Networks and Systems
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Lecture – 28
Application of Laplace Transforms (3)
System Functions and Its Significance Impulse and Step Responses.

In the last lecture we worked out several examples, illustrating the application of the Laplace transform technique, to the solutions of transients in networks. We also made special mention, of the particular virtues that Laplace transform technique brings, to the solution of such problems. We compared the Laplace transform method the differential equation approach, and pointed out the various benefits that the Laplace transform technique will bring in. we also made a note that, where the initial conditions are specified at different points of time, not necessarily all t equals 0. The differential equation approach, will be probably more beneficial than the Laplace transform technique, because there is no provision in the Laplace transform of technique, to take care of specification of conditions, at different points of time. If you have such a situation then you once again land up in arbitrary constants in the Laplace transform solution, so there is no special advantage in that. Today will start with another example of a network transient problem, in which a discontinue source is present, and then later on we move to a discussion, of the application of the Laplace transform technique through a general system.

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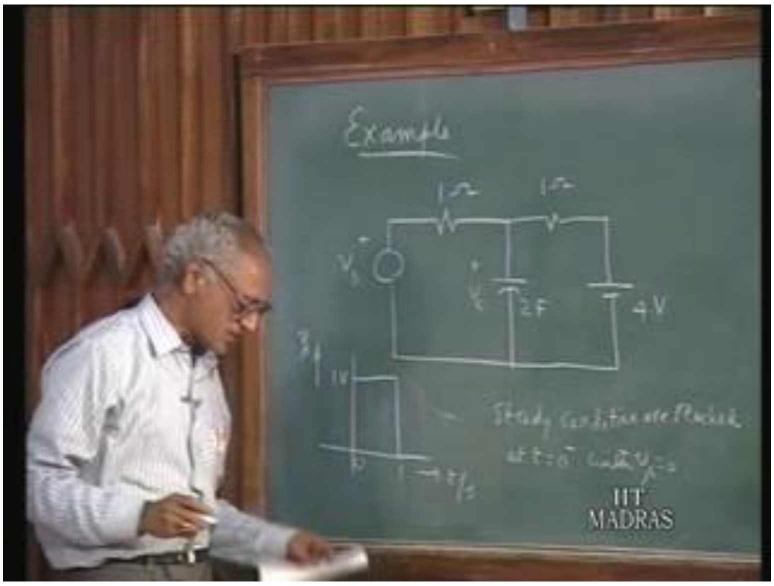

So, let this be a voltage source v s included in the circuit of this type. The capacitance of 2 F and dc source of 4 volts, and this v s is a discontinuing a source, having one volt a pulse lasting from 0 to 1 second. And we will also assume that steady conditions are reached at t equal 0 minus with v s equal 0. So, before the pulse is switched on, v s is off course 0 and that time we assume the steady state conditions are reached. The question that is asked is, find v s find v ct for t greater than or equal to 0. This is the question that is asked.

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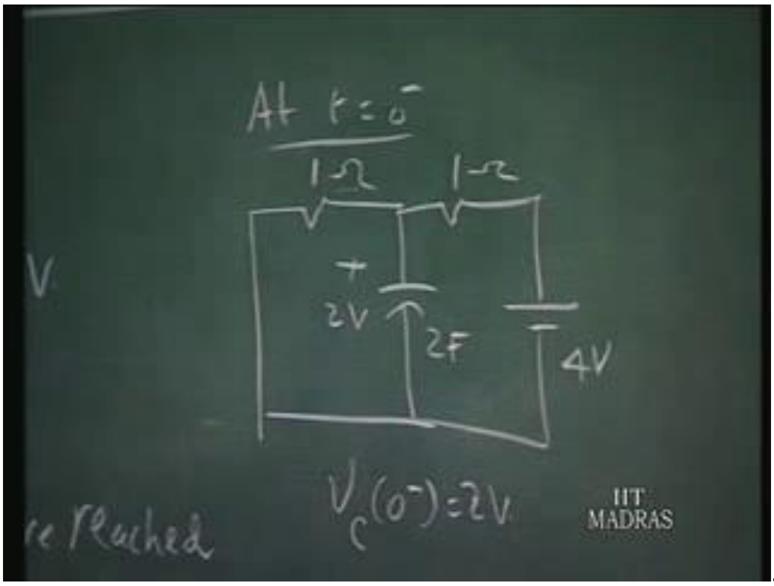

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Now, we set up the transform diagram for this. Before doing that we need to know, because there is no reactive element the capacitor we should like to know; the 0 minus value the capacitor voltage. So, to do that let us see the system, the various parameters the system at various variables in the system at t equals 0 minus. At t equals 0 minus this voltage source is 0. You have 1 ohm the systems, capacitor, another 1 ohm assistance and a dc source. This is 4 volts, this is 1 ohm, this is 1 ohm, and this is the capacitor of 2 ferrates. So obviously, if the steady state conditions have been reached in the circuit, this 4 volts drive up on 2 ampere to 1 ohm resisters in series, and there will be a 2 volts develop across the capacitor. Therefore, v c 0 minus is 2 volts. So, that is the voltage initial condition of the capacitor that you have to use in solving this problem.

