Networks and Systems

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Lecture – 23

We, have seen some joviality in the relations between the Laplace transforms from the time functions for example, differentiation time domain corresponds to multiplication by s in the frequency domain. Similarly, differentiation in the frequency domain corresponds to multiplication by t in the time domain to recall t f off t Laplace transform minus d by ds of f off s.

We have also seen that integration in time domain corresponds to, division by s frequency domain essentially, that is: integral f off t dt corresponds to f off s over s in a dual way, if you have f off t up on t it must corresponds to integration in the frequency domain this is the property which is not particularly useful. But it may be useful of this in some special situations just let us, look at this rule we have

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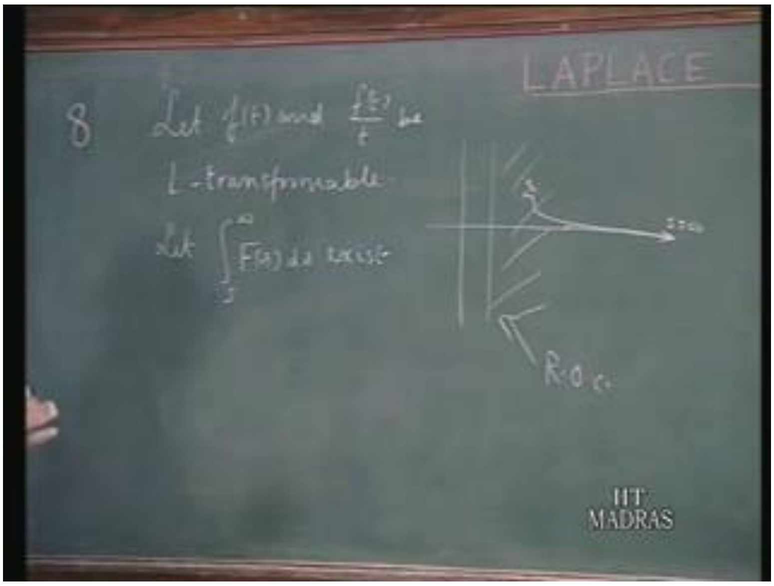

Let f of t and f off t up on t be Laplace transformable and let the integral s to infinity of f off s ds exist. So, this integral exist what we mean by, s to infinity is suppose, the region of convergence of the Laplace transform of both f off t by f off t by t electively then we take any point s here and then integrate this f off s over some con to starting from point s in the in the convergence appear infinity.

So, that the real part of the s goes to infinity; that means, then you take s is equal to infinity; that means, at least the real part of s is going to infinity. So, 1 can possibly say that the point on the x axis how get infinity is the ending point at this.

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So, if you do that, then the theorem states that, f off t up on t as the Laplace transform s to infinity of f off s ds this is the rule and since, we are talking about s. So, in as well to make it clear it can be a dummy variable z f off z dz, where z is the complex variable and once you make the integration, it is a function of s because s is the limit here.

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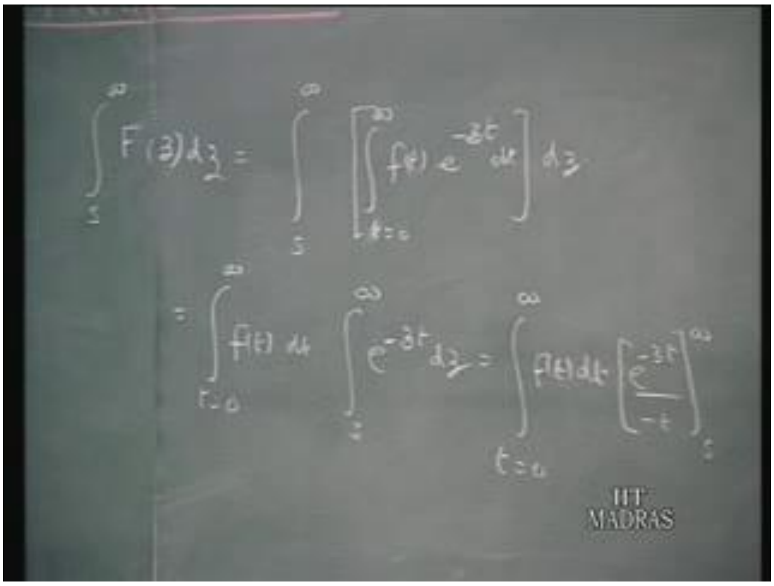

So, this will be a function of s proof of thus is the again is the straight forward s to infinity of f off z dz can be written as, s to infinity f off z dz you can write this as this is the Laplace transform of f off t therefore, 0 to infinity f off t instead of f off s, I will writing f off z therefore, I must writing e to the power of minus zt dt this will be f off z the integration is with reference to t off course t equals 0 to infinity and dz and this first integration is with reference to t second integration is with reference to z. Now, if you reverse the author of integration, I will say t equals 0 to infinity f off t. So, whatever, functions are there which are independent of z, I will pull them outside. So, f off t dt then s to infinity of you have got e to the power of zt dz and this will be, T equals 0 to infinity f off t dt this integral will be e to the power of minus zt you recall that, you are integrating with reference to z.

Therefore, t is the constant over minus t and with the limits s to infinity it is what you heard and in the upper limit e to the power of minus zt goes to 0 because you are taking this integration from s equals 0 to point at infinity along x axis that as therefore, for positive values of time because this integral involves only positive values of time therefore, when positive t is there and real part of z is goes to infinity, then this become 0 on the upper limit at the lower limit e to the power of minus s t by minus t.

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Therefore, this will be t equals 0 to infinity f off t dt. This will be e to the power of minus st by t because, we are taking the lower limit therefore, this minus sign is observed by another minus sign coming out in front therefore, this is what we are having this will be 0 to infinity of f off t up on t e to the power of minus st dt continuous is indeed the Laplace transform of f off t over t this is the Laplace transform of.

So, Laplace transform of f off t over t is this integral. So, division by t in the time domain correspond integration in the frequency domain just like; integration the time domain corresponds to division by s in the frequency domain as I mention this rule is not particularly useful to us in our context in over applications to networks. So, you will just record this as a jual rule to the rule for integration in time domain leave it at that.

In solution of networks transformations and systems sometimes we may not be interested in finding out the entire function of time from the Laplace transform variable which is Laplace transform which is available from the solution of network we would be interest in finding out the initial value of f off t or it is derivative first derivative or second derivative without having to find out the entire function of time f off t suppose, the f off s is given would like to know what is f off 0 plus or what is the initial value of the derivative of f with reference to time t equal 0 plus without our having to find out the entire f off t for all time t.

