Networks and Systems
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Lecture – 25

In the last lecture, we were considering finding out the inverse Laplace transform, through the method of partial fraction expansion.

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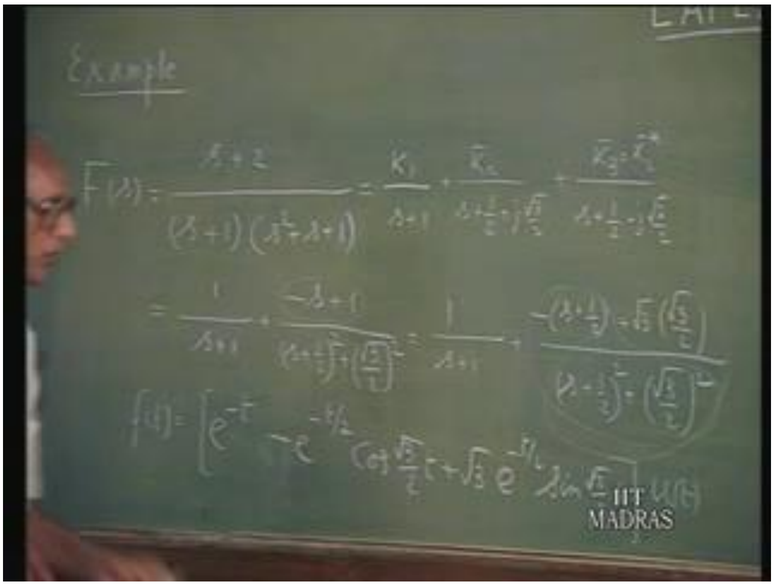

We shall continue the discussion with an example. Let us consider f of s rational function which is s plus 2 over s plus 1 s square plus s plus 1. This are; obviously, three poles; one at minus 1 and pair of complex conjugate poles at minus half plus r minus j root 3 up on 2. So, one way of handling this would be, to make it partial fraction expansion corresponding to the 3 poles. So, k 2 over s plus half plus j root 3 upon 2, and another term corresponding to the conjugate pole minus j root 3 up on 2. Now, it tells out the pole is complex, this will also the complex number, k 2 is a complex number, and since the pole, this pole is the complex conjugate of this, it terms of the k 3 will be k 2 conjugate. So, one way of filing this would be to find out the 3 residues, corresponding to the 3 poles, find k 1 which is simple, k 2 which is of course complex, and k 3 could have to spend extra time to find out k 3, because once you know k 2 the k 3 will be k 2 conjugate provide a coefficients in the rational function, coefficients of various powers of face in the rational function are real.

So, in making this in partial fashion once you find k 2 k 3 can be immediately formed out. But then this leads little bit complex algebra manipulation, and alternate wave doing this would be, to combine this 2 terms and get the result as a fraction, because after all finally, when you find the inverse Laplace transformation, you will find out that when you have a function quadratic factor in denominator, it is convenient to identify this, with the Laplace transform of e to the power of minus alpha t cos omega t type of function. So, to do that, let us write this partial fraction expansion as; s plus 1 corresponding to k 1, and the second term would be the combination of this 2, it also that will be s plus half whole square plus root 3 up on 2 whole square. So, if we can express this quadratic factor in this form, it will be convenient, because we can associate this with a well known time function as we will see as go along.

Now, to find out the residue of the pole s equals minus 1 multiply this by s plus 1 and substitute s equals minus 1 in the usual manner, and you will get the result s 1. And once you have that, to find out the term corresponding to the numerator here. You can arrive the common denominator, and identify the numerator coefficients of s plus 2 and find out the numerator here as minus s plus 1. So, this is straight forward I will not work out to give the complete details. After having down this you recall that, the Laplace transform of s plus alpha over s plus alpha whole square plus omega square, the inverse Laplace transform of that is e power of minus alpha t cos omega t. Therefore, we should like to express this, as a linear combination of s plus half one term and root 3 by 2 as the other term. If you do that then we can identify this, with the Laplace transform of terms like e power minus alpha t cos omega t and e to the power of minus alpha t sin omega t.

So, to do that, let me write 1 over s plus 1 alright, and the second term s plus half whole square plus root 3 up on whole square. Now, I must express this as a linear combination of two terms like this. So, this is the minus s here. So, I will write it minus of s plus half; that is we can recognize this to be the Laplace transform of e power minus half t cos root 3 up on t 2 t with minus sign out in front. But we have minus s plus 1; therefore instead of plus 1 you have minus half; therefore, you must introduce plus 3 by 2, but we must express this as a coefficient times root 3 up on 2. Therefore, this will be root 3 up on 2 multiplied by root 3. So, that is the linear combination of the two terms that we are looking for. Now, we have got the 3 terms which are known Laplace transform of particular time function.

Therefore, we can find out the inverse Laplace transform; that is the time function as corresponding to the first term you have e power minus t, corresponding to this term you have minus e power minus t upon 2 cos root 3 upon 2 t, and corresponding to this term you have plus root 3 e power minus t up on 2 sin root 3 up on 2 t. So, that is the of course the whole thing, is multiplied by u of t. So, that is the inverse Laplace transform of this given rational function. So, now, we have used here a trick in the sense that we are not found out the residues of all poles physically, whenever we have complex conjugate poles, the pair of complex conjugate poles. it would be convenient for as to find out a term in the partial fraction for both this terms together, with have a denominator quadratic factor with real coefficients, which case can identified with damped sinusoidal in the manner shown here.

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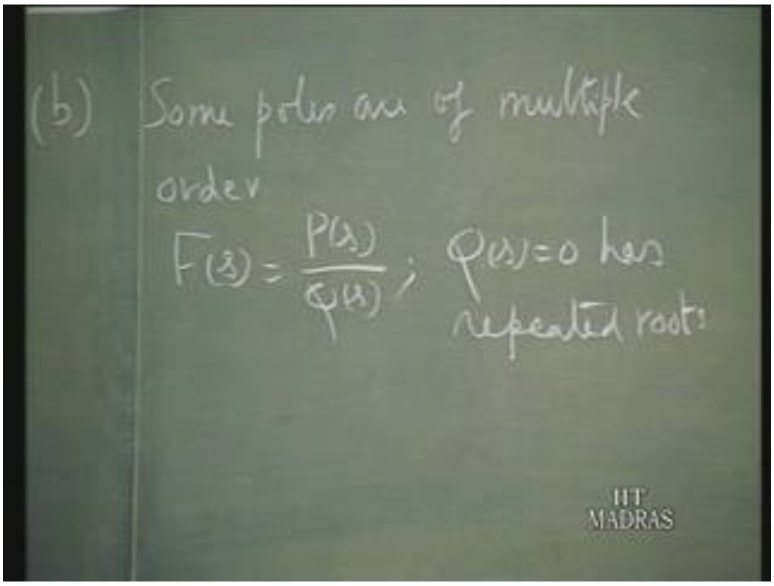

Now, let us continue our discussion with the situation, where poles are multiple order. So, far we have considered rational functions where the poles are simple. So, what happen in some poles are multiple orders. In other words if f of s is p s or q s q s equal 0 as repeated roots.