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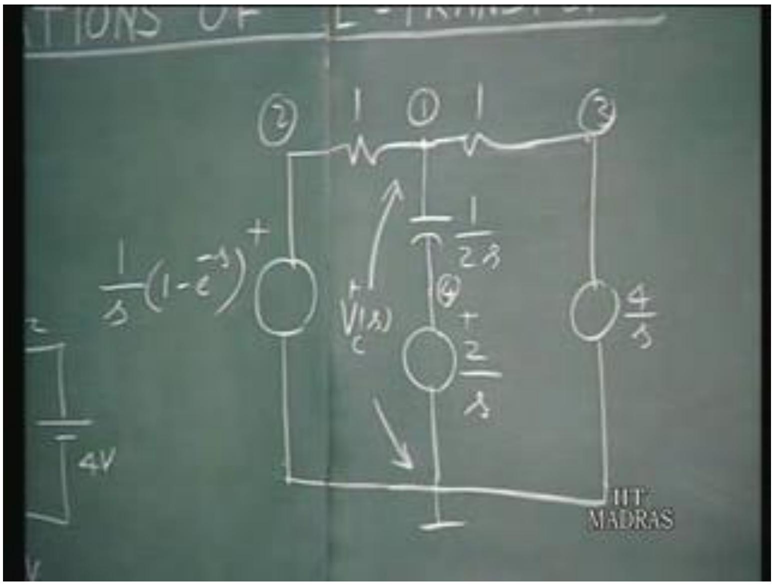

So, let us from the transform diagram of this. Transform diagram of this would be, for the voltage source, you have the Laplace transform of this voltage source. This is u t minus u of t minus 1, this is a pulse function. Therefore, the Laplace transform of this to be 1 by s times 1 minus e to the power of minus s; that is the voltage source. In addition we have this resistance another resistance, so 1 ohm, and then a capacitances whose impedance is generalized impedance 1 over 2 s, and then a source representing the initial capacitor voltage this is 2 volt source. Therefore, this will be 2 up on s, and then you have the Laplace transform of this dc voltage source, which is 4 up on s. Now, the capacitor voltage is what is required. So, it is the voltage across these 2 nodes. This is v c of s. As pointed out in the last lecture, when you interesting to finding the capacitor voltage, you should not try to measure the capacitor voltage across this portion only, because it is the totality of this 2 together, is what represents this 2 ferrates capacitors, including the initial condition source. So, when you want the capacitor voltage, it is the voltage between these 2 points, which has to be calculated. Now, a simple way solving for this would be, take the load equation approach, take this as the datum node; and call this 1, this as 2, this as a 3, and perhaps this as 4, and therefore, if you find out the voltage of node 1 that will yield you a capacitor voltage which is the voltage of course with reference to datum node, node 1 represents to datum node.

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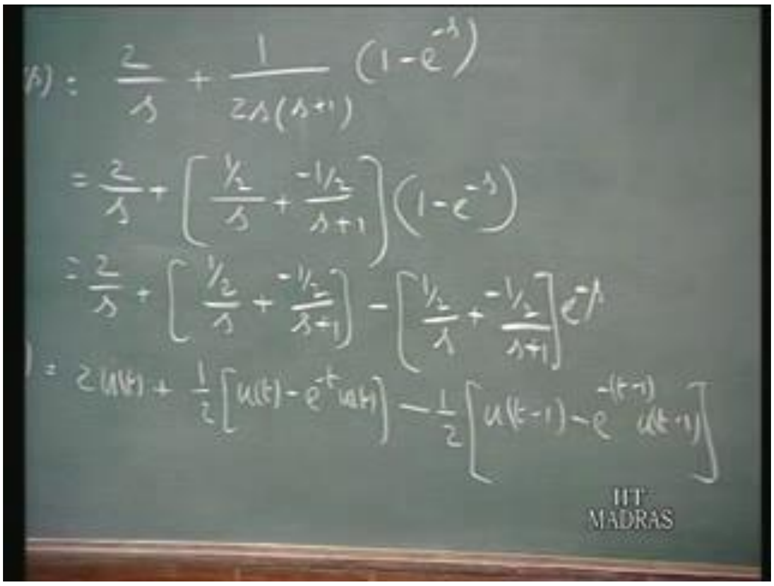

So, write in the node equation at node one v c of s times. The sum of the admittance is connected to be this node; 1 plus 1 plus 2 s minus the coupling term with reference to voltage with other nodes. So, with reference to node 2, we have 1 by s times 1 minus e to the power of minus s; that is the voltage of node 2, and the coupling admittance between node 1 and 2 is 1; therefore, times 1 minus node 3 has the voltage 4 up on s, and the admittance of the element connecting nodes 1 and 3 is 1 times 1 minus node 4 as the Laplace transform voltage 2 up on s, and the admittance between one and 4 is 2 s, generalized admittance equals to 0. So, if you the terms here are known expect v c of s. So, if you transfer this quantities on the other side, and find out the expression for the v c of s, it turns out to be 2 up on s plus one over 2 s times s plus 1 times 1 minus e to the power of minus s; that is what it will be. So, you can write this as 2 up on , and make the partial fraction expansion of this portion already, because later on we interpret, this one minus e to the power of minus s, how it reflects in the time domain.

So, will make the partial fraction expansion of this only, that will be (()) of 2 terms half and minus half times 1 minus e to the power of minus s. So, essentially therefore, you have three quantities we deal with; 2 upon s is 1 plus half upon s plus minus half upon s plus 1; that is one group of terms, and the same group of terms multiplied by e to the power of minus s. So, if you find the inverse Laplace transform of each one of this groups, and add them of that will be the v c of t. So, v c of t would be, this will be 2 times u t. Here you have half u t minus e to the power of minus t u t; that is the inverse Laplace transform of this, and you observe that this is the same thing as this except multiplied by e to the power of minus s, which means whatever time function we have here, is delayed by one second; that all means. Therefore, it is minus half of u of t minus 1 minus t minus 1 u of t minus one, so that is what it could be. You can sketch this v c of t. So, v c of t would be. This 2 u t will be a stuff function like this. This is 2 times u of t, and this is u t minus e to the power of minus t u t. So, start with the 0, and exponentially claims to a value equal to half.

So, this is half of u t minus e to the power of minus t u t, there is what could be. As for as this portion is concerned, it has a negative value, is a negative of this, but it starts at t equals one. Therefore, it will be something like this, and asymptotically reaches half. So, if you add all this you will get, some curve like this, in the case. It has this 2 will add up to 0 ultimately. So, this is your total circuit. So, that is v c of t. So, this problem illustrate, how you can handle, the solution where there is a discontinuous time function, and how the discontinuities, or time function is of course this is discontinuous, and how the finding out inverse Laplace transform, when you have the e to the power of minus s are such terms are present, how you can find out inverse Laplace transform. Make using the property, that e to the power of minus s means, that particular function of time is delayed by one second. If it is the e to the power of minus t s is delayed by t second.

