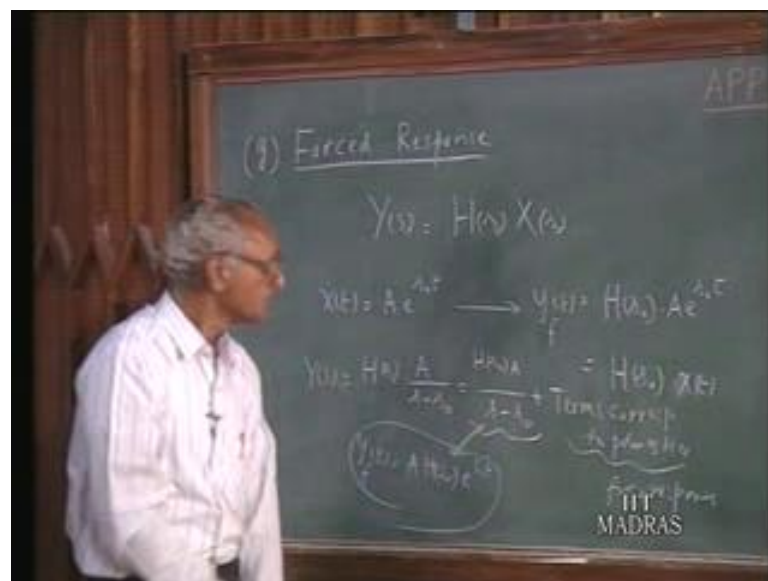
But as for the left half plane is concerned, what we illustrated for other 2 can also be extended for higher order you can have a pole on the negative real axis of order 4, 5, 6 whatever, it might be and similarly, complex conjugate, poles in the interior left of plane can be of any other what so ever, ultimately the response will decay on the imaginary axis. However, you can have only simple poles that means to conclude this discussion.

If, you want to have the free response to decayed time, then we require that the poles of h off s should be in the left of plane the interior left of plane, if they are the imaginary axis they must be simple. They must be unit multiplicity that can not be repeated and certainly we can not permit poles of any other right of plane. So, that is the meaning of the stability the system is stable bounded input must 0 the bounded response and.

So, the response can not go for can nit take of to infinite limits it and go indefinitely with reference to time and the requirement therefore, is that; the poles should be left of plane inform the imaginary axis they should be simple.

Let us, know look in some detail the forced response characteristics of system.

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We know that: $Y(s)$ equals $H(s)$ times $X(s)$.

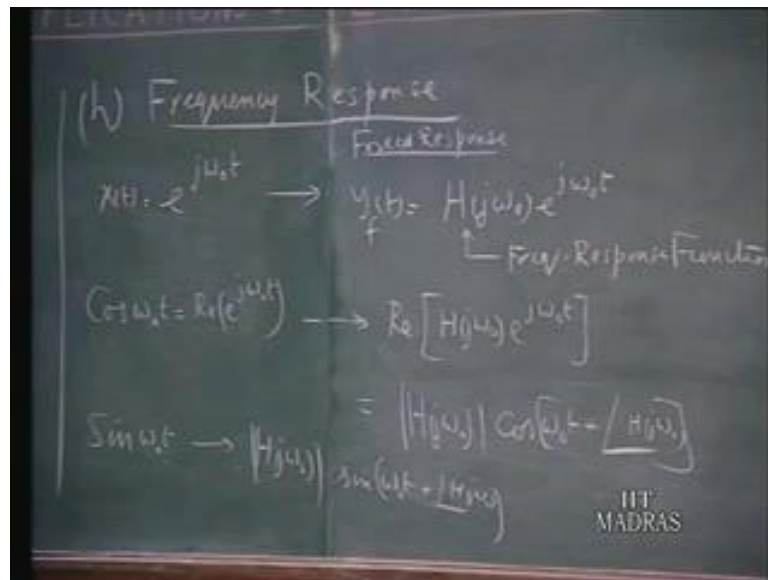
Now, we will take the particular case of the $X(s)$ which is of the form $X(s) = \frac{1}{s - \alpha}$ to the power of α not t this an exponential signal then it gives as a forced response $Y(s)$ which you have already seen in our previous lectures $Y(s)$ will be $H(s)$ times $\frac{1}{s - \alpha}$ to the power of α not t which means $H(s)$ times $X(s)$.

So whenever, $X(s)$ is of an exponential type, exponential signal with the complex frequency α not the forced response is obtained nearly multiplied by $X(s)$ by $H(s)$ evaluated at that complex frequency α substitute α not for s and that is: what you get this is something, which we already discussed. Let us see, how it is arranged we know that if $X(s) = \frac{1}{s - \alpha}$ to the power of α not t $Y(s)$ will be $H(s)$ times the Laplace transform of $X(s)$ is a divided by $s - \alpha$.

So, in the partial fraction expansion of this you have 1 term corresponding to $s - \alpha$ not which is $H(s)$ times $\frac{1}{s - \alpha}$ plus terms corresponding to poles of $H(s)$. So, there are other terms correspond to poles of $H(s)$ here you take raise to the free response. The force response is given by this gives you the force response and the force. So, from this $Y(s)$ the time domain representation of this is $\frac{1}{s - \alpha}$ times $H(s)$ times $e^{\alpha t}$ to the power of α not t . So, that is what here having that is: what exactly heard is the force response.

So, this is how it ties up in the Laplace transform theory, what we already know from your preliminary lectures whenever, an exponential signal is there the output the force response is the input times $H(s)$ not. Now arising out of this.

