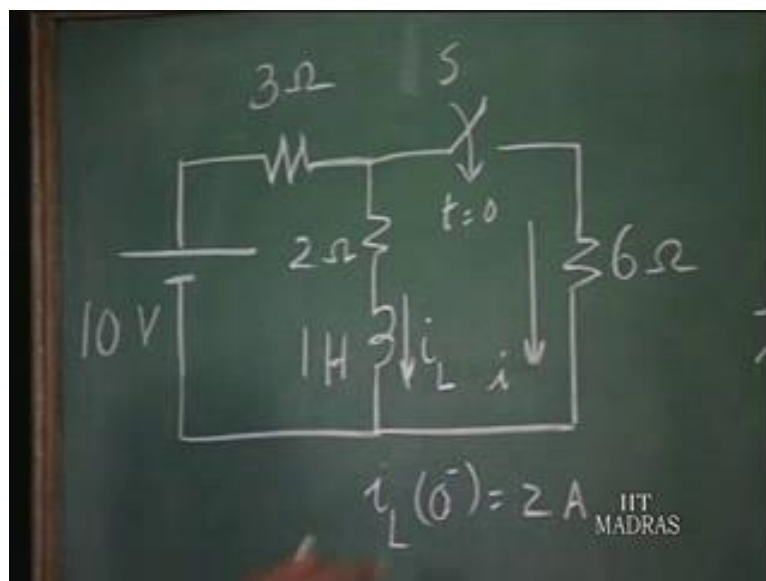


Networks and Systems
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Lecture – 27

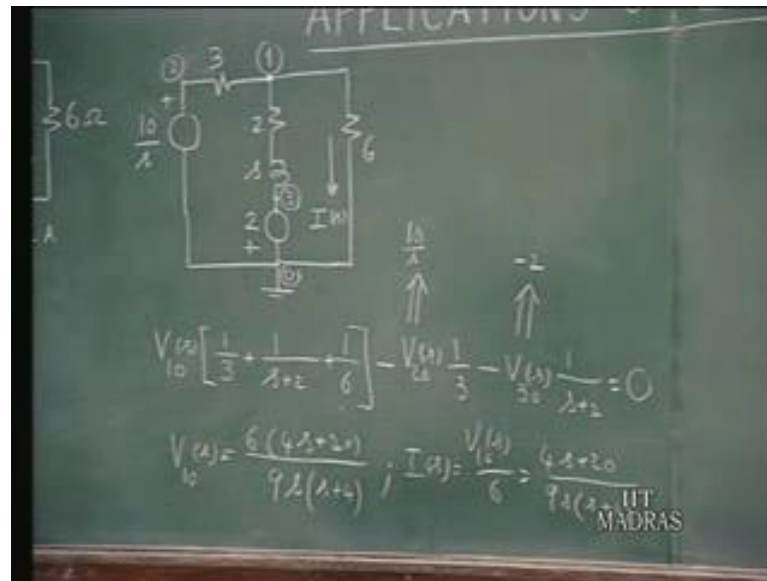
In the last lecture, we acquainted over selves with concept of transform diagrams, and how they help us, in writing down the equation are performance of a network, the Laplace transform domain.

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We took up this particular example, where the switch s is closed t equals 0, and you are ask to find out this current i . And then taking note of the current of the initial the 2 amperes. We replace that initial condition in the current by equivalent voltage source in the transform diagram, and we establish this transform diagram. As I mentioned in the last class, once we have the transform diagram, that can be analyzed in almost the same lines, as what you would observe in the case of dc circuits. Writing down the various equations, we took up in the last class; the loop current method for the solution of this network. We may as well the node voltage method. We can use super position technique. We can use thavinees theorem, whatever, all the, the whole gamete of techniques that is available, for solution of dc circuits or ac circuits can be applied to the transforms diagrams as well. Just for the sake of illustration, let me solve this same circuit by the node voltage method.

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Suppose I take this at the datum node, and if I solve for this node voltage, then you can find out the current in the 6 ohm resistor, by dividing that voltage 6. Therefore, I would like to use it node to datum voltage method. This is the node voltage to be solved for. Suppose this is node 2; the value of the node voltage with reference to the datum is 10 by s this is already known, and suppose I call this node 3, the voltage of node 3 the datum is already known. So, our aim is, to write the node equations to solve for v 1. So, there is only 1 unknown node voltage; therefore, let us say v 1 0 of s times the sum of the impedances connecting that node. One admittances, this is generalized admittances I am talking about. The impedance is 3 therefore; the admittance is 1 by 3. The admittances of this combination for entire branch; the impedance has s plus 2; therefore, the admittances is 1 over s plus 2. The impedance of 6 generalize impedance, the impedance is 1 by 6; that is the self node admittance that node 1. Therefore, the mutual terms you have to take v 2 0 of s, and admittance connecting node 1 in 2 is 3. Therefore, 1 by 3 therefore, that is the coupling term that we are having.

Further you also have coupling term with reference to 3 0 of s. So, the admittance joining node is 1 and 3 is, 1 over s plus 2. This must be the sum of the currents entering node 1, to the various currents of sources. In this particular example there is no such current source; therefore, is 0. Now, in this equation the only unknown is v 1 0 of s, because v 2 0 of is known; that is equal to 10 by s. v 3 0 of s is also known, since the polarity is plus with, the 0 as positive polarity reference to 3, v 3 0 can be written as minus 2. On

substitution of these values for v_2 and v_3 , we can get v_1 of s . v_1 of s can be calculated and shown to be, $6 \times 4s + 20$ divided by $9s^2 + 4$. But we are interested not v_1 of s , but the current in this 6 ohm resistance/ therefore, i of s obtained by dividing the v_1 of s by 6 . And when you do that this becomes $4s + 20$ divided by $9s^2 + 4$.