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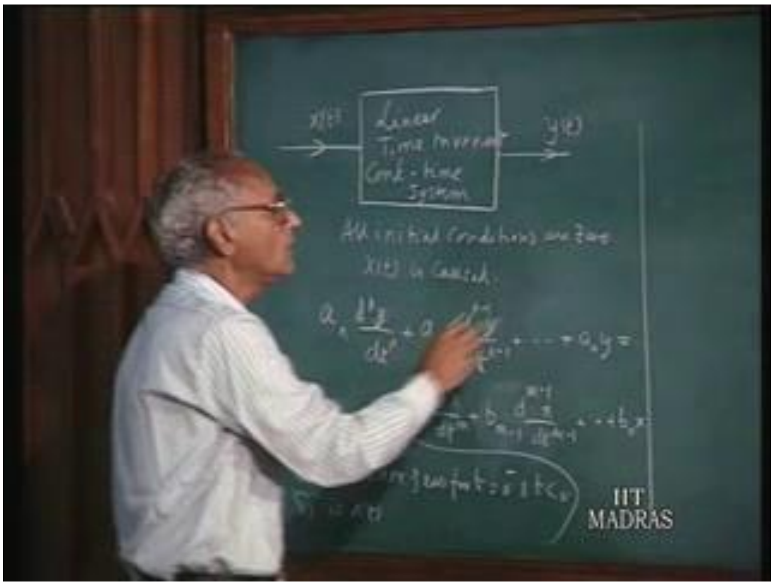

To discuss the application of the Laplace transform technique, to a general system, let us consider, a system represented by this black box. This is a linear time invariant, continuous time system with an input x t and an output y t. We shall assume that all initial conditions are 0; that is the output response quantity y t and all its derivatives are 0 prior to the application the input x t, will also assume that x t is causal; that means, x t is identically 0 t less than 0. Now, we have taken a single input single output system, and if there are really a number of inputs multiple outputs. We can find out the response to each of the individual inputs in term and use the super position principle. Therefore, there is no loss of generality involved, in our taking a single input single output system.

We have seen that such a linear timing variant continuous time system, is in general described by differential equation of the type; a n d n x d t n plus an minus 1 the n minus 1 derivative, sorry this would be y output quantity y d n y d t n d n minus y d t n minus 1 on up to a not y equals dm x d t m plus d minus 1 x d t minus 1. We have a coefficient d m minus 1 here d minus 1 x by d t m minus 1 up to b naught x. This is a n order differential equation with cost and coefficients. Now, we assume that y t and all derivatives are 0 for t equals 0 minus and t is less than 0; that means, identically the circuit is dead, as for as the response quantities is concerned, and so is x t, because x t is a caution function is identically 0 for all negative values of time. So, under these assumptions, we can take the Laplace transform of the all the terms on the left hand side and the right hand side. in doing since all the initial conditions are 0, if the Laplace transform of y is y of s, the Laplace transform of d n y d t n is simply s to the power of n times y of s.

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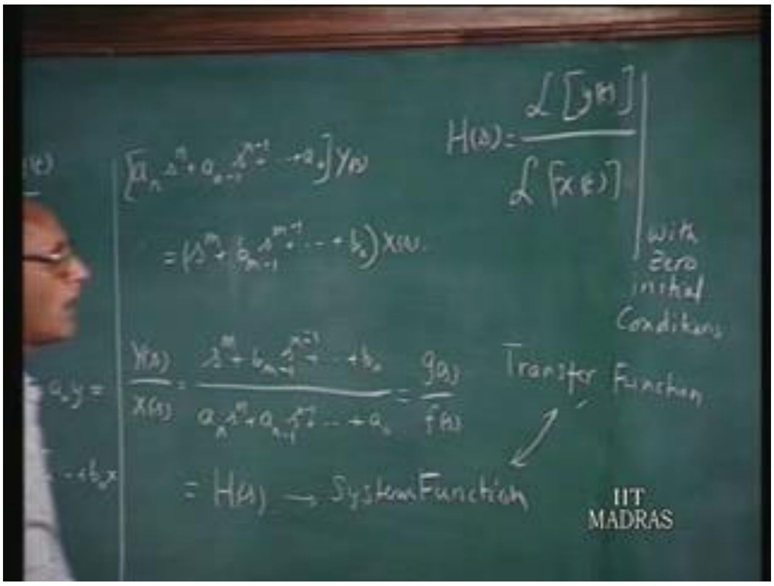

So, that makes the transforming this equation a very simple task. So, you take the Laplace transform of that, you have a n s to the power of n plus a n minus 1 s to the power of n minus 1 times a not plus a not times y of s equals the Laplace transform of the side of equation, which is s to the power of m plus a b m minus 1 s to the power of m minus 1 on up to b not multiply x of s. So, that is the differential equation, after transforming its after make in the Laplace transform of the differential equation is converted in algebraic equation, relating the transform of the response quantity to transform the important quantity. Now, the ratio of y of s to x of s, is therefore, the ratio of 2 polynomials s to the power of m b m minus 1 s to the power of m minus 1 plus b naught divided by a n s to the power of n a n minis 1 s to the power of n minus 1 plus a naught. This we can put ratio of 2 polynomials g of s over f of s.

