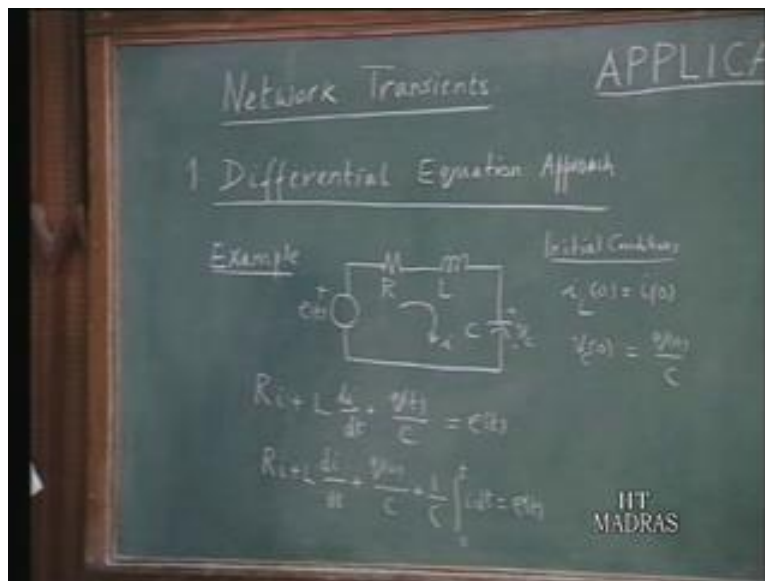


Networks and Systems
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Lecture - 26
Application of Laplace Transforms –I
Transform Diagram

After having consider the various properties of Laplace Transforms. We shall now take up Applications of the Laplace Transform technique to the solution of transients in Networks and Systems in general. To start with, let us consider solution of transient problems in networks, using the Laplace transformation method.

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In considering the network transients, we can first form the differential equation pertaining to the network. And then, transform the differential equation in to the Laplace transform domain and arrive at the solution. This is the method that suggest by itself and this is a more fundamental method. Later on, we discuss another technique, which avoids some steps to have to go through in the differential equation approach.

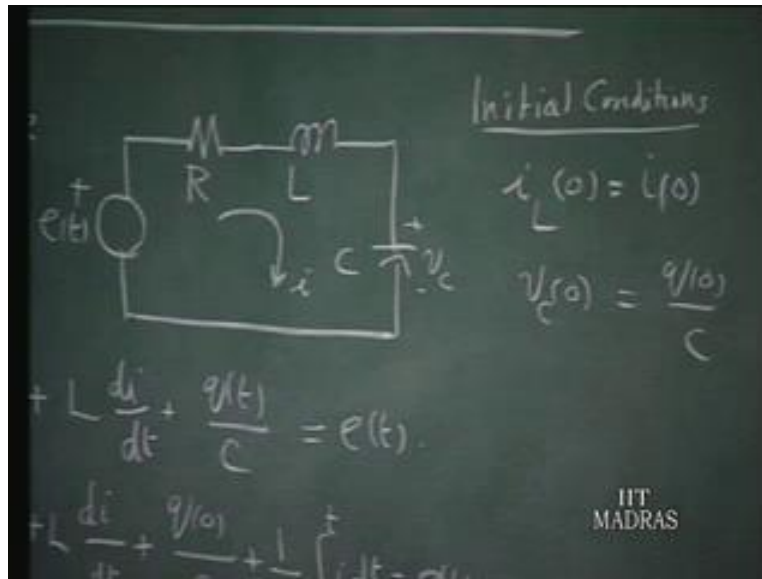
So, let us start with differential equation approach. The best way to do this is to go through an example. Let us consider a circuit in which, there is a voltage excitation. There is a resistance, an inductance and a capacitance, in series circuit comprising R L and C.

Now, you are interest in finding out, the current of the circuit I. So, the differential equation pertaining to the solution for the current given this forcing function of e of t is given as the voltage drop across resistances $R i$ plus $L, \frac{d i}{d t}$. That is the voltage drop across the inductor plus the voltage across the capacitor. That is $V c$. That can be written as q of t over c . That is the charge on the capacitor at time t divided by c . That is equal to e of t .

This can further be written as, if you take the charge q of t to be $L, \frac{d i}{d t}$ plus, there is the initial charge in the capacitor, let us say q_0 divided by c plus the additional charge that has been conduct to the capacitor plates in the interval starting from 0 up to the point t . That means $\frac{1}{c} \int_0^t i d t$. That is equal to e of t . So what you have done here is, the q of t has been thought of as comprising two parts. One is the initial charge on the capacitor, will this as positive and this as negative.

And the additional charge, that is been put at the capacitor place in the interval 0 to t $\int I d t$ divide by c . So, q_0 plus the additional charge divided by c , that is the expression for $q t$ by c is broken up into this two parts. Now, in solving in second order differential equation of this type, it will turn out to be second order will see in a moment. We need to have, no two initial conditions.

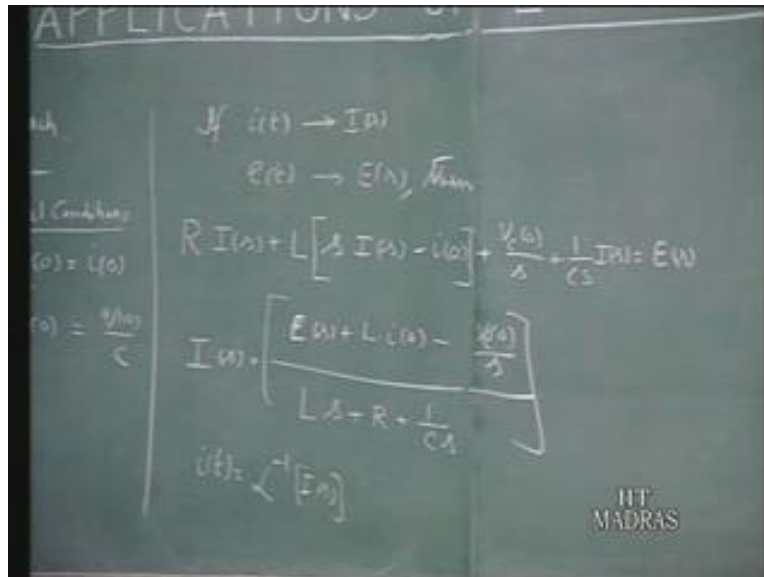
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So, the initial conditions are associated with active elements. So, one is the inductance and other is the capacitance. So, the initial conditions, we need to know to solve this problem are first of all something to do energy storage in the inductor to start with. So, $i_L(0)$ is to be found out is to be known and secondly $V_c(0)$. This is $V_c(0)$, $i_L(0)$ is same as $i(0)$. Because, after all the same current is flow to inductance first, this is $V_c(0)$, this is $q(0)$ over C .

