Networks and Systems
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Lecture – 22
Laplace Transforms (3)
Properties of L-Transforms.
Translation in Time and Frequency Domains Scaling

Looked at some important properties of the Laplace transforms in the last lecture the particular properties which are of interest are those when f off t gets multiplied by t what happens in the transform domain. If f off t is differentiated f off t is integrated there effects the transformed domain are 3 important properties which we discussed in the last lecture.

So, if you continue this discussion of the properties of Laplace transforms which will be useful when, we handle problems involving the application of Laplace transform between the solution of electrical networks and systems under dynamic conditions. So, let us consider now the next property which concerns what is called shift in frequency shift in frequency domain that is s domain which is equivalent to multiplication by an exponent multiplication by an exponent exponential function in time domain.

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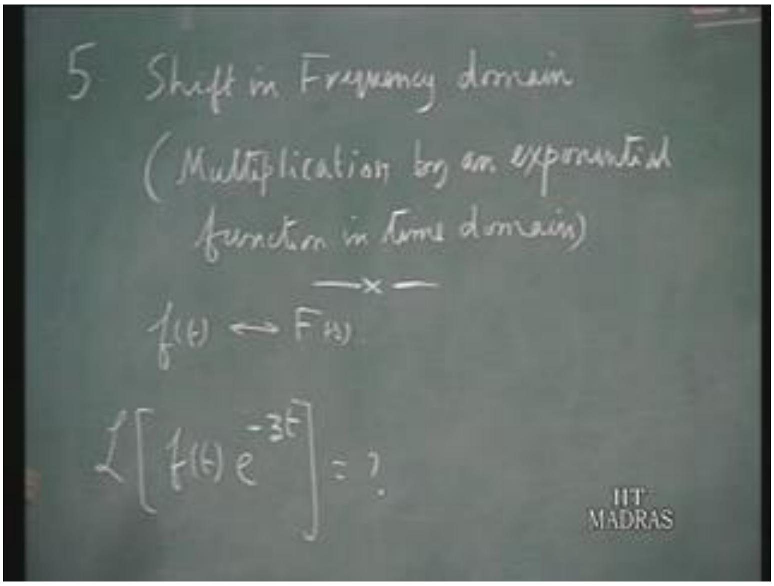

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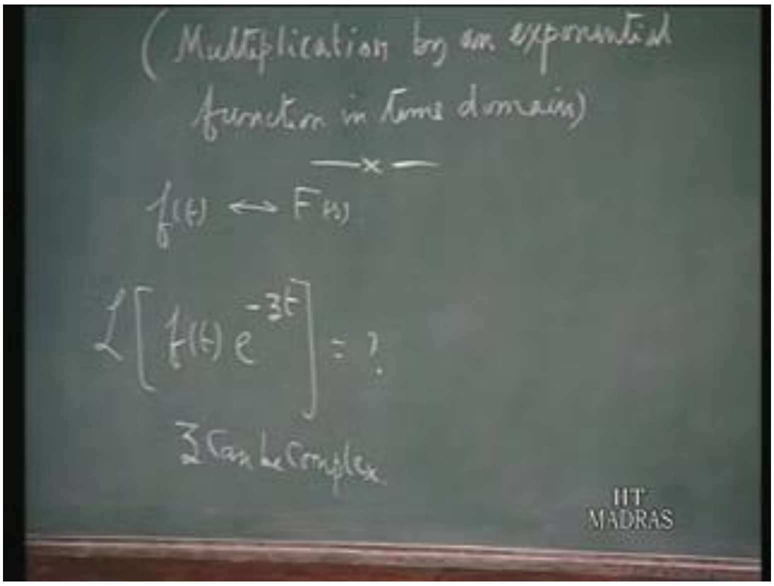

what we are going to show is that if you multiply an f off t the exponential function like, e to the power of minus alpha t it corresponds to a shift in the frequency domain that is explain are s domain. Let us, consider an f off t is the Laplace transform f off s. the question which we like to ask is what is the Laplace transform of f of t multiplied by e to the power of minus e zt assuming all this of course as assuming that f of t is causal time function.

What is that equal to? Both for generality z can be constant. So, this is the what we are going to look at? So, to using the formula for Laplace transform of the time function this particular Laplace transform would be from 0 to infinite f off t e to the power of minus zt e to the power of minus st dt.

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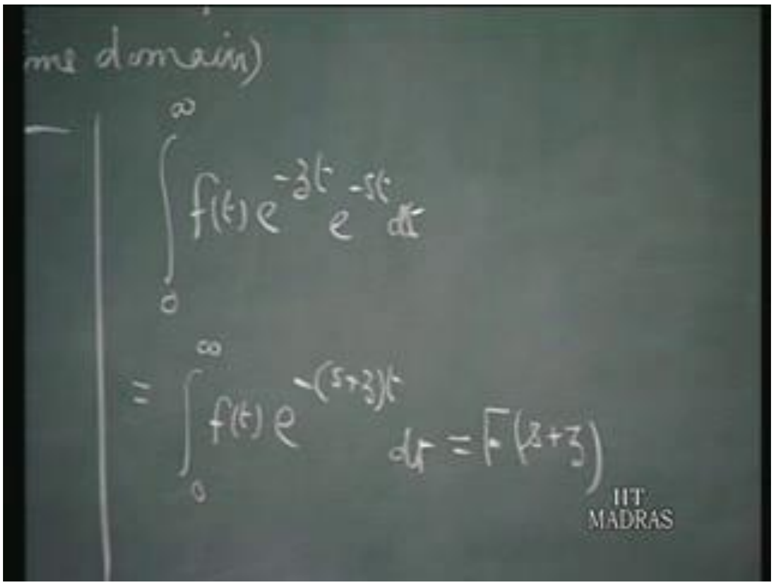

I am not putting specifically 0 minus because, depending up on our interest whether you take from the 0 minus or 0 plus depending up on what should take f off t to be the type of f off t you are interested. So, we leave it in that this of course, can be written as 0 to infinity of f of t e to the power of minus of s plus z t dt. Now, you will see that e to the power of minus st dt ft e to the power of minus st dt integral will be f off s all we have now is instead of s we have s plus z.

So, if you get the f off t e to the power of minus st dt this should have been calculate f off s this Laplace transform of f off t now instead of the variable s we have s plus z. So, this will be f off s plus z. So, we have if f off t has the Laplace transform f off s if you multiply f off t e to the power of minus zt, the Laplace transform is simply f off s plus z. That means, this translation are shift in the frequency domain.

So, this is the very important property which will before quit useful in finding out the Laplace transform of certain time function as well as, the inverse Laplace transforms of certain functions are s. And of course, in the new function if you are having the f of convergence whatever it was for f off t we will get modified. Because, of this the factor e to the power of minus zt depending up on the real value of z.

Then, the absence of convergence can be modified for the new function that is not important for us. We note of course, that is the possible the absence of convergence are the new function different from the absence of convergence for the original f off t. Let me, take an example suppose we have u off t we know that the Laplace transform is 1 over s now let me multiply u off t e to the power of minus alpha t.