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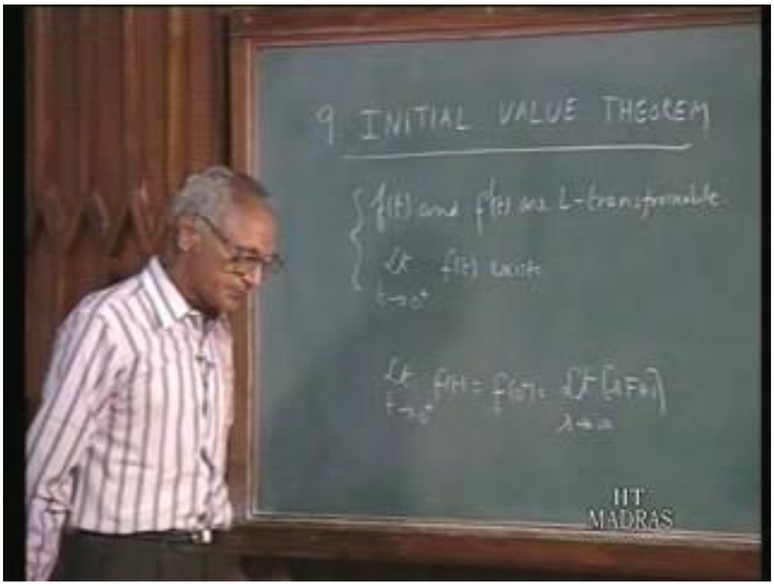

Now, this can be done using a property, how Laplace transforms which is known as the initial value of theorem. Now, let be conditions for this f off t and the derivative f f prime t are the Laplace transform. Laplace transform say gift for both this further, limit as t goes to 0 plus of f off t exists suppose, this conditions are fulfill then limit as t goes to 0 plus of f off t, which will write simply as f off 0 plus is given by limit as s tends to infinity of s times f off s this is the statement of the initial value of theorem.

That the initial value of the time function is given by limit as s tends to infinity of s times f off s this limit is quit easy to evaluate because once, to have a rational function as s tends to infinity you have to take the ratio the 2 lead in coefficients in the numerator and denominator that will be limit and that is equal to f off 0 plus. Now, what is the proof for this.

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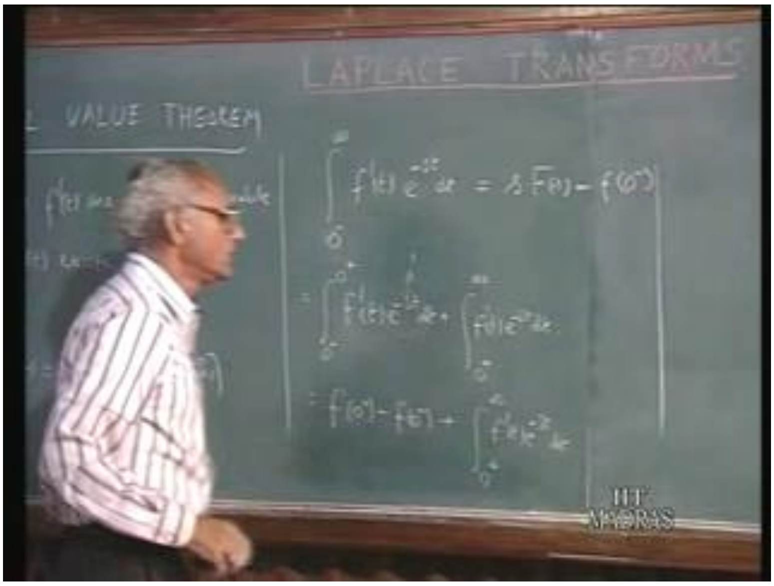

Let us say, we are trying to find out the Laplace transform of f prime p. So, if f prime p e to the power of minus st dt 0 to infinity 0 minus to infinity that is: the Laplace transform of the derivative of f off t according to what, we had already discussed this will be s f off s minus f 0 minus according to the rule for finding the derivative Laplace transform of the derivative of the time function this, what we have held the Laplace transform of the derivative of this is equal to s f off s minus f 0 minus.

Now, this can be this integral can be split in 2 parts 0 minus to 0 plus of f prime t e to the power of minus st dt plus 0 plus to infinity of f prime t e to the power of minus st that, off course is equal to s f off s minus f 0 minus. Now, as for as this integral is concerned this evaluated over a very tiny intervals from 0 minus to 0 plus. So, the value of t in this portion is equal to 0 therefore, this is equal to 1 in this interval 0 minus to 0 plus is essentially 0 f e to the power of minus st, where t equals 0 is equal to 1 therefore, we are really integrated in f prime t dt from 0 minus to 0 plus.

So, the value of this will be f 0 plus minus f 0 minus that is: what we are having in addition you have been this additional function 0 plus 2 infinity f prime t e to the power of minus st dt.

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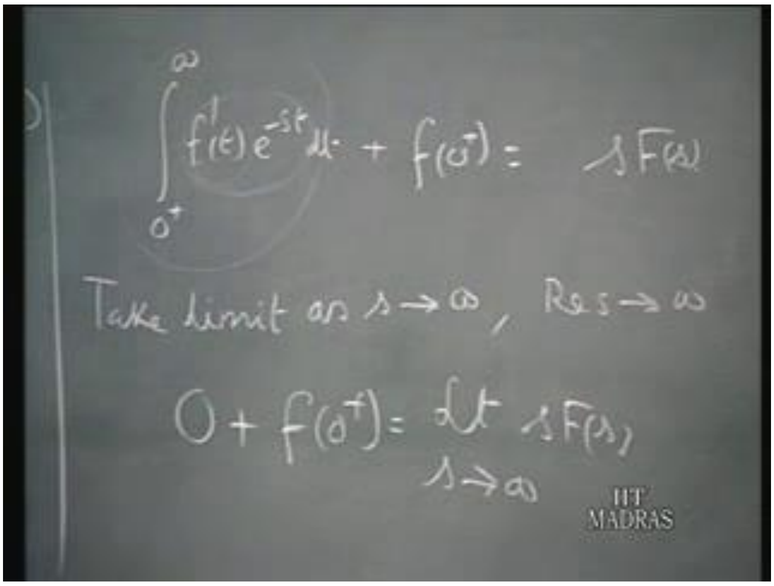

So, when you combine this 2, if f 0 minus gets canceled out then you are having equating this 2 you get 0 plus to infinity of f prime t e to the power of minus st dt equals plus f 0 plus equals s times f off s this is also, thing which we would have straight away written because Laplace transform of f prime t starting from 0 plus onwards we said Laplace transform of that is: f off s minus f 0 plus something which, we already observed when we are taking about.

So, you do not have did not have derived, but, my purpose in doing in this fashion is something, which will explain later. Now, in this let us, take the limit take, limit as s goes to infinity. Now, we take the limit as s goes to infinity such that, the real part of s goes to infinity positive infinity that, is along the say along the real access for example, then when you take the s goes to infinity because f prime t is Laplace transformable you are taking s going to infinity and you are taking about positive values of time this is the exponential order.

Therefore, this will become 0 this becomes 0 as s goes to infinity for positive t such the real part of s goes to infinity therefore, this becomes 0 therefore, this integrant becomes 0. So, this will be 0, if you do that this will be 0 and you have f0 plus equals limit as s goes to infinity of s f off s that is, what we want to prove. Now, so s must tends to infinity such that the real part of s goes to infinity; that means, if there is a reason of origins that we are having here whatever, it is s goes to infinity either in this direction or this direction.