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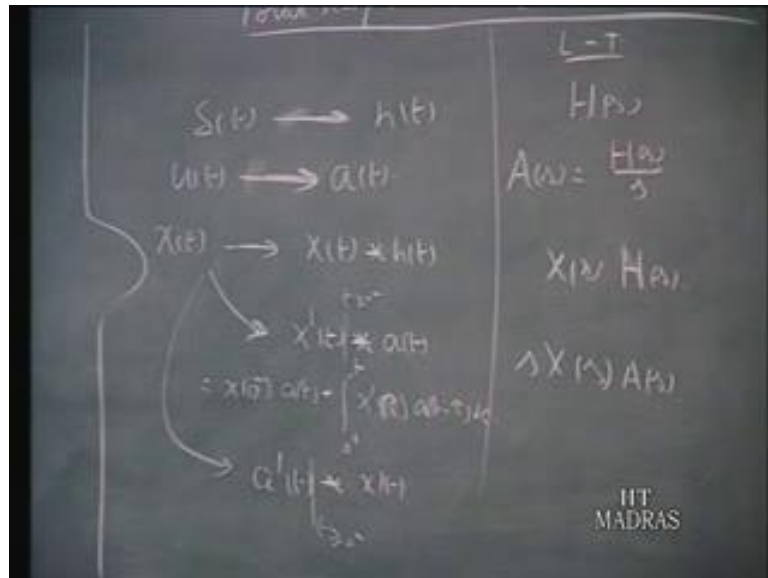
We can talk about the frequency response characteristic. Suppose, you have $e^{j\omega t}$ as the input quantity then we know the forced response of this the forced response of the system $y_f(t)$ this is $x(t)$ the forced response is $H(j\omega)e^{j\omega t}$. So, this is the frequency response function but, if you are often interested you are in exponential signals of the circuit $e^{j\omega t}$ but, something like $\cos \omega t$ let us say.

So, $\cos \omega t$ we can take this as real part of the $e^{j\omega t}$ and therefore, the response to that would be the real part of $H(j\omega)e^{j\omega t}$. So, this can be written as $|H(j\omega)| \cos(\omega t + \angle H(j\omega))$. So, the magnitude of this is $|H(j\omega)| \cos \omega t$ plus an angle which is the angle of $H(j\omega)$.

So, that will be what you are having so, depending up on if $\cos \omega t$ is the input quantity the output quantity is given by the magnitude is magnified with the magnitude frequency response function the phase is increased by the increase with the angle of the frequency response function $\cos \omega t + \angle H(j\omega)$ similarly, if $\sin \omega t$ the forced response will be, $|H(j\omega)| \sin(\omega t + \angle H(j\omega))$ thus the angle of $H(j\omega)$.

So, this is the frequency response function we have talked about this. When we discuss the Fourier transform and Fourier methods and where h off s substitution $j\omega$ will give the frequency response function. So, we can see how this ties up with our Laplace transformation theory. To summarize them, what you are talked about is.

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The total response, with 0 initial conditions. If the input is $\delta(t)$ it gives rise to $h(t)$ and the Laplace transform of that is the h off s , the Laplace transform of the output is h off s , if the input is $u(t)$ the I am sorry this not arrow this is input this is output we call that, the initial response of step response at and we indicated this as $a(s)$ that will be h off s over s .

So, we have a general $x(t)$ then in terms of the impulse response we say $x(t)$ convolved with $h(t)$, which in Laplace transform domain is $X(s)$ times $H(s)$ we can also have alternately $x'(t)$ convolve with $a(t)$ that is: the step response the meaning of this is $x(0) + \int_0^t x(\tau) a(t-\tau) d\tau$ all this we discussed, in the last class last lecture.

So, in terms of Laplace transforms we can write $X(s)$ times $A(s)$, we also have an alternative representation of this, if you recall you can write this as $x'(t)$ convolve with $x(t)$ that also have the same Laplace transform and in these 2 expressions $x'(t)$ and $x(t)$, we are talking about the situation take t starting from 0 minus starting from t greater than 0 minus. So, that is: what we total response of a system can be obtained

from the either the step response or the impulse response are in general terms this is how this equations will run.

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$$e^{s_0 t} \rightarrow H(s_0) e^{s_0 t}$$

$$e^{j\omega_0 t} \rightarrow H(j\omega_0) e^{j\omega_0 t}$$

$$\cos \omega_0 t \rightarrow |H(j\omega_0)| \cos(\omega_0 t + \text{Arg}[H(j\omega_0)])$$

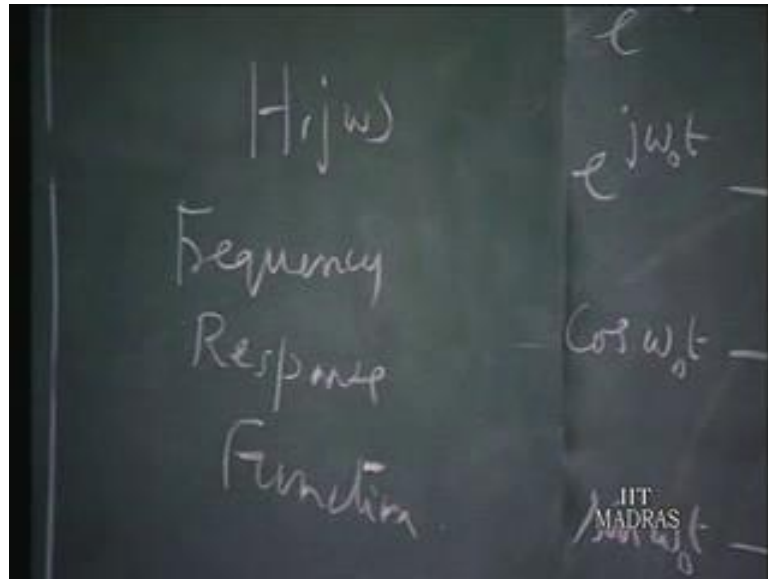
$$\sin \omega_0 t \rightarrow |H(j\omega_0)| \sin(\omega_0 t + \text{Arg}[H(j\omega_0)])$$

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Now, let us talk about setting up a similar table for the forced response. So, if the input is the exponential signal e to the power of s not t the forced response is h s not times e to the power of s not t , if it is e to the power of j ω not t it is h j ω not e to the power of j ω not t and if, it is \cos ω not t it is h j ω not magnitude \cos ω not t plus the argument of h off j ω not.