Now, I have taken the leading coefficient here to be 1. It does not entire any loss of generality, because even if there was leading coefficient here which is different from one. We can divide all the terms by the term and make this 1. Therefore, there is no loss for generality involved in taking the leading coefficient here to be equal to 1. Now, this y of s or x of s is derived in this fashion called h of the system function. So, formally we can say the system function h of s, is the Laplace transform of the output quantity y of t by the Laplace transform the input quantity x of t with 0 initial conditions. What we mean by the 0 initial conditions is, that the response quantity and all its derivatives are going to be 0 and 0 minus, and the before the application is input x t. So, in this particular case we have to extend to be a causal function which is identically 0 of t equals 0 minus, and then until the time the response quantity in all its derivatives are going to be 0.

Therefore, each of the various transformed in to Laplace transform of domain d n y d t n goes as simply s to the power of n times y of s. The system function is also referred to as transfer function in literature. Particularly in the context of networks, control systems and so on. This is also refers to as transfer function, another name for this. So, essentially when we talk about the system function in the Laplace transform situation like this. We are change that is the ratio of the Laplace transforms of the response quantity the Laplace transform of the input quantity, with 0 initial conditions appear to the application to the input. You recall that earlier we said in our introductory lectures that system function, is the ratio of the first response of the system, to the input when the input is an exponential signal. In other words earlier definition was h of s was defined as, the forced response y f of t to an input of the form e to the power of s t.

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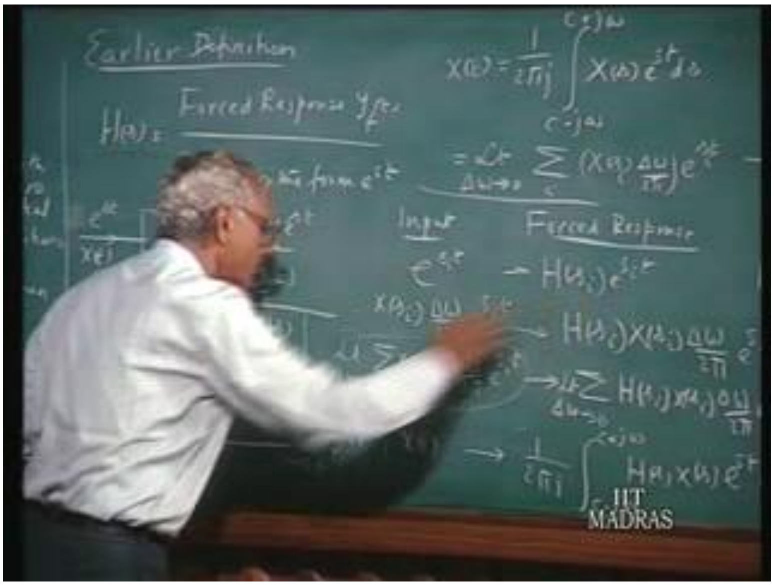

So, we said at the time, that if you had a system per which you have an input e s t, and then the x t is equal to this. Then the output, the forced response output will be h of s times e to the power of s t. Now, you defined a system function in different way, are they the same. So, you would like to now analyze this, and then tie them up, and see what to show both of them or indeed the same. Now how do know about it. we know that x of t for the Laplace transformation theory, is related to x of s by 1 over 2 pie j c minus j infinity to c plus j infinity of x of s e power s t d s. What it means is, limit as delta omega goes to 0 of summation of elementary signals x of s delta omega 2 pie x of s is the coefficient density x of s can delta omega over 2 pie is the coefficient of the signal of this type e power s t summation over all such equals lead x t.

And how do we take this elementary component, you take which belong to contour here, and take any particular equation s i and then take the delta omega, and then if should take the exponential signals corresponding to each such element, that will add up to this. In other word we can write here s i of t, s i in one particular thing and summed up on i. So, if this is the situation, then we can think of any arbitrary input x t as the sum of exponential signals of this time. So, let us see how it goes. Suppose we have the input, and the forced response. So, if the input has been e power s i t, according to our earlier discussion, the forced response will be h s i e power s i t. Now, on the other hand, we have now not a single s i of t, but a whole lot of this s i of t, each we get by a coefficient like this. Therefore, if i have x s i delta omega over 2 pie e power s i t that would give me by linear the principle, because is the linear system. We have h s i x of s i delta omega over 2 pie e power s i t.

Now, if you this is one particular complex frequency signal. We can tiny amplitude like this. Now you have whole lot of this complex frequency signal. Therefore, we take the sum on i x s i delta omega over 2 pie e power s i t that should give me correspondingly, limit as delta omega goes to 0 of x s i x s i delta omega over 2 pie e power s i t. This of course limit as delta omega goes to 0. And this is exactly what x t would be, x t we have seen from the Laplace transform theory is little omega goes to 0 of x s i delta omega over 2 pie e power s i t. So, all this tiny exponential signals, ranging from minus infinity to plus infinity along gone which contour, if we add of them that will fetch you x of t; that means this is really x of t. So, if you have this x of t, and the combination of all such signals can be put in the integral form 1 over 2 pie j c minus j infinity to c plus j infinity of. Now s is a running variable no long discrete values; x of s times x of s e power s t d s, because d s is j times delta omega. So, this is what it will be.

So, from this we have observed, if you take this entire quantity if you call that y f of t. This y f of t has 1 over 2 pie j c minus j infinity c plus j infinity of x of s h of s e to the power of s t d s; that means, the Laplace transform of y f of t will be x as h s times x s; that means, we have y f s Laplace transform is h s times x s. So, according to our earlier definition, we found that the sum of the forced responses will have a Laplace transform which is h of s times x of s. But in our Laplace transform theory, we are saying the total response y of s is equal to h of s times x of s. So, is it that is the total response y t is the same as y f of t that you have got obtained here. The answer to this is, that they turn to be the same, because this Laplace transform inverse Laplace transform integral, provided you take s in the reason of convergence of the transform h of s times x of s. It turns out from the Laplace transform theory, that plot y f of t, this is going to be identically 0 for negative values of time, and it any some other value like this, for t greater than 0. This is y f of t.