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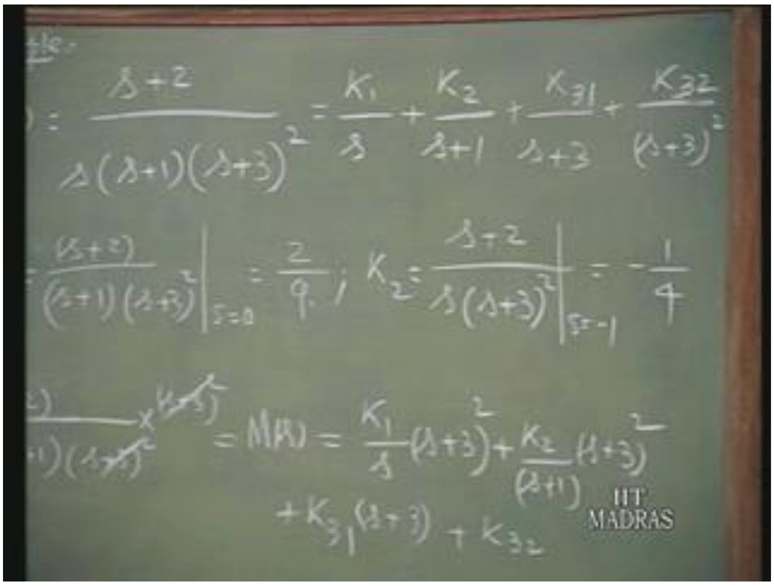

So, some factors are repeated, then we will say the factor f of s has multiple order poles, depending up on the number repudiations. Suppose is repeated twice, it is a pole of order 2. If a particular root is repeated 3 times, then we say a pole of order 3. We illustrate this again by means of example, before we work out the give the general rule. Suppose we have f of s equals s plus 2 divided by s times s plus 1 times s plus 3 whole square. So, this is an example. We will talk about the example first, then we get the general rule. Now, here we have 3 distinct poles; one at the origin, one at minus 1, and second pole is equals minus 3. Therefore, we have k 1 over s plus k 2 over s plus 1, and corresponding to this multiple order s plus 3 whole square the denominator, you can explain this as k 3 plus s plus k 4, are alternately it suits as to have express this as k 3 over s plus s plus 3 plus k 4 over s plus 3 whole square, because these are easily identified with the Laplace transform of unknown type functions.

So, it would be convenient for us to express this in this manner. To streamline this work for extend this a general case we will put it this way. This pole at 3 has got 2 repeated roots; therefore, we have k 3 1 and k 3 2. I will put in this manner k 3 1 and k 3 2. Now, it is to find out k 1, we proceed in the same style forward fashion multiply this by s and substitute s equals 0; therefore, s plus 2 over s plus 1 times s plus 3 whole square with the substitution s equals 0, and that gives me value 2 by 9. No problem at all. Similarly, if you want to find k 2, you multiply this rational function by s plus 1 which leads me to s plus 2 times s times s plus 3 whole square and substitute s equals minus 1, because that is the pole that which you want to find the residue, and that becomes minus 1 4, so no problem about for this concern.

Now you have to find out k 3 1 and k 3 2 of this k 3 t is easily found out, by multiplying this rational function by s plus 3 whole square. So, that we unsecured s equals minus 3, all the other terms vanished. Therefore, k 3 k 2 can be found out as, or maybe we will illustrate this in general way that is. Suppose I write here multiply this s plus 2 over s times s plus 1 times s plus 3 whole square. I multiply this by s plus 3 whole square. So, that this term will be canceled out, and we will call this m of s. So, this is m of s that is on multiplying this by s plus 3 whole square, I get m of s. So, on the other side you have k 1 s times s plus 3 whole square plus k 2 over s plus 1 times s plus 3 whole square plus k 3 1 times s plus 1 are times k 3 1 plus s plus 3 plus k 3 2. So, multiplying this by s plus 3 whole square the terms correspond to the partial fraction each terms get multiplied by s plus 3 whole square.

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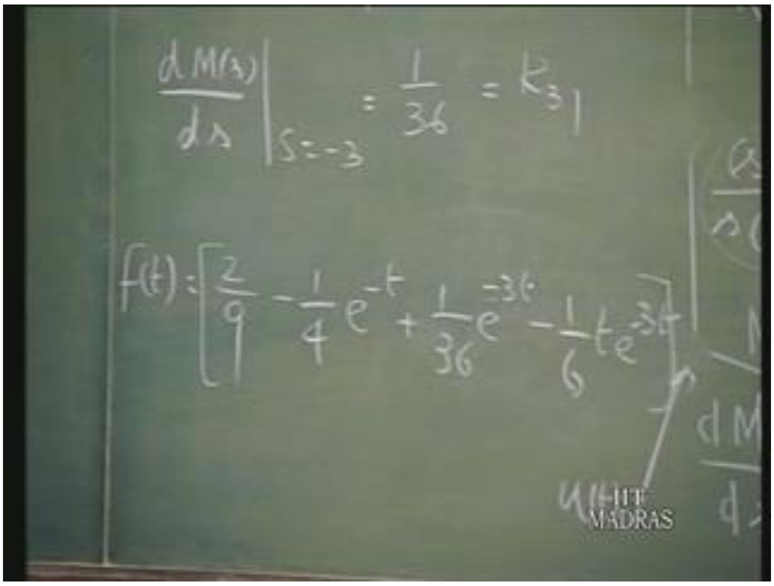

Now, if substitute s equals 3 minus 3 in this, we observe that this vanishes, this vanishes and leaves k 3 2. So, k 3 2 is found out in a again in a straight forward manner, by multiplying this by s plus 3 whole square and substitute s equals minus 3 that will k 3 2. So, we have found out this residue, this residue and this residue. When we have multiple order poles, is actually the terms is associate with linear term that is called the residue. This is really the residue pole at s equals minus 3, not this 1 in mathematical literature this is called the residual the pole is equals minus 3, and that is the one which is order to find correspond compare. Now, how do you find k 3 1. Now the tick in this a, we have got m of s is equal to s. now suppose you take the derivative of m of s with respect to s.

Then what happens is, this term drops out and this leaves you k 3 1, as far as the other terms are concerned since s plus 3 whole square is a factor here, when you take the derivative of m of s with respect to s, s plus 3 continuous be a factor, because it is a quadratic factor once you take the derivative s plus 3 continuous to be a factor, in both this terms. Therefore, we take d m s; the derivative of m s with respect to s d derivative of m of s with respect to s equals k 1. Suppose I call this x of s, I have x prime of s plus k 3 1. And now x prime of s has a factor s plus 3; therefore, we have when you substitute d m s over d s as s equals minus 3 will give me k 3 1. And what is d m s; derivative of ms with respect to s, this is after all m of s. So, if you calculate that, you have derivative of m of s with d s equals derivatives of this, s square plus s times 1 minus s plus 2 times s square plus 2; that is 2 s plus 1 divided by s square plus s and that is what we have d m s by s, and s square plus s whole square.

So, d m s over d s substitution s equals minus 3 will give me, after 1 by 36, and that of course k 3 1. So, the abstract of this is, the partial fraction, required partial fraction expansion will be; k 1 over s k 2 over s plus 1 k 3 1 over s plus 3 k 3 2 over s plus 3 whole square, where all the residuals are found out. therefore, f of t will be equal to corresponding to the first term k 1; that is 2 up on 9, 2 up on 9 or s, so for the inverse Laplace transform is 2 up on 9 times u t. u t will come again anywhere minus 1 fourth e power minus t corresponding to this term. And corresponding to k 3 1 we have 1 up on 36 e power minus 3 t and corresponding to k 3 2 or s plus 3 whole square k 3 2 is found out as. we have not work without k 3 2 will be equal to, when we substitute s equals minus 3 here k 3 2 is found out as minus one sixth; therefore, this becomes minus 1 by 6 and 6 is k 3 2 over s plus 3 whole square, inverse Laplace transform is t multiplied by e to the power minus 3 t. The whole thing is multiplied by u of t.

So, this final answer is 2 up on 9 minus one fourth e power minus t plus 1 by 36 e power minus 3 t minus one sixth t e minus t t times e power minus 3 t multiplied by u of t that is the final results. Now, after having worked out in example, let us know, discuss this more general way, when you have multiple order poles, how do you find the various terms and the partial fraction expansion. Let us look at it in a more general way, rather than given example. Let us now consider a general case of situation where multiple order poles occur.