So, the knowledge of these two initial conditions are required in order to solve this differential equation. Now, to find out the solution for the differential equation, what we do is, we take the Laplace transform of each term on the left hand side and the term of the right hand side. So, each one of the terms in the equation a Laplace transform are taken.

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So, if the Laplace transform of i of t is capital I of s . Then, $R I$; when we take $R I$; the Laplace transform of this would be R times the Laplace transform of i of s . The Laplace transform of this is L times $d i$ by $d t$. The Laplace transform of $d i$ by $d t$ is $s I$ minus I of 0 . Laplace transform of q of 0 by c is constant. Therefore, this q of 0 over c of s and the Laplace transform of this 1 over c , integral 0 to t , i of $d t$ is the Laplace transform I of s over s .

So, this is 1 over c of s times I of s . The Laplace transform of e of t is suppose of e of f , capital E of s , then this will be capital E of s . So, term by term we do the Laplace transformation. And let us say the e of t has the Laplace transform E of s . Then, term by term transformation of this term will yield as R times I of s plus L times. The Laplace transform of $d i$ by $d t$ is s times I of s minus I of 0 , plus q of 0 over c . That is q of 0 over c as, that is V of c of 0 over s , q of 0 over c is V of c of 0 over s , plus 1 over c of s times I of s . That is equal to forcing function E of s .

So, we have an equation now in I of s , which has to be solved for in terms of the more quantities. So, I can write this as I of s equals, E of s Laplace transform of the source function plus L of i of 0 minus V of c of 0 over s , divided by, the coefficient of I of s on the left hand side, which is L of s plus R plus 1 over c of s . So, what we have done is, we have got an

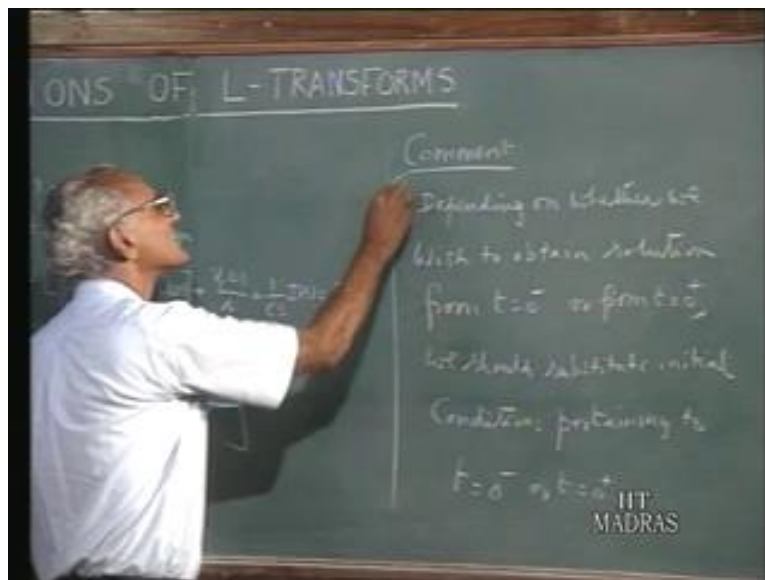
expression for the Laplace transform of the current. In terms of the known quantities, e of t is presumably known, therefore E of s can be found out.

The initial conditions in the inductor as regards the inductor current or the capacitor voltage are given, therefore these are found out. And L , R and C are parameters of the circuit, therefore we find I of s . And once we have I of s , i of t can be found out as the inverse Laplace transform of I of s . This is the approach that will have to be taken.

Now, a few comments on this, if you are interested in finding out the behavior of the current starting from $t = 0^-$ onwards. Then, the initial currents in the inductor and the initial voltage across the capacitor, you can have 0^- values. On the other hand, if you want the solution to be valid from $t = 0^+$ onwards. Then, you can substitute 0^+ values and 0^+ values for both I_0 and V_{c0} .

So depending upon what solution you want you can substitute either 0^- initial conditions or 0^+ initial conditions put that down here.

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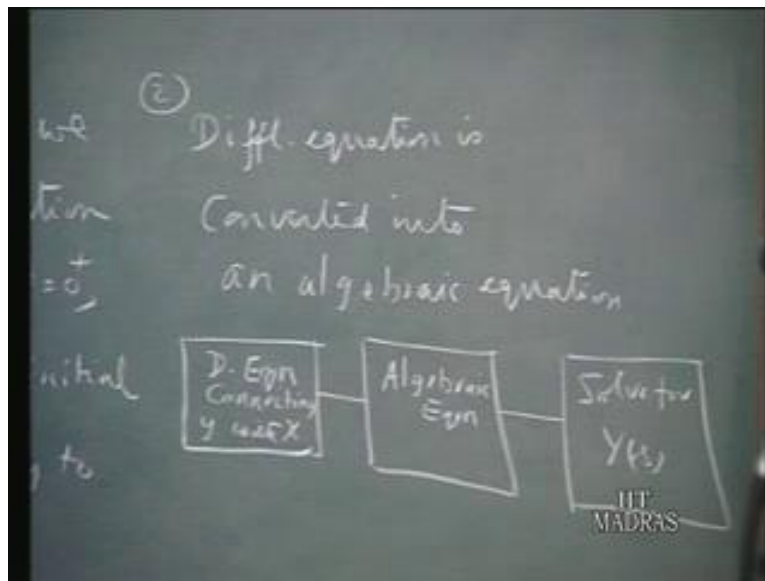
Comment, depending upon, whether we wish to obtain solution from $t = 0^-$ or from $t = 0^+$. We should substitute initial conditions pertaining to $t = 0^-$ or $t = 0^+$. Remember, in the classical differential equation approach, we need to

find out the conditions t equals 0 plus in order to solve this differential equation. But, here you do not have to do that. If you know the 0 minus conditions and if 0 plus conditions are different from that, you need to substitute 0 minus conditions the complete solution is obtained.

If 0 minus conditions and 0 plus conditions are one in the same, it does not matter, which are you substitute with make no difference. But, if 0 minus 0 plus conditions are going to be different and you want the solution from 0 minus, you can substitute 0 minus conditions. In the classical differential equation, we need to performe required to substitute the condition the 0 plus some times.

If there is transition, when 0 minus to 0 plus, we have to make separate evaluation of the 0 plus conditions. That is, avoid in the Laplace transform technique as we can see here. Now, second point to observe is, the differential equation is converted to into an algebraic equation. You are not doing any calculus part here, all the derivatives and integral sign are been converted into equivalent multiplicative factors. And so, the differential equation is converted into algebraic equation.

