Networks and Systems
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Lecture – 24

We had considered several interesting properties, of the Laplace Transformation in the last lectures. So, it is a good time now, to work out a number of examples, which illustrate the application of these properties.

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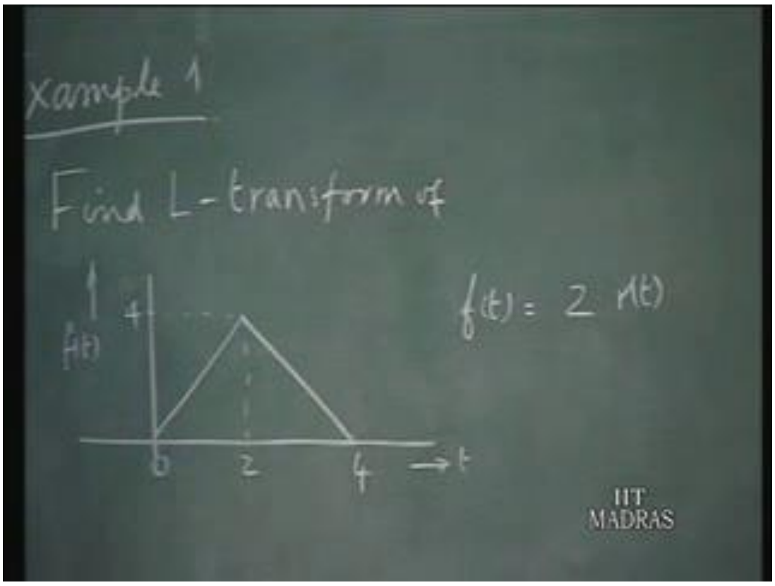

First example; Let us find the Laplace transform, of a function f of t which is, represented graphically in this form. This is t in seconds, f of t as the value of 4 units t equals 2 seconds. So, this is the function f of t whose Laplace transform we are required to find. So, we can express this f of t as. From this point to this point it claims an amount of 4 units in 2 seconds; therefore, the slope is 2 units. It is the ram function 0 for negative values of time. Therefore, 2 rt this describe the behavior from 0 to 2. At this point instead of claiming up rate of 2 units per second, it is climbing down at the rate of 2 units per second, because in 2 seconds, it comes down by 4 units. Therefore the next slope must be minus 2; therefore, you must bring this positive slope of 2 units to a negative slope of 2 units. Therefore, you must introduce a negative ram 4 units, and then that ram is introduced t equals 2 seconds. Therefore this minus 4 rt minus 2.

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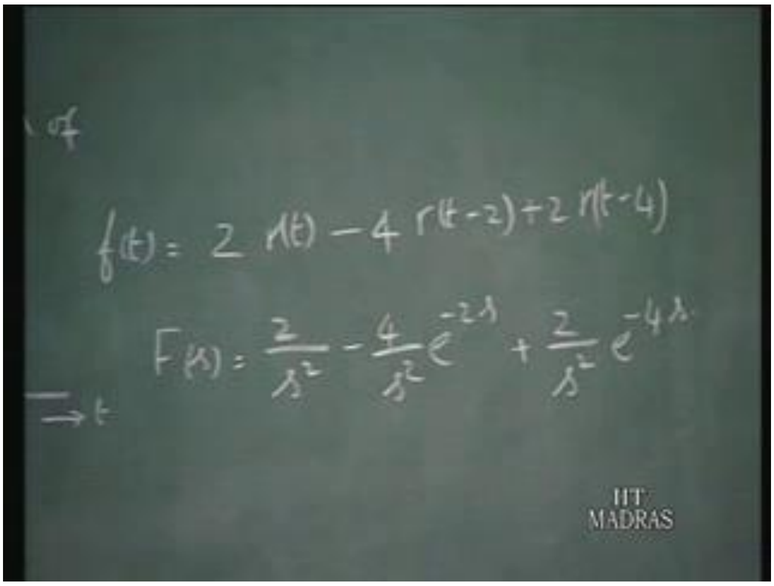

So, if this 2 rams alone are operating, then this continuous to go down like this, but at this point the downward slope must be arrested, next slope of minus 2 must be arrested and then it’s flat and out. Therefore, to introduce another slope of plus 2 units at r t equals 4. So, it is this 3 ram functions which requires this characteristics. therefore, we can find f of s as 2 up on s square minus 4 upon s square would be the Laplace transformation, if it been 4 rt, but since it is delayed by 2 units you must introduce e to the power of minus 2 s. And this as the ram of 2 units plus 2 over s square is the basic Laplace transform of the ram function. But it’s delayed by 4 seconds; you must introduce e to the power of minus 4 s. So, that would be the Laplace transformation of this. it is illustrate you to obtain this same result, by differentiating in the f of t; may be once or twice and finding out the Laplace transform of the derivative functions, and then go back and see, how it agrees with this or not.

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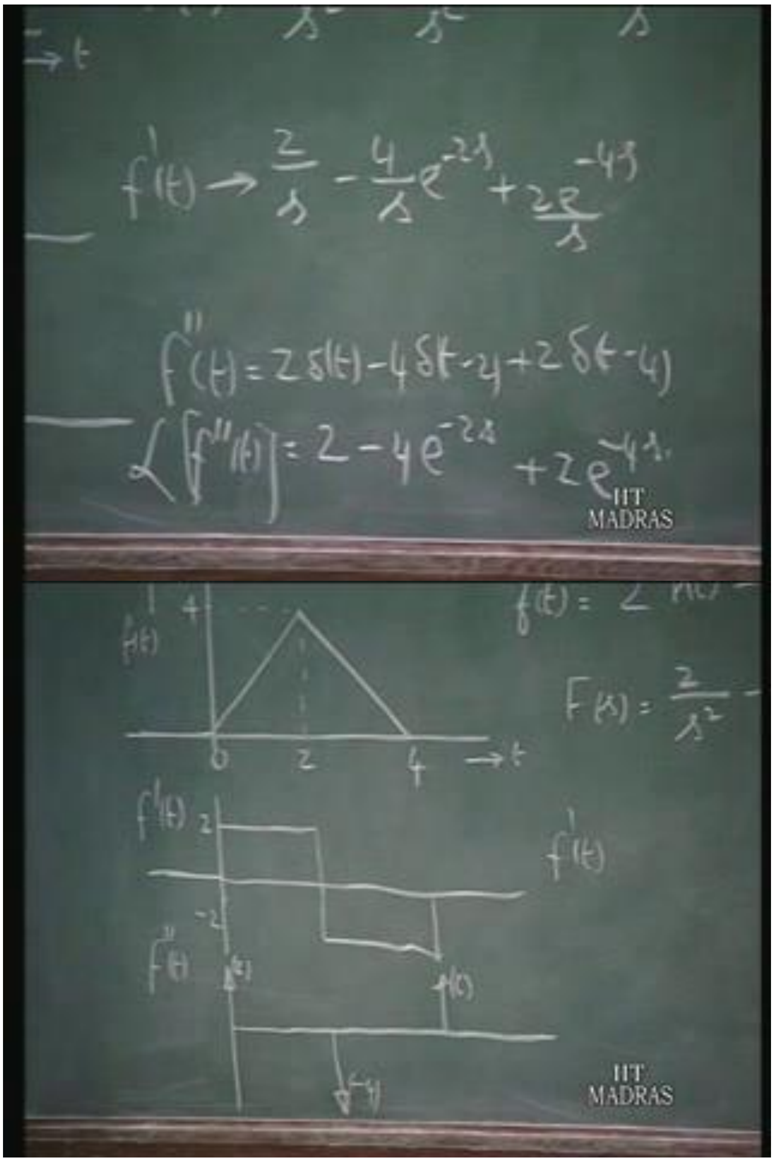

