Networks and Systems
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Lecture – 21
Laplace Transforms (2) (Time 01:03)
Properties Of L-Transforms.
Differentiation Integration And Multiplication By T In Time Domain.

We had introduced ourselves to the concept of Laplace, transformation of the function of time. In the last class and we had a look at the Laplace transforms of some important functions of time like the unity impulse function the unit step function and the 2 sinusoidal functions cos omega not t and sin omega not t. Let us, first recapitulate some of the important points that was discussed in the last class.

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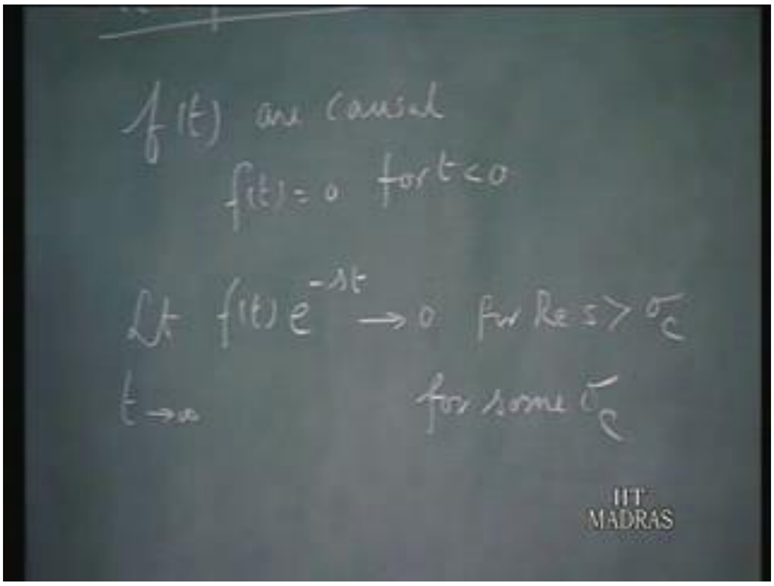

We are concerned with functions of time which are causal. So, the f off t which are concerned with are causal that is f off t is 0 for t less than 0. And further we said their exponential order; that means, limit as t goes infinite of f off t e to the power of minus st close to 0 for real part of s greater than some number sigma c for some sigma c which is the function of f off t. So, if the function of time is the exponential order that is the t goes to infinite the e to the power.

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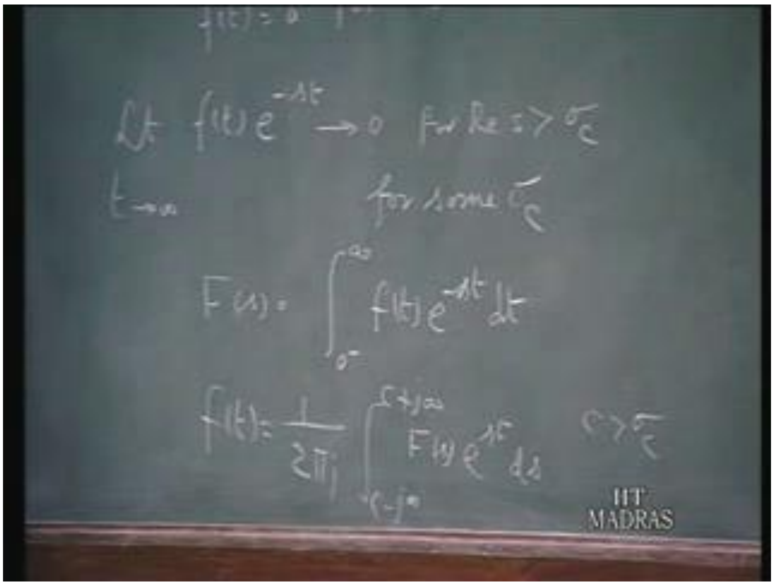

Of minus st is able to pull thus out the negligible proportion. Then for such time functions we said we have a Laplace transformation f off s. Which is given by the integral sub 0 minus to infinite of f off t e to the power of minus st dt. That is; the forward Laplace transformation formula and the inverse Laplace transform is given by 1 over ft is 1 over 2 pie j c minus j infinite to c plus j infinite of f off s e to the power of st ds this is the inverse Laplace transformation formula.

This cantor of integration which extends from c minus j infinite to c plus j infinite is the vertical line in the region of the convergence of the Laplace transformation; that means, c must be larger than sigma c. And this cantor of integration as i mentioned is referred to as the ground which cantor. Now, as the result of this the Laplace transformation that we get is usually a rational function for the type of functions of time that we consider.