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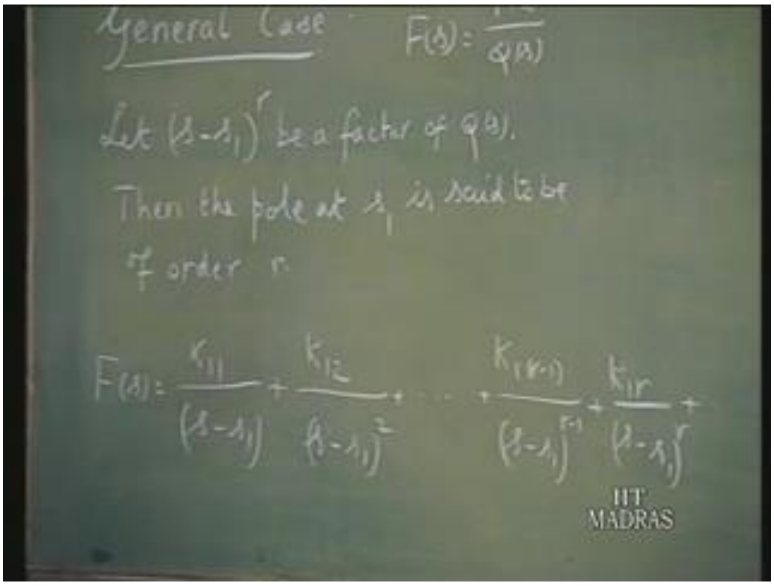

So, given f of s which is p s or q s. let s minus s 1 power of r be a factor of q of s. in this event we say that the pole at a sequence s 1, is of order r, because the particular factor is repeated r times. Then the partial fraction expansion f of s will have the form, corresponding to the pole is equals s 1. We can write this as k 1 one over s minus s 1 plus k 1 2 over s minus s 1 whole square plus. So, the denominator will have expanding powers of s minus s 1, and finally, you have k 1 r minus 1 by s minus s 1 raise to the power of r minus 1 plus k 1 r over s minus s 1 to the power of r plus terms which involve other poles of f of s plus terms which involve other poles that is poles other than let us equal to s 1. Let me call this group of terms r of s, because we really. Our focus now is on how to calculate k 1 and k 1 r. We are not interested in other terms. So, now our discussion will now sent around, methods of finding out k 1 one k 1 2 or up to k 1 r.

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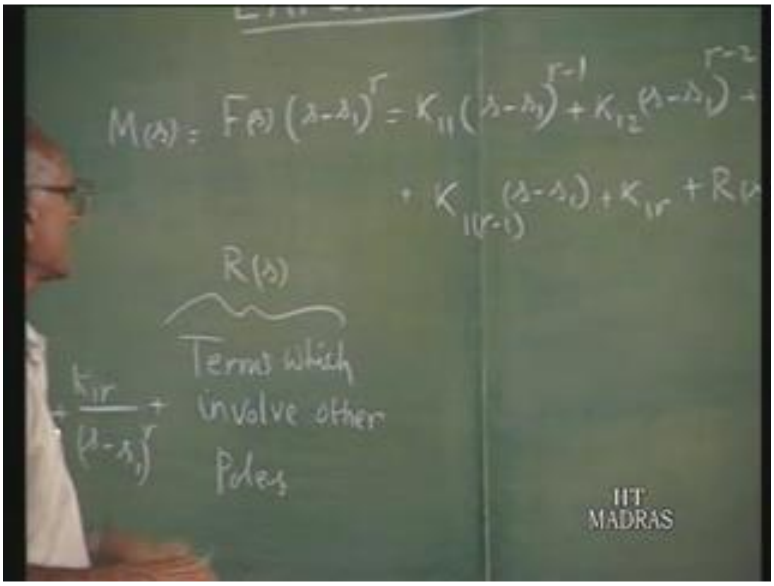

Now, taking the q from the example which we discussed just now. Let me say; m of s is obtained by multiplying f of s over s minus s 1 raised to the power of r. So, this is the partial fraction expansion f of s. We multiply this by s minus s 1 to the power of r. Then this entire group of terms will be multiplied by s minus s 1 raise to the power of r. Therefore, that will turn out to be k 1 1 times s minus s 1 raised to the power of r minus 1 plus k 1 2 raised to the power of s minus s 1 raised to the power of r minus 2 down the line plus compare to this last 2 terms k 1 r minus 1 times s minus s 1 plus k 1 r plus the entire group of term r of s multiplied by s minus s 1 raised to the power y that is how this m of s look like. So, every 1 of this term is multiplied by s minus s 1 raise to the power of r and this is what the result is. Now it is easy to see now that k 1 r is simply found out by multiply by substituting s equals s 1 in this expression, because every term manipulate expects this, when we substitute s equals s 1.

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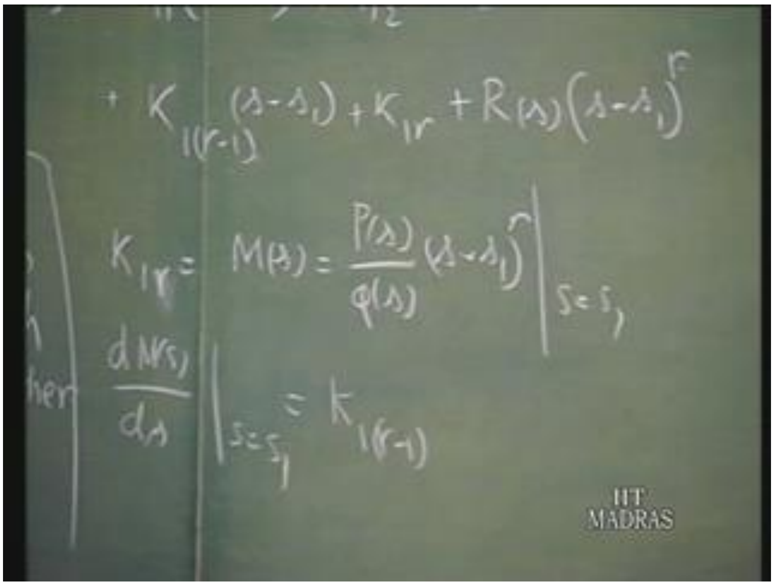

So, you substitute s equals s 1 in m of s k 1 r falls out. So, k 1 r equals m of s, which is really p s over q s multiplied by s minus s 1 raised to the power of r with the substitution s equal to 1; that is quit forward that is k 1 r. Now, to find out k 1 r minus 1, we adopt the same trick as use in this example. What you recall. We have taken the derivative of m of s with respect to s. If you do that these terms drops out, and the derivative of this is simply k 1 r minus 1. And since these are all accompanied by higher powers of s minus s 1 when you take the derivative, they still continue s minus s 1 continuous to a factor here; therefore, they all drop out. So, when you have got d m s over d s, if you do that, and substitute s is equal to s 1. Then the only term which means is k 1 r minus 1, because k 1 r disappears. All the other terms have a factor s minus s 1; therefore, we substitute s is equal to s 1 that remains; that is k 1 r minus 1. Now, suppose you take the second derivative, then the previous term will come. The previous term here you recall, it will be k 1 r minus 2 s minus s 1 whole square. So, when you take this first derivative, you get 2 times s minus s 1 when you take the second derivative, it will be 2 times k 1 r minus 2. Therefore, you have k 1 r minus 2 will be 1 upon 2 d m s over d s substitute s equal to s 1. That is a matter of fact instead of 2 I will write just 2 factorial, because that is how the progresses, at the previous term when to take k 1 r minus 3 you will get, by the final analysis 3 factorial that is how it goes.

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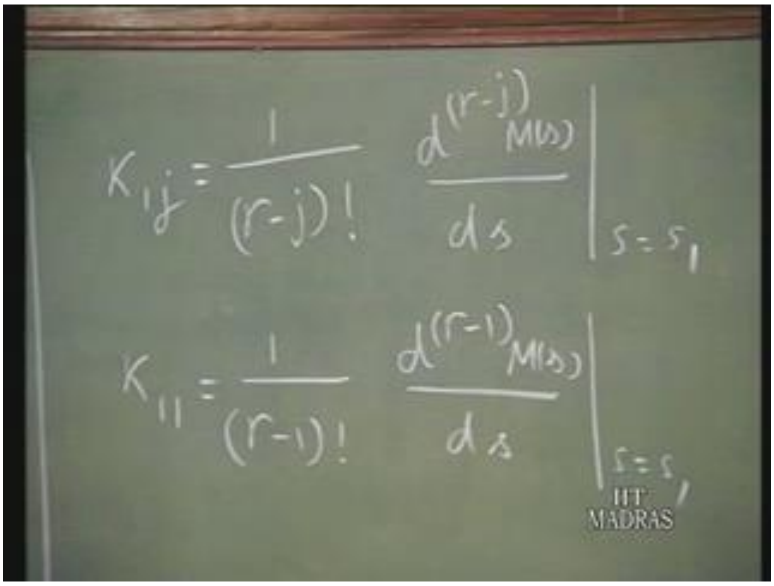

So, continue in this discussion it is easy to verify, we can show the k 1 j is obtained as 1 over r minus j factorial. The r minus j derivative of m of s over with respect to s. The substitution s equals s 1. And in fact you get k 1 1; that is the first term here k 1 one and this is called the residual of a multiple of the pole let us k 1 1, which is the hard to find out. It is obtained by taking r minus 1 derivative of the r minus 1 m of s over d s. So, in principle it is same as what we have done in the first example we worked out. Only thing is, when you have got a large number of a multiple order pole, you have to take as substitute derivative of this term m of s f of s s minus s 1 raised to power of r; that is you lock of the term the denominator which is s minus s 1 raised to the power of r, and the rest of the rational function, you take successive derivatives and substitute s equals s 1. So, you get all the terms the partial fraction expansion, starting from k 1 r write up to k 1 1 .k 1 1 is the numerator of the linear factor, k 1 r is the numerator of the factor s minus s 1 to the power of r. So, the principle is straight forward, you do not have to really remember this formulas, provided you know the principle that is involved. You can always figure out how is done.