So, let us see, if I take the derivative of this f prime t, then for 0 to 2 seconds it has plus 2 units, slope of plus 2, and from 2 to 4 seconds it is the slope of minus 2. So, that would be the derivative function; 2 and minus 2, that is the derivative function, this is f prime t. Now, if I take the second derivative of this. Once again take the derivative of this. You have here we are thinking of this is 0 therefore, this is also 0. Therefore, there is an impulse of 2 units here; 0 to 2, 2 units impulse. And at this point it jump from plus 2 to minus 2; therefore, there is a negative impulse of 4 units of minus 4 is the strength of the impulse. At this point again it claims up by 2 units therefore, this will be 2. So, you can write f double prime t, that is the second derivative of the function of time.

F double prime t as 2 delta t minus 4 delta t minus 2, because that is an impulse standing a t equals 2 seconds, plus another impulse function of 2 units at t equals 4. This is the second derivative of this. The second derivative now, consist purely impulse function therefore, its Laplace transform, is easy to find out. So, laplace transform of the second derivative is, delta t as a Laplace transform 1. So, it is 2 minus 4 delayed impulse minus e to the minus 4 e to the power of minus 2 s, and this is again a delta function therefore, this is 2, and but delayed impulse therefore, you must be introduce a term e power minus 4 s. So, that is the Laplace transform of the second derivative. To find the Laplace transform of the first derivative; that is this is obtained by integrating this. So, if you know the Laplace transform of this function, the integral will have this same Laplace transform divided by s.

Therefore, this will be adding the Laplace transform 2 up on s minus 4 up on s e to the power of minus 2 s plus 2 times e to the power of minus 4 s over s. In addition, normally we have the integral value of this function at t equal 0 minus. In this case it is 0 therefore, that does not appear here. Now, once we have the Laplace transform of this. The Laplace transform of the integral that is the Laplace transform of this, is obtained by multiplying this by 1 over s once again, and that is what we had. So, this also illustrates the possibility, that when you want to find Laplace transform of functions like this. You may find them directly, but alternately you can take the derivatives, and find the derivatives Laplace transform easily found out. Then you can use that information to find out the Laplace transform of the original function, with 2 alternative approaches. It also illustrates the rule for integration, that we already discussed.

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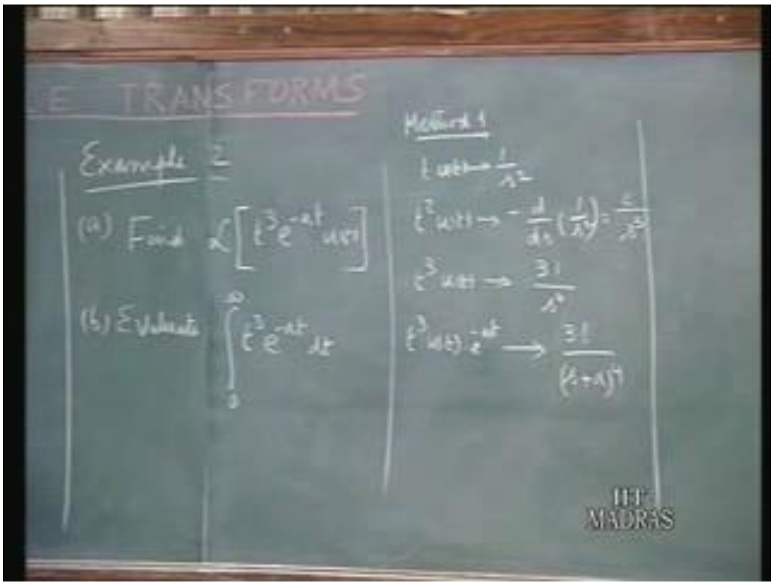

Let us take a second example; a, find the Laplace transforms t cube e to the power of minus at ut. And after having find that out, evaluate 0 to infinity of t cube e to the power of minus at ut. This is the question that is asked. Now, to find this out, we can. Let us say method 1. We start with Laplace transform of t. We know t ut that is, 1 over s square. t square ut minus d by ds of 1 over s square, because this is multiplied by t. Multiplication in the by t the time domain corresponds to the negative of the derivative of the s domain. So, minus d by ds of 1 over s square, which off course is, 2 upon s cube. Then likewise you can carry this one more step t cube ut will be, once again you can take the derivative of this. This will become 3 factorial by s to the power of 4; that is t cube ut. But now if f of t the Laplace transform of f of s. f of t multiplied by e to the power of minus at the Laplace transform of that is the obtained, by substituting s plus k for s. This is something which we already discussed. Therefore, t cube ut multiplied by e to the power of minus at, is Laplace transform is obtained, by substituting the s plus a for s. So, this will be 3 factorial or 6 over s plus a to the power of 4; that is what it is.

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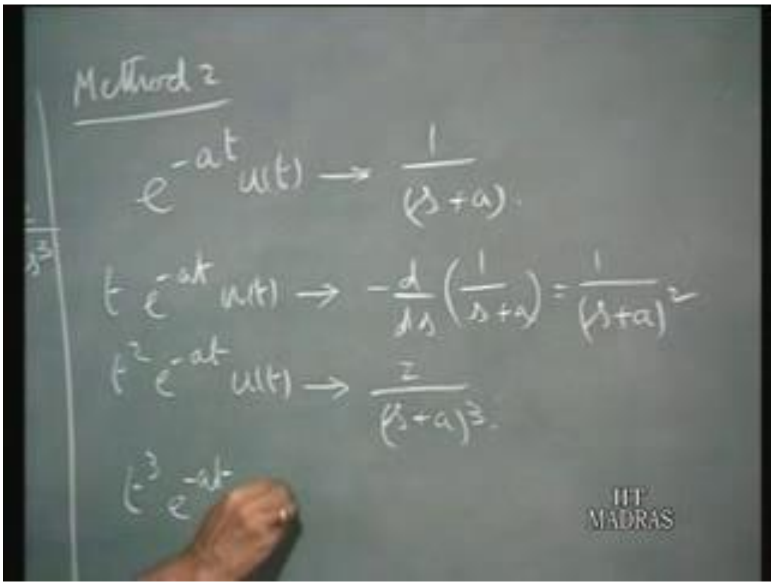

Now, let us do this by taking another approach; method 2. Let me start with the Laplace transform of e to the power of minus at ut. So, e to the power of minus at ut, as the Laplace transform 1 over s plus a. Now t multiplied by e to the power of minus at ut, its once again minus d by ds, 1 over s plus a that will be 1 over s plus a whole square. Then likewise t square e to the power of minus at ut; that will give me, by the same process taking the second derivative, again the derivative of this we reference to s and putting a negative sign, it becomes 2 over s plus a whole cube. And t cube e to the power of minus at ut will fetch me 2 time 3 factorial over s plus a power 4