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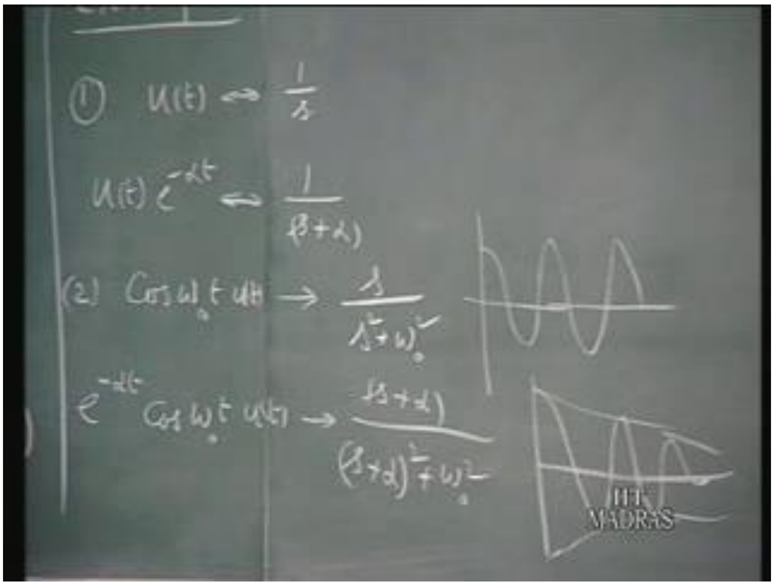

So, according to this rule instead of we have replaced s as alpha this all we have to do therefore, this is 1 over s alpha. So, in other wards e to the power of minus alpha t ut which we have derived earlier from fundamentals could have been derived using the applying this particular rule of shift in frequency domain corresponding to multiplication by e to the power of minus alp ha t.

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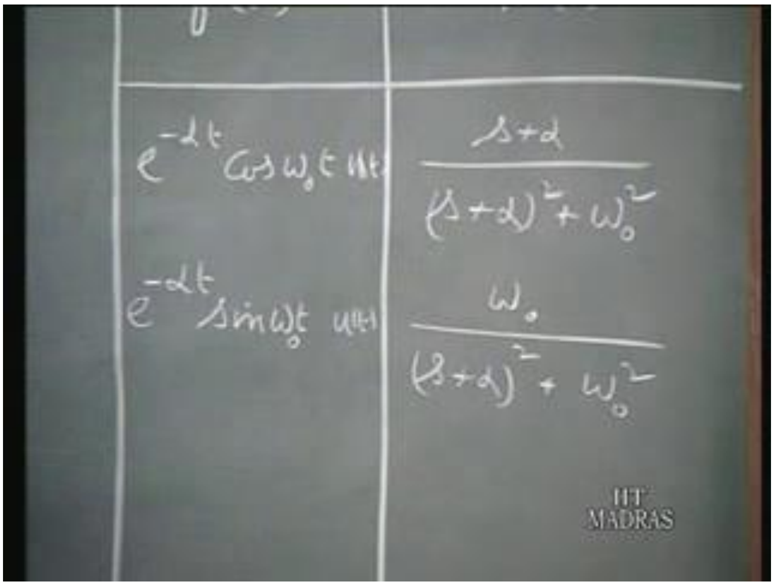

Now, let me take other example suppose I have task omega not t ut a cosine function we know its Laplace transform is s over s square plus omega not square that would be a trigonometric function which starts from t equal 0 on the other hands, suppose I take e to the power of minus alpha t cos omega not t ut this particular time function is again oscillating.

But with decreasing amplitude depending up on this value of alpha. So, its plot would be if this is the envelope so, you have something like this. Where the amplitude is decay as e to the power of minus alpha t. So, in exponential damped sinusoidal function of time. Now, all this differs from the original function is by multiplication of e to the power of minus alpha t therefore, according to this rule always we have to do is replace s by s plus alpha.

So, you have s plus alpha divided by s plus alpha square plus omega not square. So, this gives you a new function of time Laplace transform par e to the power of minus alpha cos omega not t ut will be s plus alpha over s plus alpha square plus omega not square you do not have to remember this. Because, once you know the formula the Laplace transform of cos omega not t whenever it gets multiplied by an exponential factor of that we should able to reduce this.

But more over less put this down in our table of Laplace transform and f off t pairs I will go to the other end of the blackboard even if our camera man will frond at this let me, prove that I will take that risk. So, we have e to the power of minus alpha t cos omega not t ut will have Laplace transform s plus alpha over s plus alpha square plus omega not square. In the same manner suppose, I have e to the power of minus alpha t sin omega not t ut we know that, Laplace transform of sin omega not t ut is omega not over s square plus omega not square.

So, in that particular function we have to substitute s plus alpha for s. So, omega not will remain as before the denominator will be s plus alpha square omega that would be the Laplace transform e to the power of minus alpha t sin omega not t ut. Visualizing this in this fashion will be very important when, you are taking some times inverse Laplace transform of functions where a quadratic exists in the denominator.

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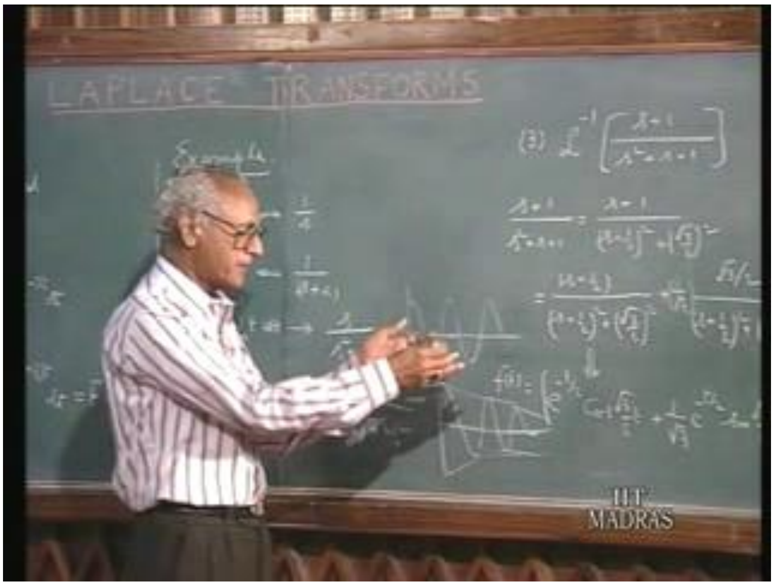

For example: Suppose I ask the question what is the inverse Laplace transform of say s plus 1 over s square plus s plus 1 suppose, I ask the question. Now, note that the poles of this the rational function are the zeros of the denominator polynomial are complex in that case, s square plus s plus 1 can be put in this form s plus alpha whole square plus omega not square try to put in that form then, you can combine you can decompose this function into sum of 2 such functions.

Then, you can readily integrities them to be the inverse Laplace transform of sinusoidal functions where decreasing the amplitude. So, let us go about that see this: so, s plus 1 over s square plus s plus 1 first of all I should write this in denominator as s plus alpha whole square plus omega not square. Therefore, I write this s plus half whole square so, that account for the coefficient of s here s square plus s plus one-fourth.