You have to take the rational fraction as such, given rational fraction, multiply by s minus s 1 raised to the power of r and you get a resultant rational fraction, function which is called m of s to take successive derivative and substitute s is equal to s 1. So, the principle of finding out the partial fraction expansion, when you have got multiple order poles, is again a straight forward extension what you have done in the simple case pole. And when you have a combination of simple poles and multiple order poles you have to use a combination of both the techniques. Simple poles is quit straight forward, multiple order poles, we have to take successive derivative of the factor m of s. A combination of this 2 has to be adopted. In the case of complex conjugate poles, if you find the residue with respect to 1 complex conjugate pole, it is enough. This second its conjugate residue corresponding to its conjugate, is the conjugate of the residues itself, this is one way of handling this. alternately as when the example that you worked out are suggested, you can combine the terms corresponding to 2 conjugation poles, and find out have a quadratic denominator, and find out a linear factor in the numerator, and associate this with terms of the type e power minus alpha cos omega t and e power minus alpha t sin omega t, that can also be done. Otherwise you can leave them as linear terms, and then find out the residues corresponding to complex conjugate poles, find out the inverse Laplace transform of this and combine this resulting time functions, which again yields finally the same result.

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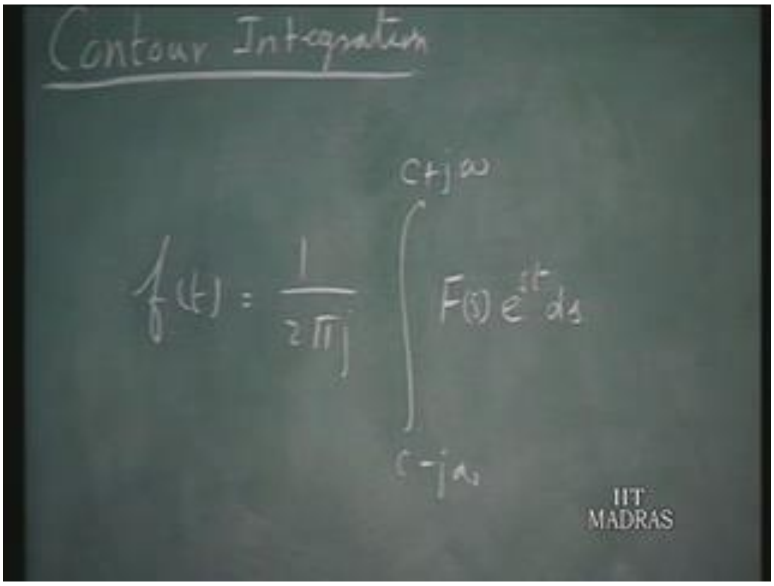

Today at the finding out the inverse Laplace transformation, using the partial fraction expansion. There is another method which is the contour integration, which is more fundamental, and which terms from the very definition of the inverse Laplace transformation. You recall the inverse Laplace transformation f t is 1 over 2 pie j c minus j infinity to c plus j infinity of f of s e to the power of s t d s.

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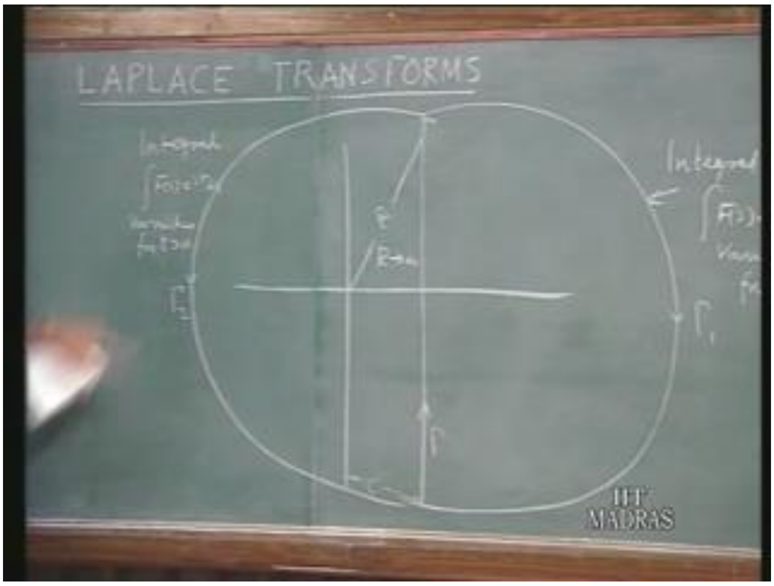

So, in the complex plane, you take the integration contour like this c minus j infinity, all them down to c plus j infinity. Let me say this contour is gamma. So, this integration is a long term, this. Now it is easy to evaluate the integral along a close contour in a complex plane, because from the theory of residues, once you have integration over a close contour that is equal to 2 pie j time, the residues of the poles inside the contour of the complex variable theory. So, what one does is, this to close this by means of, to large semicircles where r tends to infinity. Let me call this gamma 1, (()) 2. It turns out that if you have this integral evaluated around this contour for t less than 0 it vanishes. So, integral f of s e to the power of s t vanishes along this contour for t less than j. On the other hand if you take it along this contour gamma 2, this integral f of s e to the power of s t d s vanishes for t greater than 0. So, since we have, we want to really evaluate this integral along this contour, from this point to this point. But nothing is lost if you have to this contour, this particular large semicircle for t less than 0, and this large semicircle for t greater than 0.

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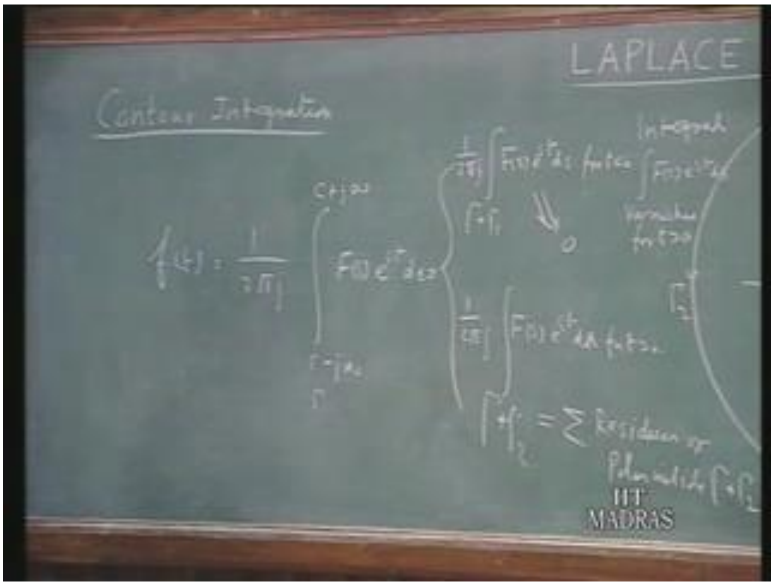

So, in other wards I can say, this is equal to 1 over 2 pie j evaluate the contour cross contour gamma plus gamma 1 of f of s e to the power of s t d s for t less than 0, because of all I made in to that, a quantity which is equal to 0, and this is equal to 1 over 2 pie j integral along this close contour for d s for t greater than 0, and it turns out that as for as this contour integration is concerned, there are no poles inside as long, because all the poles will be here; therefore. Since this close contour does not include any poles this transfer to be 0, and as for this is concerned. This will be equal to the sum of the residues of poles inside the close contour 2. So, whatever poles you are having here, you have take the residues of the poles that will be equal to f of t for t greater than 0; this is for t greater than 0, this is for t less than 0.