So, h off j ω not is a complex number and whatever, angle that h s not that is the addition to the face of the cosine function, \sin ω not t this will be like wise h j ω not magnitude \sin ω not t plus the argument of h off j ω not.

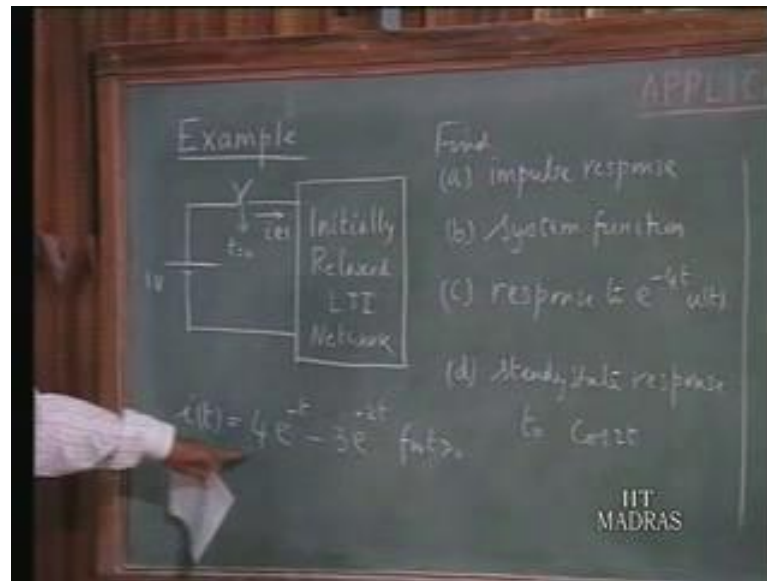
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And $h(j\omega)$ is called the frequency response function, which we had used in steady state sinusoidal analysis of good analysis of $h(j\omega)$ is something like, your impedance z and all let $h(j\omega) z$ is the function of ω $h(j\omega)$ is the function of ω .

So impedance is the ratio voltage to current. So, if you take the voltage the response quantity and the current as input quantity the system function corresponding to that z in the normally, what we talk about is z is the system function in this context. Let us now, work out in example to illustrate this ideas.

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Let us now consider example, where we have a network, which is linear timing variant and we treat this as a system the input is this whatever, voltage you give and the response quantity identifies the current it. So, when the switch is closed t equals 0 in this particular network, it is observed that the current that flows in to the network is given by this $4e^{-t}$ to the power of minus t minus $3e^{-2t}$ for t greater than 0 off course, it goes without saying it is 0 for t less than 0.

Given this data you are asked to find out. Find impulse response of the network that means; if you excitation is unit impulse, what is the corresponding output, system function, taking the input voltage as the input and the resulting current as the output, c: response to an input, which is $e^{-4t} u(t)$, d: steady state response to an input sinusoidal $\cos 2t$ suppose, a sinusoidal voltage is given to this network what kind of steady state current you would get in this terminals that is: the question that we asked.

So, all this quantities are to be obtained. Now, the input is 1 volt and the output is $4e^{-t}$ to the power of minus t minus $3e^{-2t}$.

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$$I(s) = \frac{4}{s+1} - \frac{3}{s+2}$$
$$H(s) = \frac{I(s)}{V(s)} = \frac{s(s+5)}{(s+1)(s+2)}$$
$$A(s) = \frac{(s+5)}{(s+1)(s+2)}$$

Therefore, the input Laplace transform is 1 volt. So 1 volt is given t equals 0 to this network therefore, it is a step function really v off s is 1 over s the current in the network as the Laplace transform of this 4 up on s plus 1 minus 3 up on s plus 2 4 up on s plus 1 minus 3 up on s plus 2 that is: the expression for the current therefore, I off s over v off s is the system function h off s that, will be obtained as you can work this out and show that this is equal to s times s plus 5 divided by s plus 1 times s plus 2 you can show that; this is the system function h off s .

So, that is the answer to be which we already obtained. If you look at the, this I off s can also be thought of as the step response because input is 1 unit step therefore, a off s is simply this 1. So, if this s plus 1 times s plus 2 so 4 s minus 3 is s 8 minus 3 5 so, this is a off s you can see that h off s is s times a off s that is: the result which we already know.

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Impulse response

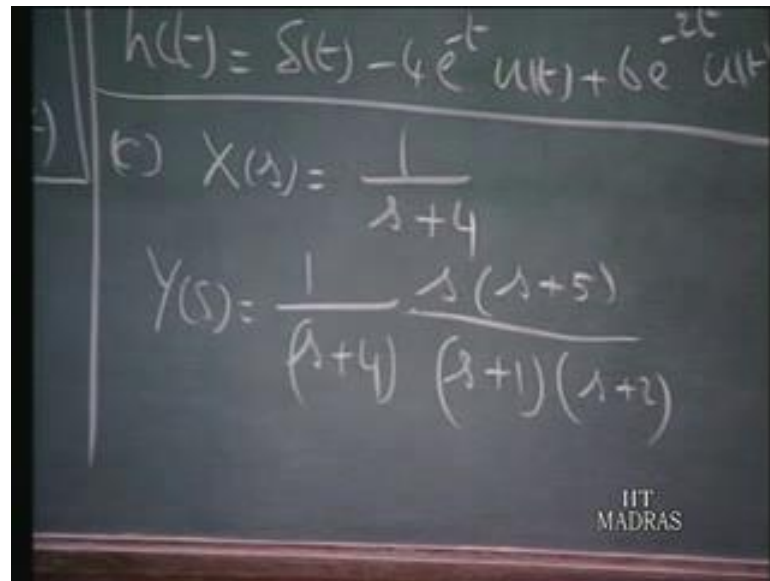
$$h(t) = \mathcal{L}^{-1} H(s)$$
$$H(s) = 1 - \frac{4}{s+1} + \frac{6}{s+2}$$
$$h(t) = \delta(t) - 4e^{-t} u(t) + 6e^{-2t} u(t)$$

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Impulse response, h of t is the Inverse Laplace transform of h of s this is h of s . So, h of s can be expanded by means of partial fraction expansion so, it will turn out to be $1 - \frac{4}{s+1} + \frac{6}{s+2}$. So, if this rational function make the partial fraction expansion of the rational fraction you get this because the numerator and denominator as the same degree polynomial s^2 divided by s^2 so, the ratio of the 2 leading coefficients is 1 that is: the first term and then you have the residual the pole as at s equals minus 1 and the pole at s equals minus 2 will be like this.