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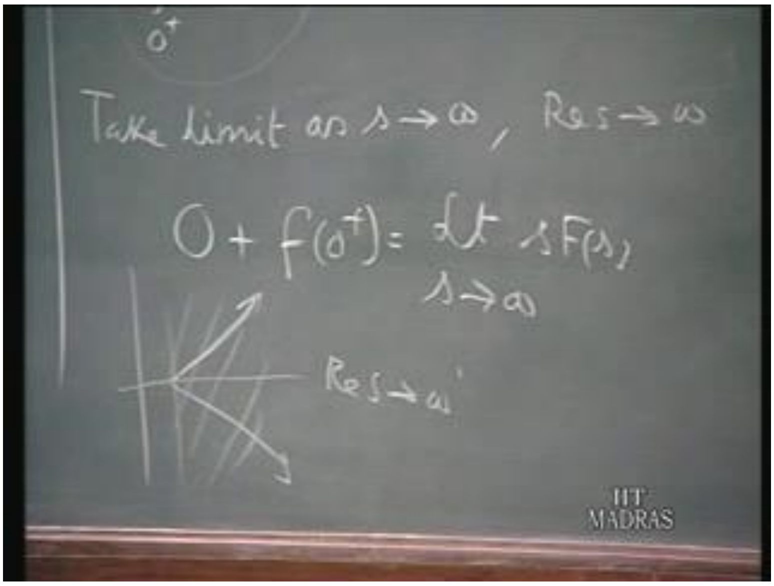

So, that the real part of s must go to infinity that also a important; that means, you must take the value of s going to infinity either in the first quadrant at the fourth part. So, real part of s must goes to infinity that is something which we have take in might. Now, for this theorem to be valid. So, f t and f prime t it must be Laplace transform already mentioned limit st goes to 0 plus f off t should exist; that means, you cannot have the Laplace transform of this.

For example, if you a constant for example, some impulse for an example, from origin then you cannot have this Laplace transform for this mean the initial value theorem will not apply for that.

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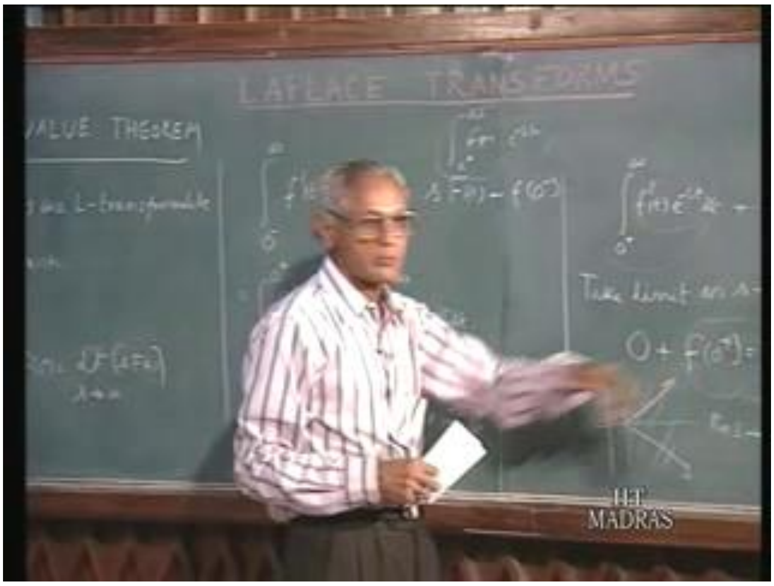

Now, the reason why I started with this integration from 0 minus a whether you define the Laplace transforms, starting from 0 minus or 0 plus the initial value theorem will always give you limit as s to infinity of s f off s f off 0 plus only it cant give you f off 0 minus sometimes people may like to think that, if the Laplace transform is defined as 0 plus to infinity of f off t e to the power of minus st then if that, f off s to take s s f off s take the limit as s tends to infinity will give 0 plus.

If you are take in the Laplace transform f off s to be starting from 0 minus then the limit as extent to have f off s s tends to infinity s times f off s give f off 0 minus no whether, you define the Laplace transform as starting from 0 minus or 0 plus the initial value can give you only this condition f off 0 plus only it cant give you f 0 minus. So, that is; why I want to make that, very clear that is; why I started with the define definition off Laplace transforms starting from 0 minus. So, even if you start from 0 minus the initial value theorem will be only 0 plus.

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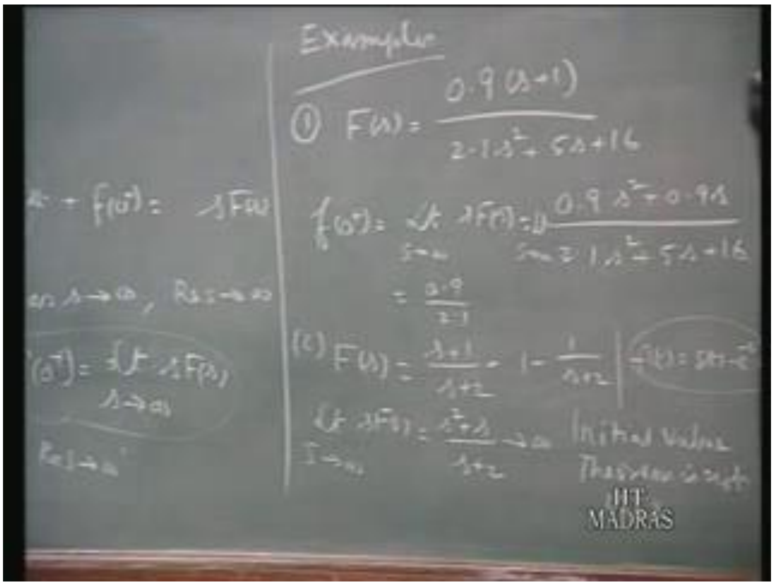

Let me, give you few examples 1 suppose. I had f off s 0.9 s plus 1 0.1 s square plus 5 s plus 16. So, this is f off s and through various techniques which we already are which we discuss later you can find f off s f off t. But suppose, we are not interesting finding f off t who want to know what is f 0 plus the function of time as the origin is approach from the positive sign f 0 plus for this, we do not have to find out the f off t this initial value theorem tells us that: limit as s tends to infinity of s times f off s which is: 0.9 s squared plus 0.9 s divided by 2.01 s square plus 5 s plus 16. And as, I said when you take limit as s tends to infinity as s f off s we can take s to be approaching infinity along the positive x axis.

So; that means, we can take the wholly the ratio of the 2 leading coefficients because this becomes insquinty compared the first term as become larger and therefore, 5 s plus 16 becomes insquinty become 2.1 s square therefore, as a s goes to infinity of again here put limit as s goes to infinity of this will be simply point mine divided by 2.1 that is all whatever, that may be. So, initial value theorem will enable as to find out the initial value at t equal 0 plus have a function of time from its Laplace transform without are having to go through the finding out f off t from this.

Let us take, another example f off s is s plus 1 over s plus 2. Now, limit as s 5tends to infinity of s times f off s equals s square plus s over s plus 2 and this goes to infinity because s goes to infinity s square up on s will become essential equal to s that, goes to infinity. Now, why lets this condition limit as t 0 plus f off t exists that; condition will not be that be violated; that means, this is actually what will happen, this as an impulse 1 minus 1 over s plus 2 f off s is 1 minus 1 over s plus 2; that means, f off t here is delta t minus e to the power of minus zt ut.

So, because this delta function, this initial value theorem does not is not valid here initial value theorem is not valid. So, you will say that whenever, delta function exist at the origin we will not have the initial value theorem applicable in such equations because it leads to infinity whatever, it might be. So, it is not very useful unless you talk about infinity of magnitude because that; take makes it little more complicated.