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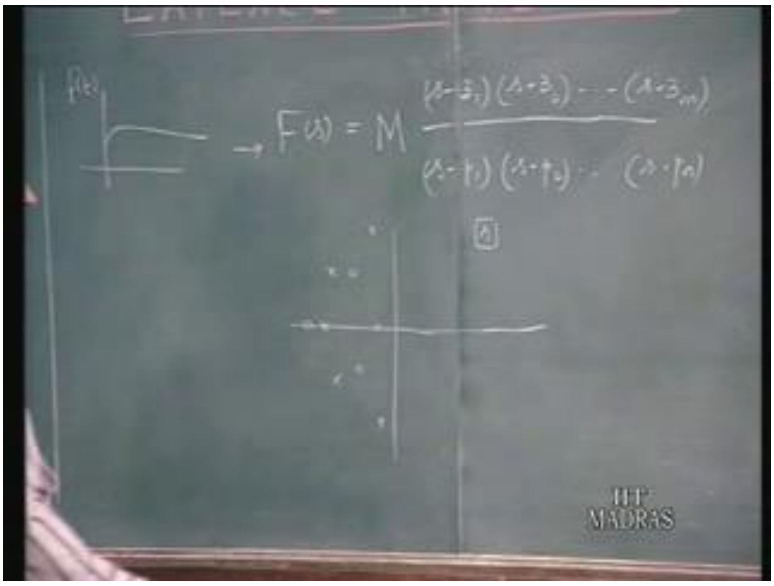

So, for the given f off t that we having we have an f off s which is the rational function. And this rational function by means, the ratio of 2 polynomial and this could be the 2 polynomial could be put in the form m is the numerator is factorized as s minus z1 s minus z2 down the line z minus zm. And the denominator likewise the factorized as s minus p1 s minus p 2 s minus pn, then the Laplace transformation epodes is defined by the values of the z value z1, z2, z3, zm and the values p1 up to pn apart from the constant multiplied factor m.

So, this f off s can be represent in the complex plane by the location of the zeros of f off s and poles of f off s. So, zeros are normally marked by means of small circles. So, this may be 0 locations poles may be are marked by crosses. So, there may be a pole here pair a poles here extra. So, the pole in 0 locations are marked in the s plane.

So, the pole of 0 locations of f of s completely defined f off s expect for the multiplied factor m. And we also lot that as long as coefficients in the 2 polynomials are real then early complex pole is accompanists by its conjugate any complex 0 is accompanists by its conjugate and the absence of convergence is.

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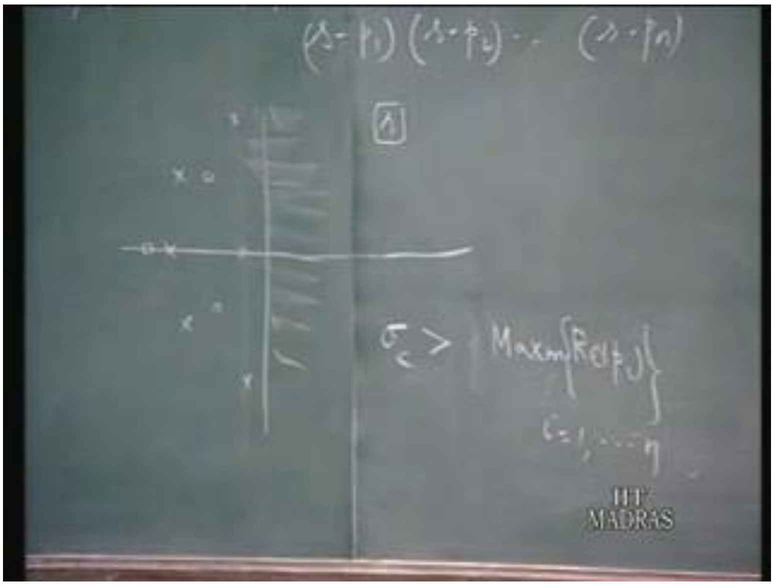

Sigma c the absence of convergence is larger than the maximum value of the real part of pi. So, whatever the pi you are having is the maximum value or the real part of pi i from 1 to n. So, that will be define the absence of convergence; that means, the region of convergence starts from the right most pole. So, if this is the right most pole anything beyond that if there is in the conversions. So, the maximum value of the real part of the poles defines the reason of convergence. And we have seen this in the case of simple functions like the unit stuff function which as the sigma c as 0 of the e to the power of minus 2t where sigma is minus 2 and so an so for, as that is what we have seen earlier a few comments about.