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And where, trying to solve an algebraic equation really. So, the step that involved here is, first differential equation. Connecting an output quantity y with input quantity x . In general, output quantity y with x . Then, we convert this in algebraic equation, where the algebraic equation, it terms of the Laplace transforms of the output y of s , y t and input x t , so y of s in x of s . From the algebraic equation, solve for the Laplace transform of the output quantity y of s .

In this case, the current is output quantity. So, we solve for I of s . So, we solve for y of s . And once, you have got that, you find the inverse Laplace transform and then, get y t . So, these are the four steps that there are involved. You form the differential equation; convert them into algebraic equation by Laplace transform in every term in the differential equation.

Then, solve for the appropriate quantity, which are interested in this case. You have our case; it is I of s in general, it is a quantity y of s and find out it is inverse transform y of t . The fact, that the initial conditions are substituted, even at the stage of finding out the algebraic equation. In the process, you have to using the initial conditions, means that there is no separate requirement for evaluation of initial conditions as you would have a classical differential approach.

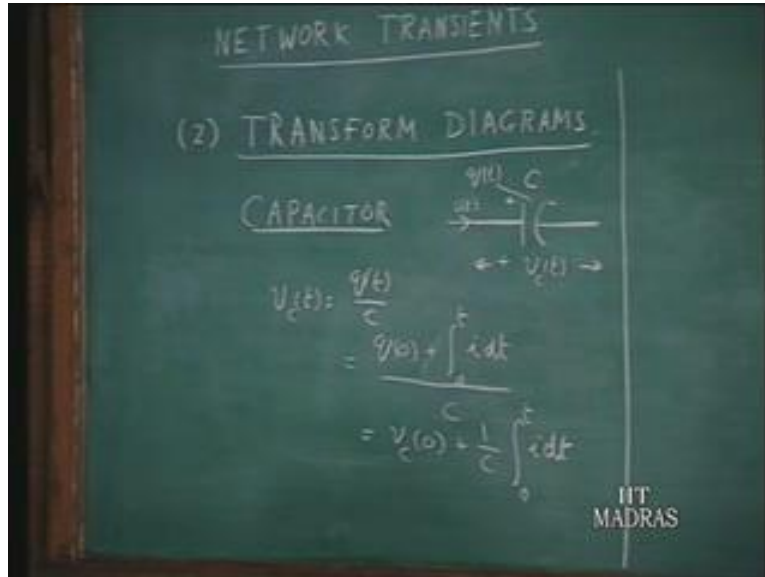
The initial conditions are fed to the data at the beginning of the problem itself. And the evolution of the arbitrary contents, which is trouble some feature in the differential equation of approach in completely avoided in the Laplace transform approach. So, this is the way, which one can handle.

However, you would like to simplify this further. Since, we are interested in finding out the electrical network transients. And this is the problem, which are quite frequently; who would like to streamline this procedures. And try to arrive at the formulation of the algebraic equation this manner, straightaway, without having to go through the it intermediate differential equation.

This is what, lead to us, what is called transform diagram approach. That is what will take up later. Essentially, what we will try to do there is, try to write down this equations

straightaway, without having to go to the preliminary of setting up the differential equation and trying to Laplace transform that. So, this is the approach that will take up. And that goes under the term, transform diagram approach and this is what will take up next.

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The second approach will take for the analysis of network transients is, what is the called the method of using transform diagrams. Before, we actually illustrate this method. Let us look at some preliminaries. To start with will take the capacitor, we know that the terminal equation is over capacitor can be put in the form, that v_c of t is q of t over c . So, if the capacitor here, parameter c , this is v_c of t .

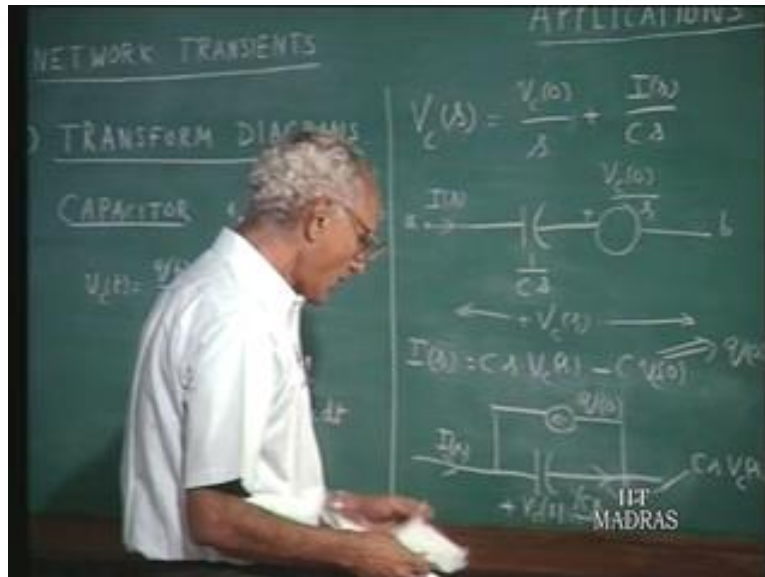
And the charge and the capacitor, suppose is q of t , the current is i of t . Then, a ready point of time, v_c of t is q of t over c . We can write this q of t as, the initial charge and the capacitor at time 0 plus the extra charge. That has been deposited on the capacitor place in the intervals from 0 to t . So, 0 to t of $i dt$; that is the additional charge on the capacitor in the interval from 0 to t . That is the total charge 0 to t divided by c .

Now, this can be further broken as q_0 over c is the initial voltage across the capacitor. Because, that the initial charge divided by the capacitor parameter c . Therefore, I can

write this as $V_c(0) + \frac{1}{c} \int_0^t i dt$. So, in time domain, we have the capacitor voltage $v_c(t)$ is $V_c(0) + \frac{1}{c} \int_0^t i dt$ this is the fundamental equation.

Now when we are analyzing a circuit, which a capacitor is situated. We should like to look at the relation between the currents and voltages in the transform domain. Therefore, you like to express these quantities, not in terms of time expressions, but in terms of Laplace transformed variables. So, what we like to do is, transform these quantities into the corresponding Laplace transforms.

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So $v_c(t)$, suppose as the Laplace transform of v_c of s . Then, that is the Laplace transform on the left hand side. On the right hand side, $V_c(0)$, that is a constant. Therefore, its Laplace transform will be $V_c(0)/s$. Now, the second quantity $1/c$, the integral $i dt$, this is this is definite integral from 0 to t , which means, I of s is the Laplace transform of I . Then, integral 0 to t of $i dt$ of the Laplace transform I/s up on s . And of course, we have $1/c$ also here, therefore I of s over $c s$.

So this is the equation in terms of the Laplace transform of the terminal current and the Laplace transform of the terminal voltage. We have an additional term here v_c of s and I

of s are connected, not only by the capacitor parameter. But, also, we have an additional term depending upon the initial voltage across the capacitor.