So, we will say whenever, this limit leads to infinity, which means f off s impulse functions here it will not the initial value theorem will not be valid. So, let us now consider, the dual rule of this which gives the final value of a function of time without our having to find out the inverse Laplace transformation we have seen in the initial value theorem that, the behavior of the f off s at s equals infinity essentially, down as the value of off t equal 0 has a dual to this the behavior of f off s at s equals 0 will essentially decade the value of f off t, when t goes to infinity. And that is, given by this statement of what is called the final value theorem.

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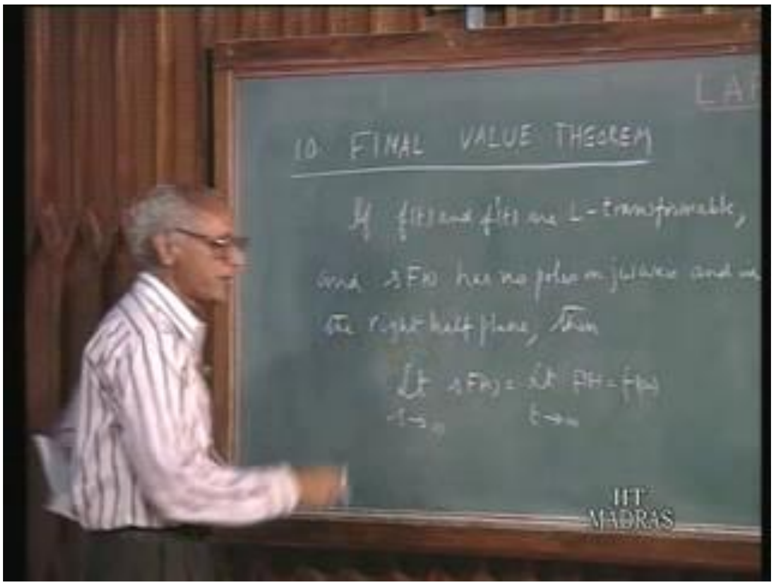

It is says like this, if f off t and f prime t are Laplace transformable both are Laplace transformable. And s times f off s has low poles on j omega axis and in the right half plane I will wrote this in the right half plane then limit as s tends to 0 of s times f off s equals limit as t tends to infinity f off t or you can say that is the, final value of the time function f off t.

So, you take the limit as goes to 0 f times f off s that will give the final value of the function of time f infinity.

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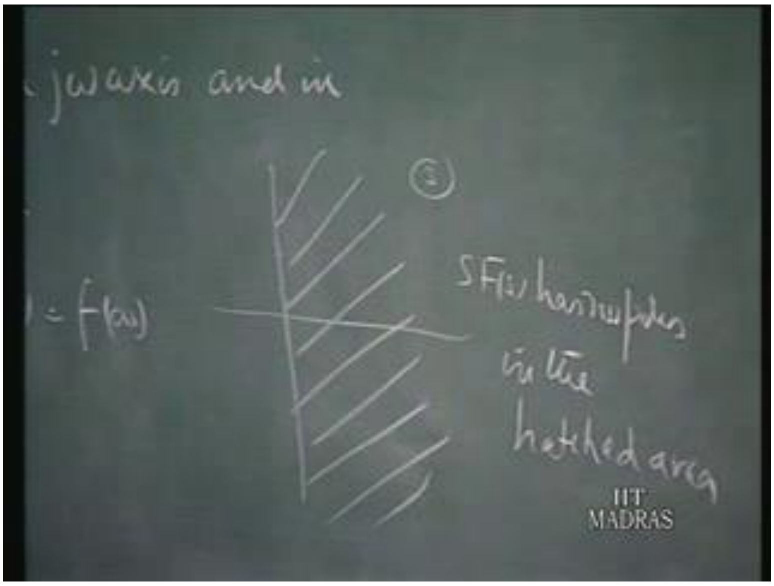

Now, we say s f off s has no poles in the j omega axis in the right half plane; that means, if you plot in the complex plane this must be the region of enlisted of s times f off s. So, s off s f s times f off s has no poles in the hatched area, if you have the pole of the imaginary x axis r in the right half plane the particular theorem is no longer valid.

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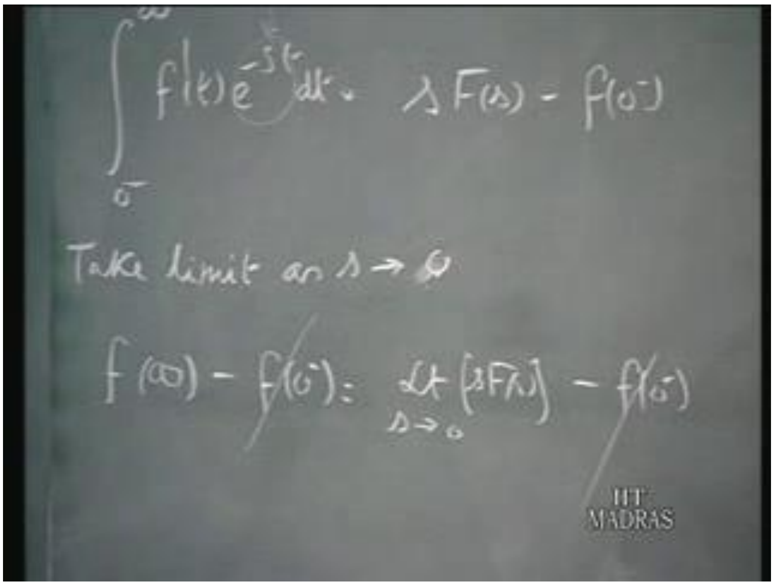

Proof is again straight forward you have 0 minus to infinity of f prime t e to the power of minus st dt that is: the Laplace transform of the derivative of f off t is s times f off s minus f 0 minus according to our rule. Now, take limit as s goes to 0 this will be then s goes to 0 this will become 1 this will become 1. So; that means, you have essentially you are integrating f prime t from 0 minus to infinity.

So, when you are integrating f prime t this becomes f off t you are taking the limits between infinity and 0; that means, f infinity minus f 0 minus this is, what you are getting here and on the other hand you are having limit as s goes to infinity s goes to 0 of s times f off s minus f 0 minus. So, if you take the limit as s goes to 0 on both sides this is: what you result what results and you cancel this 2 terms this is: what you are having f infinity as t goes to infinite limit as t goes to infinity of f off t, if you call f infinity is limit s goes to 0 of s times f off s.

This is again a dual rule to what we had earlier reserved as initial value theorem.