So, in the inverse Laplace transform you will get f of t identically 0 of t less than 0, because of this, and for t greater than 0, it is equal to sum of the residues of the poles inside this. Our discussion here is lesser the (()), because this is not the method that we used for normally evaluate of the inverse Laplace transformation. Just giving this to you the shape of completeness, and the partial fraction expansion is. So, much simpler and that takes care of most of our needs. We do not really have to the contour integration. But, because this is basic one, and this will have to be resorted to, for special types of functions which we may not know the inverse Laplace transform simply, so I included this. So, we will not pursue this topic, just mention the contour integration is alternative method to finding out the Laplace transformation.

So, far we have discussed the definition of the Laplace transformation, its properties and also method of finding out the inverse Laplace transformation. We have notice that many of the properties of the Laplace transform parallel to those that we have already studied under the Fourier transform method, and we also note is, that for certain functions the Laplace transformation and Fourier transform are quit related to each other, in sense that we substitute j omega for s you get the Fourier transform in the Laplace transform. For example, you have e to the power of minus alpha t u t; the Fourier transform is 1 over j omega plus alpha. The Laplace transformation is 1 over s plus alpha. So, for substituting j omega for s you get the Fourier transform, from the Laplace transformation. On the other hand, suppose you have u of t itself, the Fourier transform of this is 1 over j omega plus pie delta omega. On the other hand, the Laplace transformation is 1 over s. So, here there is a discrepancy. So, what would like to know, under what conditions will be the Fourier transform obtained the mere substitution of j omega for s from the Laplace transformation, and under what cases will be differ. This substitution will not yield as the appropriate Fourier transform. This is what, this is the question with to like to address.

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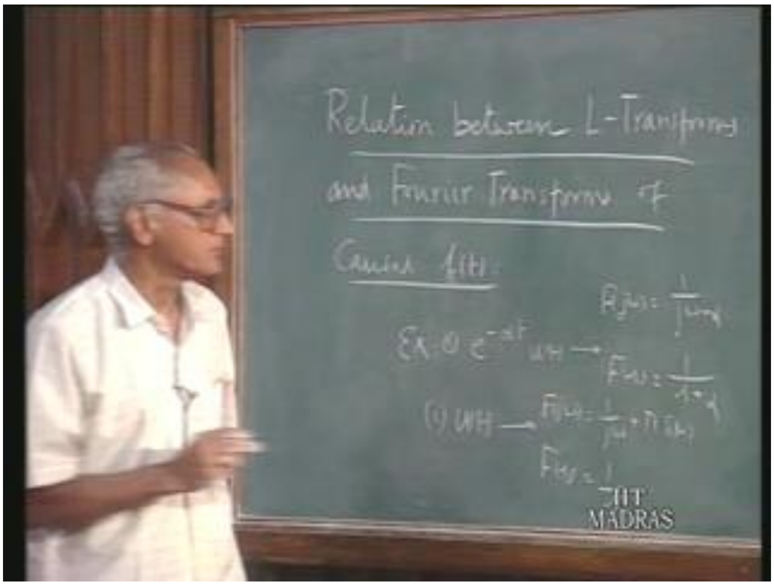

Let us consider three cases; a, in the Laplace transformation the apse of convergence is less than 0. Now, before this it must be clearly mentioned, that when we compare the Laplace transform in the Fourier transform, we can do it only for the causal time functions, because the Fourier transform integrates for minus infinitive can plus infinitive the time axis. So, f of t values for t less than 0, there is no reasons for the Laplace transform in Fourier transform to be related to each there. So, whatever comparison we can make, can be done only for casual time functions. So, the first case corresponds to situations, where the apse of convergence is less than 0, which means that f l of s has all poles in the left of plane. Examples e power minus alpha t u t has the Laplace transform 1 over s plus alpha example a b e power minus alpha t cos omega naught t will have the Laplace transform of s plus alpha over s plus alpha whole square plus omega naught square; that means, here poles will be the left of plane. In all this cases, the Fourier transform f of j omega is obtained from the Laplace transform of the corresponding function with substitution s is equal to j omega. So, we have the Fourier transform can be obtained to be the Laplace transforms by the mere substitution of s by j omega. Conversely, the Laplace transform is obtained to be the Fourier transform by substituting s for j omega.

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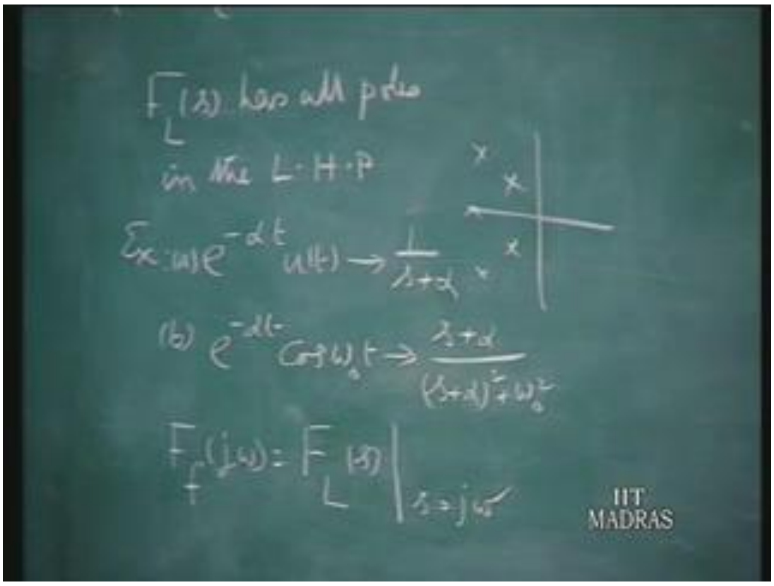

So, when the apse of convergence is less than 0, Fourier transform and Laplace transform are very closely related, 1 is obtained from the other by substitution of s for j omega r j omega for s. Suppose, I have the apses of convergence to be greater than 0; that means, there are some poles in the Laplace transformation of f l of s in the right half plane.

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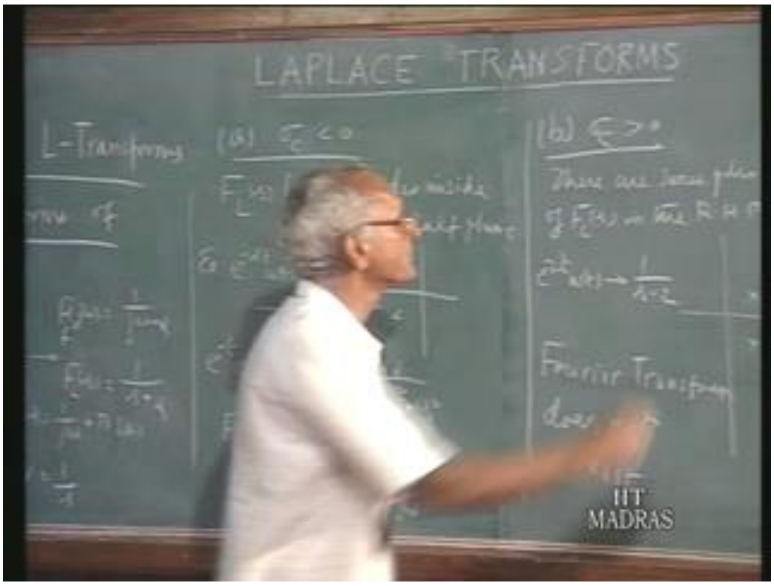

So, may be you are having some poles here. So; that means, that is why the apse of convergence is a positive number. Example of such situation will be e to the power of 2 t u t will have the Laplace transform 1 over s minus 2. Now, all we can say is, for such functions Fourier transform does not exists, because the integral does not converge. You have the f of t e to the power of minus j omega t d t that integral cannot converge, because you have exponentially increasing time function. In such cases Fourier transform does not exist. So, there is no way in which we can relate the Laplace transform and the Fourier transforms. Simply a Fourier transforms does not exist for such functions.