So, according to the Laplace transform theory y f of t is going to be identically 0 for negative values of t and it have only values for positive value, non 0 values for positive values of t. And in our system, we also have y t which is identically 0 for negative values of time. Therefore, y t aggresses with the y f of t identically for negative values of time. Therefore, there is not be any other term we introduced that spoil the initial conditions of the problem. Therefore, this y f of t is also equal to y t. Consequently there cannot be any other extra term that is present. So, this y f of t is indeed equal to y t therefore, y f of s which is h s times x s, is indeed the same as y f of s h s times x s; therefore, y f of s obtained in this fashion is the same as y of t. So, the discussion therefore, gives us this information, that the sum of the forced response as calculated in this manner, will also yield the total response. Total response in sense there cannot be any other term as far as the negative values of time is concerned.

But as far as positive t is concerned, whatever you calculate for y f of t will include the finite number of transient terms, and the finite number of force frequency terms appropriate to the given h of s and x of s. There are finite numbers and in the conventional sense what we talk about what we meant by transient term transient response, and forced response pertaining to x t and ht, as for as t greater than or equal to 0 will always be obtained y f of t. Even though your calling this forced response, the forced response y f of t includes the transient and the steady state responses, that terminate from t equals 0 onwards. The reason why we are saying this force response, because all this tiny exponentials here, start from minus infinitive onwards. So, if there all starting from minus infinitive all this exponentials are starting and each of them produced response, and the some other force responses, agrees with the total solution for negative values of times; that means, there cannot be any other additional terms starting from t equals minus infinity onwards, but y f of t which is obtained here as for as this part of the solution is concerned, it includes both the transient and steady state terms, which we talk about next conventional fashion. So, h of s that was described earlier h of s, is defined now one and the same therefore, we do not have to make a things between this 2. And the h of s place very important role in the system studies. It has got various useful properties and various interesting properties, and it will be our task now to take up a discussion, of the various important aspects of the system function h of s, and that will take up next.

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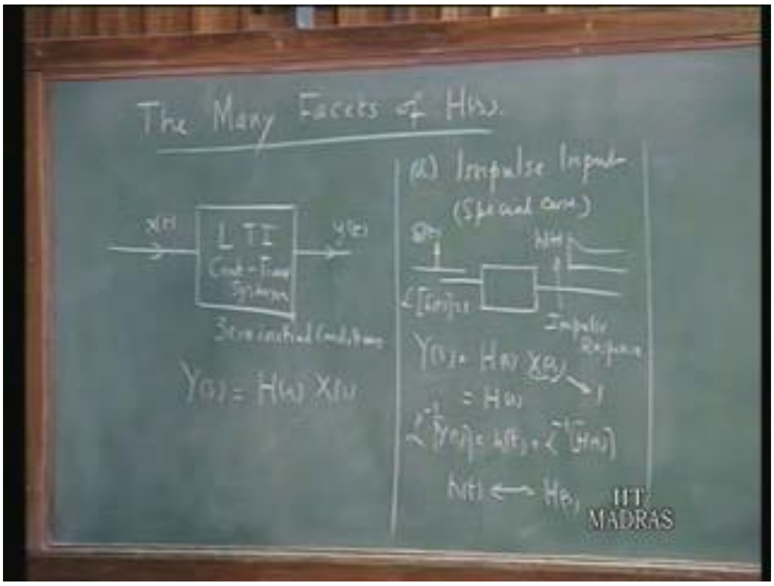

So, the system function h of s as many important connotations. So, it will be our task now to look at these connotations. So, there is a many facets to this the system function h of s, and that is what you like to steady. So, we are talking about a linear time invariant continuous time system with an input x t and an output y t, and the system function enables us, to relate the output to the input, not by means of the differential equation, which is complicated, but by a simple algebraic equation in the Laplace transform domain. We also assume here that 0 initial conditions; that is important. So, once you have the 0 initial conditions, the output and input are related by a pure algebraic equations, where h of s is in general a rational function; a ratio of 2 polynomials. And that differential equation is converted an algebraic equation of this type, is the central advantage of the transform technique, because which Laplace transform is very prominent technique.

Now, let us look at the various examples; a, suppose we have impulse input; a special case, not a general expect t, but let us say we have any impulse input. So, what we are talking about here is, input is delta t, and you get an output which perhaps like this, will call that h of t. So, corresponding to a impulse input impulse at the origin, you get an output h of t h of t is called the impulse response, which we already we observed earlier. Now, applying our formula here, y of s whatever output you get here, is related to the input (()) by the h of s the system function times x of s, x of s happens to be 1, because x t happens to be delta t x of s has 1. The Laplace transform of delta t as we know equals 1. So; that means, the output is simply h of s, but the output you are calling h of t; therefore, Laplace transform, or I will write this y of s itself is h of s. So, the inverse Laplace transform of y of s, which around particular case is h of t, is the inverse Laplace transform of h of s.

In other words the impulse response h t and the system function form the Laplace transform pair, and it is anticipation of that, you use the symbol h for the impulse response, and the H for the system function. So, this is a very important result. The system function h of s of the Laplace transform of impulse response; that means, you apply unit impulse to the system, is 0 additional conditions; unit impulse station like t equals 0. Whatever response we get call to the h of t, that will be the inverse Laplace transform of h of s, or if you know impulse response of h of t, you can find out h of by taking the Laplace transform of h t. This 2 form Laplace transform pair.