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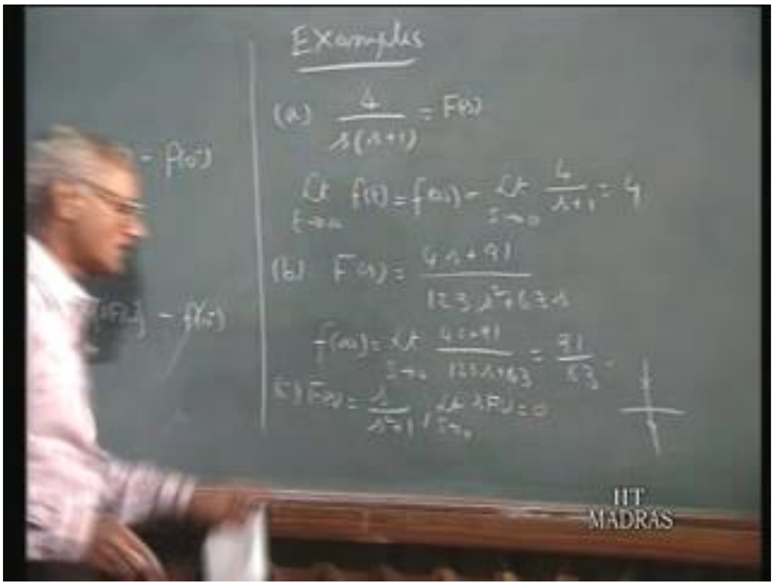

Examples a: 4 up on s over s plus 1 this is f off s. Now, we will like to know whether we can find f infinity without having our finding out f off t. So, limit as t goes to infinity of f off t which we write for continence as f infinity simply this will be limit as s goes to 0 of s times f off s which means 4 up on s plus 1 this is 4 that is all.

So, the final value of the time function is 4 units, we did not have for really find out f off t. Let us take another example b this tells as at the advantage of theorem is without having our 5 to find out a particular analytical expression for f off t from f off s sometimes can be complicated, we can straight away find out the final value without having to go through this intermediate step of finding f off t.

Let us take, f off s as 4 s plus 91 divided by 123 s square plus 63 s once, again we which to show that, f infinity can be found out without having explicitly finding ft with this accrued numbers. So, f infinity now, is limit as s tends to 0 of s times this which is 4 s plus 91 divided by 123 s plus 3 because this is multiply by a s this square will be come with s and that other s square dropped out this becomes 63.

So, this becomes 91 by 63 third example f off s equals s over s square plus 1 suppose, you this is off course cos t you will know that cos t ut.

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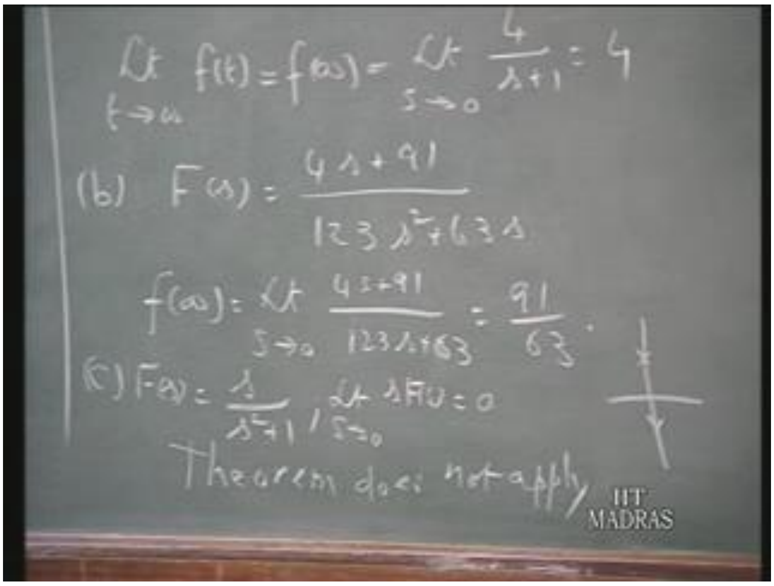

The final value theorem, if you like to apply for this you must multiply this by s and then take the limit as s goes to 0 this will tell you limit as s goes to 0 of s times f off s is off course 0. But then what is the final value of cos t we can say, what it will be, it will becomes going on oscillating between plus 1 and minus 1 write. So, if you take the average of that it may be 0, but, then the point 2 observe here is this does not satisfy the statement of the theorem s f off s has no poles and j omega axis.

So, if you take s times f off s, this will be s squared over s squared plus 1 s square plus 1 s squared over s square plus 1 will have poles at plus or minus 0 1 therefore, initial the theorem does not apply. So, the final value theorem does not apply in this case and therefore, whatever limit you will get may or may not be true therefore, we cannot use this wherever, s times f off s poles either in j omega axis or right half plane.

So, this is situation the theorem is not well similarly, if you have 1 over s square plus 1 than also the theorem, is not well because for the same reason that, sin t you cant find out the final value of sin t we can only assume that 0 will average, but, that is not very regress statement. We shall now, consider another important property, which is useful in system studies particularly when you want to find out the response to an arbitrary input, when you know the response to an impulse input through the medium of convolution integrals.

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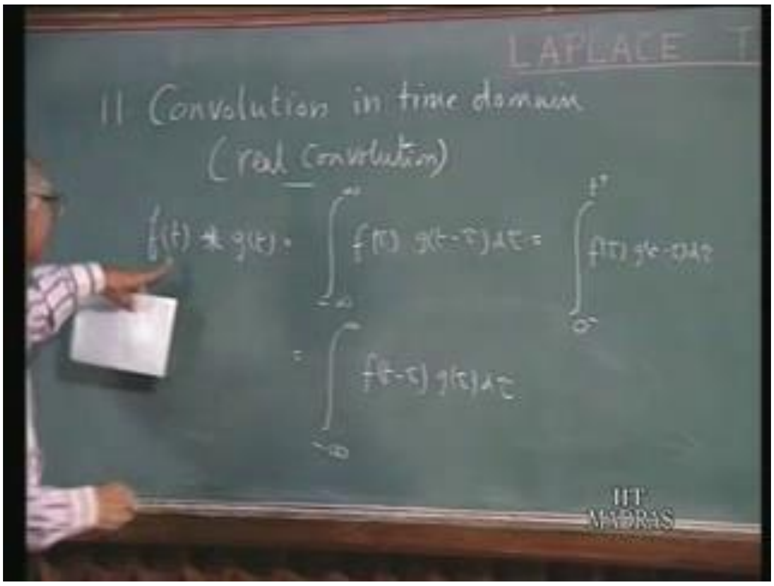

So, you would like to know, how the convolution in time domain how it transforms itself in the s domain. Convolution in time domain, this is some times referred to as real convolution. This is said to be real convolution in the sense that carrying out this convolution in time domain which is the real variable not in the transform domain which is the complex value.

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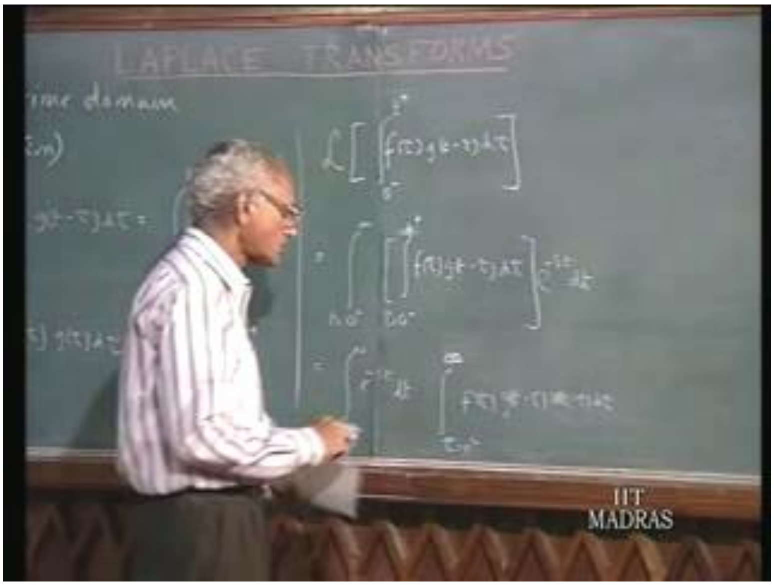

So, this is real convolution you recall that ft convolve it g off t is the definition is f off tow g off t minus tow d tow you can also write this as ft minus tow g tow d tow this are both are equal and you take, the variation of tow the minus infinity to plus infinity in the general case. And for causal functions, where ft and gt are causal time functions, you take the limits because, f tow as the value only from tow equals 0 onwards possibly f off tow as f off t as an impulse the origin will take 0 minus.