So, if you take the inverse laplace transform of this $\delta(t) - 4e^{-t} u(t) + 6e^{-2t} u(t)$ you can write this $4e^{-t} u(t) + 6e^{-2t} u(t)$ that is: the impulse response.

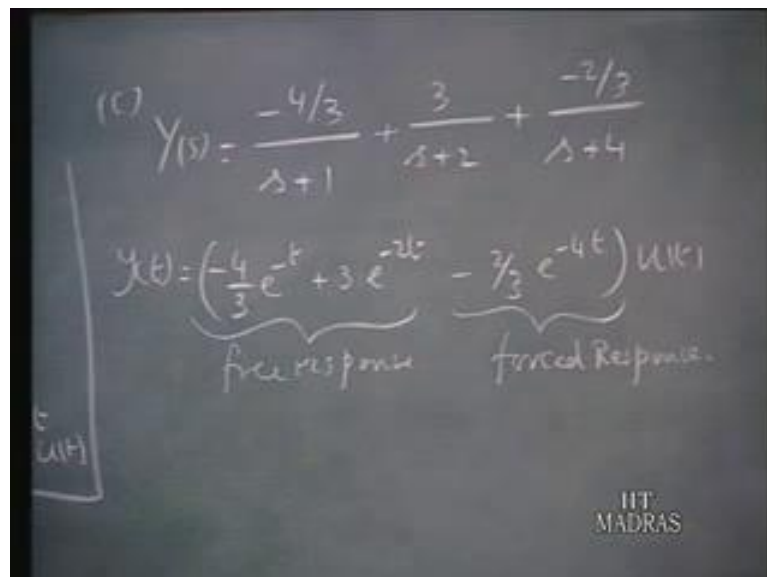
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$$h(t) = \delta(t) - 4e^{-t}u(t) + 6e^{-2t}u(t)$$
$$X(s) = \frac{1}{s+4}$$
$$Y(s) = \frac{1}{(s+4)} \frac{s(s+5)}{(s+1)(s+2)}$$

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Now, response to $e^{-4t}u(t)$. So, the input is $e^{-4t}u(t)$. So, the output is $e^{-4t}u(t)$ multiplied by the transfer function. So, the output is $\frac{1}{s+4}$ multiplied by $\frac{s(s+5)}{(s+1)(s+2)}$. So, you make the partial fraction expansion of that.

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$$Y(s) = \frac{-4/3}{s+1} + \frac{3}{s+2} + \frac{-2/3}{s+4}$$
$$y(t) = \left(\underbrace{-\frac{4}{3}e^{-t} + 3e^{-2t}}_{\text{free response}} - \frac{2}{3}e^{-4t} \right) u(t)$$

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So, the partial fraction expansion of this we contain 3 terms those of the 3 terms. So $y(t)$ will be minus 4 by 3 e^{-t} plus 3 e^{-2t} minus 2 by 3 e^{-4t} multiplied by $u(t)$.

plus $3e^{-2t}$ minus $\frac{2}{3}e^{-4t}$ times $u(t)$. The input is e^{-4t} and the output contains e^{-4t} .

This is the forced response, the forced response contains exactly the same complex frequencies as the input. The system function has got 2 poles at $s = -1$ and $s = -2$ corresponding to this you have got these 2 terms e^{-t} and e^{-2t} . So, this corresponds to these poles of $X(s)$, so, this represents the natural response of the free response.

So, as we mentioned earlier, the Laplace transform theory enables us at the total solution in 1 step the free response part and the forced response part you can apply get $y(t)$ using convolution integrals also all that you know $h(t)$ and also you know this itself the it itself is at you can use the convolution and find out $y(t)$ either from the impulse response or the step response wind up in the same result that you can work out exercise.

Now it will be interesting to see whether, you can get forced response independently suppose, you want only the forced response. So, the input is e^{-4t} .

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$$y(t) = \underbrace{\left(-\frac{4}{3}e^{-t} + 3e^{-2t}\right)}_{\text{free response}} + \underbrace{\left(-\frac{2}{3}e^{-4t}\right)}_{\text{forced response}} u(t)$$

$$e^{-4t} \rightarrow y_f = H(s)e^{-4t} = H(-4)e^{-4t}$$

$$= \frac{(-4+5)}{(-3)(-2)} e^{-4t} = \frac{-2}{3} e^{-4t}$$

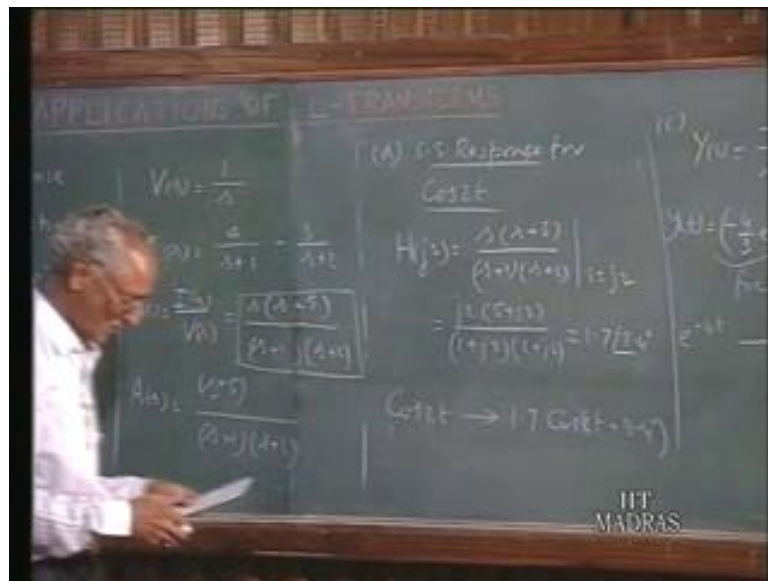
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So, the forced response of that is: $h(s)$ not where s not is -4 times e^{-4t} . So, if you substitute this is really $h(-4)$ e^{-4t} we know $h(s)$ is $s(s+5)/(s+3)(s+2)$. So, when you

substitute minus 4 for in this for s you get minus 4 s plus 1 divided by minus 3 times minus 2.