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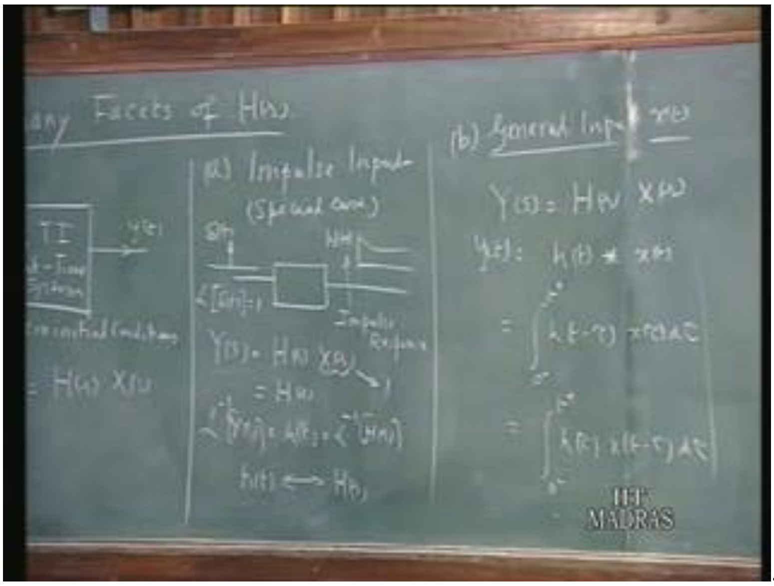

Now, using this information general solution in time domain, general input x t, suppose you are having, and you want to find out the output in time domain. We know that y of s equals h of s times x s. Now, by the convolution property, we know if y of s is the product of two quantities h of s and x of s. Then y t we know, is the convolution of the time domain quantities which are the inverse Laplace transform of this two. The inverse Laplace transform of h of s is h t the inverse Laplace transform of x of s is your x t. So, in other words the output is the convolution of the impulse response, and the input quantity, which in fact you already observed in our preliminary discussion. We already noted this. I am just trying to relate this to the Laplace transform equation in this manner. What does it mean this is the convolution integral. It means 0 to t h of t minus tow x tow d tow. Alternately if you wish, you can write as 0 to t h tow x of t minus tow d tow. this are you can interchange roles of this, both this, or the equivalent to each other, and as I mentioned earlier on if it may turn out, that x of t you have impulse such origin h of t is the impulse the origin.

So, to take care of the impulse of the origin, in general it would be advisable to take 0 minus to t plus. If to take care of the possibilities that x t are h t may have impulse. If you have do not have impulse there is a n does not matter whether you take 0 to t a 0 plus or 0 minus. Now, these integrals are called convolution integrals. They are also called literature super position integrals, because after all your super posing the effect of x of t by treating them as a summation of the different impulses, something which have already discussed. So, the significance of the convolution integral as a super position of, the input considered as a sum of impulse something which we already discussed. A third aspect; I also mentioned in our earlier discussion that step response is also a way of characterizing a linear system.

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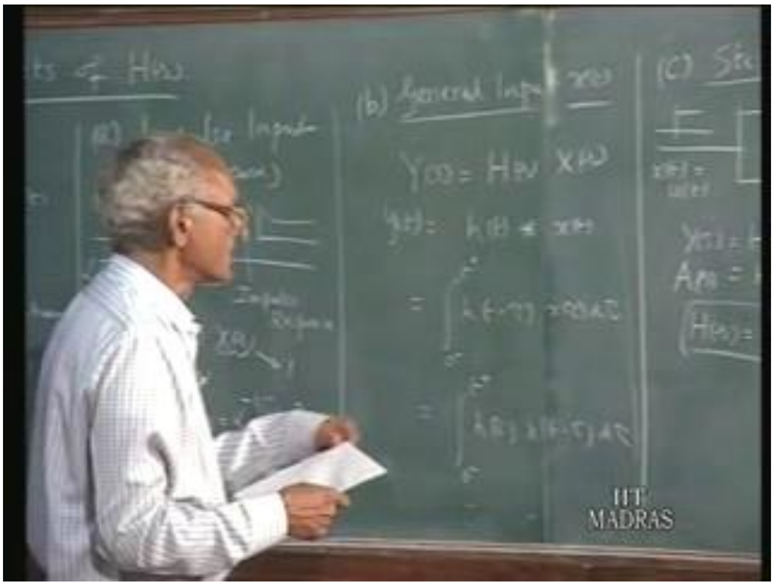

So, let us see how it is related to a system function h of s. So, in step response we are thinking of unit step applied as a input. So, x t is u t, and the corresponding output whatever you are having here, is given a special symbol a t. And this a t is called the step response, sometimes it is called indicial response. Now, using this general equation y s equals h s times x of s. So, in our particular case this h of s and x of s happens to be the 1 over s, because the Laplace transform of a unit step is 1 over s. And y of s the particular Laplace transform of the step response that we are getting (()) called that a of s; that means, if a t has the Laplace transform of a of s, then a of s is h s times one over s or h of s equals s times a of s. So, the relation between Laplace transform of the step response and the Laplace transform of the impulse response is given like this; h of s times 1 over s is a of s, or s as times is equal to h of s.

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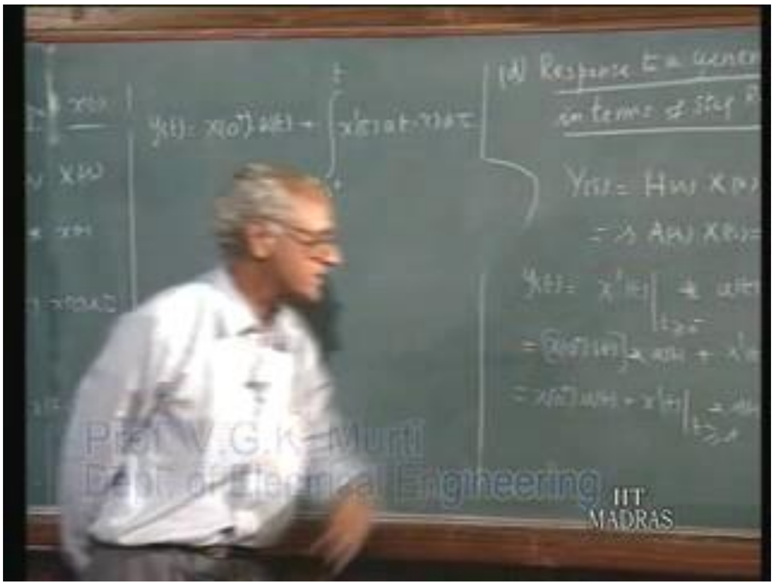