Then, when tow exceeds t this becomes negative therefore, there is no point in carrying out integration. Beyond t equals tow they are tow equals t therefore, we take this as t then f tow gt minus tow and take care of possibility of an impulse, if g off t the origin we take this as t. So, this is what we arte having by means of the meaning of f off t convolve with g off t. Now, let us try to find out the Laplace transform of this. So, Laplace transform of f tow t minus tow d tow 0 minus to t plus that is: what we are seeking.

So, this will be 0 minus to infinity off this quantity 0 minus to infinity a t plus f tow gt minus tow d tow and this integration is carried out on tow and the whatever, results is there function of t you multiplied by e to the power of minus st dt this integration is with reference to t. So, this is the definition of Laplace transform of the convolution of the 2 time functions ft and gt which are assumed to be causal.

Now, I would like to do the integration must with reference to t and then go to tow later therefore, to do that I must, I have also a function t plus s. So, to avoid that what can do is I will have e to the power of minus st dt because I would like to do the integration with reference to I will do the integration with reference to tow itself start with toe equal 0 plus to t plus. Now f off tow gt minus tow suppose, I introduce ut minus tow d tow t equals 0 minus to infinity.

Now, ut minus tow is going to be 0. When tow exceeds t, when tow exceeds t ut minus tow is 0 therefore, this integrant will become 0 when tow exceeds t therefore, I have instant having t plus I can as well infinity. Now, that I introduce the symbol ut minus tow here I may as well take the limit of integration of tow from 0 to infinity this is 0 minus infinity.

So, to ensure that; the same integral valid even here same the values of the 2 integrals are the same I will introduced purposely ut minus tow and take the limit as up to infinity because the value of ut minus tow is going to be 0 for tow greater than t the reason why I did this because, we will like to relate them to the Laplace transform integrals therefore, the limits must be 0 to infinity.

So, in order to have that kind of property, I have to purposely introduce this alright f off t minus t d tow.

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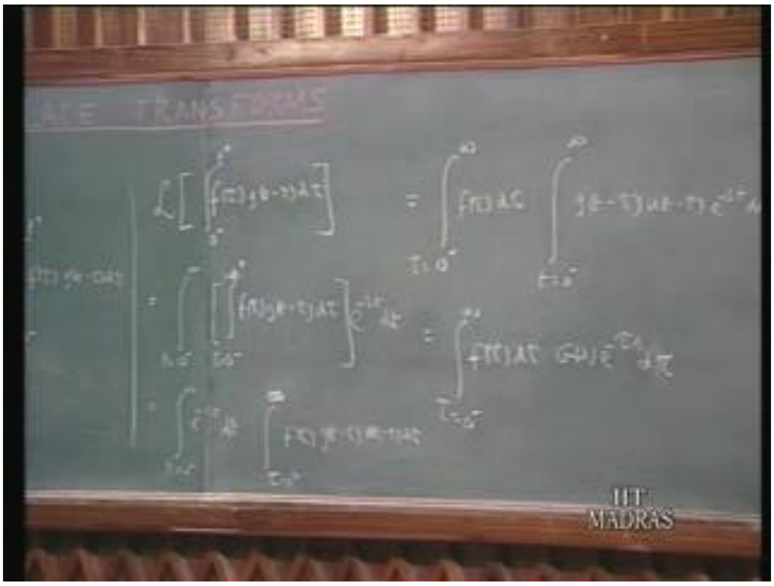

Now, this will be equal to 0 minus to infinity. Now I would like to interchange the limits of integration this is originally the first integration is refers to tow next integration with reference to t. Now, suppose I reverse the roles. So, I do the integration with f off tow later and then do the integration with reference to t first.

So, whatever we are having as constants whenever, we integrating with reference to t must be got outside which is f tow for 1 example, d tow can be brought outside. And all functions which are involved with t are to be taken into account in the first integration that will be gt minus tow ut minus tow e to the power of minus st dt. Now, this is the Laplace transform of the delayed function of gt delayed by tow seconds.

If the Laplace transform of g off t is g off s Laplace transform of the delayed time function t minus tow is e to the power of minus s tow time g off s if, g off t has the Laplace transform g off s g off t ut as the Laplace transform g off s g off t minus tow ut minus tow as Laplace transform g off s time e to the power of minus tow s that is; something which we already k.

Now, therefore, we can write this as tow equal 0 minus to infinity of f tow d tow g off s times e to the power of minus s tow d tow sorry d tow come from here. Now, in this g off s is the constant.

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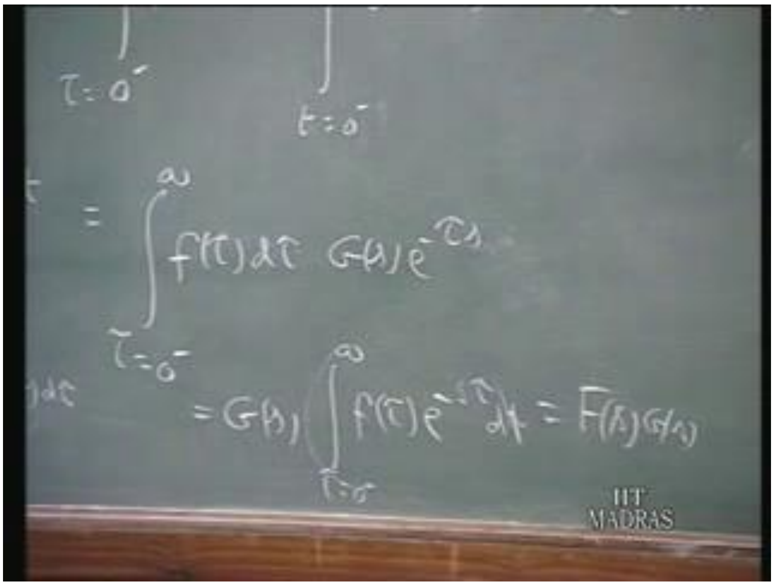

Therefore, g off s times toe equal 0 minus to infinity off f tow e to the power of minus s tow d tow, I have writing d tow twice here is not necessary. And this is indeed f off s f off t e to the power of minus t dt integrated from t equal 0 minus to infinity is f off s instead of t we have the dummy variable tow therefore, this is f off s multiplied by g off s.

So, the neat result that we are having here is finally, that ft star gt star the convolution of 2 time functions ft star gt star as the Laplace transform the product of the 2 corresponding Laplace transform.