This exactly the same expression that we got for the current i of s , if the loop current methods, and once we get to this point further work is the same, as what we have done in the case of loop current method, so I will not do that, so from this you can get i of t . The point is that once we have the transform diagram, you can analyze the transform diagram, by any one of the known techniques that is available to you, whatever you (()) in the case of dc circuits can be applied to this transform diagram as well, except in terms of resistances we have generalized impedances, except in terms of tight dc voltage sources, we have transform voltage sources instead of dc current sources you transformed current sources. So, everything is in terms of s , but otherwise the principle, the technique is the same.

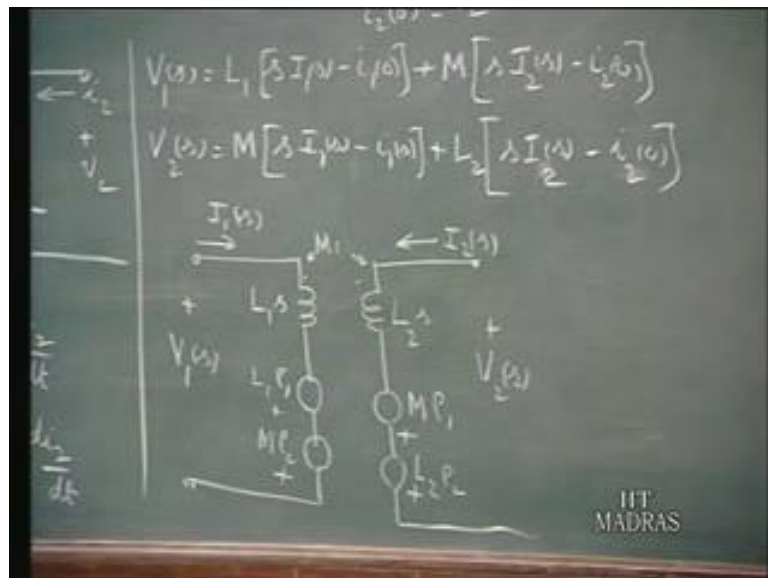
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Now, next question that you like to ask ourselves is, how do deal with mutual inductance. We have so far addressed ourselves, to finding out the generalized impedances r and l and c elements. What we do when we encounter mutual inductances. So, suppose I have two couple coils, having self inductance l_1 and l_2 . These are the dot

points, and I have a mutual inductance m between them, and let me say the voltages and the currents, are the 2 coils are indicated as follows, as given here. Then in time domain the equations are performance of this mutually coupled pair of coils is $v_1 = L_1 \frac{di_1}{dt} + m \frac{di_2}{dt}$, because the two currents, both the currents into the dot points therefore, beside of the mutually induced m is the same as the self induce m ; therefore, this is plus this is also plus v_2 likewise, is m times $\frac{di_1}{dt}$ plus $L_2 \frac{di_2}{dt}$. These are straight forward standard equations, governing the behavior of the pair of couple coils which (()). When we transform this equations, in term by term we transform. So, v_1 will have Laplace transform $V_1(s)$ of s .

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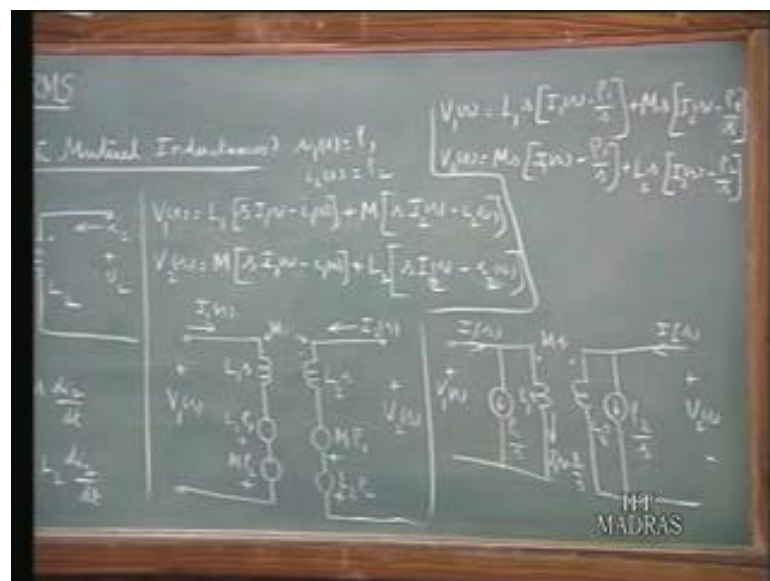


When you transform this, this will be $V_1(s) = L_1 [s I_1(s) - i_1(0)] + M [s I_2(s) - i_2(0)]$; that is the Laplace transform of $L_1 \frac{di_1}{dt}$. Similarly, the Laplace transform of $m \frac{di_2}{dt}$ be $m [s I_2(s) - i_2(0)]$. Second equation will be $V_2(s) = M [s I_1(s) - i_1(0)] + L_2 [s I_2(s) - i_2(0)]$. So, this is straight forward application of the transforming this equations. Now we have to draw a transform diagram, which repeats these equations. So, what you have now is, $V_1(s)$ of s is $L_1 s I_1(s) + M s I_2(s) - L_1 i_1(0) - M i_2(0)$. Therefore, if I have in the transform diagram; for the first coil $V_1(s)$ of s is the transformed voltage, terminal voltage the coil, and the coil has the self inductance L_1 and it has got a generalized impedance $L_1 s$ of s ; this is self impedance of the first coil. And you have 2 sources voltage sources represents the initial currents in the inductors therefore, this is $-L_1 i_1(0) - M i_2(0)$.

write; as rho 1 as i 2 0 as rho 2 to simplify my notation. So, l 1 rho 1 and minus mi 2 0 therefore, you have another term m rho 2. So, you have 2 voltage sources, representing the initial currents in the inductor, and the second coil likewise we have a representation like this. Where v 2 of s is the terminal voltage and the current is i 2 of s, and the laplace transform of the current here is i 1 of s, l 2 of s and then here you have once again minus mi 1 0, l 1 of s minus i 1 0 that is m times rho 1 and here you have l 2 times rho 2.