Now this can be modeled by means of a circuit representation like this. Suppose, I have a capacitor, let us say at the moment, we have the impedance $1/c s$ and a current I of s is passing through it. Then, if this is the generalized impedance, it develops the voltage here, which is times $1/c s$. So, v_c of s is I of s over $c s$ plus also we have additional quantity, which does not depend on the current.

Therefore this can be considered to be source. A voltage source, whose value is $V_c 0$ over s and this, is the total terminal voltage across the capacitor. So, we see observed that the terminals of the capacitor is suppose this a two terminal a and b . In the transform diagram in the domain, we can think of the capacitor to represent by means of a circuit like this.

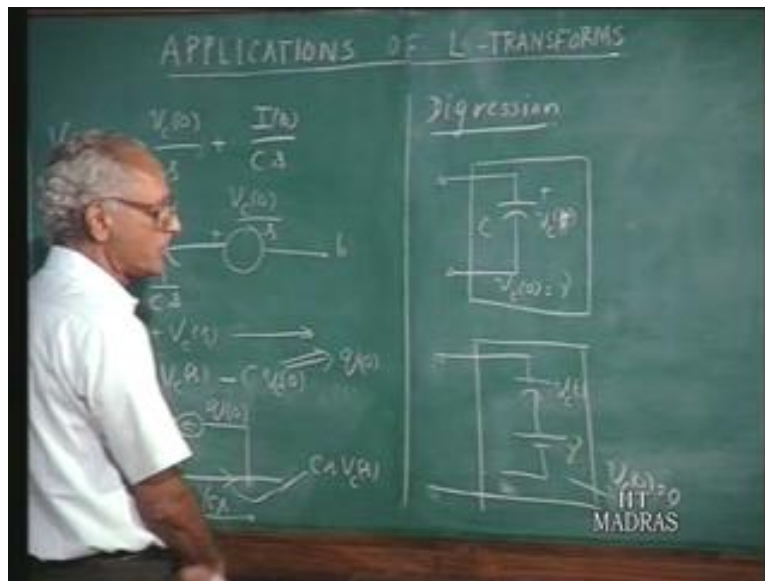
Where, this capacitor has generalized impedance $1/c s$ in the sense. The voltage across these capacitors in the transform domain divided by the current in the capacitor in the transform domain is $1/c s$. So, I of s passing through $1/c s$ develops the voltage $I s$ over $c s$. In addition, we have voltage source, $V_c 0$ over s , together will constitute the capacitor voltage in the transform diagram.

We can express the current in the capacitor in terms of the voltage. So, from the same equation you can write I of s equal $c s$ times v_c of s , minus $c V_c 0$. So, the current is now expressed terms of the voltage like this. This can be represented by a simple diagram like this. You have a capacitor, which voltage is v_c of s . Let me create a little more room here, I will write this as I of s , $c s$ times v_c of s minus $c V_c 0$, which of course is also $q 0$.

Therefore, I can represent this equation by means of the circuit like this. Whose impedance is $1/c s$ and in parallel like this; we have a current source, whose value is $q 0$ in the transform domain. And the current here I of s and the voltage across this is v_c of s . So, a voltage v_c of s occurring across an impedance $1/c s$, will have a current here, which is $c s$ times v_c of s .

So that $C s$ times V_c of s minus q_0 is the total current time, the current I of s . And this is the circuit representation of the capacitor in the transform domain. You observe that, we are now a capacitor in terms of a generalized impedance 1 over $C s$. Just as we have in the ac circuit analysis, you talk about the impedance of the capacitors 1 over $j \omega C$. In the transform domain, we have thinking of the impedance the capacitor to be 1 over $C s$ more about this.

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Now what is the corresponding situation, before for an inductor? But, before that, let me take a small digression here to stress a particular point. Suppose, I have a capacitor, which is charge, suppose I have the capacitor here, the initial charge on the capacitor, this is V_c of t . The initial charge of the capacitor, suppose is γ . I enclose this capacitor to the black box.

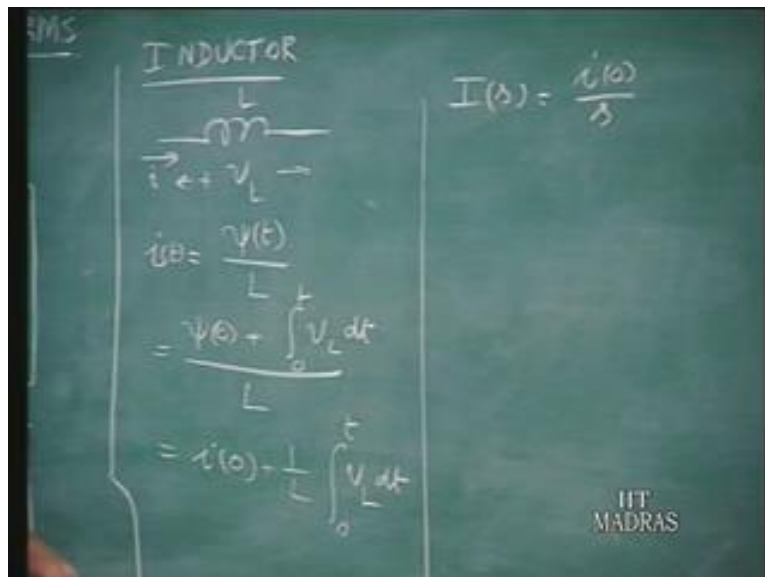
Now let me take another situation, where we have an uncharged capacitor, in series with a voltage source equal to γ , this voltage source. So here this is V_c of t and we have V_c of t is 0 . In this case, V_c of 0 is 0 . That is initially uncharged V_c of 0 is 0 . So, we have two situations, one a capacitor, which is charge initially a γ volts. And another, a capacitor in series with the voltage source, whose value is γ , but the capacitor itself, initially uncharged.

So, if these two, a capacitors systems are given to you in two black boxes. They provide identical terminal relationship into whatever to circuit in which it are connected. Or, in other words, if you for any external measurement, you will not be able to distinguish between these two. You have the identical terminal relation for all possible experimental situations.

In other words, these two are equivalent for every possible consumable extensible. And therefore, if you have a capacitor, which is carrying an initial charge and therefore, initial voltage γ , we can represent this an uncharged capacitor in series with a source. And if you take the Laplace transform of this and this capacitor c as 1 over $c s$ and the dimension for generalize impedance and a source, which is γ over s . So, that is an exactly this equation that you are having.

Initial voltage for the capacitor can be represented of that source and that source; $v c$ source will have a transform like this. So, this two apply this. Now, we can also think of an equivalent current source representation that will lead to this but we will not do that.