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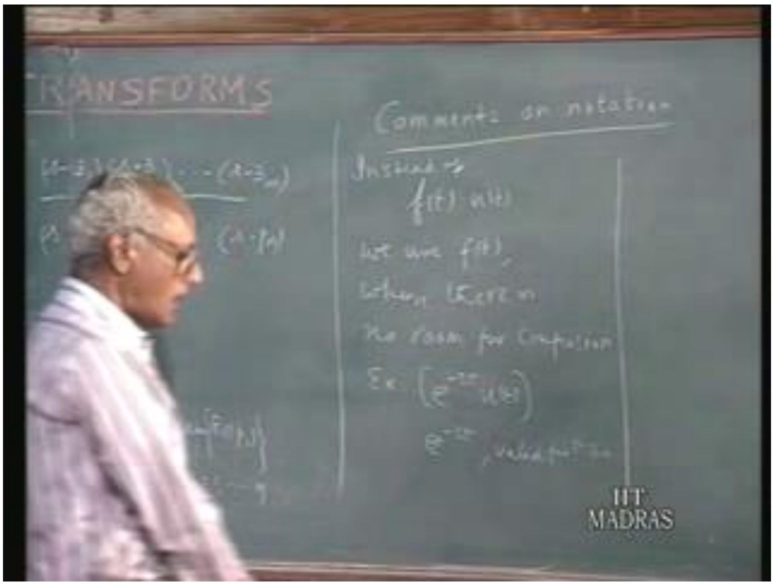

The notation as i mentioned the Laplace transforms exists for unilateral Laplace transform that we are talking about exist only for causal time functions. So, if f off t by definition has the value for negative values of time as well, to make sure that we are talking about the truncated part of that function of t we normally write f off t to ut. To make sure that f off t the composite function is 0 for negative values of time. Often instead of f off t ut we use simply f off t then there is no cause for consumption when there is no room for compusion.

In other words example, suppose e to the power of minus 2t this is f off t that you are talk that you cant find Laplace transform such a function. We have to find the Laplace transform and e to the power of minus 2 t times ut. So, when you are got 1 over x plus 2 the inverse Laplace transform of that e to the power of minus 2t ut. Instead of that we write e to the power of minus 2t itself, with the understanding that we are talking about e to the power of minus 2 t valid for t greater than or equal to 0.

So, as long as we understanding that whatever time function we are having is valid only for t greater than or equal to 0 and it is 0 for negative values for time. Instead for e to the power of minus 2 t ut we offer by e to the power of minus 2 t to save or write. Another point is suppose we have a some constant.

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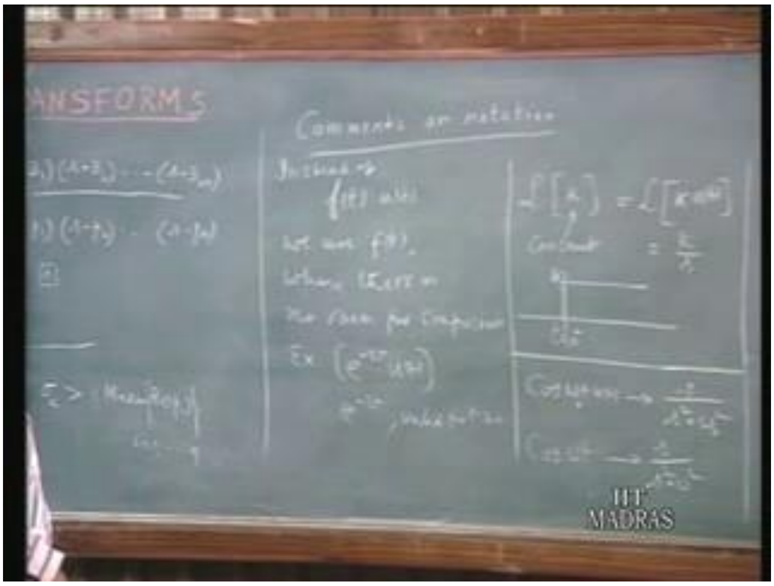

Laplace transform or some constant k again by definition by constant k exists for all values of time, but in the context of Laplace time form. We assume the distance function is the value k starting from t equal 0 minus. Because we can teak only functions which are 0 for negative values of time in the Laplace transformation consideration; therefore, even if you teak the going to Laplace transformation constant. We regard that constant that will have the function will have the value k starting from t equal to 0 minus and 0 for values of t less than this.