In addition we have a coupling impedance ms. so; that means, once you have write down this equation, the transform diagram in this fashion. We say the voltage terminal voltage v 1 of s are this coil is i 1 of s passing through l 1 of s we create a l 1 of s times i 1 of s. In addition i 2 flowing to this coil will have a mutually induced voltage here, whose Laplace transform is ms times i 2 of s. So, you have not only l 1 of s l 1 s times i 1 of s plus also ms times i 2 of s. L 1 of s i 1 of s plus ms times i 2 of s. In addition representing the initial current in inductors we have 2 voltage sources. Likewise the same situation is there in the second coil also. This is very similar to what you do in the case of study state circuit analysis ac circuit analysis, where instead of self inductance is represents as impedance j omega l 1. The mutual impedance is j omega m. So, instead of j omega we have s; otherwise it is exactly the same, that the situation that we have in the case of ac circuits. Now, you may as well think of v plus in the initial currents, through current sources. So, if you do that, you can write this equation in this form.

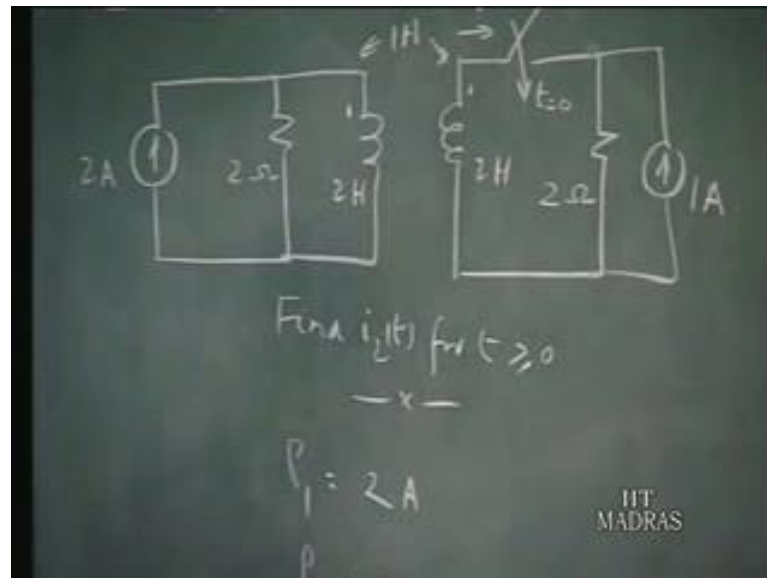
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You can write v_1 of s , you can write as $l_1 s$ times i_1 of s minus ρ_1 by s . So, instead of writing minus $l_1 \rho_1$, I can write l_1 of s times minus ρ_1 by s . So, this term here, is put in this form plus $m s$ times i_2 of s minus ρ_2 by s . and you can write v_2 of s exactly going to the same method; $m s$ times i_1 of s minus ρ_1 by s plus l_1 of s time i_2 of s minus ρ_2 by s ; that is what we handle. So, is you look at these two equation, you can represent them in this fashion. You can have a current source here. So, let the current source be of strength ρ_1 by s . So, the current in the actual inductor here, is i_1 of s minus ρ_1 by s , and that current passing through l_1 of s will set up a voltage drop l_1 of s times i_1 of s minus ρ_1 by s . In addition you will have a mutually induced voltage $m s$ times, the current in this coil, and what is the current in that coil, in a symmetrical way you have i_2 of s here this is v_2 of s , and you have another current source here, which is ρ_2 by s . So, the current in this coil is i_1 of s minus ρ_1 by s , and the current here is i_2 of s minus ρ_2 by s .

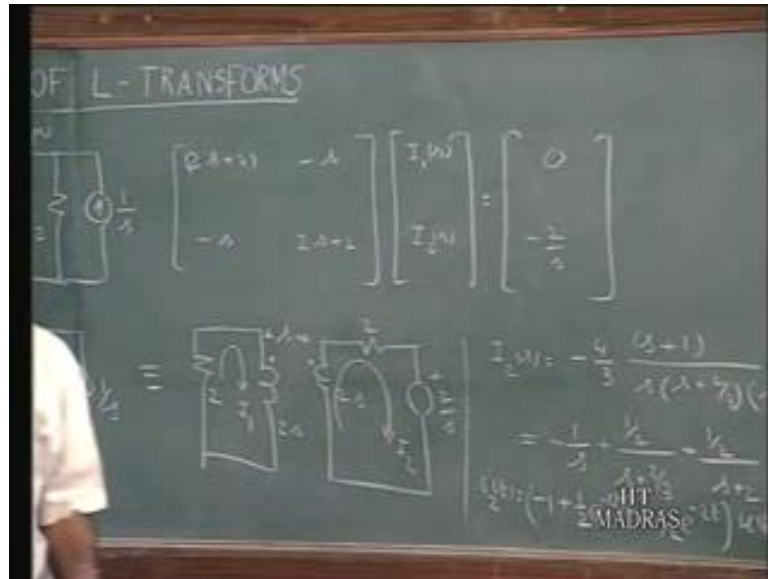
So, now you see that this voltage v_1 of s is obtained as the drop in this inductor. What is the drop in this inductor to self induced $v_m f i_1$ of s minus ρ_1 of s times l_1 of s . and the mutually induced voltage is $m s$ times the current here, which is i_2 s minus ρ_2 by s . So, this exactly the term that you have having here. Similarly the voltage induced in this which is equal to v_2 of s , is the self induced voltage i_2 of s minus ρ_2 by s by l_2 of s , and the mutual induced voltage this is $m s$ times i_1 of s minus ρ_1 by s . So, this are 2 alternative representation of the initial conditions the mutual inductor, if you low with the voltage sources use this diagram. If you like to use the current sources you can use this diagram. The advantage of the current source representation is, that one side you have only ρ_1 only figures here, when you put the voltage source both ρ_1 and ρ_2 come on both sides; that is the minor advantage. So, this is the way in which we can analyze the transient problem, involved in mutual inductances. Let us work out in example to clarify these ideas.