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Now, let us look at the situation with reference to an inductor. We have an inductor L , carrying a current i and let us say, the voltage across this V_L . We know that, the current

in inductor, say i equals flux linkages in the inductor at any point of time divided by L . That is the differing parameter in inductors.

See, the ψ of t , the flux linkages can be written as the initial flux linkages plus the additional flux linkages that come up in the inductor as the results of an applied voltage, which is $V L dt$ divided by L . After all voltage is the rate of change of flux linkages, therefore flux linkages are related to voltage by an integral relation like this. This can be written as, ψ_0 over L , that is the initial value of the current in the inductor.

So, I_0 plus 1 over L integral 0 to t , $V L dt$. So, that is the terminal relation of an inductor, the current in terms of the applied voltage and the initial value of the current that we have here. Now, you would like to represent this by means of a circuit diagram, similar to what we had in the capacitors.

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So, taking Laplace transform of this quantities, we have I of s , the Laplace transform of i is I of s , i_0 is the constant. Therefore, I write this I_0 over s , that is the transform of this. And again, we have a definite integral from 0 to $V L dt$, therefore in $V L dt$ has the Laplace transform of $V L$ of s . This will be $V L$ of s over s and we have 1 over L also. Therefore, we have $V L$ of s over $L S$. That is the equation.

That describe the terminal relations of an inductor in the transformable. Obviously, therefore, you have an equivalent circuit like this. In the transform diagram, which represent this equation, suppose we say the inductor has generalize impedance L of s . L s and then, terminal voltage is V L of s . So, the current in this portion is V L over L S . But, the terminal current is different from by an additional current here. Therefore, we have a current source, which is equal to I_0 over s and this is the terminal current I of s .

So, terminal current I s is related to V L of s in this manner. You have parallel circuit, one contains the impedance the inductor, who is generalize impedance is L s . Therefore, V L of s divided by L S the current here. In addition, we have this source current here. Now, on the other hand, if you like to express V L of s in terms of I s , I can write this as V L of s equals from the same equation L as times I of s , minus L times I_0 .

So, this equation, let us itself to an interpretation in the transform diagram in this manner. So, L s is the generalize impedance. You have current I of s . So, that develops the voltage, L s times I of s . In addition, you have another voltage, which is independent of I of s , therefore we can think of this as a currents voltage source L i_0 with this pollard. So, the total voltage is V L of s . So, we have V L of s is I of s passing through L s generating through voltage, L s times I of s minus L i_0 , what we are having.

Now, just as we have an inductor capacitor, we can initially charge can be represented by uncharged capacitor is with the voltage source. We can also interrupt at inductor current to be an uncharged. An inductor is does not carry an initial current in parallel with the current source. So, that is leave it as you for you to work it out. And you can do the same interpretation in the case as well.

Now, we therefore for both the capacitor and the inductor, we have reach two equivalent representations, one in terms of voltage source and other in terms of current sources. Similarly, here in terms of voltage source, in terms of current source. We can use either of these equivalents for either of these equations. And depending up on the total circuit computation, one of the other will be found to be more advantages in many instances.

For example, you have low current basis. Then, you would like to reduce the number of loops. Then, a voltage source representation may be favorable. On the other hand, if you are using the node voltage method, a current source, like this. So, the total number of nodes 2 here, it will be 3. So node voltage that in that situation, you may find use of the current source more favorable, that of course depends up on the particular problem.

Now, few another common theories, that if you have a and b the terminals of the inductor. Here, also this must be the terminal a and b. You should not think that, this of the two terminals of the inductor. If these are the terminals of the inductor in the original circuit, the transform diagram. We should not think these are the two terminals of the entire branch, including the voltage source, which transfer the inductor in the transform diagram.

Few commons about, what you have done so far. We see that, unlike the Ac circuit analysis, where we have impedances, $j\omega L$ and $1/j\omega C$. We have not only such impedance generalize impedances. But, also we have sources. If you did not have the sources represents the initial condition, the inductance and the capacitor are represented by impedance Ls and $1/Cs$ respectively.

So, it is the initial sources which coming to the picture, initial conditions of the reactive elements, which are equivalent to sources. On the other hand, we can say that, if the capacitor has an initial charge, inductor has an initial current. They can be replaced by equivalent sources and therefore, here afterwards, we can think of in the circuit analysis problem. The effect of the excitations or effect of source on otherwise the circuit is what we are looking for.

Because, initial currents in inductor an initial charges the capacitor can be represented by equivalent sources also you have seen. Therefore, without any loss of generality, we can think of a situation, where we are considering the affect of sources on circuit elements, which do not carry any initial current or initial charge. Because, those initial condition can be replaced by equivalent sources.

And once, we have that, then the terminal voltage and the terminal current of an inductance capacitance transform domain. Have proportional to relationship this V_c of s equal I_s over $c s$ and V_L of $s L s$ times I of s . So, we have the concept of generalized impedances for both the inductor and capacitor in the transform domain. The transform of the voltage has the proportional relationship the transform the current and so on and so forth. So, this are called generalized impedances and that what we have to make note on.

This generalize impedances are after all, extensions of the impedances that we have in the Ac circuit analysis. We know that in the Ac circuit is 1 over $j \omega c$ is the impedance of the capacitor; $j \omega L$ is the impedance of the inductor. Here, is the $j \omega$, we have s in the Ac circuits, we are thinking of an excitation function, which is $e^{j \omega t}$. It is on that basis, we have derived the impedances in the Ac circuit analysis.

But, here it turns out that thinking of excitation function the type, $e^{j \omega t}$. Therefore, it is not surprising, therefore we land up an impedance function, generalize in a way has 1 over $c s$ and $L s$ respectively. So, this is the generalization of the impedance that we use in the case of Ac circuit analysis. A third point, which like to make note of is, when you take the initial conditions. You can liberty to choose either 0 plus conditions are 0 minus conditions.

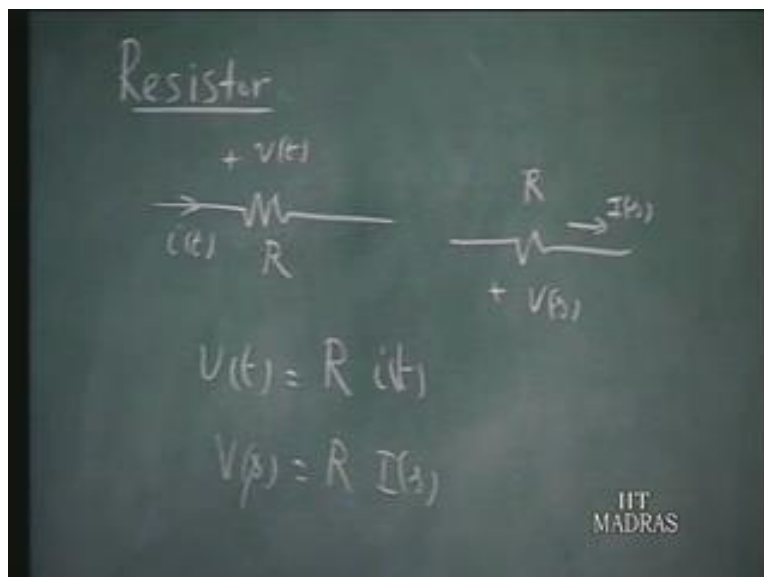
That is why; all this analysis, I did not commit myself to stay, whether it is 0 , 0 plus or 0 minus. You can use 0 plus or 0 minus, if you want to find out the system behavior, starting from 0 minus onwards. Then, you have to use all the initial conditions pertaining to 0 minus. On the other hand, if you are satisfy with the determination of the network response starting from 0 plus onwards. You can substitute 0 plus conditions and carry out the analysis.