So, that will become minus times e to the power of minus 4 t that becomes minus 2 up on 3 e to the power of minus 4 t. So, this is exactly what you got here so, verification of the force response you can see that, the force response can be obtained by substituting s not for s, where s not e is the complex frequency in the input exponential signal the last part is: what is the steady state response for cos 2 t.

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So the sinusoidal has the input sinusoidal frequency 2 so, $H(j2)$ equals our $H(s)$ is s times s plus 5 divided by s plus 1 times s plus 2 this substitute s is equal to $j2$. So, when we substitute s is equal to $j2$ in this you get $j2$ times 5 plus $j2$ divided by 1 plus $j2$ times 2 plus $j2$. If you simplify this rationalize in simplify this will turn out to be approximately 1.7 at an angle of 3.4 degrees.

So, when you substitute this $j2$ for s and evaluate this the frequency response function $H(j2)$ terms at 1.7 times 3.4. So, if $\cos 2t$ is input the steady state response of that; will be 1.7 this is the magnitude the frequency response expansion $\cos 2t$ plus 3.4 degrees. So, this frequency response function it cancels all the amplitude of the sinusoidal of the phases of the sinusoidal modified in the as for as the steady state response is concerned we will $\sin 2t$ to still be 1.7 $\sin 2t$ plus 3.4 degrees.

So, this is how I can use the frequency response function evaluate the steady state response the sinusoidal trial in forces. In the course of the last 4 lectures, we have seen how the Laplace transform whether can be portably employed to solve for the transients in networks and systems taking the case of network transients to be specified we saw that in the Laplace transform method, how the governing equations for the network in question or formed a algebraic form either from the transformation differential equation or to the transform diagram.

How the response quantities laplace transform to the quantities can algebraically arrive that and then from through inverse transformation, how the corresponding time functions can be found out. There is no algebra involved for them are the initial there is no calculus involved is only the algebra that is; involved further more the initial conditions are plugged into the formation of the algebraic equation, even at the very beginning itself and know separate evaluation of the arbitrary constants is called for as we had in the case of differential equation approach.

Perhaps the greatest difficulty in the solution of the laplace transform method lies in finding out inverse transformation particularly, when the denominator polynomial is of a high order not only we have find the residues in the various poles. But, the determination of the poles itself, will become tedious if the denominator polynomial is of a high order. But, this is the problem which is attendant for large size problems by in any method even the differential equation approach and certainly the blame can not be laid at the door of the Laplace transformation method.

We have also seen, how this system function can be performance of a general system, the system function embedded itself various properties pertaining to the system, we can use this system function the steady state response in exponential input by means of fusion corresponding complex frequency in the h off s . We can see, how the system function can be used to find out the impulse response you can find, how the system function can be used to find out the step response and how the system function is almost equivalent to the frequency response function.

So, the system function has number of versatile property which we are seen and in a general situation, we can see how the system, how we can the once we know the frequency response the impulse response of the step response, we can find out the

frequency response conversely. If you know the frequency response over a large entire range of frequency there also methods available by means of which we can find out the transient response.

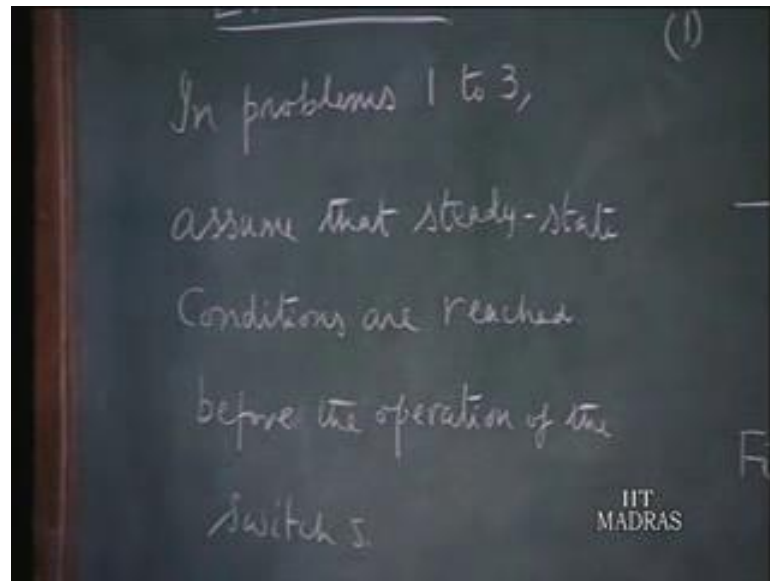
In otherwards, linear system if you know the transient response by that; I means, there are response to a step input or impulse input we can find out the frequency response for any sinusoidal input and vice versa there are algebra procedures and traffical procedures available where analytical expressions can not be found out for either the frequency response or the transient response.

After cravings that all this, the Laplace transformation method must also say as some disadvantages certainly it can not be used for systems which are either non-linear or time varying, even for the simple case of the product of 2 time functions the corresponding Laplace transform can not be easily found out it involves complex convolution for other types of non-linear it may difficult to finds the Laplace transformation. Similarly, when you time varying equation the convolution integral that you talked about can not be used in the same form it becomes much more complex.

So, for the general non-linear time varying systems, Laplace transformation is almost useless the differential equation is more fundamental and perhaps it may permits only feasible method of solution in such situations, even though this may involve complicated calculations and perhaps even numerical methods for the solution. But, then differential equation method is more fundamental and it can be used for a general situation where as the Laplace transform method can not be used in such situations.