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In this example we have a circuit with a pair of couple coils, having this arrangement 2 henries, and let this also be of 2 henries self inductance, and let the mutual inductance i in these 2 coils be 1 henry. Let the elements value is as mark here. These two are current sources, this is the current sources; 1 ampere and 2 ampere, and the switch is kept open for a long time and close the t equals 0. You are asked to find i_2 of t , find i_2 of t for t greater than 0; greater than or equal to 0. So, let us see, as for as this inductor is concerned it does not carry any initial current t equal 0, because the switch is open. Therefore, if you call the ρ to be the current here it is 0. As for as the first induct is concerned, this 2 ampere current source dc current source close entirely to the inductance, because for steady state the voltage here is 0. Therefore, this 2 ohms is short circuited; therefore, ρ_1 2 henries. So from the statement of the problem we know, that ρ_1 is 2 amperes that is the current in that inductor equals 0 minus, the current in the second inductor is 0, so that is what we are having.

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So, if you set up the transform diagram for this, you have corresponding to the 2 amperes dc source, you have $2/s$; that is the current source in the transform domain, and you are having the $2/s$ generalized impedance of the resistance, and you having self inductance $2/s$. And since the initial current here, is 2 amperes, I can replace this by an equivalent current source of $2/s$, in the second representation which we have just now discussed. And these two inductors, have the same self inductance and mutual inductance, which is equal to s in the transform domain, and once the switch is closed here, you have this $2/s$ ohms and then the current source here. You should not put the current source as $1/s$ here, but $1/s$, because the dc source, and we are talking about the transform domain, all the variables have the transform domain. Now in this we are interest in finding out $i_2(t)$, this is what we are after (t) . Observe here the $2/s$ and minus this $2/s$ coming together the same node, cancel each other out; that is if you have the current source here of some i/s and another current source of i/s here. So, both of them cancel each other out, and therefore that current in the, whatever which is corrected is 0. So, using this principle here; after all, this current source and this current source are parallel to each other, and equal and oppositely headed.

So, the effect of rest of the circuit is negligible is 0; therefore, I can write this simply as $2/s$ this is $2/s$, and the coupling of s between this $2/s$, and you have $2/s$ and you have $1/s$ of the current source. This is $2/s$. This is $2/s$. So, that is what here i having. So, suppose I like to use loop current method analysis. They can convert the current source, in parallel

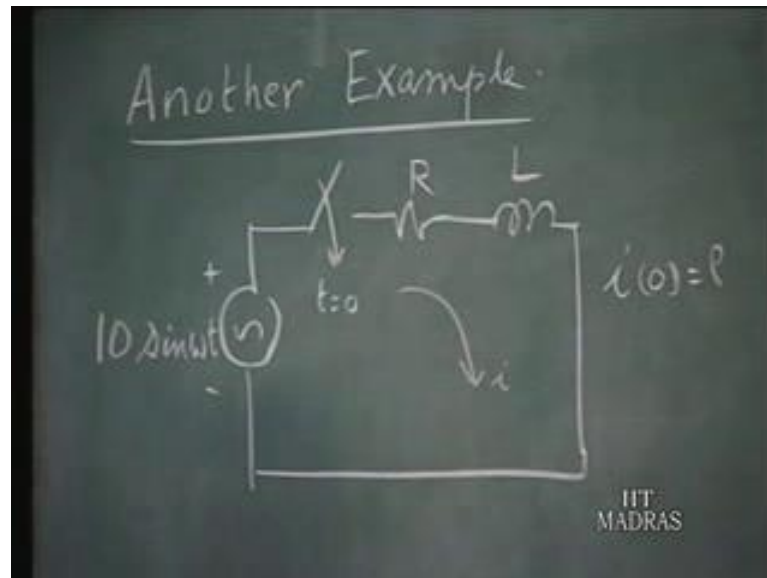
with the resistance, as a voltage source in with the resistance. So, this equivalent to 2 ohms 2 s. This is 2 again, generalized impedance 2 ohms 2 s coupling to s . Now a current source in parallel with resistance can be replaced by an equivalent voltage source, in series with assistance, and the strength of the voltage source assists we use as equivalent that is in dc circuits, the multiplication of this 2 is becomes 2 by s . So, you have a voltage source 2 by s in series with a generalized impedance 2 . So, if I write the 2 loop equations as; loop currents as i_1 and i_2 . The current here is what we are interested in that is same as write 2 the current in this coil. Therefore, the identical the current is retained, and you write the loop equations for this. So, I can write this as 2 s plus 2 . I write these loop equations in matrix form, because this is quite simple and easy to visualize.