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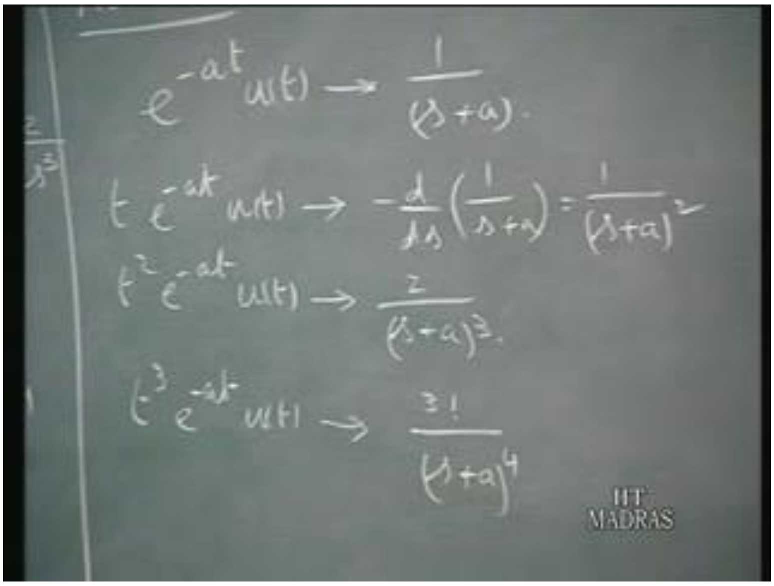

So, that is the same result that we obtained earlier. So, this problem, over this out. Taking 2 different approaches; starting with the Laplace transform of t cube ut first, and the Laplace transform e to the power of minus at first. These are two alternative, we are doing this. Now, for the second question. Now let b let g of t be t cube e to the power of minus at ut. It has got the Laplace transform; let us say g of s, which is this.

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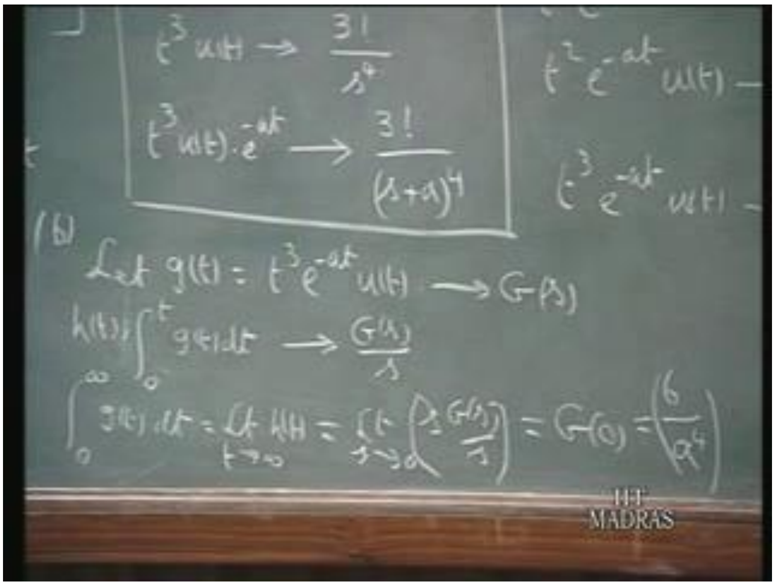

Now, if I take the integral from 0 minus to t of gt dt will say 0, because there is not. The 0 plus value of g of t is also going to be 0 therefore, whether you take 0 minus as 0 plus makes no difference, so simply write 0 to t of g of t dt. This get the Laplace transform gs upon s, because the initial value of the integral state to be 0, therefore, it is gs up on s. Now, what is 0 to infinity of. Suppose this is ht or some, what we want is not 0 to t, but 0 to infinity. Therefore, 0 to infinity of g of t dt can be regarded as, the final value limit as t telling to infinity of h of t. So, the final value of this function, who wants to find out this function, has got this Laplace transform gs over s. To find out the final value of this function. This is equal to limit as s tends to 0, of s times g of s over s; that is the final value theorem. You know particular function as the Laplace transform gs over s, or whatever it is, h of sit is if you like. Then the final value of h of t h infinity is obtained by, taking the limit as goes to 0 of s times h of s. Now, in this case s times g of s over s. So, this indeed s cancels out g of 0. So, the final value of integral, from 0 to t gt dt it happens to be a simply g of 0, where g of s is the Laplace transform of this quantity. In our k g of s is equal to this. So, the final value; that means, when s is equal to 0; this is 3 factorial 6 upon a to the power 4. So, that is the answer for second part 6 upon a to the power of 4. In other words what we are having is, the principle that we are using here is.

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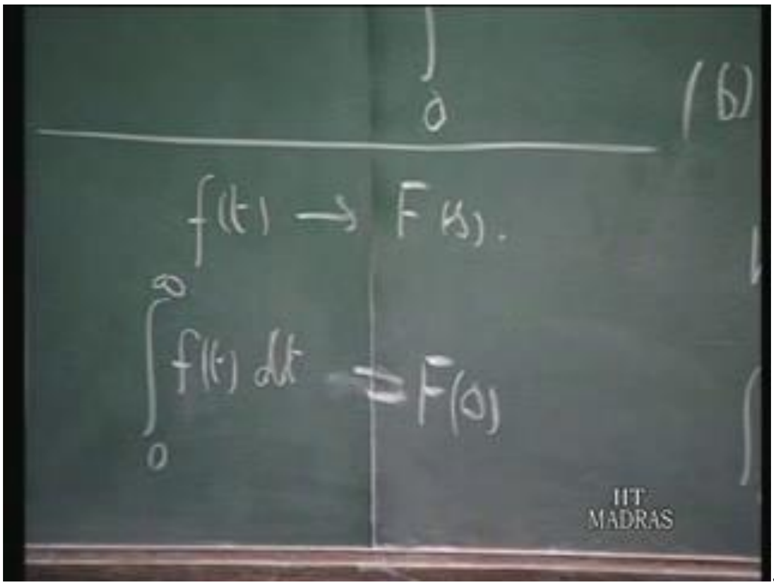

If f of t has the Laplace transform of f of s. What we are saying is, 0 to infinity of f of t dt is equal to f 0; that is what the principle that we used here. This is quit easy to see why it is. So, because after all f of s equals 0 to infinity of f of t e to the power of minus st dt.

When you substitute s equals 0 in this, this becomes f of 0, and this is 0 to infinity of f of t dt, because this become 1. When s is equal to 0 this becomes 1, and that is the same thing that we used here. So, all this are tied up in some fashion are other, but what we wanted to do here is, to illustrate the rule for integration, and then there also want to illustrate the application of the final value theorem in working out the solution. As third example, let us consider the convolution of function of time with delta function or unit step function.