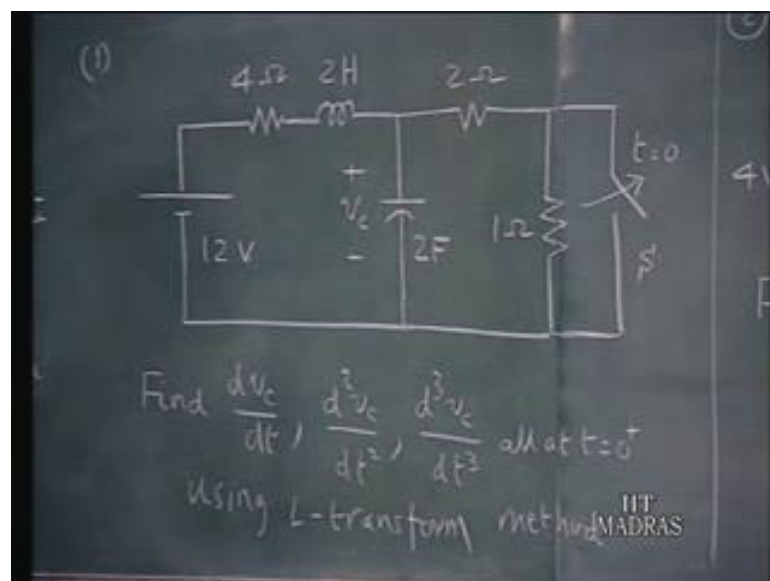
So, with that we close our discussion of the application of Laplace transformation method to networks and systems and now we look, some problems which form an exercise on the topic which we are covered in the last 4 lectures.

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In the first 3 problems in this exercise we find the transients in a circuit operation of certain switches. So, we assume that in problems 1 2 3 assume that; steady state conditions are reached before the operation of the switch s. So, whether the switch s is opened or closed in the first 3 problems we assume that; before the operation of the switch the conditions in the circuit have reached steady state. So, that will enable us to find out the conditions at t equal 0 minus in the various circuits that we are going to investigate.

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First problem. So, in this problem you are having a 12 volts v_c source connected to circuit consisting of a 4 Ohms resistor 2 Henry inductor 2 Ferrate capacitor in shunt and the 2 Ohm resistor and a 1 Ohm resistor. Now, this switch s is kept closed for a very long time since, steady state is reached and then it is opened t equals 0. Now you are asked to find out the capacitor rate of change of capacitor voltage $\frac{dv_c}{dt}$ $\frac{d^2v_c}{dt^2}$ the third derivative $\frac{d^3v_c}{dt^3}$ not as a function of time.

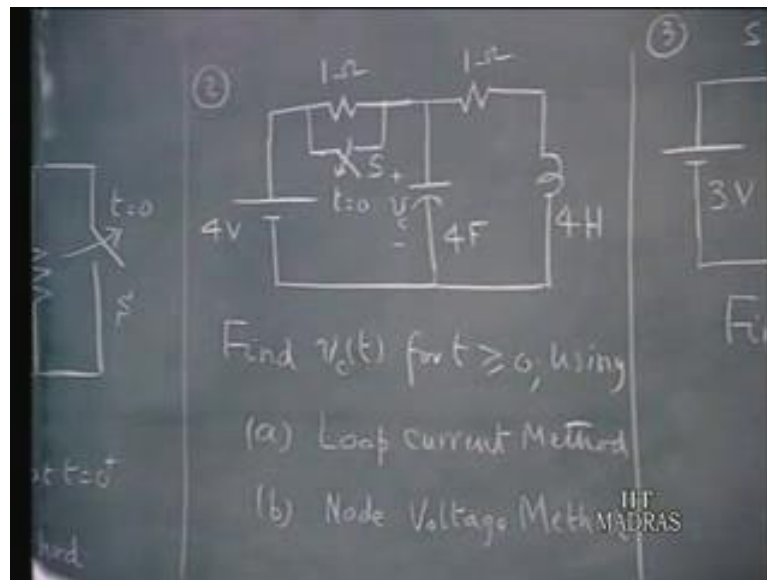
All of them evaluated t equal 0 plus and how do you do this use the Laplace transform method. So, we are not interested in the time domain behavior of v_c of the derivatives we are interested in the values of the first second and third derivatives of the v_c evaluated t equal 0 plus. Now, if you have to do this in the classical differential equation approach in time domain becomes little trepan complicated, but.

The Laplace transform method gives as very simple way in which you can calculated this and this problem illustrate this technique. So, all you have to do is using the t equals 0 minus conditions of v_c set up the transform diagram for this and arrive at the expansion for v_c off s the Laplace transform of the capacitor voltage once, you have the laplace transform of the capacitor voltage you can also find out the Laplace transform of the derivative of the capacitor voltage, the second derivative of the capacitor voltage, third derivative of the capacitor voltage.

All in the laplace transform domain. But, you do not have to find out the Inverse Laplace transform to evaluate v_c or v_c prime v_c double prime or v_c triple prime only once you got the Laplace transform of the respective quantities you apply the initial value theorem and find out the values are t equals 0 plus. So, this is an application of the initial value theorem so, as we said the initial value theorem will always give you t equals 0 plus values.

So, you have to do apply that theorem and find out respective values you do not have to find out the Inverse Laplace transform of any 1 of this quantities.