So, I write this i_1 of s and i_2 of s , and the other side I write the forcing function; therefore, the first loop equation is 2 s plus 2 times i_1 of s . The mutually induced voltage i_1 is the entering the dot point and i_2 is leading the dot point. therefore, the sign of the mutually induced voltage is negative, and the multiplication factor is s , because this is the mutual impedance; s times i_2 is the magnitude the mutual induced voltage, but you have to have a negative sign therefore, minus s times i_2 of s , and the total driving voltage in this loop is 0 . Now, for the second loop the self impedance of the loop is 2 s plus 2 , and the mutually induced voltage, again i_2 is leaving the dot point and i_1 is the entering the dot point; therefore, the side of the mutual induced voltage is negative. The mutual induced voltage is the Laplace transform s times i_1 , but the sign is negative; therefore, minus s times i_1 2 s times i_2 of s , that must be the net voltage in this loop, trying to drive a current in the direction of i_2 , but the total e m f in the circuit is 2 by s in the opposite direction; therefore, I must write the minus 2 by s . So, these are the equations which describe the behavior of the circuit and the loop current basis.

You can solve for the, and you get equation for i_2 of s solution of this 2 equations will give you i_2 of s equal minus 4 up on 3 s plus 1 over s times s plus 2 by 3 times s plus 2 , which the partial fraction expansion will yield minus 1 by s plus half divided by s plus 2 by 3 plus half s plus 2 . So, that is the partial fraction expansion of i_2 of s , and therefore, you can write i_2 of t as minus 1 plus half e to the power of minus 2 t by 3 plus half e to the power of minus 2 t , and the entire quantities of course, multiplied by u , that is the expression for i_2 of t . So, you can see the solution for the transient problem of the start

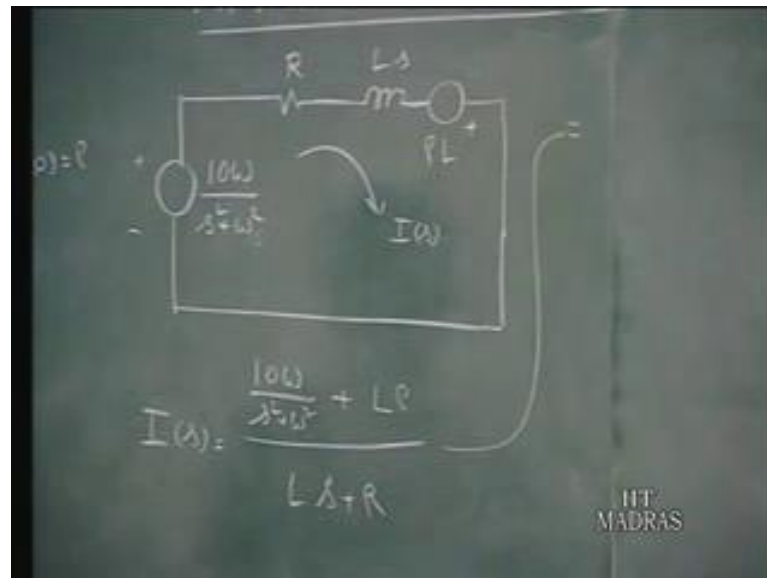
once again, can easily be handled with transform diagram approach. You do not have to write a differential equation. And once you have the transform diagram, you write the equations up on forms in s domain, in the same way as you do for dc circuits. And once you get the solution for the required quantity in terms of transforms, you have to find the inverse transform, to find the quantity in time domain.

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Let us now consider another example, in which the Laplace transform techniques applied, to the circuit containing sinusoidal sources. So you have a sinusoidal voltage source of $10 \sin \omega t$, and the voltage and the switch is closed t equals 0. Let there be 2 elements in the circuit r and l , and we are interest in finding the current in the circuit for i greater than or equal to 0, and it is given that i_0 is ρ , because of some previous arrangements the current in the inductor is i_0 at the time of switching. So, we make the transform diagram for this.

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So, the voltage source has the transform 10ω over $s^2 + \omega^2$; that is the Laplace transform of $10 \sin \omega t$, and you have the resistance r that is the generalized impedance of this. As far as inductor is concerned, you have a generalized impedance $l s$, and in addition you have a voltage source, representing the initial current in the inductor, which is ρ times l . The voltage of the voltage source is ρ times l over s , so that is what we are having. So, that completes the transform diagram, and in this we are interested in finding out the current I of s . So, if you write this after all the simple loop circuit therefore, I of s can be written as 10ω over $s^2 + \omega^2 + l s + r$; that is the total voltage driving in the loop divided by $l s + r$; that is the total impedance of the circuit. Now, this can be written as. After all you make the partial fraction expansion for this. So, I can be written as; first of all $\frac{\rho l}{s + r}$; that is straight forward; therefore, I get ρ over $s + r$ by l , that comes from this plus in addition I have 10ω over $s^2 + \omega^2$, the ratio of the first 2 terms I will write $s + r$ by l , and divide this by l .

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$$\frac{10\omega}{s^2 + \omega s + r} = \frac{\rho}{s + \frac{r}{L}} + \frac{10\omega L}{R^2 + \omega L^2} + \frac{10\omega R}{R^2 + \omega L^2}$$

$$f(t) = \rho e^{-\frac{rt}{L}} + \frac{10}{R^2 + \omega L^2} [R \sin \omega t - \omega L \cos \omega t + \omega L e^{-\frac{rt}{L}}]$$

S: terms

So, 10ω over 1 divided by s square plus ω square times s plus r by 1 . So, you make the partial fraction expansion of this, you will get ρ over s plus r by 1 as before for the first term. And for the second term you have; s plus r by 1 term and that will turn out to be 10ω divided by r square plus ω square 1 square. You can carry this out, I do not spend time in working out the details, but one can easily arrive at this values. And you have for the s square plus ω square term $10\omega r$ over r square ω square 1 square divided by s square plus ω square minus 10ω divided by r square plus ω square plus 1 square times s square plus ω square. Actually when you have s square plus ω square in the denominator you have a s plus b time in the numerator. I split up in to these two portions so that this can be recognized to be the Laplace transform of the sin function. This has recognized to Laplace transform cosine function. So, i of t is, for the first time is concern ρe to the power of minus rt over 1 ρe to the power of minus rt over 1 ; that is the Laplace transform of this. as for as this is concerned you have 10 over by r square plus write let me say ten by r square plus ω square 1 square, that is the common factor write through, so I will keep that separately 10 have r square you have ω over s square plus ω square gives you $\sin \omega t$.