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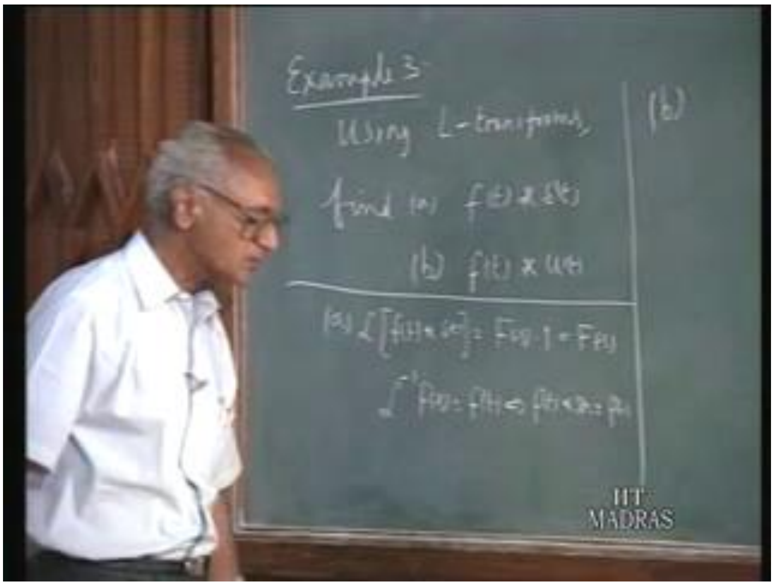

So, using Laplace transforms, find what this functions among to; f of t convolve with delta t first. B, f of t convolve with u of t. If you recall, the results we already know. When we talked about the convolution property in the introductory lectures, we said whenever a function is convolved with delta t that function is reproduce itself; that means f of t star delta t f of t itself, because the delta t stands this functions as its moves along, and it reach at any particular point, the value of the product of delta t and that f of t. The displace delta t and f of t will be f of, the value the function at the particular point of time that is the magnitude the impulse, and when you integrate that will be f of tow times f of tow whatever it is, wherever it is situated. Therefore, the convolution of this will be must result in f of t itself. And when we talked about the integration rule, under four year transforms, recall that f of t convolved with u of t, has been shown to be the integral of f of t. We will see this result here, using Laplace transform. So, a, we know that the Laplace transform of f of t convolved with delta t, is the product of the Laplace transforms of the 2 individual functions. This is f of s multiplied by 1. The Laplace transform of delta t is 1; therefore, this is f of s. And the inverse Laplace transform of f of s, is equal to f of t. Therefore, f of t star delta t is f of t itself; that is what we already know. Second question; f of t convolved with u of t, as for its Laplace transform f of s multiplied by 1 by s. So, this is f of s over s, and we know that f of over s, has the inverse

Laplace transform 0 minus to t of f of t dt. So, when we convolve with f of t with unit f function, it is the integration of ft. In the four year transform theory, you may taken this from minus infinitive onwards, but in the Laplace transformation because you are dealing with causal time functions. We take the limit from 0 minus to t, because f of t is assumed to be 0 for negative values of time; that is the difference.

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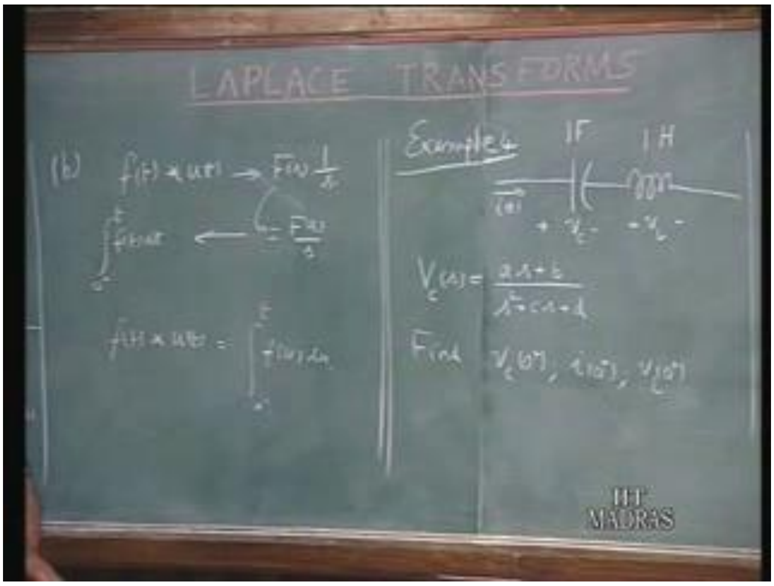

So, the integral of f of t dt from 0 to t, is f of s over s, and that is indeed the product of Laplace transform of these u function. Therefore, we conclude that f of t star ut is 0 to t of fu du if you wish, because it is dummy variable integral of f of t. assuming that f of t is the causal time function; therefore, we are starting the limit integration from 0 minus onwards. This results are already known, but this is only a confirmation of results that we already derived, in earlier context. example four; Let us consider now, a section of a circuit, in which we are inductance l of 1 henry and capacitance of farrant, and let the voltage is across the 2 elements we described as vc and vl. In some circuit analysis, we have obtained vc; the Laplace transform of vc of t Laplace transform. The voltage across the capacitor, is found out to be as plus b over s square plus cs plus d, through some analysis we have obtained this. Using this information we are asked to find, vc 0 plus, the voltage of the capacitor immediately after 0. The current in the circuit 0 plus is the current it. And the voltage across the inductor 0 plus. These are the three quantities that are require to be found out. Now, since we are interested in finding out the 0 plus values in all this situations. We can assume that the Laplace transform of that we are talking about. The Laplace transform defining integral start from 0 plus 0 onwards, not from 0 minus, because you are after all interesting 0 plus value; that means, we ignore any jumps in functions from 0 minus to 0 plus. We assume that all our functions starts from 0 itself, and therefore, we can use the 0 plus value, instead of 0 minus value wherever its necessary.

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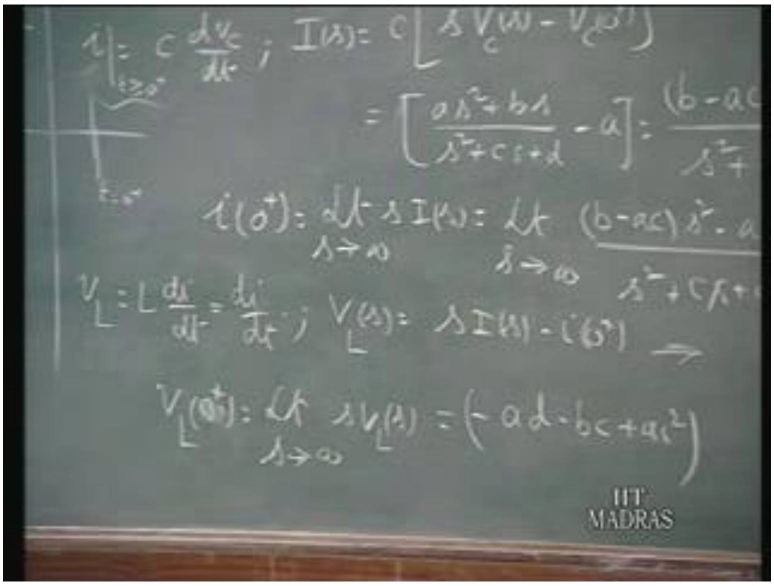