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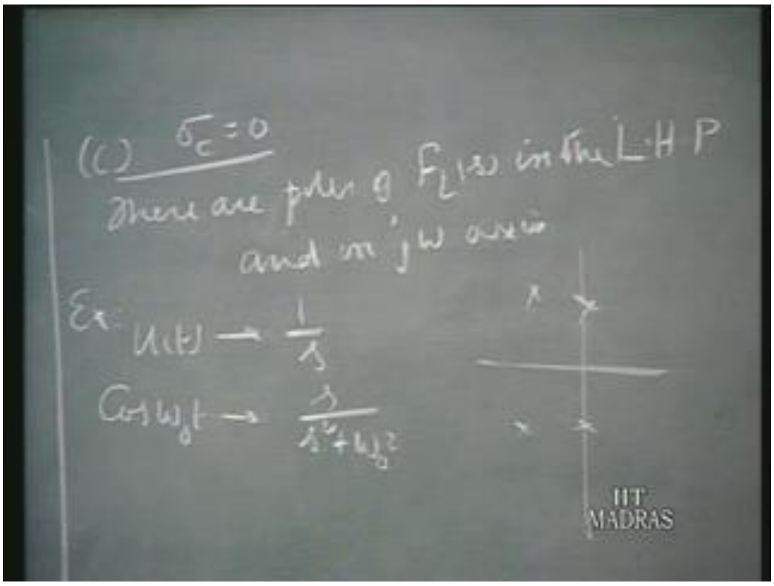

The third case will be the apse of convergence is equal to 0; that means, there are poles of f l of s in the left half plane, and on the j omega axis. The apse of convergence is 0, because you have poles on the imaginary axis as well example u t 1 over s cos omega naught t s over s square plus omega naught square. So, all these are cases, where you have poles on the imaginary axis.

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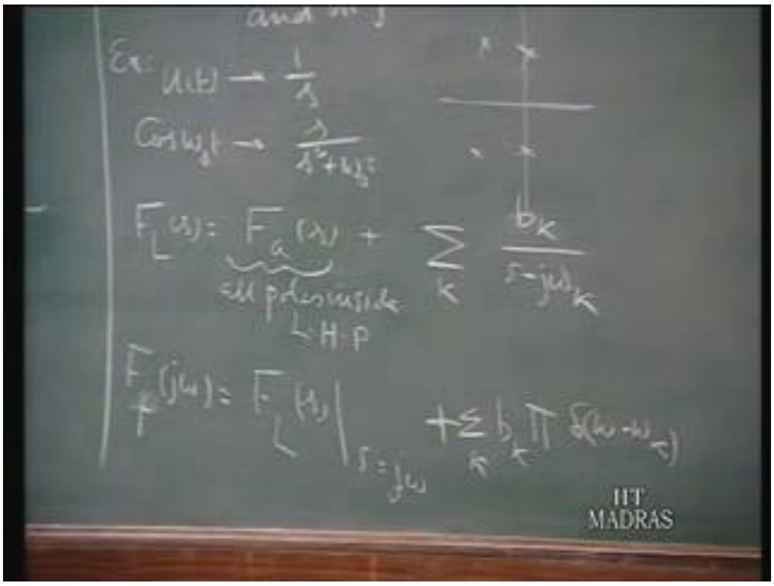

What we do in such cases. The general rule is, f l of s in general will have f a of s which has got all poles inside left half plane plus some poles here on the imaginary axis. Let us say b k over s minus j omega k. So, it will have b k over s minus j omega k where k; the summation is on k; that means, the Laplace transform f l of s can be thought of, as the combination of the terms which correspond to poles in the imaginary axis plus terms which do not all poles on the imaginary axis, which all poles inside the left half plane corresponding to case a. In such equation, it can be easily visualize, that before a transform of this, will turn out to be the substitute in j omega for this f l s s is equal to j omega plus corresponding to each one of this poles of the imaginary axis you have got b k pie delta omega minus omega k b k pie b k delta omega minus omega k sum of k. So, it tells out that in such equation for every pole and imaginary axis and corresponding residue, you have an extra delta function, and that is why when you go to u of t 1 over j omega pie delta omega you are getting, the corresponding residues is 1.

So, pie delta omega you are getting. So, this is what it would be. So, to summarize them wherever you have the apse of convergence is less than 0, then the Laplace transform and Fourier transform are very closely related substitution s is equal j omega will get one from the other. But if you have poles on the imaginary axis, you have delta functions in the Fourier transform. You do not have delta functions in the Laplace transformation. We will close this discussion of properties of Laplace transformation at this stage, and in the next lecture will continue with the application of Laplace transformation technique to system analysis and circuit analysis. But now I thing is appropriate time to break at this point, and look at an exercise on the topic that I already discussed, related to Laplace transformation.

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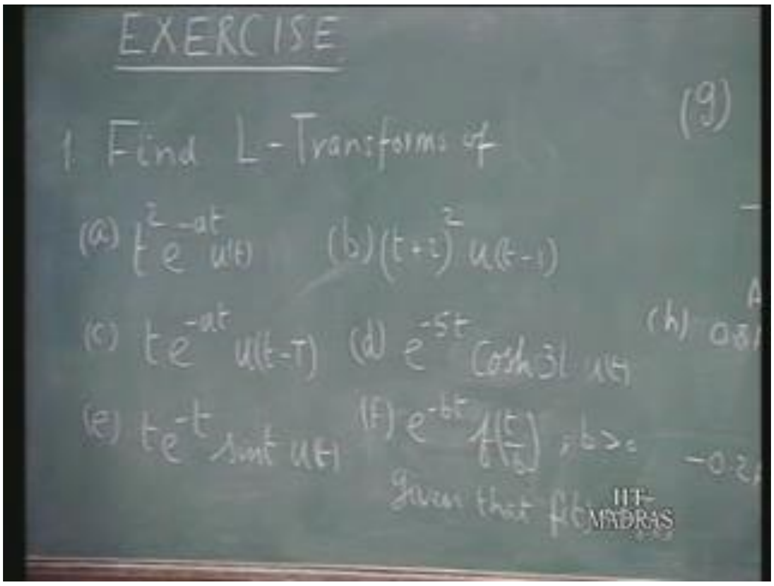

The first question is, find the Laplace transforms of the set up time functions that are given here; a, t square e to the power of minus a t u t. b, t plus 2 whole square times u of t minus 1. So, this time function really starts t equals 1 onwards. The third function; time multiplied by e to the power of minus at t e to the power of minus a t multiplied by u of t minus t; a delayed step function. So, once again this functions starts from t equals T onwards, its (()) from values of t less than capital t. d, e to the power of minus 5 t times hyperbolic cosine cosh 3 t u t. We are not talked about Laplace transforms hyperbolic functions, but you can always express them, as a sum of exponential functions, and then combine this e to the power of minus 5 t and you can find the Laplace transform of that. e, t e to the power of minus t sin t u t. here again you have the combination of the factors. We have worked out examples where e to the power of minus t sin t, we found Laplace transform, but that multiplied by t. So, whenever you multiplied by t equivalent to taking the derivative in the transform domain. So, you have to use those properties, to find the Laplace transform of this. f, given that f of t has the Laplace transform of f of s, what would be the Laplace transform of another function derived from f of t in this manner; e to the power of minus b t times f of t minus b**.** So, its scale by a factor b, in addition you multiplied by multiplied it by exponential term e to the power of minus b t u t. Take b to be a positive number, and then find this out. Find the Laplace transform of this, in terms of b and f of s.