Therefore, you have $r \sin \omega t$. here again 10 over r square plus ω square 1 square taken out, s by s square plus ω square will yield me $\cos \omega t$; therefore, I have minus ω $1 \cos \omega t$ plus. Here $10 r$ square by r square plus ω square 1

square taken out the common factor; therefore, plus omega l times e to the power of minus rt by l, and of course u of t. So, this is your, all this multiplied by u of t. You notice that t equal 0. This is 0. This cancels out with this plus cos omega t and e to the power of minus rt by l are both equal to 1 this is 0; therefore, this term is equal to 0, and this is equal to 1 and the initial value of the current it is rho which is what we know. Now, in these you observe, that this term and this term can be combined, if you wish, is decay in transient minus rt by l. Therefore, this term and this term together are the transient terms that decay with time, actually both of them can be combined. Whereas these two terms are the steady state term. You have driving this particular circuit with sinusoidal forcing function; therefore, this is steady state solution, and this is what we are having.

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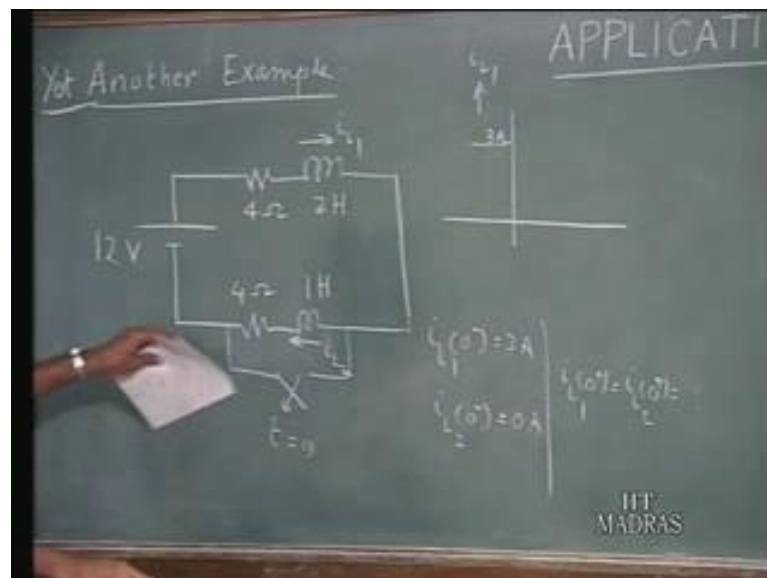
$$i_{ss} = \frac{10}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \theta), \tan \theta = \frac{\omega L}{R}$$

$$i_{tr} = \left(\rho + \frac{10 \omega L}{R^2 + \omega^2 L^2} \right) e^{-\frac{Rt}{L}}$$

You can show this steady state term to be 10 by root of r square plus omega square l square; that is the impedance circuit; sin omega t minus theta, where tan theta equals omega l over r. So, this is the steady state solution. you have here in ac circuit in which having a driving force 10 sin omega t, and you have a r and l in the circuit; therefore, the impedance is square root of r square plus omega square l square, and the power factor angle equals 10 inverse of omega l over r, so that is the steady state solution. You combine this two and you can show very easily, that the sum of these two indeed this. the transient power is the solution, equals rho plus 10 omega l over r square plus omega square l square e to the power of rt by l; that is the transient power of solution; rho e to

the power of minus rt by $1 + 10$ by r square plus ω square l square time $\omega l e$ to the power of minus rt by l , this is the transients solution. So, in this we can see that once we have a solution, we can identify the terms which corresponds to this steady state, behavior of the current, and what are the terms which correspond to the transient portion of the response. So, both this are simultaneously brought out in the final solution. You can combine these terms and show it to be (i) . And off course, once you have the total solution that is valid only for t greater than are equal to 0 ; therefore, u of t will always there, to specify that i of t that expression whatever we have, is valid only for t greater than are equal to 0 .