So, let us see vc 0 plus the initial value of the capacitor voltage, is limit as s goes to infinity of s times vc of s; that is initial value theorem. Therefore, this will be limit as s goes to infinity, of as plus b upon; say s square plus cs plus d multiplied by s. This must be multiplied by s and take the limit as s goes to infinity. Therefore, that will be as square plus bs over s square plus cs plus d. And when you take the limit as s goes to infinity and as I mentioned in the last lecture. We can take s going to infinity in the, along the real axis, in the positive time axis; therefore, we can. Ratio of the 2 leading coefficients only, because all the other terms where in significance cs plus d, hence with significance in s square, bs becomes negligibly small compare with as squared; therefore, it is a ratio the 2 leading coefficients; therefore, this is a. So, the immediately after 0, the capacitor voltage at 0 plus as you approach 0 from the positive direction will have value a. Now, what about the current. We know the capacitance i equals c dvc dt; that is the relation between the current in a capacitor, and the voltage across the capacitor for any general variables it and vc. Therefore, i equals c d vc dt.

Therefore, we can say the laplace transform of the current i of s is c times. If vc as the laplace transform vc of s, the derivative d vc by dt is s times vc of s minus. Now, we are taking stock of all values from starting from 0 onwards, 0 plus onwards; therefore, I may write here vc omega plus. If you are starting all over account in from 0 minus you would put it as 0 minus; that means, I am trying to take the value variation of current. Suppose this variation of current. I am taking the star from 0 plus onwards only. I am not considering the jump from if any from here to here. That means there is any jump in vc; suppose vc 0 minus is this and vc 0 plus is this, the current would have impulse here. But those impulses I am ignoring. I am talking about the variation of the current; i for t greater than r equal to 0 plus. Only that is the type of current which i am interested. So, that is why i am putting this 0 plus. So, this will be see in our cases 1 farrant. So, this will be s times vc of s. vc of s is a s square plus as square plus bs over s square plus cs plus d.

Therefore, s times vc of s is as square plus bs over s square plus cs plus d minus vc 0 plus as just be evaluated that is a therefore, when you do this, then it become b minus ac times s, minus ad divided by s square plus cs plus d; that is the Laplace transform of the current i of s. To find out i 0 plus, we apply the initial value of theorem, and say that this is equal to limit as s goes to infinity of s times i of s, which means limit as s goes to infinity of b minus ac s square minus ad times s divided by s square plus cs plus d, which by the same arguments that before, is the ratio of the 2 leading coefficients, which will be b minus ac. So, this is answer. So, the initial value of the current b minus ac mps. This is the Laplace transform of voltage across the capacitor. Now, again to find out the value, initial value of the voltage across the inductor vl; vl equals l di dt. In our case the inductance value l is 1 henry therefore, this is di dt, which now tells us that vl of s is s times i of s minus i 0 plus.

Once again we are taking the variation the voltage that was inductance from t equals 0 plus onwards, where ignoring any changes from 0 minus to 0 plus values. So, we again multiply the expression for the Laplace transform of it which is this 1 multiply this by s substitute, and remove from that i 0 minus; which is be minus ac. if you do that, I will not go into the details of this this will turned out to be give some expression here, and then vl infinity is limit as s goes to I am sorry vl; 0 plus that is what you are interested, limit as s goes to infinity of s times vl of s. So, after carrying out this work, and take this limit as s goes to infinity of s times vl of s you will get the answer minus ad minus bc plus ac square. So, that. So, many rules is the value of the voltage across the inductor.

This examples shows, that once we have the Laplace transform of quantity find out the initial values of different quantities associated with the capacitor voltage in the capacitors. You really did not have to carry out the numerical the value of the capacitor voltage vc t at all in symbolically in terms of ad dc where evaluate this. So, in other words if the particularly and will the numbers say 97 35 9.5 whatever it is. You do not have to do the numerical work, try to find out real value of t, and from that vc of t and then try to find out i of t, and then from that vl of t, and then find the initial values of this respective quantities. You can straight away use the initial value theorem, and get all the initial value of the quantities which you are interested in.

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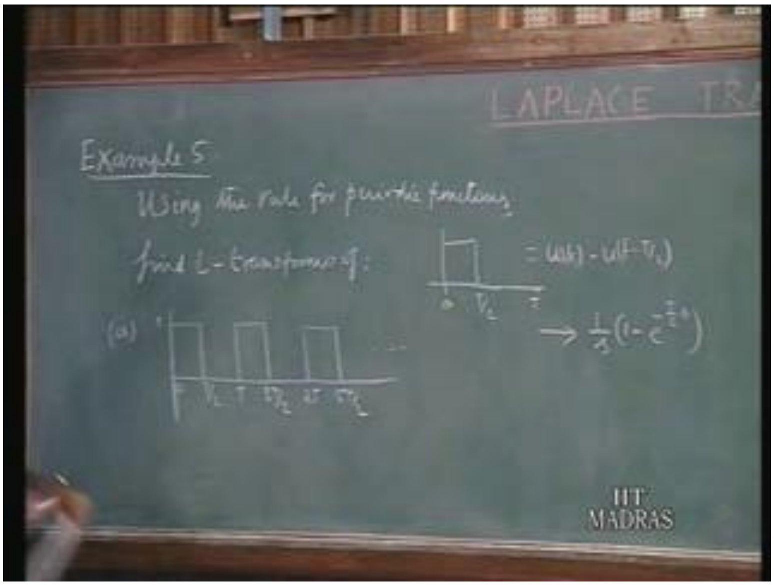

Let us consider one more example; using the rule for periodic functions, find Laplace transform of two functions. A, a pulse train like this, like that it goes; 0 p upon 2 t 3 t upon 2 2 t 5 t up on 2 extra, it is an unit amplitude. B, a single pulse of a sinusoidal 0 to t upon t not upon 2, where omega naught equals 2 pie up on t naught. So, this will be; suppose this is 1, this is sin omega not t, in the interval 0 t not up on 2. So, I can write this as ut minus ut minus t naught upon 2. This is something which we already discussed earlier, but you would like to arrive at the results in a different fashion. Now take this, we have a pulse train here. So, if you know Laplace transform of 1 single pulse, we can find out the Laplace transform of the pulse train, because if the periodic repetition of the same event, whatever is happening from 0 to T, is repeating itself. So, if you want to find out the Laplace transform of, just this portion. We know that this is equal to u of t minus u of t minus t up on 2. This is the function is simply a step function minus delayed step function, staring at small t equals t upon 2.