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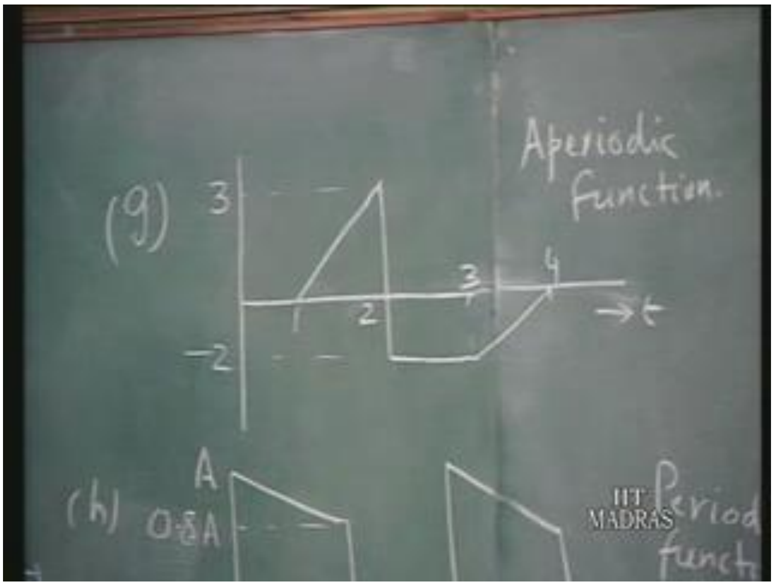

Then we are given the wave forms of two time functions; one is a periodic function. This is all the function there is, for outside this range from t equals 1 second to 4 second it is 0. So, within this interval t equals 1 to 4 it behaves its manner, it linearly raises from 0 to 3 units, and then claims it down claims down to minus 2 steeply, and then remains at minus 2 from 2 second to 3 second, and then once again comes back to 0 at 4 and stays there for p greater than 4. So, it is a periodic function, described in this manner.

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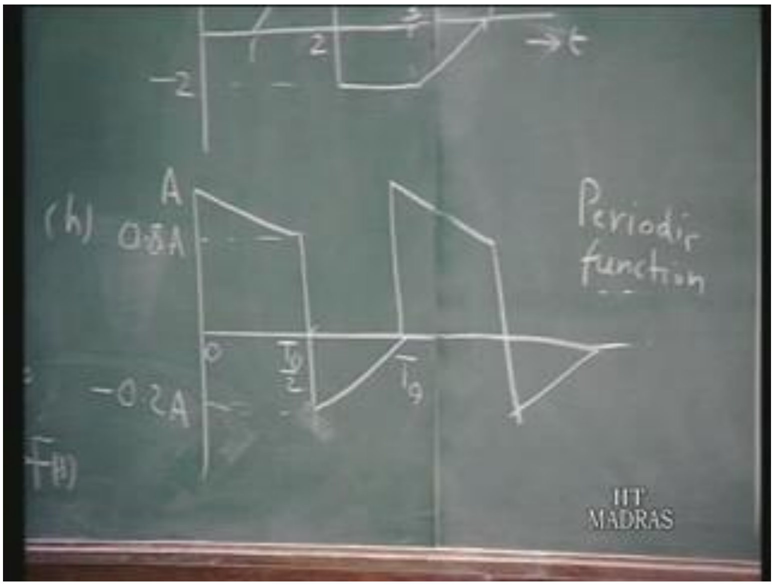

H is a periodic function; the variation for 1 period 0 to t not is described as here. It starts from a and comes down to point 8 a in half a period linearly, and then it jumps to minus point 2 a at this point, and again increases linearly and reaches 0 at equals t naught. So, this is the basic variation in one period, and using the, find the Laplace transform of this variation in one period, and then use the property of Laplace transforms of periodic functions to find out, the entire function of the Laplace transform.

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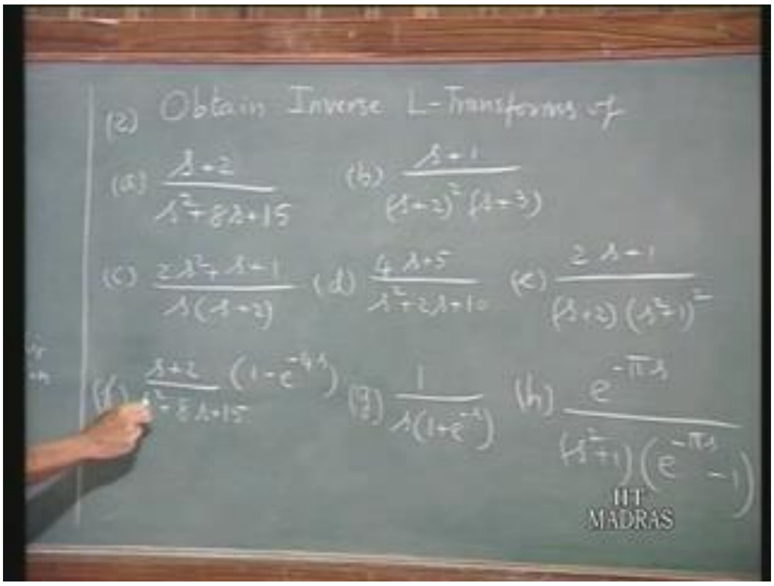

The second example listed here a set of Laplace transforms, find the time functions to use the correspond; that means, find the inverse Laplace transforms of A; s plus 2 over s square plus 8 s plus 15. B, s plus 1 over s plus 2 whole square times s plus 3; that means, the repeat at pole here, so you have to use the rule for finding out the residues when poles are involved. C, 2 s square plus s plus 1 over s times s plus 2. You observe here that the numerators have the same as the denominator; therefore, apart from the factor which correspond to k 1 over s plus k 2 over s plus 2. You also have a constant term 2 which is the ratio of the 2 leading coefficients. So, 2 plus sum k 1 over s plus k 2 over s plus 2 is the partial fraction expansion of this. So, corresponding for the 2 the inverse Laplace transform will be 2 delta t.

So, whenever the numerator degree (()) from the same as the denominator degree, you have constant term in partial fraction expansion and the inverse Laplace transform you have a delta function. D, your 4 s plus 5 over s square plus 2 s plus 10, these are lead to complex conjugate poles. So, instead of finding out the residues of each of the complex conjugate poles, we can do it; that is one way of doing this. Other way would be to treat this as s plus a whole square plus omega square, and then try to find out the inverse Laplace transform, in terms of cosine and sin functions with an exponential decay amplitude. Then you have 2 s plus 1 over s plus 2 times s square plus 1 whole square.

Here you repeat poles and the imaginary axis slightly more complicated, but you can handle this in same way as you have done earlier. Then F, s plus 2 over s square plus 8 s plus fifteen times 1 minus e to the power of minus 4 s.

In tackling this problem you already know the inverse Laplace transform of s plus 2 over s square plus 8 s plus 15. So, all you have to do is use this information here, because e to the power of minus 4 s means whatever f of t you have obtained here, is the same f of t delayed by 4 seconds is what corresponds to this. Therefore, use this information that you already obtained here, to find out the f of t corresponding to this. 1 over s 1 plus e to the power of minus s. So, if you are 1 minus e to the power of minus t s we recognize this to be, the Laplace transform of periodic function of time. So, all you have to do is multiplied both the numerator and denominator by 1 minus e to the power of minus s. So, you have 1 minus s 1 times 1 minus e to the power of minus s by s times 1 minus e to the power of minus 2 s.

Therefore, you can recognize this to be the Laplace transform of some periodic function with the Laplace transform for the basic period being 1 minus e to the power of minus s over s, you can find that out. Similarly here you have e to the power of minus pie s by s square plus 1 e to the power of minus pie s minus 1. So, you multiply both sides numerator and denominator by minus 1. You have 1 minus e to the power of minus pie s. So, you can remove that, then find out the Laplace transform of the remaining function, and once you introduce it becomes the Laplace transform of periodic function of time. So, using all this properties that we derived, you find the inverse Laplace transforms of all this 8 functions of s.