But still you have 1 here therefore, to cover that I will put this root 3 by 2 whole square. So, you observe that 3 quarters plus 1 quarter is equal to 1 and therefore, we have this the complex conjugate the poles the denominator of the function are clearly displayed as minus half plus or minus root 3 by 2. So, the numerator has been s plus half you would identified this to be e to the power of minus half t cos root 3 by 2.

Therefore, let us do that so you have s plus half divided by s plus half whole square plus root 3 by 2 whole square this is 1 term and what is left behind is another half here if the numerator has been constant root 3 by 2 then, you would put this as e to the power of minus alpha t sin omega not t by 2. So, if this had been root 3 by 2 divided by s plus half whole square plus root 3 by whole square. Then, we would have set this to be the Laplace transform of e to the power of minus half t times sin root 3 up on 2 t. But had only half here therefore, to cover that put 1 over root 3 Then, you have s plus half this term we know is the Laplace transform of e to the power of minus t by 2 cos root 3 by 2 t and this term will be 1 over root 3 e to the power of minus t up on 2 sin root 3 by 2 t.

So, of course, the entire thing is multiplied by u off t that is the f off t which corresponds to this Laplace transformation. So, this rule are what happens when, you multiply the time function by an exponent will come in handy not only in find the Laplace transform as a certain time functions. But when you are finding the inverse Laplace transform off quadratic factors where the denominator zeros are complex conjugates can be easily be handled in this fashion.

The rule we have considered just now, is that if f off t is multiplied by exponential function of time that corresponds to a shift in the frequency domain who have a dual rule which corresponds to shift in time domain and that corresponds to multiplication by the exponential function in the frequency domain.

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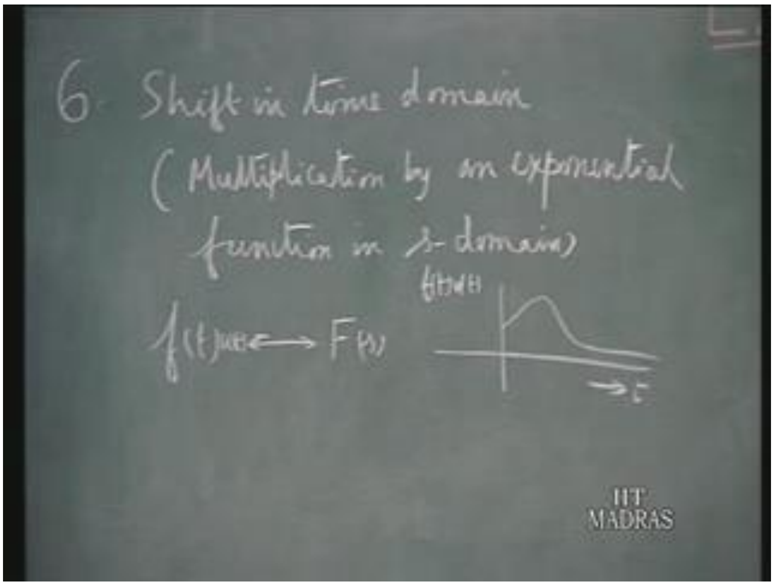

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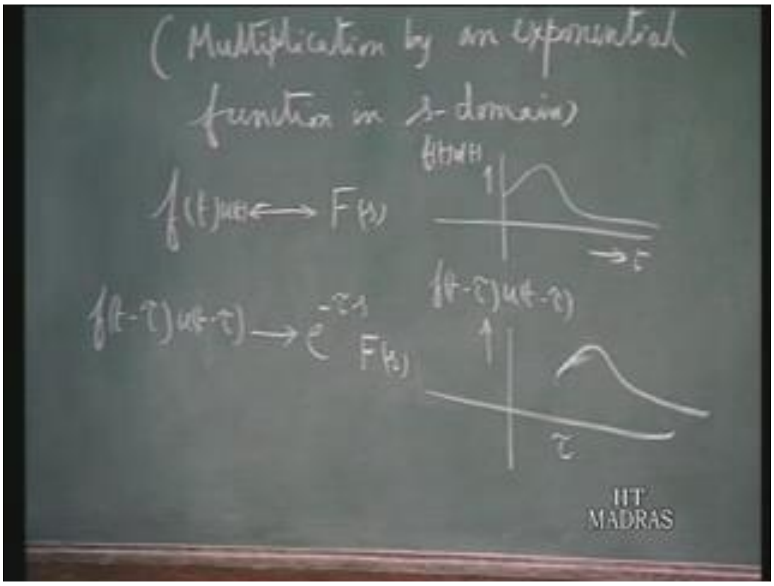

This is corresponds to multiplication by an exponential function in the frequency domain that is the complex frequency domain or s domain. Let me, first put down the rule. And then, will try to demonstrate that suppose f off t and f off s format as compare let me, assume that I will also put u off t to make sure that it is a causal function. Suppose, I have this is f off t ut now let me, say that I translate this I delay this function of time by the amount of term seconds.

So, you are starting from t equal 0 it starts from here and it goes like this so this is tow. The same function is delayed among tow therefore, we can describe this as f off t minus tow u off t minus tow, this is the description of this function. Everything is delayed by toe seconds whatever, value this functions takes at t1 this will take at this t1 plus tow second in the seconds is delayed. So, what is the Laplace transform given the Laplace transform of this is f off s who ask the question what is the Laplace transform of this.

So, f off t minus tow u off t minus tow has the Laplace transform it hands out to be e to the power of minus tow as f off s. So, a shift in the time domain corresponds to multiplication by exponential factor in the frequency domain delay by tow seconds involves multiplication the frequency domain by e to the power of minus tow s. We take tow to be greater than 0 that is this corresponds to delay.

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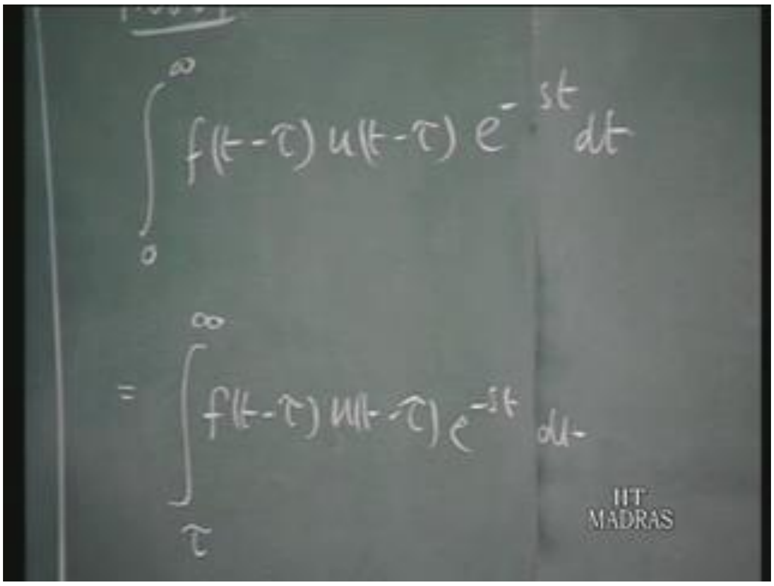

Now, the proof of this is quit straight forward. So, we are interested in filling the Laplace transform of f off t minus tow u off t minus tow and this to be multiplied by the e to the power of minus st dt that is what you are interested in final. Now since, the variable of integration is t as long as t is less than tow u off t minus tow is zero. So, as long as t is less than tow the argument is ready to this makes the 0 therefore, I can as well start the integration from tow infinite of f off minus tow u off t minus tow e to the power of minus st dt.