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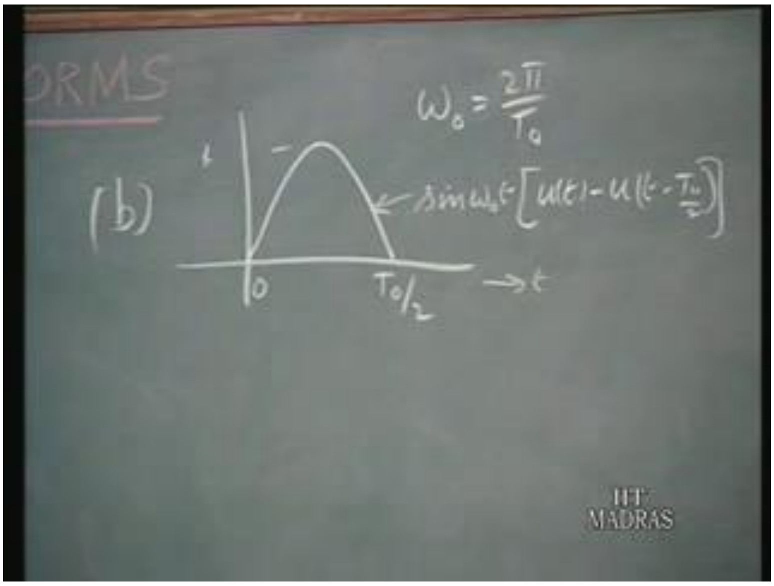

Therefore, Laplace transform of this is 1 over s. This is the Laplace transform of this is 1 over s. The Laplace transforms of this the same 1 over s, but multiplied by e to the power of minus t upon 2 times s; that is the Laplace transform of this. Now, all we are having is, the same pulse is repeating itself, identically every capital t seconds. Therefore, this is the periodic phenomena, and we found out the Laplace transform of the phenomenon in one period. So, the entire periodic function will be obtained by 1 over s 1 minus e to the power of minus t by 2 s. The basic function, divided by 1 minus e to the power of minus ts, according to the rule that we had earlier derived for a periodic case. So, the answer is 1 over s, you can put it in the form if you wish or 1 over s times 1 over 1 plus e to the power of minus t by 2 s, because this is a factor here. So, that is the answer for this the Laplace transform of this periodic pulse time.

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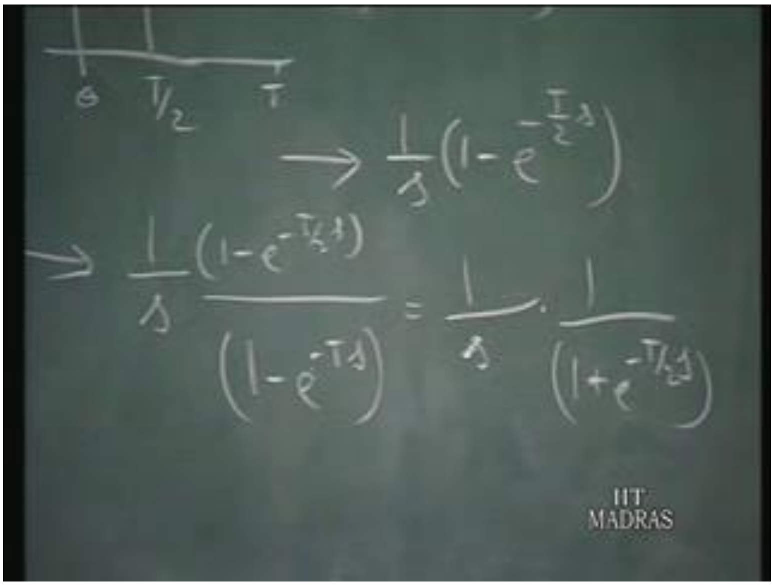

Now, when you go to B, we have derived this in a different fashion, but now I would like to, derive it using the property of the periodic function. Suppose I call it g of t. Then let me say that this as Laplace transform g of s.

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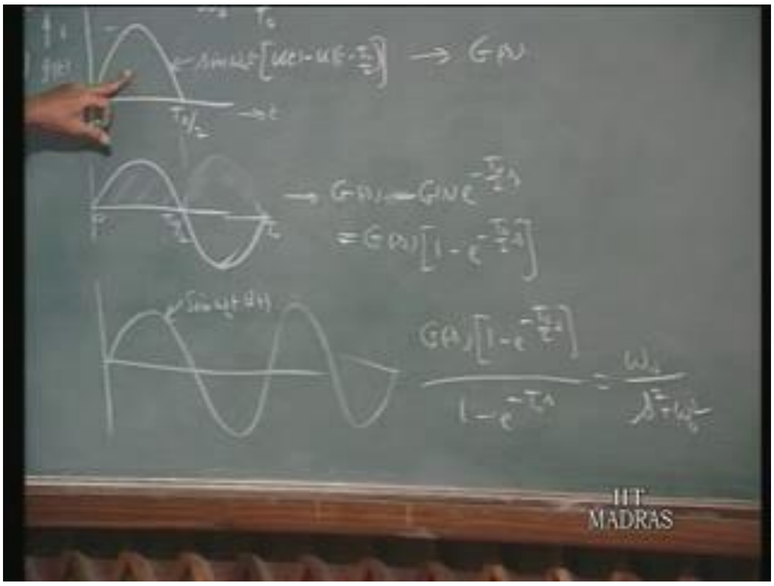

Now, suppose I extend this like this. I complete this sign function for one complete period. If the side Laplace transform g of s, what would be the Laplace transform of this. This portion could have the Laplace transform g of s. Suppose this portion can be clipped over like this. Its Laplace transform would have been g of s multiplied by e to the power of minus t naught by 2 s, because the same thing is delayed by t naught by 2 second. So, the Laplace transform of the simple, a loop like this would have been g of s times e to the power of minus t not upon 2 s. But know instead of this you are having a negative going thing. Therefore, this will be plus minus g of s times e to the power of minus t not by 2 times s, in other words the Laplace transform of this 1 period of the sin way would have been g of s times 1 minus e to the power of minus t not by 2 times s; that would have been the Laplace transform of this, in terms of the Laplace transform of this. Now, suppose the sinusoidal is repeatedly numbered. So, that is the a complete sin omega not t ut, repeating endlessly. Since we know the Laplace transform of one period. You can find out the Laplace transform of the entire periodic function by dividing this by 1 minus e to the power of minus t naught s. This is the Laplace transform of this. But we know the Laplace transform of this, this is after all sin omega not t ut, and we know the sin omega not ut as the Laplace transform. Omega not over s square plus omega not square. So, the Laplace transform of this which derived in terms of the Laplace transform of the single loop, must be equal to omega not over a s square plus omega not square. Therefore, using equating this 2, we get the result that g of s equals. Omega not over s square plus omega not square, multiplied by 1 minus e to the power of t not s divided by 1 minus e to the power of minus t not s up on 2, which is simply omega not over s square plus omega not square into 1 plus e to the power of minus t not s up on 2.