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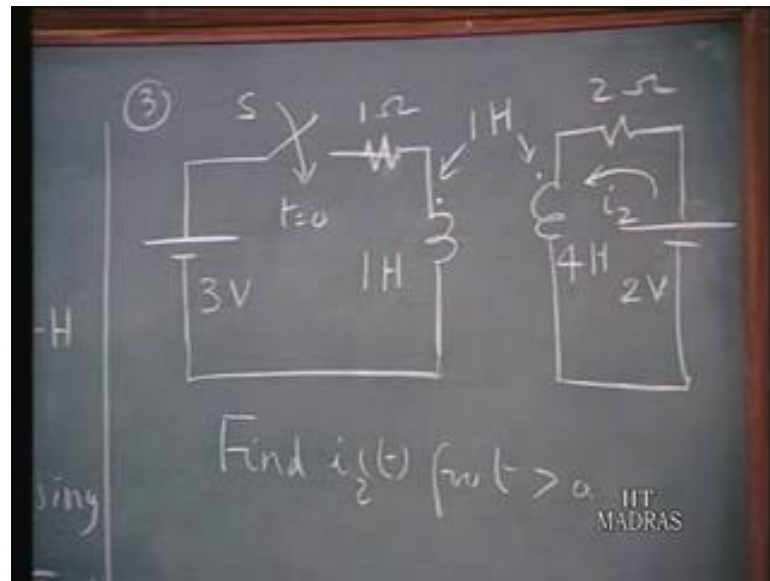


Second problem is this: in which we have a 2 loop circuit consisting of an inductor and a capacitance so on. The switch is kept close for a very long time that enables you to find out the initial condition respect to the capacitor as well as the inductor. And you use them the transform diagram it 4 volts 1 Ohm, 4 Ferrates, 1 Ohm, 4 Henries. And you are asked to find out find v_c off t for t greater than are equal to 0 use 2 methods once, you have the transform diagram.

We analyze the transform diagram on the loop basis using the loop current method or the node voltage method. And we get solution for v_c off t by both methods conclude and both should be the same when you are using the node voltage method as I mentioned when you replace this initial conditions on the capacitor by means of an equivalent source is the voltage across those 2 nodes which represents capacitor voltage in the transform domain. So, you keep that in mind; in calculating the capacitor voltage in node voltage method.

So, this is an exercise in using the transform diagram, for solution of another response quantities which happens to be v_c off t in this particular example. So, you can take this down.

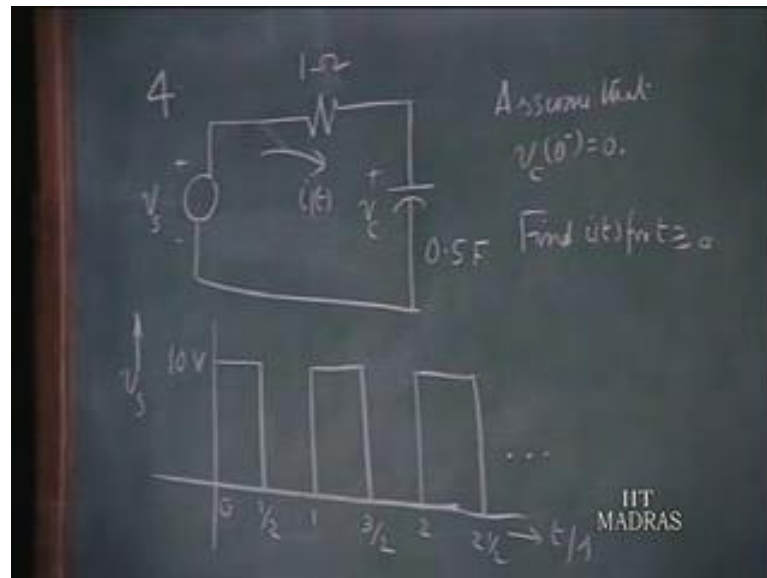
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Now, the third example, is concerns use the transform diagram approach where mutual inductance. So, you have a 3 volts source connected to a series circuit of 1 Ohm and the primary coil having a self inductance of 1 Henry it is couple to a secondary coil of 4 Henry self inductance the mutual inductance of 1 Henry, this is the mutual inductance, this is the self inductance of foil 1, this is a self inductance of coil 2 and the secondary circuit is close to a 2 volts dc cell this is the source and the 2 Ohm resistors this is the situations and when the switch is kept open naturally only current will go through this when we know current here.

So, $i_2(0^-)$ you can find out and put appropriate initial conditions in the transform diagram. And once, the switch is closed for this circuit is also completed you have got a current here and current here and you write down the appropriate loop equation and solve for i_2 off s from that; find $i_2(t)$ for $t > 0$. So, you have to use the replacement of initial conditions in couple coils equivalent sources you can have use current sources or voltage sources and find out the appropriate solution.

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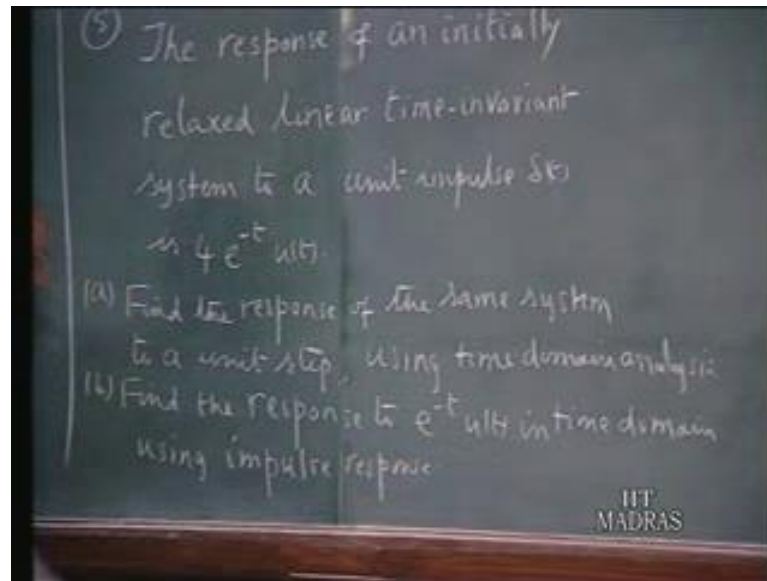


Problem number 4, we have a series circuit containing a resistance of 1 Ohm and a capacitor of 0.5 Farads and this is connected to a voltage source v_s which has a discontinuous character, it is a set of periodic pulses and so on, so for this is the variation of this v_s , it has the pulses have an amplitude of ten volts this is t in seconds.