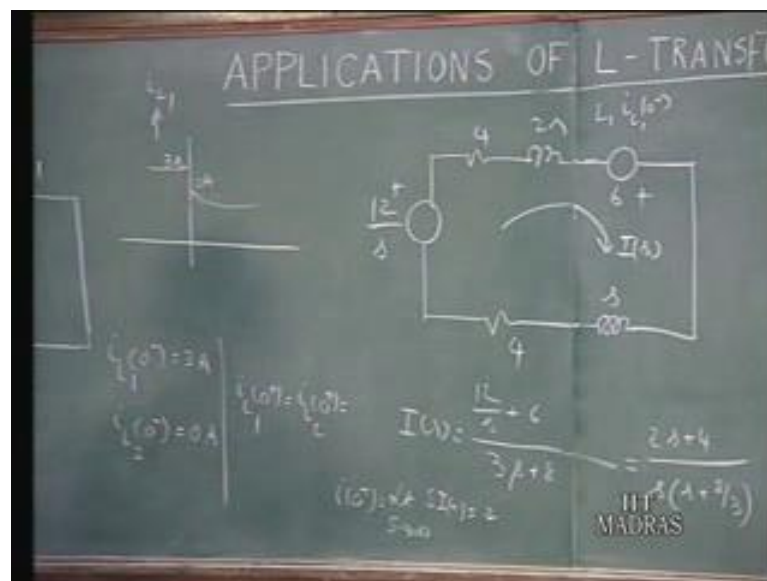
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Let us work out one more examples. Let us now consider this example, which we had earlier discussed in the context of the classical, whether differential equation approach to the solution of transients. We have set up here in which the two inductors. The two inductors are originally carrying different currents, but once the switch is opened both of them are perforce, required to carrying the same current; therefore, there is the case of discontinuous in inductor currents, in the transient from t equals 0 minus to 0 plus. In the case of differential equation approach, we have calculated the t equals 0 plus conditions separately. Now, let us use the Laplace transform other than show, that once you circuit t equals 0 minus conditions 0 plus conditions, almost immediately follow in the normal routine. You do not have to make separate calculations. So, let us use 0 minus conditions, and solve for this circuit. You recall that the current was originally 3

amperes, when the switch is closed, the current in the i_1 is 3 amperes, and i_2 is 0 amperes. But once the switch is opened i_1 and i_2 are going to be the same, and you would like to know what the value is, and we use the constant fluxing case theorem, or some similar effect to find out the new current in the inductors. Let us use Laplace transform techniques here. So, let us draw the transform diagram for this.

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You have 12 by s is the transform of the voltage source. you have 4 2 s, and as for as the inductor is concerned we use the 0 minus values, 0 minus value is 3 amperes, so 3 times 2 6 1 1 i 1 1 0 minus; that is 6; and down here you have inductance of value s impedance s. and the initial current is 0 t equals 0 minus; therefore, you may as well do not have that, and you are having 4. So, what is the current here? This is very simple circuit, so i of s will be 12 by s plus 6 divided by 3 s plus 8; that is what you are having. So, this will be 2 s plus 4. Simplify this you get 2 s plus 4 times s plus 8 by 3. Now, we can make the partial fraction expansion find out the i of t, but before that find the initial value the current; i 0 plus equals limit as s tends to infinite of s times i of s. So, when you multiply this by s, and take the limit as s tends to infinite that becomes 2, 2 s square by square; therefore, this is 2; that means, this current from 3 amperes, we have jump to 2 amperes at t equals 0 plus and something else equals. So, automatically you find out the i 0 plus.

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The image shows handwritten mathematical work on a chalkboard. At the top, the current $i(t)$ is given as $i(t) = \left(\frac{3}{2} + \frac{1}{2}e^{-\frac{8}{3}t}\right)u(t)$. Below this, the inductor voltage $V_L(s)$ is derived as $V_L(s) = 2sI(s) - 6 = -2 - \frac{8/3}{s + 8/3}$. A partial fraction expansion is shown as $\frac{-4}{s + 8/3} = \frac{3/2}{s} + \frac{1/2}{s + 8/3}$. Finally, the inductor voltage $V_L(t)$ is given as $V_L(t) = -2\delta(t)$. The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

So, if you want to find $i(t)$ separately, you make the partial fraction expansion, this becomes $\frac{3}{2s} + \frac{1}{2} \frac{e^{-\frac{8}{3}t}}{3}$. So, you get $i(t)$ in the loop as $\frac{3}{2}$ plus half of e to the power of minus $\frac{8}{3}t$ by 3 $u(t)$. So, that current was originally 3 amperes in $i(0)$, jump to 2 amperes, and then finally, indicates to $\frac{3}{2}$ amperes. Now, what about the inductor voltage; suppose I want to find out the inductor voltage $v_L(t)$ of s . When you want to find $v_L(t)$ of s , now that we have got $i(s) = \frac{3}{2s} + \frac{1}{2} \frac{e^{-\frac{8}{3}t}}{3}$; that is the drop here minus 6 . So, when you want to find the inductor voltage, you must take the voltage between these 2 terminals, not across the inductor alone this portion only. You must take the initial current also in account, because these are the known terminal of this inductor in the circuit. This inductor has been replaced by not only an inducting impedance, but also initial current source. So, when you want to find out $v_L(t)$ of s , if this is $v_L(t)$ of t the corresponding Laplace transform, is the voltage in the transform diagram existing between these two terminals. You should not make the state of computing mainly this portion. So, $2s$ times $i(s)$ minus 6 ; that is $v_L(s)$, and you can show that this will be minus 2 minus $\frac{8}{3}$ divided by $s + \frac{8}{3}$, which means $v_L(t)$ is minus $2\delta(t)$. I will write this separately here.

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$v_{L_1}(t) = -2 \delta(t) - \frac{8}{3} e^{-\frac{8t}{3}}$
 $v_{L_2}(t) = 2 \delta(t) - \frac{4}{3} e^{-\frac{8t}{3}}$
 $\leftarrow -\frac{8}{3}$

$v_{L_1}(t)$ is $-2 \delta(t) - \frac{8}{3} e^{-\frac{8t}{3}}$, and likewise we can calculate $v_{L_2}(t)$; that is the voltage across this, between this 2 points $v_{L_2}(t)$ likewise we calculate, you will get this result $2 \delta(t) - \frac{4}{3} e^{-\frac{8t}{3}}$. So, this both the currents are jump from $t = 0$ minus 2 0 plus values in no doubt, there will be impulse functions. But taking the L_1 and L_2 put together, the impulse functions can cancel each other out; that means, even though the inductors have impulse voltages in them. In the local loop the impulses get canceled; therefore, all the other voltages are finite, and therefore, the total voltage law is still satisfied, because the impulse voltage generated here, is cancelled by the impulse voltage are generated here, because this will be, now $v_{L_2}(s)$ this $v_{L_1}(s)$ define these 2 points, and the sum of this transient be 0. So, we have worked out, a number of examples illustrate in the Laplace transform techniques. As I mentioned once we have the transform diagram, we can use the whole range of network analysis technique; that is available to us, in the context of d c circuits or a c circuits. We can use the node analysis, the loop current method analysis.