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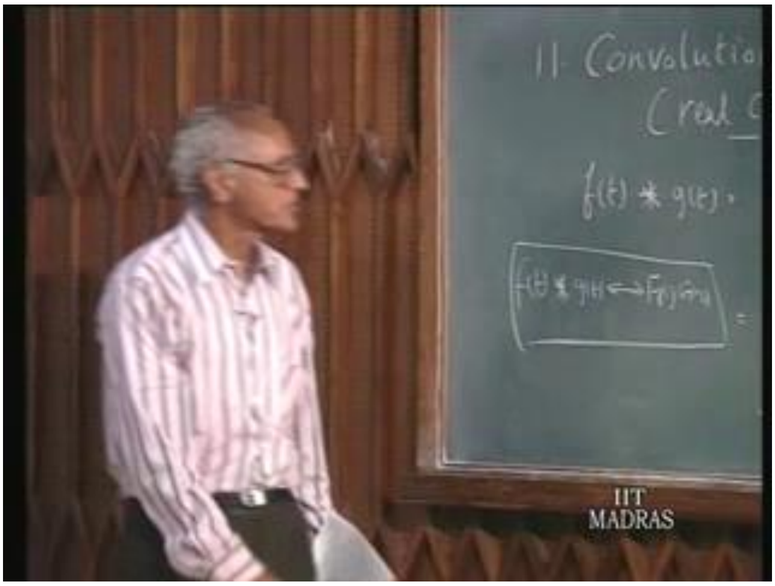

So, this is the important facility convolution, the time domain corresponds to multiplication of the respective transforms in the frequency domain are: the complex frequency domain s domain this is quit neat. And this is a rule which will be useful for our as a side in a system studies, where when we know the impulse response h off t to find out the response for arbitrary function of time all we have to do is the Laplace transform of the output will know to be the product of the Laplace transform.

So, the impulse response and the input function example. Let us consider, an example suppose I have 1 over s plus 1 times s plus 2. Now, we can spilt this 1 over s plus 1 multiplied by 1 over s plus 2. So, this f off s is the product of 2 functions and suppose, I call as 1 over s plus 1 the inverse Laplace transform of that is; e to the power of minus t ut. Let me call this as f 1 ft are simply f off t the inverse Laplace transform of this 1 over s plus 2 is e to the power of minus 2 t ut. Let me call this g off t.

So, the product of this 2 Laplace transforms must corresponds to the convolution of these 2 time functions f off t and g off t because convolution product in time domain is equivalent to multiplication the transform domain. So, the inverse Laplace transform of that: is ft star gt ft star time gt ft star gt. So, that will be 0 to t there will be no impulse special may have start away 0 and t 0 minus and we can simply say 0 minus t of f off tow gt minus tow d tow.

So, 0 to t f tow e to the power of minus tow u tow and gt minus tow g is e to the power of minus 2 t ut therefore, e to the power of minus 2 t minus tow ut minus tow. Now, in the range of integration when tow goes from 0 to t this is equal to 1 u tow is equal to 1. Because tow takes only positive value obtained tow positive values and ut minus tow also the argument the t minus tow is going to be positive. Because tow is going to be less than t therefore, this is also equal to 1 you need not regard them we need not pay any attention to them.

Now, the integration is only with reference to tow therefore, e to the power of minus 2 t is the constant as for the integration is concern. So, we can full this outside in the integral sign. So, you have 0 to t of e of minus tow multiplied by e of plus 2 tow therefore, it is e tow e of minus tow multiplied by e to the power of plus 2 tow. So, e tow t tow; that means, this e to the power of minus 2 t the integral of this is e to the power of tow itself by 1 with the limits 0 and t.

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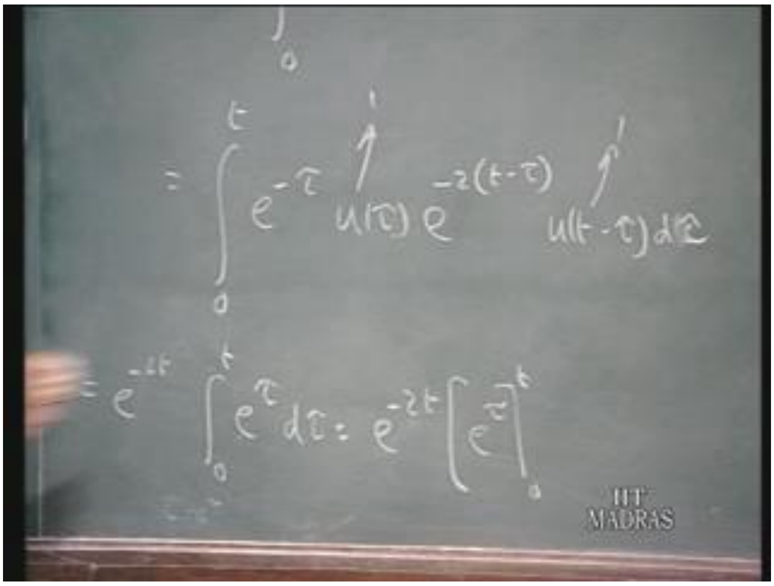

So, that will be e to the power of minus 2 t e to the power of t minus 1 and that will be e to the power of minus t minus e to the power of minus 2 t. And this is valid for t greater than 0 in all over arguments. If t is greater than 0 only we assume, to take ut minus u tow to be equal to 1 therefore, t greater than 0 it is e to the power of minus t minus e to the power of minus 2 t and therefore, this particular Laplace transform has got by the simplest Laplace transform e to the power of minus t minus e to the power of minus 2 t ut.

Now, we wanted to illustrate this as the application of the convolution theorem you can get at this result even simply by considering this by making partial fashion expansion which will talk about in get a detail in nature. But this can always be 1 over s plus 1 thus 1 over s plus 2 and you can write this as 1 minus 1. So, you consider this you have common denominator you have s plus 2 minus of s plus 1 that will be 1 this is equal to this and we know that, the Laplace inverse Laplace transform of this e to the power of minus t ut.

The Inverse Laplace Transform of 1 over s plus 2 is e to the power of minus 2 t ut. So, this exactly the result we obtain here we have more in fashion the simpler fashion. But I would wanted to illustrate the application of the convolution theorem, by considering this as a product of 2 transforms and in the time domain it will terms of to be convolution of time functions this is verification of our result.

So, we have seen, the convolution in time domain corresponds to multiplication of the respective Laplace transform in the frequency domain as a dual mode, as a dual rule we have convolution in complex domain convolution in s domain this is called complex convolution. I will just mention this rule, I leave it at that we will not pursue this because we say the difficult to apply and we do not have really in the use this greater deal.

Therefore, I will simply say that, it to multiply time functions ft and gt multiplication in time domain corresponds to convolution the frequency domain f off s convolve with gs, what is meant by this convolution the frequency domain is an integral, which by this 1 over 2 pie j c minus j infinity to c plus j infinity of f off z times g off s minus z dz. So, you do the integration of product fz times gs s minus z, where z is running variable.

And therefore, the entire integral will be a function of s and that would be the Laplace transform of the product of f off t. And we have to take see the absence of convergence from this sigma c must be greater than abs of convergence function f off t Laplace transform of f off t Laplace transform of g and Laplace transform of delta f sigma f plus sigma g. The abs of convergence the product f off t gt Laplace transform must be larger than all this.