Now, using this step response, let us see how we can find out the response to a general input x t in terms of step response. Now, if x of t is the general input, y s will be h of s x of s, is a fundamental rule that we are having, but h of s is s times y of s. So, I can write this as s times a of s times x of s; that is what we are having. I can write this as s times x of s times a of s. Now, y of s therefore, is the product of 2 Laplace transform; s times x of s and a of s. the time function corresponding to this is the indicial response at. What is the time function corresponding to this, is d x by d t, the derivative of this, taking into account that starting from t equals 0 minus. Therefore, y t can be written as x prime t. When you take the derivative of x prime t, starting from t 0 minus; that is if x t is like this, this is x 0 minus, this is x 0 plus. When you taking the derivative you must take this transition also into account, and that is in meaning of this x prime t consider from t equals 0 minus onwards, convolved with a of t which is the inverse Laplace transform of that.

Now, you note that x prime t, if this is the variation x of t this is x t, x prime t can be written as in the 0 minus 0, because we always talk about causal inputs. So, the causal input 0 jump to 0 plus; therefore, the derivative you get x 0 plus times delta t plus the rest of the derivative taking starting from t equals 0 onwards 0 plus onwards. So, x prime t starting from t equals 0 minus, can be thought of as an impulse function, which is comes because of this jump plus derivative from t equals 0 onwards. So, I can write this further as x 0 plus times delta t. this convolve with at plus x prime time which will considered only from 0 plus onwards, convolved with a; that is what we are having. And we earlier observed that any function of time convolved with delta t, means is equal to the same function itself. Convolving with delta t does not change the function; therefore, I can write the as x 0 plus times a t.

This you can also visualize easily, because the Laplace transform of delta t is 1. Laplace transform of this is a of s, then the convolution as the Laplace transform 1 times a of s the must Laplace transform gives back as a of t plus x prime t t greater than equal to 0 plus convolved with a t. So, this is the result that we get. Now, let me put this down in the form of integral more explicitly. So, what we are here is, y t therefore, is x 0 plus times a t plus this one I will put in the form of the integral staring from 0 plus to t x prime tow a t minus tow d tow; that is what you have; that is the meaning of that; that means, you response to any arbitrary input x t can be obtained if you know the indicial response for this step response, you can find it out other this comes from of course from this Laplace transform s a s times x of s.

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Now, one can also alternately later have some. First of all before that physical meaning of that, I will just briefly outline without finding too much time on that. Suppose this is x of t you can thing of this x of t as a step function, to start with this is x 0 plus times u of t plus a number of small steps occurring later on values of time, and each of this steps will have something related to x tow delta tow delta tow, something like it will be there.So, we have the given x of t can be decomposed, as a number of steps starting with x 0 plus u t, and subsequent steps you have a height, depending upon the derivative and the interval that you are taking. So, each one of the x prime t delta tow, is the height of the step, and that occurs the point tow; therefore, it means for delayed step response. So, summation of all those things will be representing by this integral, and the response to the initial step is represented by this term. So, that is the physical significance of this particular formulation of the output in terms of the indicial response. So, this is the physical significance of this.

Now, there are impulse as in the step response as the origin; suppose a of t has the impulse and origin. To take that in to account input as t plus, t plus to take into account impulses. May be I will write this separately, because instead of clipping the picture you can say, upper limit taken as t plus to include impulses at the origin if any in a t. So, it is possible the step response may included at the impulse at the origin to take that complete impulse in the origin, to take that impulse into account the upper limit is taken as t plus. And alternative version, after all i associated with s with x of s. I could have taken associate s with as instead of x of s I could (()) s times a of s. So, in alternative version would be y t will be a 0 plus times x of t plus 0 plus to t plus a prime tow x of t minus tow d tow.

So, that is an alternative way of representing this, because after all x a s the symmetrical in a s and x s. So, whatever role x t plus here, a t will also play here. So, these are alternate. Now these 2 integrals what we have here, are also referred to super position integral literature. They also refer to Duhamel's integrals. So, this are alternative names for that, even the convolution integral terms of impulse response is also called super position integral, that a literature this is called the super position integral. This are also super position integral are Duhamel's integrals Duhamel's integrals is of course particularly with a step function. Another property of the system that you like to make a note of. This is the significance and poles of poles and zeros of h of s.

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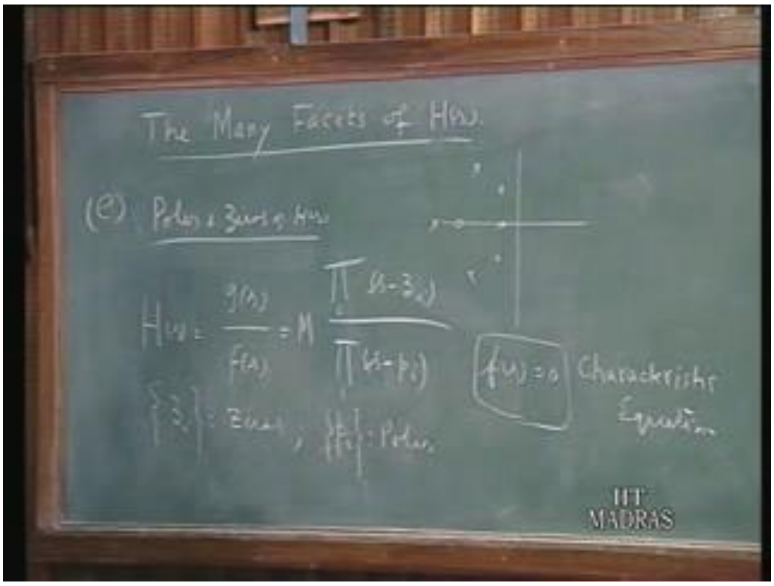