Because, this factor made the integrate 0 for values of t up to equal tow therefore, f off t minus tow u t minus tow e to the power of minus st dt and for values of t larger than tow than this is going to be 1. So, you may as well limit like that, are you may even if you want keep it in no harm in there.

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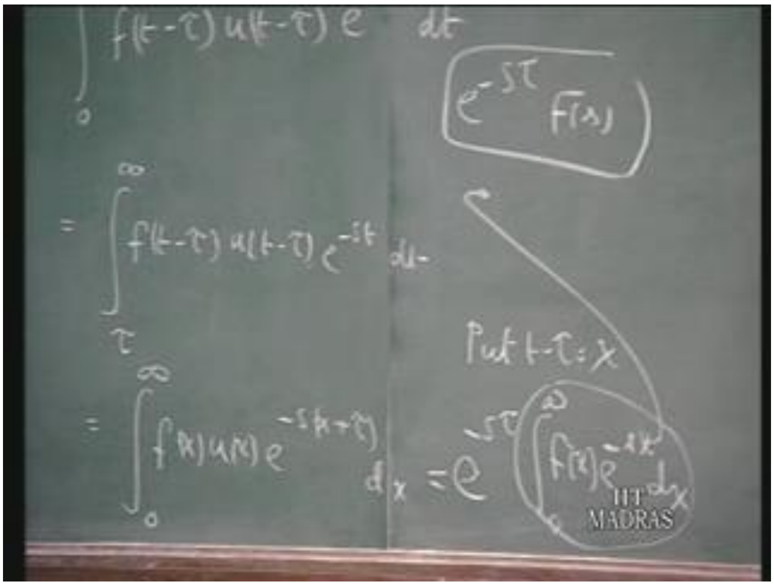

Now, let me put here p minus tow as let us say x then, this is f off x u off x e to the power of minus s t equals tow plus x x plus tow and dt equals dx. And the range of integration now when t becomes tow x become 0. So, it start from 0 and when t equals infinite x also be infinite because, tow is finite that is what you are having now, this integral involves the variation of integration x this tow is the constant.

Therefore, I can write this as e to the power of minus s tow 0 to infinite of f off x e to the power of minus sx dx and what you are having here is indeed you can also consider this 0 to infinite f off t to the power of minus st dt. So, that is the Laplace transform of f off t therefore, the result is e to the power of minus s tow this is what We mention here as the Laplace transform of f off t minus tow that is the proof of this.

This particular rule is very important because, it comes in very handy in finding of the Laplace transforms how various typical functions that we handle like, pulse functions and so on as will take some examples illustrate this Let us, see let work out some examples. Let us, consider first of all a pulse function 0 to d it exits from 0 to d value a here this is f off t.

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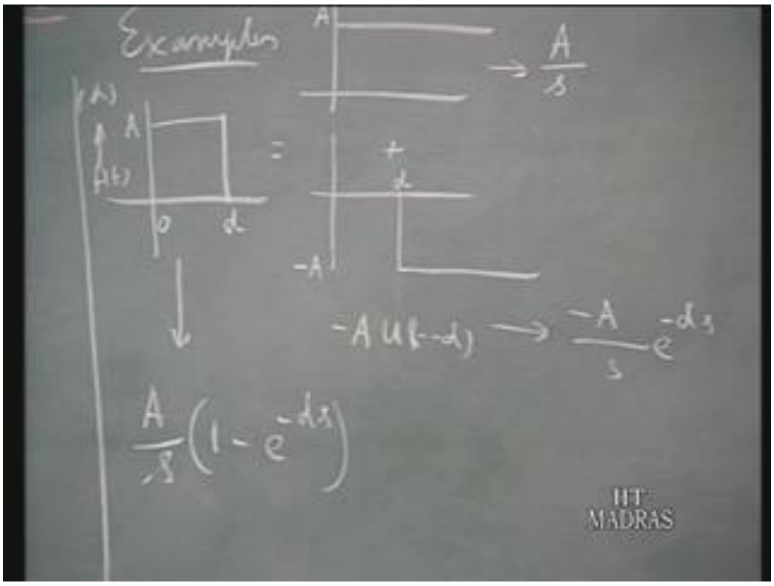

Now, you can always consider this pulse to be a step function of amplitude a starting a t equals 0 plus another step function of amplitude of minus a starting at time p equal d. If you add this 2 step functions: 1 regular step function starting a t equal 0, another a delayed step function starting a t equals d adding of this 2 you will get this and we know the Laplace transform of this will be this is a not a unit step function. But the step function of amplitude or magnitude a therefore, the Laplace transform of this is a over s.

Now, the Laplace transform of this is how do you describe of this is minus a u off t minus d this is the expression for this second function where the first function is a u off t. And therefore, it has the Laplace transform all we are now doing is step t delaying this seconds. Therefore, the Laplace transform u off t minus d is 1 over s e to the power of minus ds you have to multiplying this a minus a.

Therefore, the Laplace transform of this is minus a by s this would have been the Laplace transform if this is delayed did not occur. Because, the delay we have e to the power of minus ds therefore, the Laplace transform of this is the sum of these 2. So, you have a up on s to 1 minus e to the power of minus ds. So, you are observe that whenever we have distinguish is in the function.

Then, type of pulse functions like, this they can quit conveniently handle by using this property of the shift in time domain what it difference is make transform domain is all that means, is we have multiplying by exponential factor e to the power of minus s times the delay that is involved. Let me, take another example b suppose I have 1 half cycle of a sinusoidal just this 0 to t not up on 2 and of course, will assume that omega not equal 2 pie up on t not.