So, in all this conditions $I(0^-)$ $V_c(0^-)$ and so on, can be either 0 plus or 0 minus consistently write through. After having set that, we know that in transient problem some switching takes place, let us say $t = 0$. And we are given or at least we can find out the conditions in the circuit, before the switching operation takes place. That means 0 minus conditions are known to us in most of the situations. Therefore, it will be convenient for us in substitute 0 minus conditions.

And the solution that we get will include the transient between 0 minus to 0 plus. We do not have to purchase additional trouble, to find out the 0 plus conditions, because they are not normally known to us before hands. Unless, this continuity between 0 minus and 0 plus. So, we do not have to take extra effect to calculate 0 plus conditions and substitute in the equations. It is enough to take the 0 plus conditions.

And if there is any transient between 0 minus conditions and 0 plus conditions, that is automatically coming out in the final solution and this is one of the advantages. So, the Laplace transform technique as you will get to know, when you have got circuit problems.

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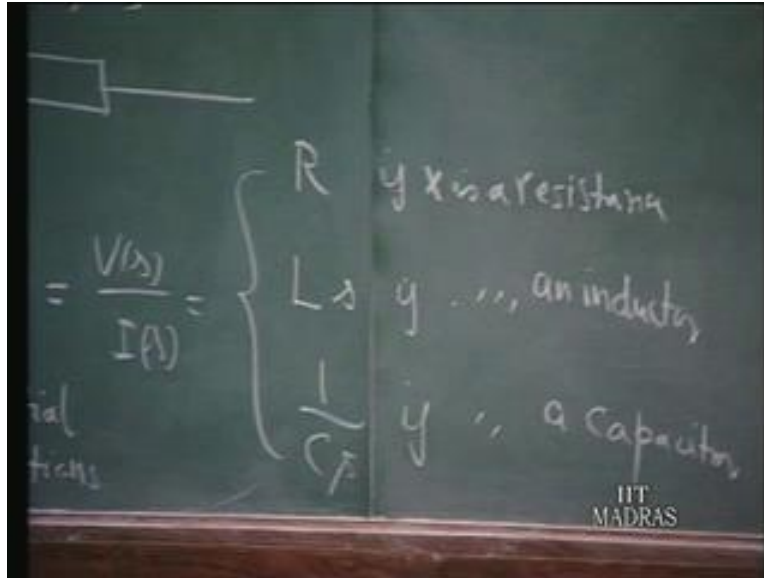


We have considered the terminal relations of an inductant capacitor in terms of the Laplace transform variables, what about resister this is of course quit simple. If I have a resister R , carrying a current $i t$ and the voltage across this is $V t$. Then, straightaway, we know that, V of t equals R times i of t at all points of time. Therefore, in terms of Laplace transforms V of s equals R time I of s .

So, in the transform diagram as well you have an R , this is V of s and the current is I of s . So, there is nod difference between the transform diagram and the normal time domain

representation of a resistor. The ratio of the time domain voltage to the current is R . The ratio of the Laplace transform voltage to the current is also R .

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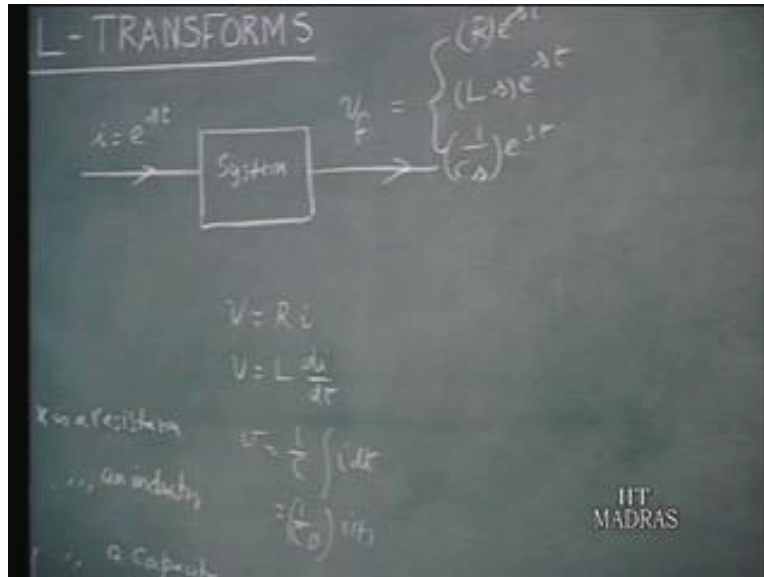
So, in general, if you have a general element like this and this is I of t and this is V of t . In the transform domain, we have the Laplace transform current indicated as the current variable. And the Laplace transforms voltage as the voltage variable. And the ratio of the Laplace transform of the voltage to the Laplace transform of the current, let us say with 0 initial conditions.

That means, we are assuming in the case of the capacitor and current, capacitor and inductor. There is no initial charge on the capacitor. There is no initial current in the inductor, 0 initial conditions. So, Laplace transform of the voltage to Laplace transform of the current 0 initial conditions, if this are V s over I of s respectively. This ratio will be equal to R , for the case of the resistor. If this box contain the resistor R ; that will be V s over I s is R .

If this box represents the pure inductor, this will be L s and the box represents the capacitor it will be 1 over c s . So, if this box, if this is x , R , if x is the resistance, if x is a inductor and 1 over c s , if x is a capacitor. So, we can think of this as a generalized

impedance of this element, either R or $\frac{1}{Cs}$. And this of course, can be extended a combination of element. Suppose, the inductance assistance or in series, we can say the generalized impedance R or $\frac{1}{Cs}$ and so on and so forth.

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We can also think of this, from the system point of view, suppose I have represents this element of a system and we have a current, which is given by e to the power of s t . And you want to know the output the force response of the system, for this input condition. This will be turn out to be, if the system happens to be a resistance current passing through a resistance. If this i , e to the power of s t is the excitation to the system and the force response is equal to after all V equals R i is the equation for the capacitor.