So, various network theorem we can apply all this, follow in a straight forward duty in fashion. Later on we return to, a discuss on network theorem and some point, and then we can pick up this thread at the time, but at this time, from what we have seen. Let us know to see what are the advantages of the Laplace transform domain, in the transient analysis of the electrical networks. We find there are number of advantages compared

with the classical differential equation approach. First, when you have functions of time which are discontinuous, or which are discontinuous derivatives; like your step function, or ramp function. Then the Laplace transforms of these time functions can be simple expressions, whereas if you look at the classical differential equation. Suppose you have the function is described by different electrical expression in different regions of time. So, you have to find the solution for one interval of time, find the final solution, and put this for the initial condition for the next interval and so on and so forth, that becomes quite complicating. Whereas in the transform domain, for the entire range of time you have one single simple expression, for the Laplace transform domain. Secondly, we observed that we do not have to do involve ourselves with calculus operations; no integration, no differentiation. All the equations describe in the behavior of the system, are put in purely algebraic form, algebraic equations are involved.

So, calculus is converted in to a kind of algebra, because differentiation corresponds to multiplication by s , integration corresponds to division by s . So, calculus is converted to algebra. A third feature is, that you get the total solution; the transient part the solution, and the steady state part of the solution both together in 1 unit. Whereas in the class steady the differential equation you have the particular integral solution, the complementary function. So, there are found separately and combine them, but here you do not have to do that. You find out the total solution in 1 step. But once you got the total solution, you can identify the steady state part and the transient part in the final solution, using common reasoning which is, and this terms will become evident, if you see what are the terms which come to forcing function, and what are the terms which come from the system itself. This will be clearer when we discuss the system function and its ramifications later on. But the main advantage of the Laplace transform technique is, that the initial conditions pertaining to the system are plugged in into the solution in the equations at the very beginning itself.

Therefore, you do not have to evaluate separately the arbitrary constants, that you have to do, in the case of differential equations. And this is very important, because if the order of the system, the number of reactive elements becomes large. Suppose there are 4 or 5 reactive element, and you have the fourth order differential equation. Then you need to know 4 initial conditions, not only the initial value, but the values of the first 3 derivatives. At this becomes quite complicated, because they are not evident, they are not

given the problem, and we have to calculate this, I have to find out the initial derivatives second derivative third derivative, to separate manipulations. All that unnecessary as far as the Laplace transforms technique is concerned. All the initial conditions pertain to the reactive elements, are use in the transform diagram, are equivalently in the differential equation, we substitute them and you get the solution. So, you separate evolution separately arbitrary constant is not necessary at all, in the transform approach. And in this context it is seen that, you do not have to find out the 0 plus conditions separately from 0 minus conditions.

If you have 0 minus conditions, you straight away plug them in u 0 minus conditions, and the solution that you get is 1 that starts from 0 minus, automatically it will proceed. And if you in fact interest in finding out 0 plus conditions, you can use the initial value theorem, refine the expression and find out what 0 plus conditions are. So, all these are the advantages of the transform approach, to the solution of transients in networks, to summarize once again, because this is the very important aspect, let me recapitulate the very less advantages that we have mentioned. First of all, functions which are discontinuous and which are discontinuous derivatives, are easily handled in the transform domain, because you get once a single expressions valid for all time. Secondly, we find that calculus is converted into algebra. So, the manipulations are that much easier. Thirdly we find the transient and steady state solution come together in one package, but you can separate them at the final solution into the 2 separate components if you (). And most important of all, is the fact that the initial conditions are used in the problem, to start with the data regarding initial conditions in fact in the problem the very beginning itself.

So, separate evolution of arbitrary constant, is avoided. This troubles of feature as for the classical differential equations solution is concerned. After having said all this, must also be fair to the fundamental approach of the differential equations. What is the differential equation can do, which Laplace transform perhaps cannot do. See the differential equations very fundamental solutions, fundamental approach. Suppose you have given some data, not the initial condition t equal 0, but you want to specify some conditions on the response, initial value equals so much at t equals 0. The derivative is equal to so much at t equals one second, the second derivative so much at t equals two seconds. So, suppose the data which regarding the arbitrary constants are specified in a different way.

Not all a t equal 0, then the Laplace transform domain, is not very convenient to handle such type of data, if all the initial conditions t equals 0 or given then it is nice. But if it is not so, then the differential equation is much more unable to giving as the proper solution, and it is certainly is a more fundamental approach.

So, one will also have keep in mind, one should also know how to handle transient problems, using the classical differential equation approach, even though Laplace transform has lot of advantages, and we for the most part (()) Laplace transforms, one should not completely ignore or forget, the differential equation approach to the solution of the transient problems. Particularly the simple cases. You can write down the differential equation solution by the mere inspection. You do not have to transform those equations every time. So, is a powerful technique find out, but in very simple case. The straight forward application of differential equations will fetch the result quickly, and also with little bit of inside in to the working (()) problem. So, we have covered in this last 2 lectures; the application of Laplace transforms, to the transients in networks. Now in the next lecture we will take up the question of how to use the Laplace transform technique, to general analysis of system. We go back to the system function that we already talked about in the preliminary set of lectures, and see how you connect it up the Laplace transform technique. This will be the topic in the next lecture.