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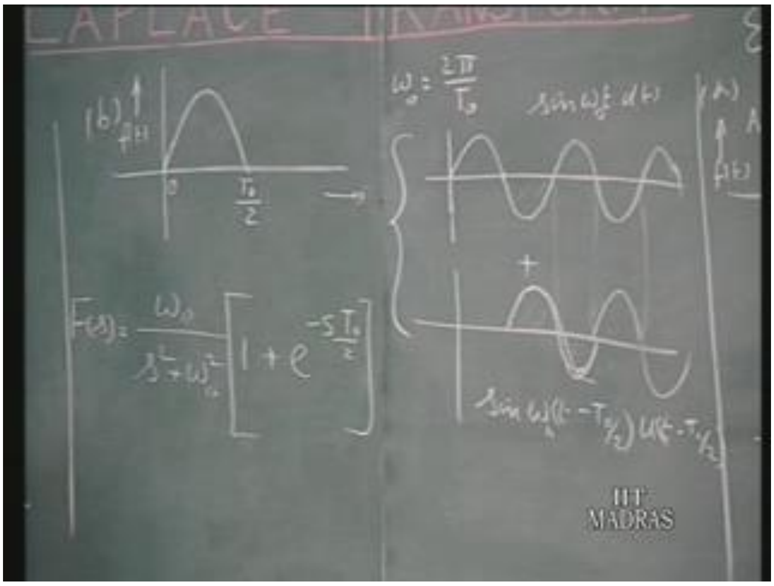

So, you want to find out Laplace transform of this a single loop of a sinusoidal. Now, this can be considered to be a regular sinusoidal omega not t ut plus suppose, I delay this sinusoidal by 1 half period. so, I get another sinusoidal like this and this would be sin omega not t minus t not up on 2 delayed by the amount u off t minus t not up on 2 that is delayed sinusoidal. Now, if you add this 2 this negative loop is cancel buy the positive loop, this positive loop have canceled by the negative loop and so on.

If you add this 2 this is what you get write. And therefore, if you add this Laplace transforms are have this 2, you will get the Laplace transform of this by the linearity principle if you talked about. Therefore, the Laplace transform of this single loop will be the Laplace transform of this which is omega not over s square plus omega not square plus the Laplace transform of this which is obtained by multiplying the Laplace transform of this by e to the power of s times minus s times delay that is involved.

Therefore, e to the power of minus s t not up on 2 that is what we have that is the Laplace transform of the single loop you do not have to do any integration the advantage of knowing all the properties is that whenever, you are having functions of this type if you cleverly manipulate this, you do not have to carry out this integration to find out the

Laplace transform from the Laplace transforms the known functions you can construct the Laplace transform of this purely to algebraic integers as shown here.

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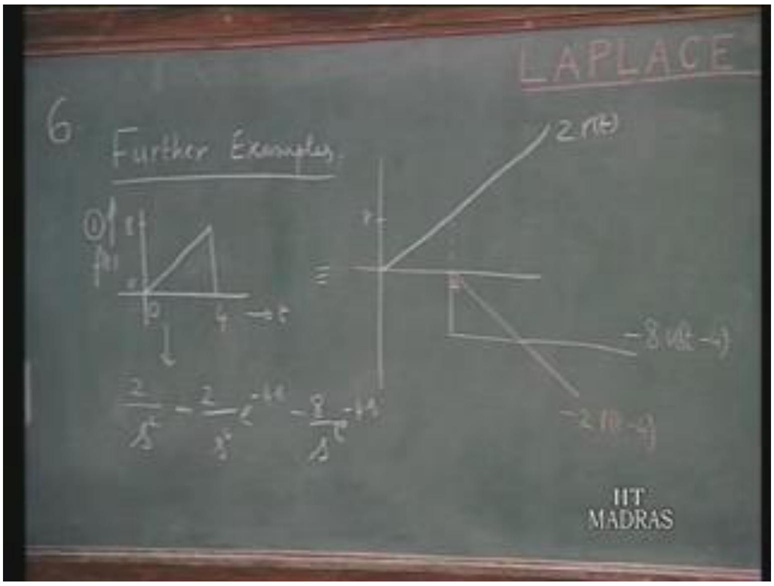

So let us, move on let me consider 2 further examples: 1 suppose, I have a triangular pulse function 0 to 4 seconds it reaches the values of 8 as t equals 4 and you like to find out the Laplace transform of this. Now, once again you do not have to any integration because, do you know the Laplace transform of ram function and f functions. So, you like to decompose this function in terms of appropriate step and ram functions

Let us, see how we do that suppose we describe this portion by a ram function then you have a ram starting at 0 reaching the value 8 at time t equals 4 and this would be described as 8 units it jumped by 8 units in 4 seconds. Therefore, it is a slope of 2 so, this is 2 times r t that would be the ram. Now, this ram describes the actual behavior of f off t up to t equals 4 seconds. But at this point we must nullify whatever is left beyond 4.

So, to do that we introduce a negative ram here at 4 seconds of the same slope so that, the growth is arrested. Therefore, this would be having the negative slope of minus 2 and then, its pair are t equals 4 seconds. So, that would be description of this curve minus 2 r t minus 4 would be negative going ram with the slope of magnitude 2 starting that t equal 4. If you added this 2 you what your get is this 1 plus a constant here because, whatever increase here is cancel by the increase here.

So, what is left by adding this 2 curve is not only, this triangular pulse. But also a kind of constant a step here. So, we should cancel the step as well and the magnitude of the step is 8 minutes and starts it 4. So, in addition we need to have a step here a negative step. So, this will be described as minus 8 u off t minus 4 so if you added this 3 characteristics. Then, you would get this pulse you have a ram function to start with negative ram growth.

Then, afterwards among to which ever going to must alps be canceled out that will be minus 8 u to the power of t minus 4. Now, you know the Laplace transform of this 3 functions. So, the Laplace transform of this would be the Laplace transformer of the ram which is 2 up on square. And then, the Laplace transform of this ram is if this is 2 up on s square, this is 2 up on s square times e to the power of minus 4s. Because, the same ram with an negative sign delayed by 4 seconds.

Therefore, minus 2 up on s square e to the power of minus 4s that would be the Laplace transform of this delayed ram the Laplace transform of this step function is minus 8 it would have been started a t equals 0 it would have been minus 8 by s. But since has been delayed 4 seconds you have to write minus 8 by s e to the power of minus 4s that is what you are having 2 up on s square times 1 minus e to the power of minus 4s minus 8s power e to the power of minus 4 this is the how it goes.

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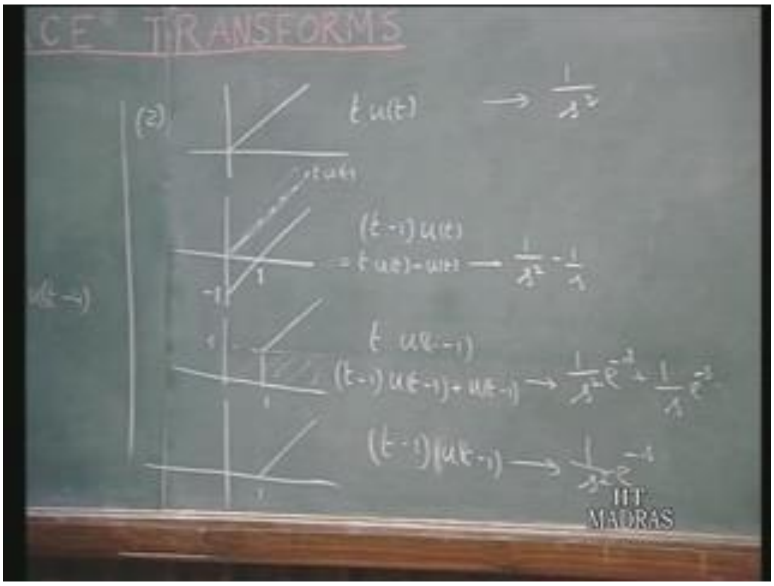

Second example now, you must be careful when we are talking about the rotational for the delay time function that you f off t minus tow times u off t minus tow use an proper expression for the delay time function to distinguish between other possibilities. Let me, take a set of example this would be tut suppose f off t is t then, this is but suppose I write t minus 1 u off t what does it mean? This is not a delayed function because, u off t is not been replaced by u off t minus 1 t minus 1 ut would be this 1 this will be at p equals 0 it would have been minus 1 that is what it could be the same slope of course.