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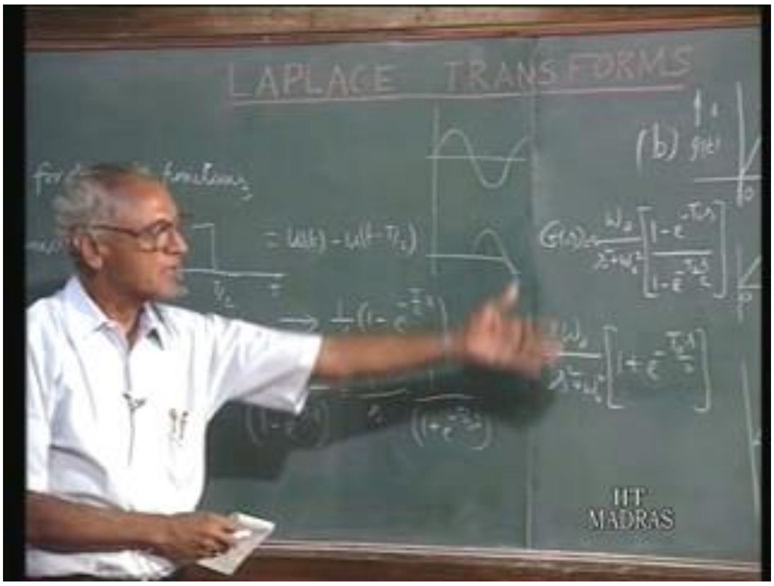

The result which we already derived in the last lecture also, using a different property what we did was, if you had a sin wave like this, and another sin wave which is delayed t not of seconds. If added this 2 you produce this pulse. So, from this we are arrived the same result in a different way, but now we got the same result using the property of the periodic functions. We had. So, for talked about the transformation in the forward direction; that is given function of time, we are trying to find out the Laplace transforms. But in the solution of network transient, we also have a occasion, to have to find out the inverse Laplace transformation. Given f of s you should like to find out what the f of t the corresponds to it. This is called the inverse Laplace transformation. Now, this inverse Laplace transformation can be approach from 2 points of u; 1 is what is called the partial fraction expansion; that is what will discuss primarily. It can also be approached through the defining integral relation, of f of t, getting f of t from f of s of the inverse transform integral relationship, which goes as remember 1 over 2 pie j integral from c minus j infinity to c plus j infinity of f of s e to the power of st ds. The second approach is, little complicated, and it can be used only for a special occasions arise. But for ordinary purposes, its enough we know how to find the inverse Laplace transformation to the partial fraction expansion.

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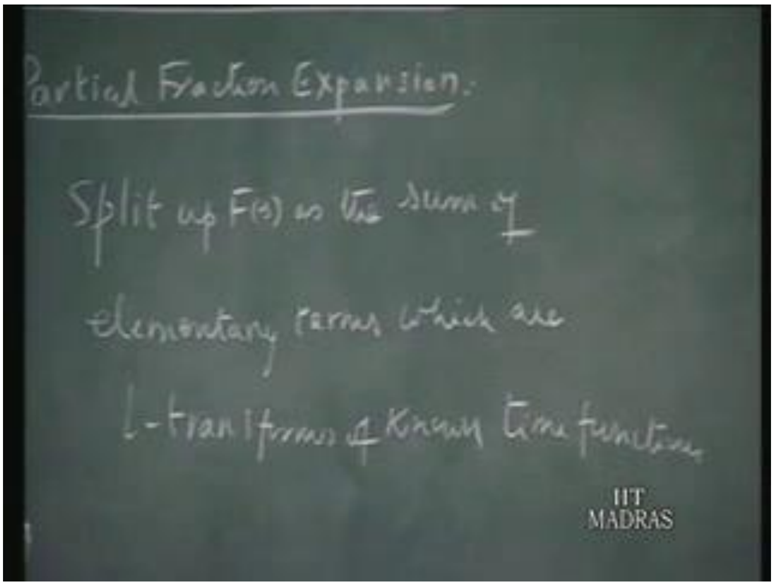

So, let us consider the partial fraction expansion as the, method of finding out the inverse Laplace transformation. The approach that we take up here is. So, what analog as what we do, when we take up the integration of functions. You know when we have to integrate something, the integrant is split up into components which are recognized to be the derivatives of some functions. So, once we know recognize the derivatives, then the function whose derivatives they are we know, and then some of all those functions will be the integral of the various derivative function that we know, more over the same approach is take up here also. In other words, normally the f of s that we have to deal with is the rational function. So, we split up this rational function, as the sum of small elementary functions, which are Laplace transform of known time functions. So, if you do that, then the sum of all this elementary function, will have the inverse Laplace transform which is the sum of the corresponding time functions which we recognize as the made of the elementary functions; that is what we do. So, split up f of s; the philosophy is split up f of s as the sum of elementary terms, which are Laplace transforms of known time functions. This is the philosophy.

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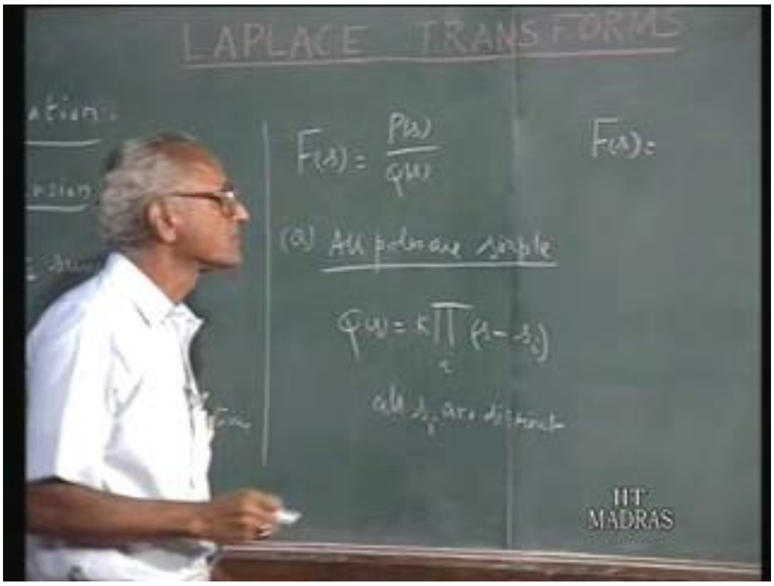

So, let me illustrate this; let us take f of s as the rational function, which is the ratio of 2 polynomials in s qs. And we know the values of s which make the numerator 0 are called the zeros of this function. And the values of which make the denominator 0, are called the poles of this rational function. So, let us take the case all poles are simple. in other words q of s is the product of some constant k times s minus si where all si are distinct; that means, if you take i from 1 to n whatever the factors here, all of them are different values. No two repeat themselves, none of this poles repeats itself; that means, no two si are the same.