For the resistance, V equals L d i d t is the equation for an inductor, V equals $\frac{1}{C}$ integral i d t the capacitor. Therefore, using this three fundamental relations, if the forcing function is the e to the power of s t at the voltage will R times e to the power of s t . That is the force response of the resistance. If the system happens the pure inductor, the relation between the voltage and the current is L times d i by d t . Therefore, this is L s times e to the power of s t .

And if the system is the pure capacitor the relation between the force response voltage to the current, suppose treat this as a kind of differential equation and I is the input e to the power of s t . This is 1 over c d operating on i of t . Therefore, if i of t , is the exponential input the force response for the voltage is 1 over c s times e to the power of s t . So, these generalized impedances are R , L s and 1 over c s , can also be thought of as the ratio of voltage to the excitation.

When, when the current excitation is form e to the power of s t . So, this is an alternate way of looking at this. So, now that we have got this, we would like to set up the transform diagram of a whole circuit or a whole network, in terms of the generalized impedance. To aid as in this process, let us recapitulate, what you did in the case of Ac circuits under sinusoidal excitation function.

We have the general circuit that given to you. So, instead of dealing with i of t and v of t sinusoidal voltage and currents. We represent in the circuit diagram, the impedances of the various elements j ω L , 1 over j ω c or as the case may be. And instead of the raw currents and voltages time domain, we represent them by means of phases. So, the whole entire circuit diagram can be written in terms of the variables being the phases of the respective quantities voltages and currents.

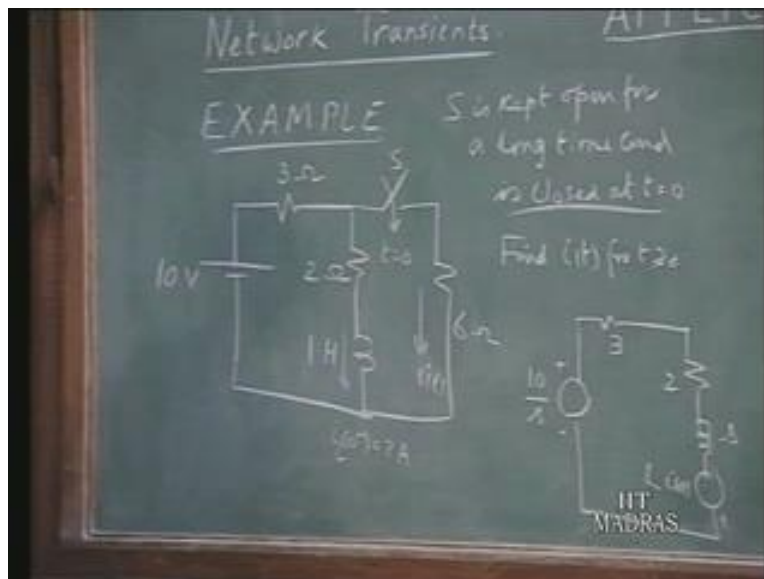
And the element values, instead being R , L and c ; we write in terms of R j ω L , 1 over j ω c the steady state impedance values. We do much the same thing in the transform diagram. What we do is, we write up represents the circuit in the same form manner as we have in the time domain. Except that, we put down the generalized impedances of the various elements.

And certainly, the variable that we are taking about are instead of the time domain expressions will talk in terms of the Laplace transformed variables. The Laplace transform the various current, the Laplace transforms of the various voltages. A third point, which is new here is, wherever the inductance carrying initial currents and whenever the capacitors carry initial charges or initial voltages equivalently. Then, we represent those initial conditions by equivalent sources in the transform diagram.

So, we can have a voltage source; that represent in the current inductor, initial current in inductor or a current source. Similarly, we can voltage source representing the capacitor voltage or a current source equivalently. So, the only difference is, that we have initial currents represented by additional sources, excitation sources. Otherwise, the diagram approach is very similar to what we do in the case of Ac circuit analysis.

We deal in terms of transformed variables in some phases is you deal with generalized impedances, instead of sinusoidal steady state impedances. That is, wherever $j\omega$ occurs, we use s instead of $j\omega$. That is the only difference. So, this will be illustrated by means of an example, which will now take up, we illustrate of the technique of the Laplace transform to the transient analysis are problem in the transform diagram.

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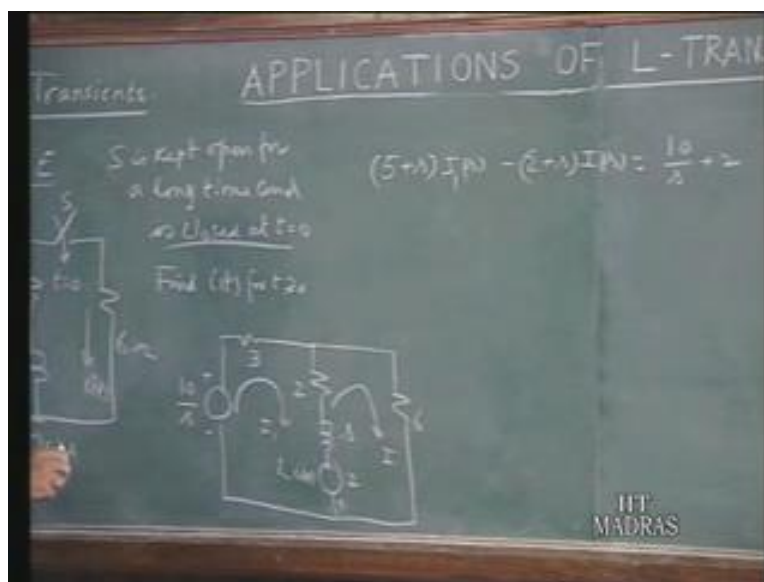
Let us take an example, we have 10 volts source, operating in a circuit comprising a resistance, 3 Ohms, 2 Ohms, 1 Hendry is the switch. This is 6 Ohms and the switch is closed, t equals 0. After having it been kept, open for a long time, S is kept open for a long time and is closed at t equals 0. So, we are asked to find out, i of t, find i of t for t greater than are equal to 0. So, we what you need to do is, set up the transform diagram for this.

So, you have for the 10 volts in straight away take the Laplace transform of this. So, that becomes $10/s$. Because, after all this is source, the transform of a constant 10 volts will be $10/s$. The resistance 3 Ohms is represented by 3 Ohms again is 3 Ohms. You do not have to write Ohms, it generalize impedance of the resistance is 3. Now, this is 2, resistance here is 2. As for the inductor is concern it generalize impedance is s , because is 1 Henry inductance therefore Ls is s .