But it does not start t equals 1 this is not this is not delayed version of this important recognizer that. Suppose, I have t ut minus 1 so it is this is t ut, but the it t is multiplied by ut minus 1 that means, the function t is multiplied by a step which is starting a t equals 1 at this point. Therefore, this would have been this would have been t, but this portion as in cut off. So, that would be t ut minus 1.

So, this is 1 and that is t ut on the other hand, if you have t minus 1 ut minus 1 this would be whatever sequence of values this had it takes 1 second later therefore, it is a proper delayed function this same function is delayed by 1 second therefore. So, you see you have presumably this delayed function. But there not the delayed function of this. So, you must be careful indistinguishing between the various function that covered by this expressions.

Now, the Laplace transform of this is off course 1 over s square very clear now this is the delayed function therefore, you can immediately right 1 overs square e to the power of delay is 1 second that 1 over s square e to the power of minus s there is problem of more that. Now, what about this t minus 1 ut can be written as t ut minus t ut; that means, you have t ut u remove 1 unit value at every point of time that this is what we get t ut this would be parallel to this.

So, if you remove u off t from that this curve is as therefore, Laplace transform of this is 1 over s square minus 1 over s Laplace transform of t ut os 1 over s square Laplace transform of ut is 1 over s. So, as for as this is concerned t ut minus 1 you can thing of this as t minus 1 ut minus 1 plus ut minus 1 add this 2 that is what you get. So, t minus 1 ut minus 1 is a proper delayed function of this.

Therefore, this is 1 over s square e to power of minus s and ut minus 1 then, we have the Laplace transform 1 over s e to the power minus s. So, you can see that this 3 functions now different Laplace transforms they are not the same time functions off course. Therefore, we except Laplace transform to different you can also see this as this can be thought of as a delayed step plus a delayed ram that is what we have delayed step right. Now, before we proceed to a discussion of another property let me, caution you about 1 possible bit fall that 1 might fall into.

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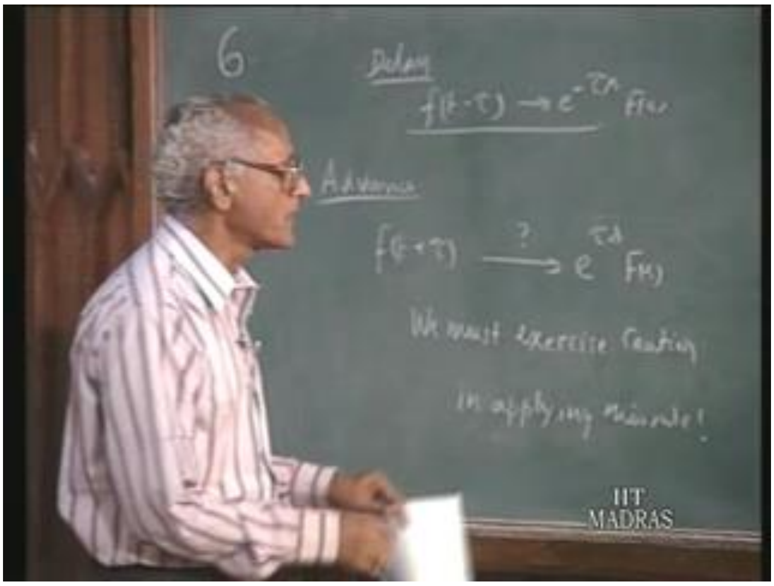

We agreed that, if the function f off t is delayed by tow seconds f off t minus tow this will have to the power of minus tow as f off s that is the Laplace transform of this. Now the question that, ask you what happens to advance in time. Suppose, we have a function f off t you advance it so that means, f off t plus tow tow is once again consider to be positive does it mean that this will be e to the power of tow s f off s.

So, this the question that you like to ask we must exercise caution in applying this rule why? When you advance a time function what happens is when, you push some part to the time function to the negative values of time. Then, a part of the function can get last therefore, it is not a if delayed them the same functional variation whatever we had f off t ut continuous. But you advance it in the negative direction that is the negative direction of a time that means, the part of the function can be get last.

So, therefore, whatever is left may not have a complete replica of the original f off t ut therefore, this rule may or may not be work. So, 1 must be carefully this as for as Fourier transforms are confirm the delay if the delay are advance tow seconds is multiplied by the e to the power of j omega tow or e to the power of minus j omega tow as the case may be without any difficulty. Because, the Fourier transforms angle time functions would exists from negative values of time as well, but not Laplace transforms. So, let me illustrate by means of an example suppose f off t is like this is 2 seconds and then, I have let us say g off t which is obtained as f off t plus 1.

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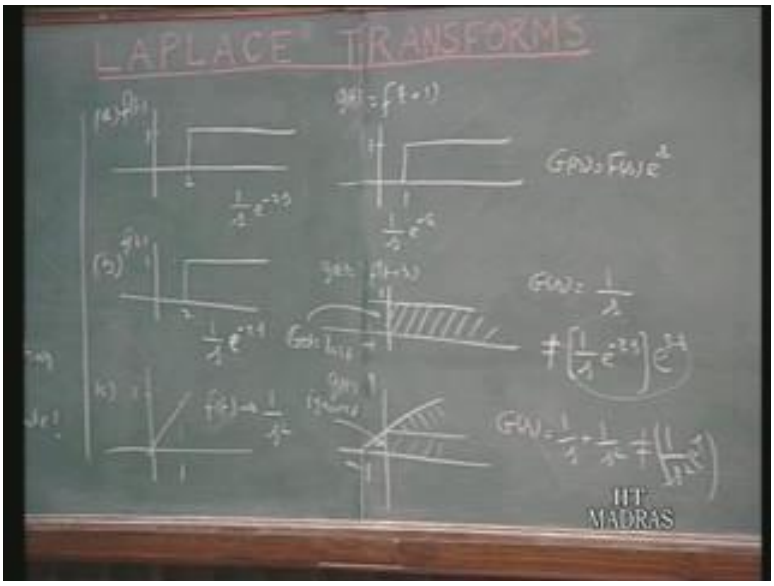

So, f off t is a delayed step function starting a 2 units and g off t is advanced by 1 second is f off t plus. The Laplace transform of this is obviously, 1 over s e to the power of minus 2 s the Laplace transform of this is 1 over s e to the power of minus s. Therefore, we can see in this case the g off s is f off s multiplied by the e to the power of s corresponding to 1 second advance therefore, this rule works quit well here.

But because, you are advancing them as nothing of this function is getting loss right. Now, suppose you have the same f off t and now I say g off t now is f off t plus 3 that means, this is advance by 3 seconds; that means, it is like this when you take the Laplace transform of g off t how what we ignore this portion for negative value time and take account only of this. So, this portion is gets neglected it gets lost this portion.