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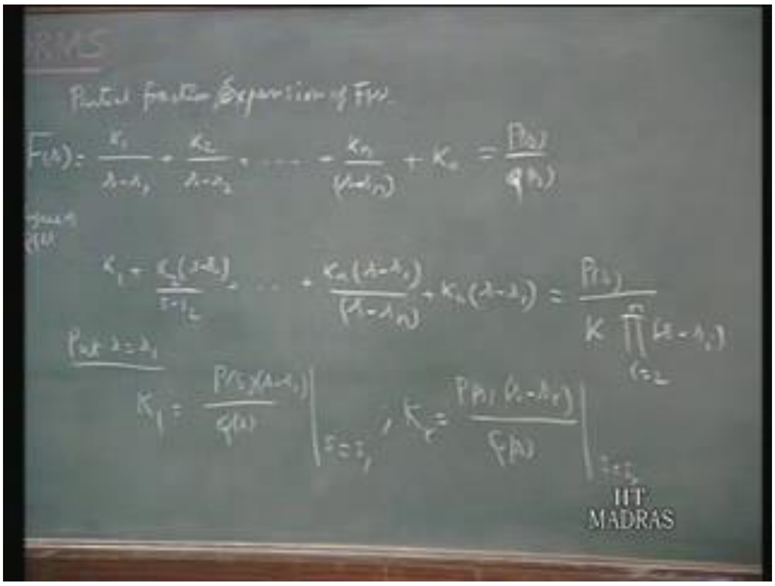

Now, to handle this, we can write this f of s as; k 1 up on s minus s 1 plus k 2 up on s minus s 2 down the line kn over s minus sn. If there are n such factors in q of s; say i from 1 to n, then there are n such factors. And if the numerator degree same as the denominator degree you also have a constant term, k naught. We will assume in our case that the degree of p of s, is not greater than the degree of q of s, which is usually the case, so we do not have to worry about cases where you have stopped with k naught, you do not have the terms like ks ks square and so on. So, this is f of s. now if this is the partial fraction expansion of f of s, where all poles are simple. So, given qs we have to put this in this form. If you do that, then we can find out the inverse Laplace transform of each one of these elementary term quit conveniently. So, our first job is, to express ps over qs to be like this. Now how do you find the values k 1 to k n. Now, you observe suppose I multiply all these terms, this side the equation and this side the equation by s minus s 1, then I get k 1 plus k 2 over s minus s 2 times s minus s 1 etc plus k n times s minus s 1 over s minus s n plus k naught times s minus s 1 equals p s over. As far as denominator is concerned, because s minus s 1 is the multiplying factor the numerator, one particular terms get dropped out. So, you have some k the product of s minus s i i starting from 2 to n only, because s minus s 1 got dropped out.

Now, if in this put s is equal to s 1. Suppose substitute s is equal to s 1 in this, then all this will vanish, and on this side only k 1, and on the other side you have therefore, p s over q s what you have really done is, we have multiplied by this s minus s 1; therefore, this s minus s 1 got cancelled out in q of s and that left this. So, I can put this in this form p s times s minus s 1 by q of s with the substitution s is equal to s 1. So, that is how you can evaluate k 1. in general any one of this suppose you want to find k r, all you have to do is p s multiplied by s minus s r divided by q of s; that means, this s minus s r term is cancelled out q of s. Symbolically you represented in this fashion, then amplitude s is equal to s 1. So, that is how we can find out all this factors. These are called the residue of this poles k 1 is the residue pole it equal to s 1, k 2 as the residue as the pole it s 2, k n is the residue at the pole in s n. So, we can find out all this residues, and once we have all this residues, we can find out the corresponding f of t. So, once we have this f of s, corresponding f of t, is easily obtained, by finding the inverse transform of each one of this. We recall the k 1 over s minus s 1, as corresponds the time function which is equal to k 1 e to the power of s 1 t. Similarly k 2 e to the power of s 2 t and so on and so forth.

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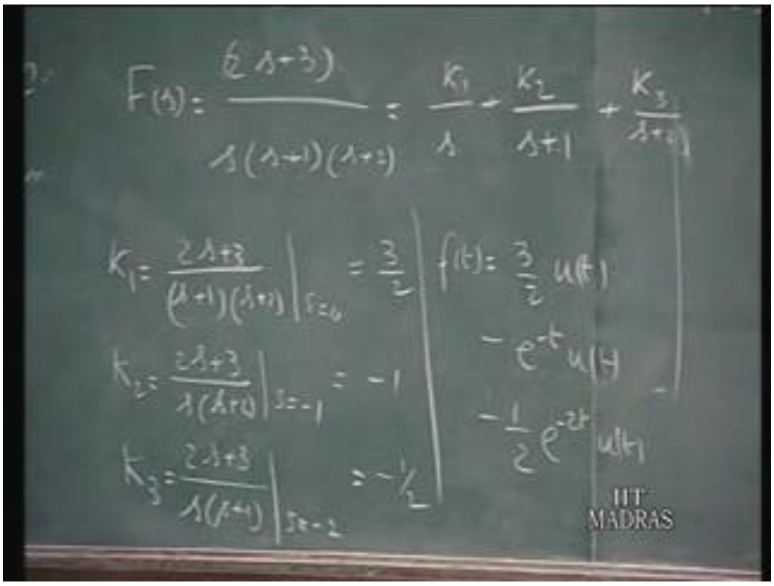

One quick example we illustrate this, and then we will be done with that, as for this lecture this concerned. Example, suppose f of s equals 2 s plus 3 over s times s plus 1 times s plus 2. So, it has got 3 poles. So, I will have k 1 over s plus k 2 over s plus 1 plus k 3 over s plus 2. So, these are the three terms that evaluate. To find out k 1, you multiply this function by s. So, once you multiply this function by s, you have 2 s plus 3 over s plus 1 times s plus 2 and substitute the value s equal 0, because pole is at equal to 0, this is 3 upon 2. Now, as far as the k 2 is concerned, you multiply this function by s plus 1. So, 2 s plus 3 over s times s plus 2 substitute s is equal to minus 1, because s plus 1 made equal to 0. So, s is equal to minus 1. If substitute s is equal to minus 1 in this, you get the answer minus 1. And like wise k 3, you multiply this by s plus 2; therefore, we have left with 2 s plus 3 divided by s times s plus 1 and we substitute s is equal to minus 2, and the answer for that happens to be minus half. So, the partial fraction of expansion of f of s leads to this three terms; k 1 k 2 k 3 which are evaluated like this. So, from this f of t can be obtained as k 1 by s 3 by 2 up on s, it corresponds to 3 by 2 u t plus k 2 up on s plus 1. This corresponds to k 2 we to the power of minus t k 2 is minus 1; therefore, e to the power of minus t u t; that is the inverse Laplace transform of k 2 up on s plus 1.

The inverse Laplace transform of this is, k 3 e to the power of minus 2 t k 3 is minus half. So, minus half e to the power of minus 2 t u t; that is the final result 3 upon 2 minus e to the power of minus t minus half e to the power of minus t u t. So, this is how one can find out the partial fraction expansion in the case of simple poles, and from that you can find out the time function, to which the original given f of s corresponds to. So, in this lecture, we worked out various examples, to illustrate the properties of Laplace transform that we studied earlier, and then we started a discussion on the, whether of finding out the inverse Laplace transformation of a given function of s; that is to find out identify f of t to is the given f of s corresponds, and in this direction we started out in the partial fraction expansion of the given function of time, given function of s. And as the first case we have taken the situation where all poles are distinct and simple; that means, no poles is repeated, and we can find out the rescues of the poles in every compact way, by multiplying ps up on qs the rational function, by s minus s i where s i is the pole, at which you have to find out the residue. So, once these residues are find out. All the k factors are found out, the inverse Laplace transformation is quite straight forward, and that is what we have illustrated by means of an example. We will continue this discussion of finding out the inverse Laplace transformation by partial fraction of expansion, in the next lecture where we will consider, the situation where some poles are repeated, may be for the 2 or 3 as the case may be.