But, you must also indicate have here, $L i(0)$ a voltage represent the initial current in the inductor. And what is the initial current in the inductor, when the switch is kept open; 10 volts raise the current as $3 + 2 = 5$ Ohms. Therefore, the current here $I(0)$ is 10 divided by $3 + 2 = 5$, 2 Amperes. Therefore, $L i(0)$ is 2, the strength of the voltage signal is L times $I(0)$. $I(0)$, which is 2, where circuit 0 minus values, in this case 0 minus 0 plus or 1 in the same.

But, we are substitute in 0 minus values, because come naturally it was and then you are having 6. So, we solve this equation in the terms of the loop currents. After all, once we have the transform diagram the equations are return down in exactly the same manner as you do for dc circuit. So, you write down the loop equations for the two loops. In terms of the variables I_1 of s and I_2 of s , I of s is what we are interested.

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So, the first loop, you have got total impedance is 3 plus 2 plus s, 5 plus s times I of s. The mutual impedance between loops 1 and 2 plus s and the 2 currents are oppositely directed in this. So, minus 2 plus s times, this is I 1 of s, this is I of s. Equals the total driving voltage in the first loop is in the direction of I 1 is 10 plus s plus 2, 10 over s plus 2.

And as for the second equation is concerned the total self loop impedance is 6 plus 2 plus s 8 plus s times I of s. And the mutually impedance is loops 1 and 2 is the s plus 2 as before minus 2 plus s times I 1 of s. The total emf in the second loop in the direction of I is minus 2, because this voltage drives to current in the opposite direction, therefore this minus 2.

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The image shows a chalkboard with the following handwritten work:

$$sI_1 + (8+s)I_2 = -2$$

$$I(s) = \frac{\frac{20}{s} + 4}{9(s+4)} = \frac{4s+20}{9s(s+4)}$$

$$= \frac{5/9}{s} - \frac{1/9}{s+4} \quad i(t) = \left[\frac{5}{9} - \frac{1}{9}e^{-4t} \right] u(t)$$

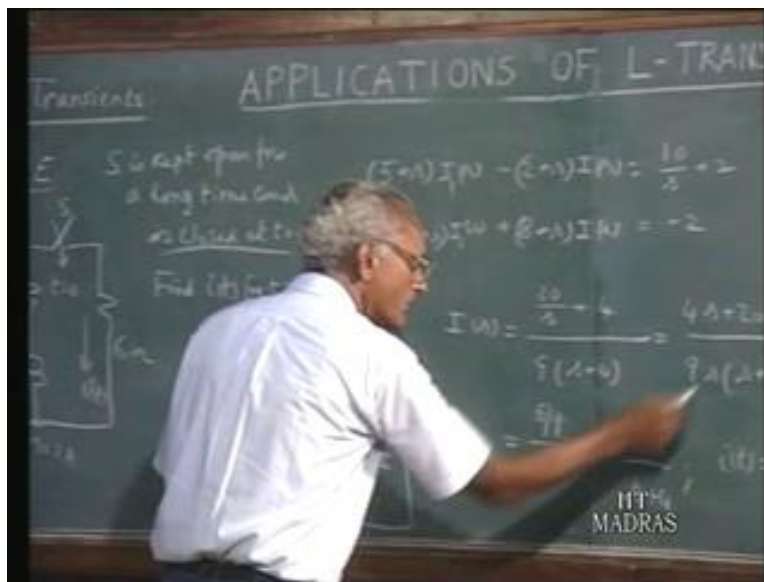
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So, you have two algebraic equations involve in I 1 and I, you can solve for I of s, through elimination of I 1 to determinate. So, whatever method you having and you arrive at the expression for I of s as 20 upon s, s 4 divided by 9 times s plus 4. And this can be written further as 4 s plus 20 divided by 9 s times s plus 4. So, at his stage, you make the partial fraction expansion for I of s in order to find i t.

So, you carry out the partial fraction expansion, it becomes $\frac{4}{s} + \frac{5}{s+9} - \frac{1}{s+4}$. So, in other words, I of t would be once we have I of s , i of t would be 5 by 9 minus 1 by 9 , e to the power of minus $4t$. That is the expression for the current i t . So, let us see, what we have done. We started with this circuit diagram; replace this by a transform diagram.

And from the transform diagram, you set up this algebraic equations so the pertain to the transform diagram. And solve for the current I of s required variable and find out the inverse Laplace transform.

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So, the steps that involve would be to set up the transform diagram in the transform diagram. You also with initial current sources with initial condition sources, with initial condition replaced by sources. Appropriately, the voltage sources or current sources from the transform diagram it set up algebraic equations. Then, get the Laplace transform of the response quantity, whichever you are after.

And then, find the inverse Laplace transform response in time domain. For this, you do the inverse Laplace transform. That there is the way, which we handle the network transient problems, during the transform diagram. So, we avoid writing down the

differential equations pertaining to this. By once, we have the transform diagram the solution runs on very similar parallel line to Dc circuit analysis. The equations are written in the same fashion, expect of this Dc quantity, we write in terms of the.

Functions of s , what currents and voltages are Laplace transform variables and instead of pure resistances, we write in the terms of functions of generalize impedances. This also very, very similar to, what we do steady state circuit analysis. Again, instead of phases we use Laplace transform here. And instead of impedances, $j\omega$, you write generalized impedance in times of s .

As a matter of fact the writing down the equations is simpler than in the case of Ac circuit analysis, because, we avoid complex number. Even though, this is complex expressions we have only using s . S is the complex variable, but the coefficients all the f and the higher powers are real coefficients. Therefore, we do not see the complex numbers explicitly on the, you writing down these equations.

So, this the j terms, which as turn all over the place in the case of Ac circuit analysis are avoid or repeat slot seeing here. And therefore, it proceeds in a very smooth fashion. The major difficulty of course is, the finding out the inverse Laplace transform of this. If I of s have a complicated rational function, then the partial fraction expansion and finding out the inverse Laplace transform is that much more complicated. But, however the procedure is straight forward.

So, today in this lecture, we took for the consideration the application of Laplace transform in network transient's problems. We saw the two different approaches; one is to start from the differential equation. This is the very basic approach and general approach and we should always keep this as the back over mind, because fundamentals are always are very important.

But, then we would like to simplify the initial labor, that is involve, because we have may to do this type of problem again and again, not it simplify the work. Instead of writing down the Laplace transformed oceans of the differential equations, which turn out to be

algebraic equations. We try to find out short cut by the writing down the algebraic equations.

Then, we develop the team of the transform diagram, which is alternative representation the circuit diagram. And from the transform diagram, we straight away write the algebraic equations performance of the network. And from that, we get an expression in Laplace transform the response quantities and you find out the inverse Laplace transform. To find out the response quantity in time domain, that consequent the solution of the transient problem of require; more about this in next lecture.