So, we would say g off s is a Laplace transform this off course I take this all the period magnitude. So, we will take the this the Laplace transform of unit f function this is 1 over s. Now, f off s is 1 over s e to the power of minus 2 s the function is advanced by 3 3 units therefore, this is not equal to 1 over s e to the power of minus 2 s which is the

Laplace transform of f off s the advance corresponds to t s. So, this would be what would have expect if the same rule applied for advance functions as it would applied for delayed functions. So, if here multiplied by e to the power of 3 s you got 1 over s e to the power of s, but it is not equal to 1 over s. Therefore, this is not equal to this this is in correct expression for this as for the Laplace transform this concern.

Let me, take a third example suppose you have f off t which is a ram function. So, f off t here will give me 1 over s square. Suppose I deal advance this by 1 second so, this I will call it g off t. Now, once again now you are considering a Laplace transform of this, this portion is ignored and you are considering already this portion and that is equal to a step function plus a ram function of low.

Therefore, the Laplace transform of the g off s would be 1 over s plus 1 over s square and this is certainly not equal to 1 over s square multiplied by e to the power of s corresponding to the advance. Therefore, this is not equal to this so when we are advance in time functions 1 has to be very careful whether, this particular rule like this will apply or not if the advance corresponds a situation low portion of the original function of s has been truncated as it has been in this case then that rule will apply.

But in other cases like this which is more usual the situation this rule will not apply. So, as for this rule we talked about primarily intended for delay time functions. When, it is advanced 1 has to take it with a pinch of salt. The next property will talk about is scaling what we mean by this is f off t and f off s form Laplace transform pair. Then, f off at where a is a positive constant that means, you scale the time events we make them appear faster f off at this corresponds to a Laplace transform 1 over a f off s over a is the rule.

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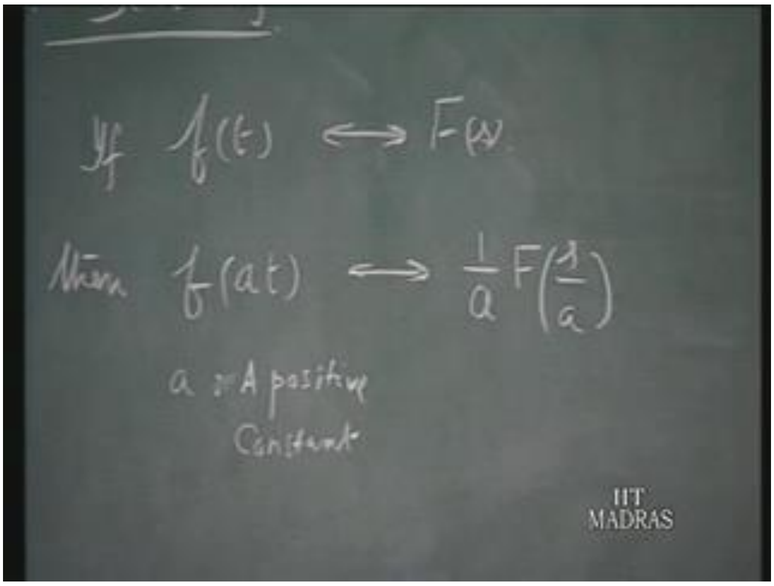

So, scaling in time domain also corresponds to scaling in the frequency domain, but in the inverse fashion. F of you make events faster that will be slower and wise versa, But off course with the multiplying factor here proof 0 to infinite of f off at e to the power of minus st dt that will can be written as if you put at as x this will be f off x e to the power of minus s x up on a dt equals dx up on a.

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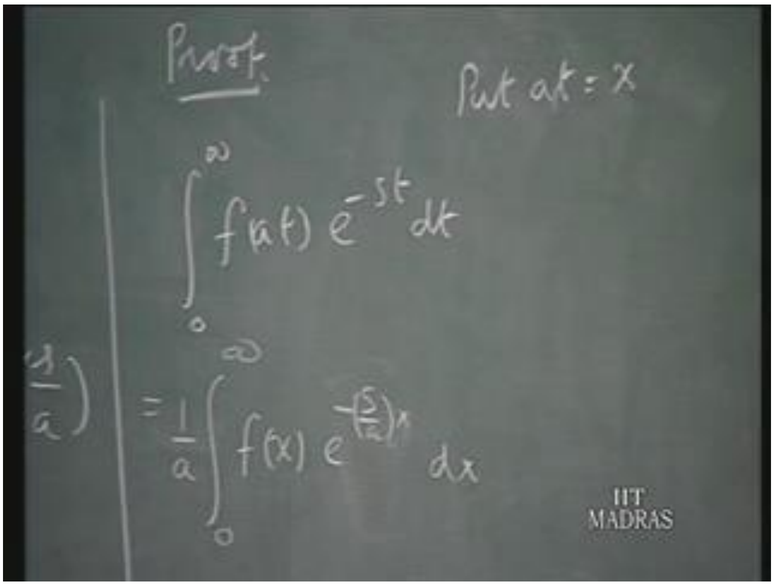

So, put this once and when t equals 0 x is 0 when t equals infinite x is also infinite. So, always having this f off x instead of e to the power of minus x you have x up on a or s up on a instead of s t you have s up on a time c x s up on a time p x is the done the variable we can replaced by t as well therefore, what we are having here is instead of s you are really having s up on a times this x which is the variable of integral. So obviously, this is the Laplace this is if f off x is the Laplace transform of f off t this would be f off s for a. Therefore, 1 over a f off s by a that is what we have do here or you can put this a times f off at as the Laplace transform of f off s over a it is the same thing.

Now, this particular rule comes in handle sometimes in numerical working like this that means, illustrate the given example. When, you have large numerical constant sometimes we can avoid this by suitable scaling. Find the inverse Laplace transform of 1 over 10 to the power of 4 s square plus 10 to the power of s plus 1 off course I ask this question.

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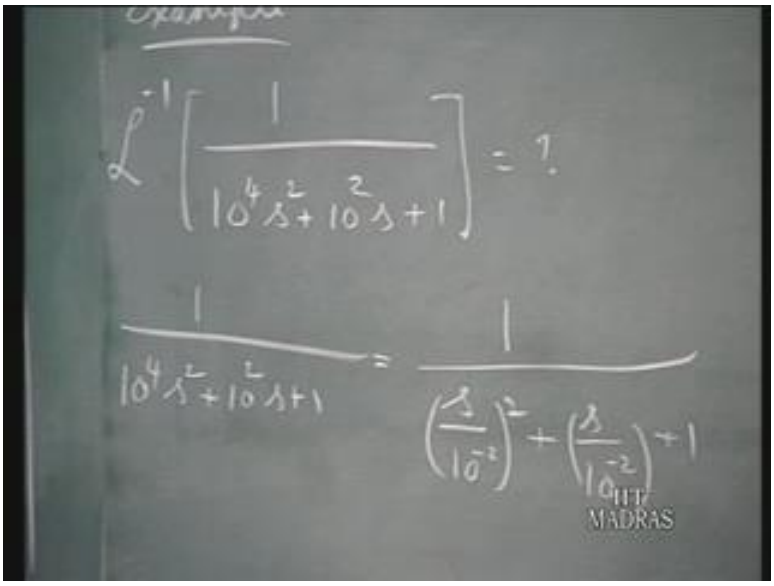

What is the inverse Laplace transform of this who want to find this out. So, you can scale this in the frequency domain in this fashion 1 over 10 to the power of 4 s square plus 10 to the power of s plus 1 can be written as 1 over s up on 10 to the power of minus 2 whole square. Suppose, I divide this by the same thing s square divide by 10 to the power of 4 or 10 to the power of 4 you can have the denominator is 10 to the power of minus 4 this same thing and especially in this fashion.