So, whichever is the maximum that, should be smaller than a sigma c. And this contour integration c should be larger than sigma f and then should be sigma minus sigma g; that means, you are actually what it turns; how it is you have to take this contour integration in a narrow band separated by 2 regions; what we said will not peruse this call, it we can say is that; multiplication time domain corresponds to convolution of the frequency domain those of your interested look up the reference books on this are you will get for the details on this. But you will not assumed this.

Now, take up an interesting application of the Laplace transforms which we will also thing of this is the property of the Laplace transforms of periodic functions of time Laplace transforms are periodic functions of time.

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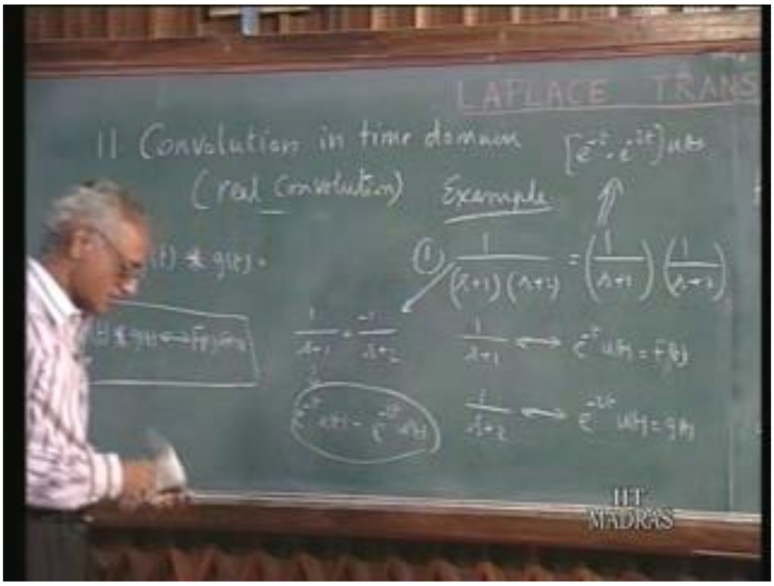

Suppose, I have a periodic function of a time, which is given by like this 0 to t not 2 t not 3 t not this is f off t. Now, consider what period of this blank this, I will call this ft multiplied by ut multiplied by minus u off t minus t not; that means, you are multiplying this a gate function u off t and u off t minus t not that is just like a pulse like this unit amplitude you can multiply this if this you can this.

So, this will call that f1 off t. So, if you take a single pulse the variation of the f off t over 1 particular period only if, you know the Laplace transform of this you can find the Laplace transform of this easily it goes like this suppose, f1 off t as the Laplace transform f1 off s.

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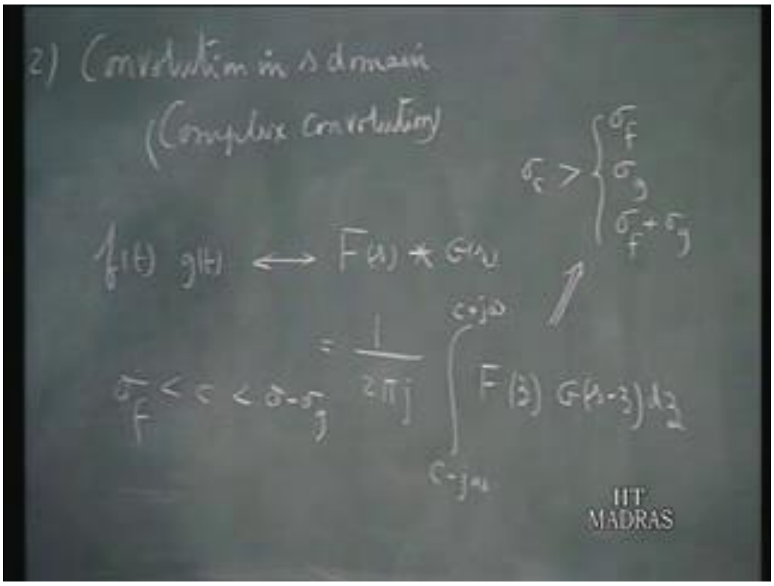

That is the single pulse it have the Laplace transform of f1 off s then what is the Laplace transform of f off t, f off t is this pulse plus, second pulse plus, third pulse each pulse is this place by this by an amount equal to t not the Laplace transform of this is f1 off s the Laplace transform of the second pulse duration over the second period is f1 off s times e to the power of minus t not of s the Laplace transform of the third pulse is f1 off s times e to the power of 2 t not s and so, on so.

So, the entire periodic pulse function will have 1 plus e to the power of minus t not s thus e to the power of 2 t not s extra and this can be put in the form f1 off s over 1 minus e to the power minus t not s. So, if you have the periodic function of time you want to find out the Laplace transform all unit to do this consider, the Laplace transform of over 1 period of the variation over the 1 period.

If it is Laplace transform of it is f 1 off s the Laplace transform of the entire periodic function is f1 off s over 1 minus, over the e to the power of minus t not s, where t not is the period of the particular wave form

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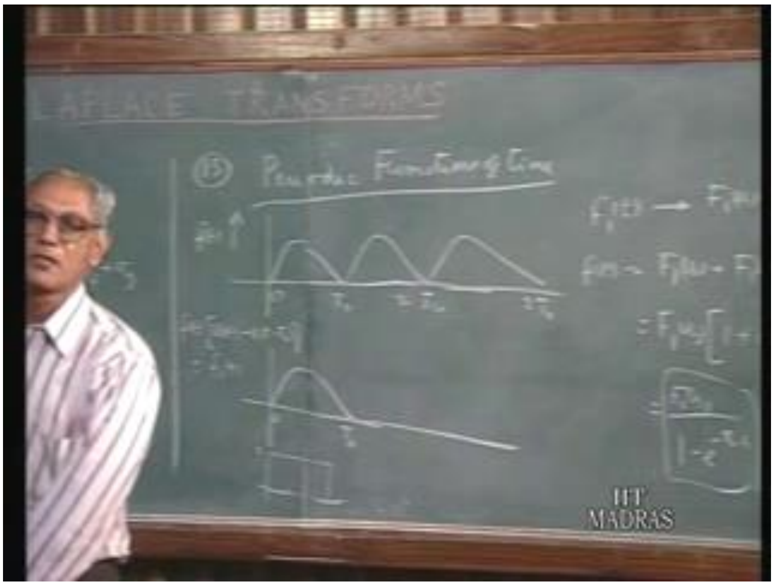

To some up we have in this lecture, consider further interesting properties of the Laplace transform in particular. We have considered, the Laplace transform of ft up on t given the Laplace transform of the f off t how we can find the Laplace transform of f off t over t than we consider, the initial and final value theorems, which enable as to find out the initial value and final value of function of time without from there Laplace transforms without having to find out f off t explicitly.

Then we also talked about the, convolution property, convolution in time domain corresponds to multiplication frequency domain of the 2 particular transforms and the convolution in frequency domain correspond to multiplication in time domain the later 1 we said is a limited interest was then. So, we pursue this in some detail we also observed that, if you have the periodic function it s enough to find the Laplace transform over 1 period.

Then you can use that information to find out, the Laplace transform of the entire periodic function by multiplying that, the Laplace transform of 1 period by 1 1 minus e to the power of minus t not off s, where t not is the period of the periodic function of time. In the next lecture, we will work out several examples illustrating, the application of the various properties, that we have discussed so, far.