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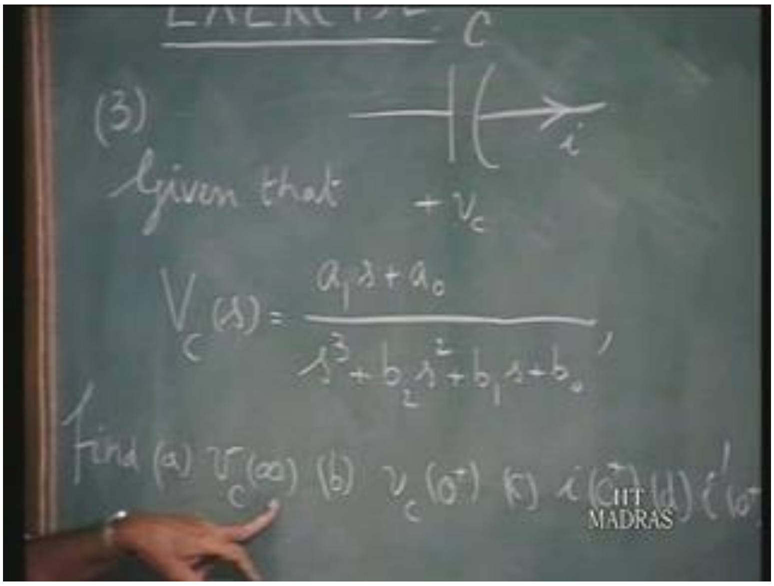

Question number 3. You have a capacitor circuit, current I is current passing the capacitor, and v c of t is the voltage across to the capacitor. Suppose the solution of the network we found out, the Laplace transform of the voltage across the capacitor v c of s. Let it be in symbolic form a 1 s plus a naught over s cube plus b 2 s square plus b 1 s plus b naught. That is the Laplace transform of the voltage of the capacitor. Now, using this information, in terms a and b constant. Find the final value of the capacitor voltage v c infinity. b the initial value of the capacitor voltage, which is v c 0 plus immediately at after t equal 0. Also find out the initial value of the current i 0 plus. And lastly find the initial value of the derivative of the current i prime 0 plus. This means this is d i by d t evaluated t equals 0 plus. So, in application of the initial value theorem, final value theorem, and the derivative rule; all this are needed for solution problem. In other words you do not have to explicitly find v c of t using this initial value theorem, final value theorem in the derivative rule. You should be able to find out all this quantity in terms of the a constants and b constants.

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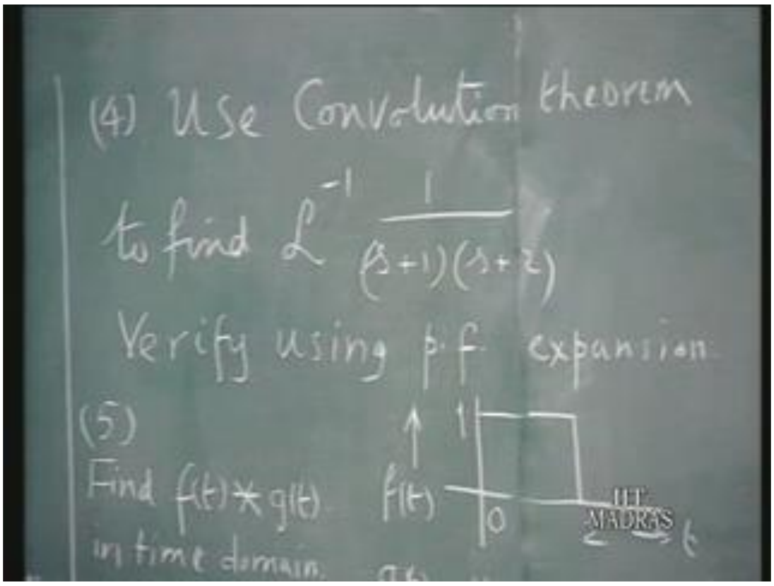

Question number 4. Use convolution theorem to find the, inverse Laplace transform of 1 over s plus 1 times s plus 2. Verify using the partial fraction expansion. So, this Laplace transformation can be thought of, have the product of 1 over s plus 1 time 1 over s plus 2. You can find out the time functions corresponding to this two factors, and since it is the product that is involve the transform domain, in the time domain, is the convolution. So, it is the convolution of the time function corresponding to 1 over s plus 1 and 1 over s plus 2. Find out the corresponding time function. Now, you can also find out the inverse Laplace transform, using partial fraction expansion verify, using partial fraction expansion. And compare the 2 answers, then both of them must naturally agree. So, this is an exercise in not only partial fraction expansion, but also the using the properties of convolution.

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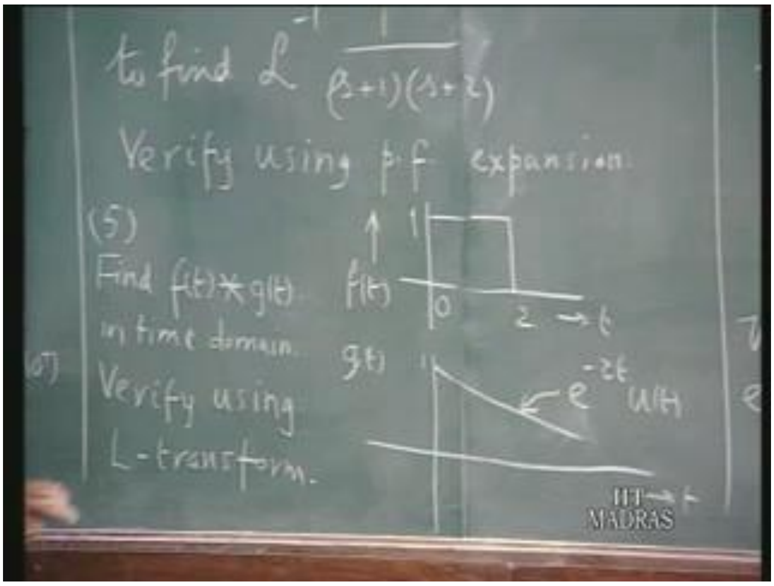

Question number 5. You are given 2 time functions f of t is a pulse which loss from 0 to 2 seconds, having an amplitude 1. And g of t is an exponentially decay time function e to the power of minus 3 t u t. Find ft star g t; that is the convolution of f t and g t. Both are causal time functions, both are emendable to Laplace transformation .Find this in time domain, purely working out in time domain from the basic definition of the convolution integral, you find out the ft star g t in time domain. As an alternative verify using Laplace transform. In the second step is what you have to do find the Laplace transform of f of t, find the Laplace transform g of t. Since we interest in the convolution of this two; that whatever is the result of this convolution must have for its Laplace transform f of s time g of s.

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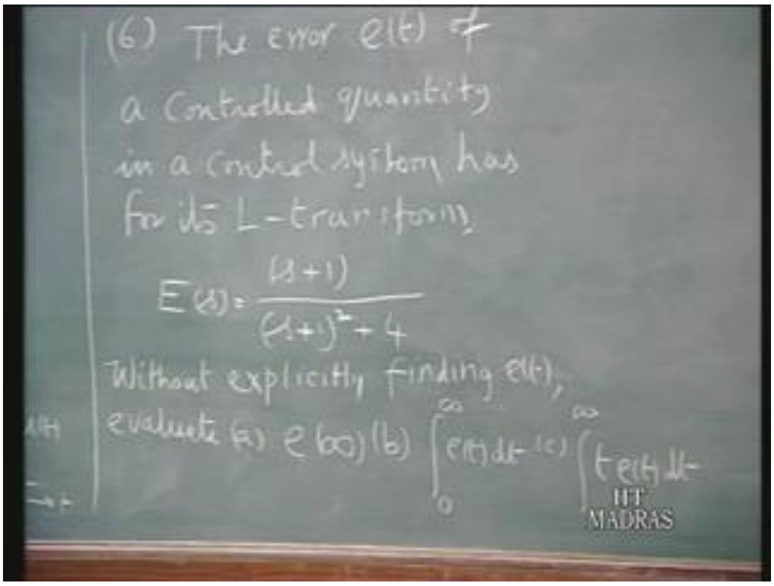

So, you find the Laplace transform of this, find the Laplace transform of this, both are simple to find out. Find the inverses Laplace transform, you get f t star g t. Lastly, you have the error e t of a controlled quantity in a controlled system, has for its Laplace transform. This expression e of s equals s plus 1 over s plus 1 whole square plus 4. The error et of a controlled quantity in a controlled system has for its Laplace transform this expression e of s equals s plus 1 over s plus 1 whole square plus 4. Without explicitly finding e of t, you do not have to find inverse Laplace transformation of this to find e of t, without explicitly doing that evaluate e infinity; that the final value of this error. B, integral 0 to infinity e t d t. The integral of that error over the period 0 to infinity. C, 0 to infinity t e t d t; that is time multiplied t e t d t. This you should be able to do it without actually finding out e of t, using initial value theorem, the rule for integration so on and so forth. You can also extent this to, I can ask you find out integral 0 to infinity of e square t d t; that is the integral of the square error. But that requires a convolution in the complex domain, since we have not discussed that we will not include that type of question. So, this exercise covers the various properties of Laplace transformation that we discussed in the earlier 6 lecture. In the next lecture, we will discuss the application of Laplace transformation technique to evolution of transients in networks and systems.