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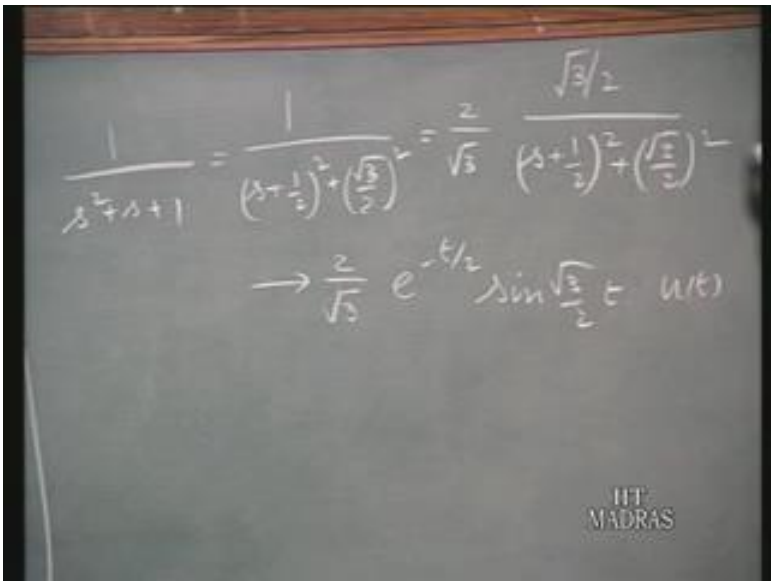

And then, this can be written s again s up on 10 to the power of minus 2 which is same thing as 10 square s plus 1. So, if I had scale this s by the factor so that s up on 10 to the power of minus 2 is replace by s then, this is 1 over s square plus s plus 1. We know the Laplace transform 1 over s square plus s plus 1 once we know that, we can find the inverse Laplace transform of this by applying this scaling rule.

Let us, see what we how we do that if you this had been simply 1 over s square plus s plus 1 instead of this s up on 10 to the power of minus 2 who would have written this as we have already mention as s plus half whole square plus root 3 up on 2 whole square and this would have been 2 up on root 3 times root 3 up on 2 over s plus half whole square plus root 3 up on 2 why I put this root 3 up on 2 here is because, this is form omega not over s plus alpha square plus omega not square.

So, this particular Laplace transform would be adding its inverse Laplace transform 2 up on 3 e to the power of minus t up on 2 sin root 3 up on 2 that is the off course, u off t will always t this is the inverse Laplace transform of this. But now, if this is f off s what we have having here is instead of s we have s up on 10 to the power on minus 2. So, we have f s over 10 to the power of minus 2 that is what this expression is.

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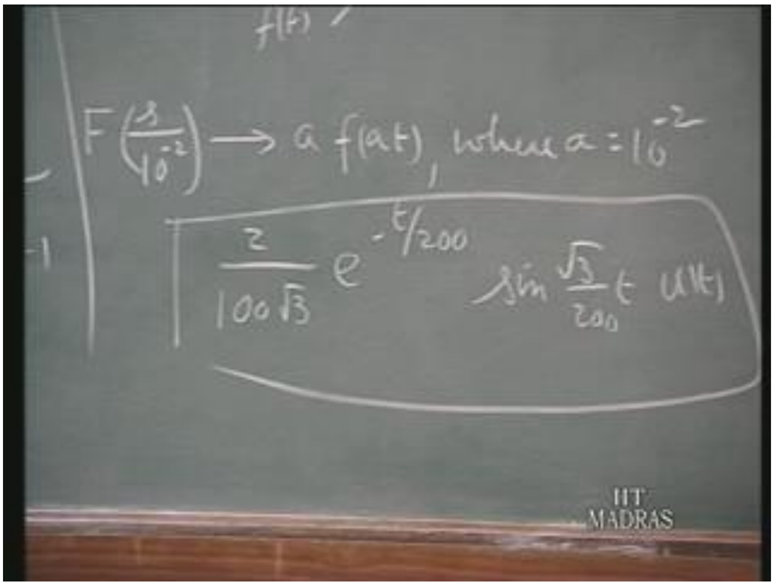

Now, if you go to this rule here f off s up on a will be a times f off at a in our case ten to the power of minus 2. So, since we know f off t we can find out a times f off at. This corresponds to a times this is f off t if this is f off t the a times f off at where a equals 10 to the power of minus 2. So, if you substitute that you have 10 to the power of minus 2 2 up on 100 root 3 e to the power of minus t up on 200 at sin root 3 up on 200 that is the inverse Laplace transform of this.

So, this scaling rule helps as to make the coefficients change the scale of the coefficients. So, that you can handle convenient numbers instead of sometimes and will be the numbers like this we can often advantage all this rule. Remember this, in accordance with what we discuss in the case of Fourier transform. If you make events faster in time, this is equivalent to increasing the frequency. That is if you scale t in 1 direction the frequency goes in other direction, if t is replaced by 2 t s is replaced by s up on 2.

So, if you compress events time to take them accrue at a faster time the frequency spelt from increases. So, you need to encounter higher frequencies on the other hand if you spread out in the time domain then, the frequency domain its gets compressed the higher frequency components come become smaller. So, in this lecture we covered 3 important properties of the Laplace transform: the first 1 we considered what when f off t function of time is multiplied by the exponential function of time e to the power of minus alpha t. For example, the in the frequency domain it corresponds to a shift in the frequency variable. That is s will be change is to s up on alpha. And the second important property if discussed was when, you shift a time function in time delayed by tow seconds the corresponding effect the transform domain is multiplying multiplication by f off s by e to the power of minus s tow. And we saw the ram in this case of this, if you try to use this to apply to advanced functions we said, when you advance the time function tow seconds it may or may not correspond to a f off s which is multiplied by e to the power of s tow.

In some case, it may in most of the cases may not because part pf the time function may get lost in the process. Lastly, we consider the scaling in time domain which corresponds to also a scaling in frequency domain if you scaling up in time domain with to be scaling down in the frequency domain by factor a off course, there will be also change in the multiplying constant.

In all this 3 cases we have considered various examples which illustrate the usefulness of this properties. When, you are evaluating either the forward Laplace transforms or inverse Laplace transforms we shall pick up at this point in the next lecture.