So, h of is the ration function as mentioned as the ratio as 2 polynomials. Therefore, it is ratio of g s over f of s. So, it is in general in constant multiplying factor, and product of several such factors sum down i and in the denominator we have products of factors sum down i. And the set z i are called zeros of this h of s, and the set of p i values are called the poles of this something which we already mentioned. So, h of s is specified by the locations of zeros and poles, as well as the constant multiplying factor m. So, the poles in zeros are locate are general impulse in the complex plane, by the z location z i location are indicate by zeros, pole locations by crosses. And it has to be kept in mind that; since g of s and f of s are polynomials with real coefficients of s. whenever a complex pole occurs its conjugate also must be present. Similarly a complex 0 is always accumulate conjugate; that means, complex zeros and poles occurring conjugate pairs not singly. As far as the real poles are concerned, there is no problem, it can appear with a unit multiply be seated. Now, an important factor that one would notice in solving a system is. Suppose we have different types of output that you looking for, and naturally you have different system functions, relating the different outputs for corresponding input. it turns out suppose h one of s h 2 of s h 3 of s depending up on the y’s that you are talking about; y 1 y 2 y 3. All of them have got the same f of s in the denominator. So, f of s equals 0 is what is called the characteristic equation of the system. This is the characteristics of the system.

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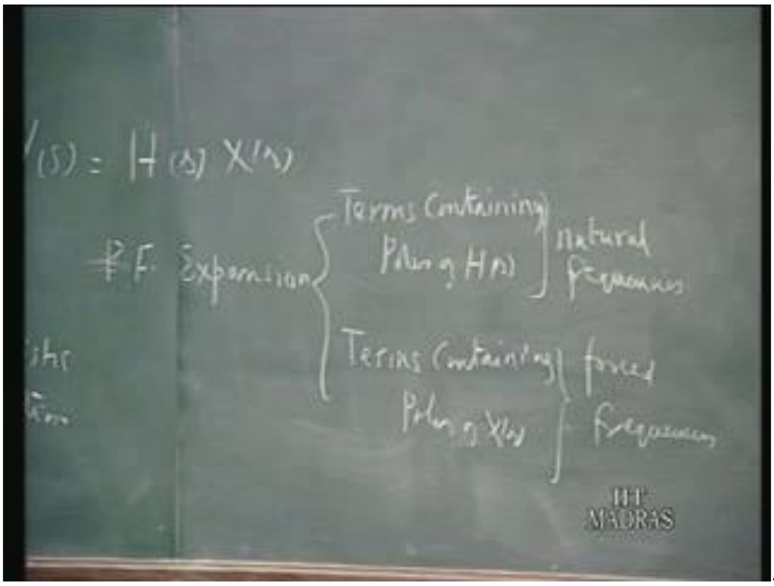

So, no matter what response quantity you are looking for, the denominator here f of s is the same, for all transfer functions, for all system functions, for a given system this called the characteristic function equation. And when you talk about y of s here your h of s times x of s, and you make the partial fraction expansion of this, in y of s. Then the partial fraction expansion of y of s, will involve terms containing poles of h of s, terms involving poles of x of s. In the partial fraction expansion what we have is, terms containing poles of h of s are representing the natural frequency system. They are the PICL. There are the roots of the characteristics equation. So, the natural frequency of the system, are the roots of the characteristics equation. when I talk natural frequency I am talk about frequency complex domain there roots of the characteristics equation, they are the poles p i. On the other hand you are h the terms containing, poles of h of s x of s are the forced frequencies.

They are the frequencies that come as the result of the particular input. So, when you have got y of s, when you make the partial fraction expansion, you get some terms corresponding to the poles of h of s, sometimes corresponding to the poles of x of s. The terms contain the poles of x of s, if you add the time domain response of all of them, this is the force response. On the other hand if you take into account only the poles of h of s, they represent natural frequencies, the summation of all the corresponding time domain expressions, represents the natural response the system, and the total response is what you obtain take the inverse Laplace transform of this. So, whenever you want aggregate the total response is natural response and force response. We can always look at this, as emanating respectively from the poles of h of s by the poles of x of s respectively. As far as zeros are concerned, they do not have any particular connotation like this, the only vary the amplitude of this various components.

Suppose I have one particular response quantity e to the power of s t a. It is the amplitude of the multiplying factor, of the complex frequency term; that is dictated by the zeros. The poles however represent the nodes, the time expressions, the complex frequencies, dictated by the poles, where as the zeros will vary the amplitude of the various terms. So, in this lecture what we have done so far is, acquainted ourselves the definition of the system function in terms of the Laplace transform. You recall that the system function is defined as the Laplace transform of the response quantity, to the Laplace transform of the excitation quantity, with 0 initial conditions, prior to the application input and the system function is general the ratio of 2 polynomials, for the type of system that we are taking about number of parameter systems, which is f of s g of s over f of s. And we looked at various property of the system function.

The system function is the Laplace transform of the impulse response. We also saw the system function is also related to the Laplace transform of the step response. We saw the various types of convolution integrals, how to get the output for the general input in terms of the impulse response h of t, or the step response a of t. We also finally, looked at the nature of the ratio of two polynomials, which comprises, which specify the locations of zeros and poles of complex plane, and we mention that the poles of the h of s indicate the natural frequencies of the system. They are invariant for a particular system, and whatever type of output that is looking for in the system. They do not depend up on the input quantity. On the other hand x of s we have some poles which are characteristic of the particular excitation function that you have, dealing with at the point of time.

So, y of s which is the product of h of s have some x of s, will have poles partly coming from h of s, partly coming from x of s. The poles coming from h of s presents the natural frequency, and the associated response as the natural response. The poles coming from x of s are the forced frequencies, and the associated times response, is the forced response. More about the system function will be taken up in the next lecture.