So, it is a pulse frequency of 1 Hertz and the pulse duration in each period is half a second, 3 by 2 and so on, so for extra. So, this is the v off s and assume that; if this is a capacitor voltage $v_c(0^-) = 0$ that is, the pulse is applied at $t = 0$ first pulse at the time the capacitor voltage is 0. Now, you are asked to find out the current. So, you are asked to find out an expression for the current in the network on the application of the discontinuous voltage source this is the periodic voltage source starting from $t = 0$.

So, you must find out the Laplace transform of that; use that to get an expression for i off s and interpret the i off s you will have e to the power of s factors also in that; when you find out the Inverse Laplace transform you get some delayed versions of certain quantities interpret them suitably and get an expression for i off t in the final expression in the final result.

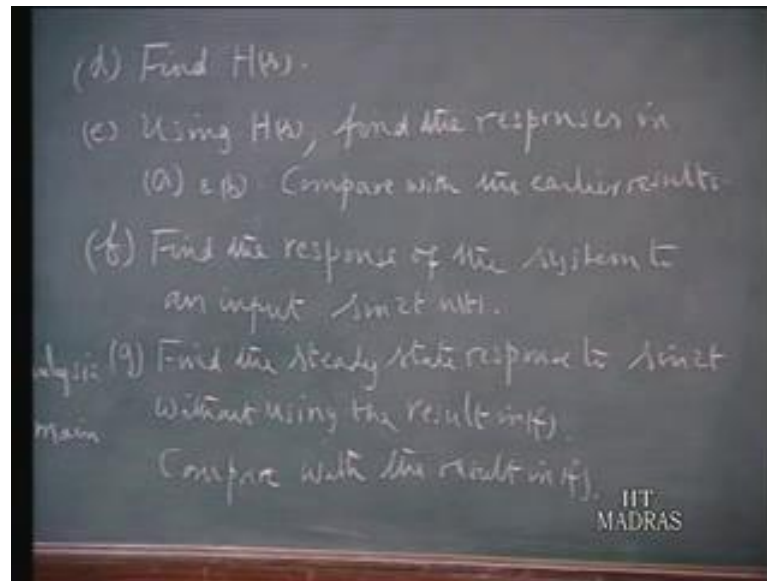
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Fifth problem involves the application some of the system concepts that been talked about. So let me, write this down there are the response of any initially relaxed linear time in variant that means; constant parameters system to a unit impulse $\delta(t)$ that is applied a t equal 0 is $4e^{-t}u(t)$ that means; the impulse response the system is given.

And you are asked to find a series of quantities fins the response of the same system, find the response of the same system that is: once again assuming to be initial relaxed to unit step using time domain analysis that is: you try to find out the convolution of the step and the impulse response b find the response to $e^{-t}u(t)$ in time domain that is: do not use Laplace transform. Find the response to this in time domain using impulse response as the difference.

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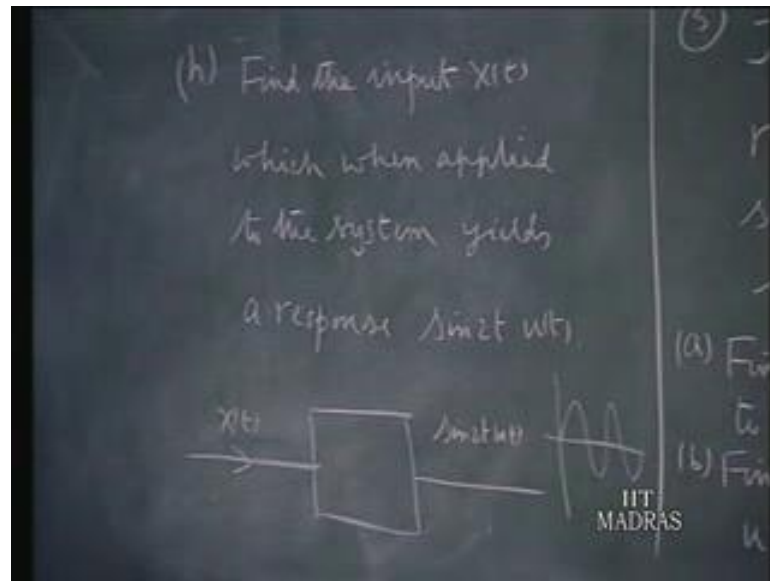


So, you use the convolution integral using h off t then c . Find the solution in b using step response. So, in time domain once again, we use the convolution integral involved in this illusion response a off t d find the system function h off s e using the system function h off s , find the responses in a and b that is: use Laplace transform method this time earlier you will use the convolution integrals. Now, using h off s find the response in a and b and compare with earlier results, f find the response of the system to an input $\sin 2t$ ut.

So, the input is a sinusoidal applied t equals 0 find the response of the system to be input offcourse, you use Laplace transform method for this quite convenient to use g , find the steady state response to this input $\sin 2t$ without using the result in f . So, you are asked to find out forced response for the system $\sin 2t$, with the forced response with an input $\sin 2t$ also happens to be steady state response. Because is the steady the sinusoidal is the steady state characteristics in does it the amplitude is maintain.

Therefore, the forced response to $\sin 2t$ is also the steady state response and that portion of the total response that; obtained in f you obtain independently using the frequency response function and compare with result in f that means; whatever, in the total response the particular portion corresponding to the steady state behavior must agree with this compare with the result in f then lastly h .

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Find the, input $x(t)$ which, when applied to the system yields a response $\sin 2 t ut$. So, what you want is you determine what type of excitation should be given to the system. So, that the output is $\sin 2 t ut$ that means, it must start from 0 like this. So, that is the output so, what type of input should be obtained in order to find out get this output.

So, this last problem illustrate: the ideas of system use the Laplace transform to the system analysis that has discussed in the last 2 lecture. It is quite a comprehensive package of questions that you are having. It covers more or less all the aspects that you discussed.