So, when we are talking about the Laplace transform the constant. We treat this as laplace transform of k ut at say this is k apart s. So, we do not sometimes exhibits specifically this u off t which understood just to say, a right. Another feature of common notation is you recall that i said cos omega not t ut has Laplace transformation s over s square plus omega not square. We have taken a particular angular frequency omega not and truncate it a t equals for negative values of time and arrive get this Laplace transformation. Now, s is equal to sigma plus j omega. The imaginary part of s is also given the simple omega it is a running variable and to distinguish that running variable omega with this omega.

So, we have given as a special simple as a omega not. But in our manipulation we very rarely decompose s sigma plus j omega looks at the imaginary part of s. So, where there is no confusion we even right simply cos omega t having a Laplace transform. As s square plus omega square as this is return in this manner in this notation we understand that this omega is not to be confused with the imaginery part of the s. This is the omega a particular value of omega associate a trigonometric function this should not be confused with the imaginery component of s. And secondly, as we mentioned earlier actually we are talking about cos omega t ut.

But this additional function u off t is often dropt. Assuming that, we are talking about a function which has this functional notation only t equal to 0 plus onwards 0 minus onwards. So, often instead of righting like this we simply said cos omega t is the laplace transform s over s square plus omega square. These are minor deviations in the notation we often find to simplify the righting of the various expressions this type of simplification is distorted 2. But basically, we must note that whatever function we are talking about is assume to be 0 for values of t less than 0 minus.

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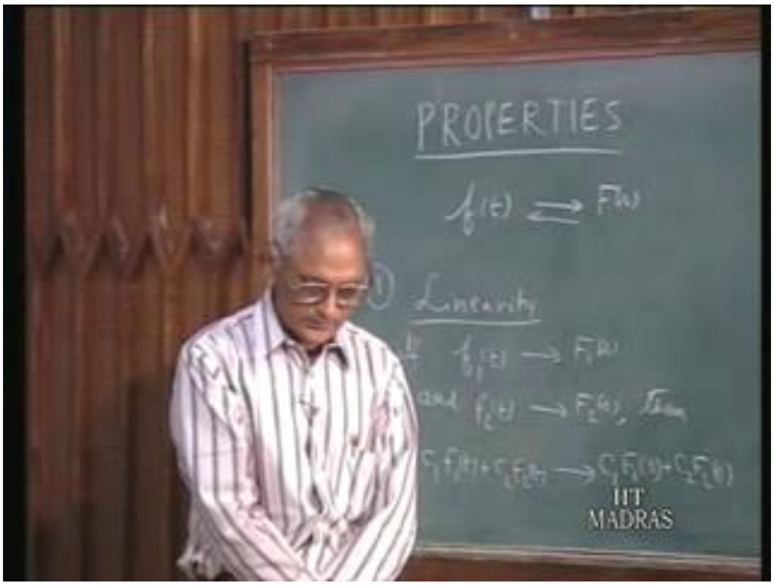

Let us now, take a look at some of the important properties of the Laplace transforms. We use, the notation that f off t has the Laplace transform f off s and f off t inverse the Laplace transform of f off s F off s. Most of the properties that we are going to discuss parallel goes which we have already discussed in the context fourier a transform. So, as we go along, you compare the properties that we are having discussing here with those that we discussed earlier in the context of fourier transform. The more or less are similar except for minor variation which arise as a result to the fact that the Laplace transform concerns itself essentially with causal time functions.

Whereas, the fourier transform deals with functions from which can exists from t equals minus infinite t equals plus infinite. So, basically, that is the difference and that create some differences otherwise the properties are quit similar. First property is something which we already assumed is the linearity property. What we mean is; if f1 off t has the Laplace transform f 1 off s if f1 off t is the Laplace transform f1 s. And f 2t has the Laplace transform, F2 of s then a combination of c1 f 1 off t and c 2 f2 off t where c1 and c2 are arbitrary constants the linear combination of this 2 is having Laplace transform c1 of f1 off s plus c2, f2 off s is nothing much poor about this.

You plug this find out the Laplace transform of this plus this in to the expression for the Laplace transform defining integral break it up the 2 parts and you can show it. There is no difficulty doing this. As a matter of fact where assume this relationship, if you recall when we are trying to find out the Laplace transform cos omega not t. We said cos omega not t is 1 half of the e to the power of j omega t and plus e to the power of minus j omega t and therefore, there we are already made use of this area to property.

So, we read that discuss it any for them this is quit obvious and it can be really demonstrated.

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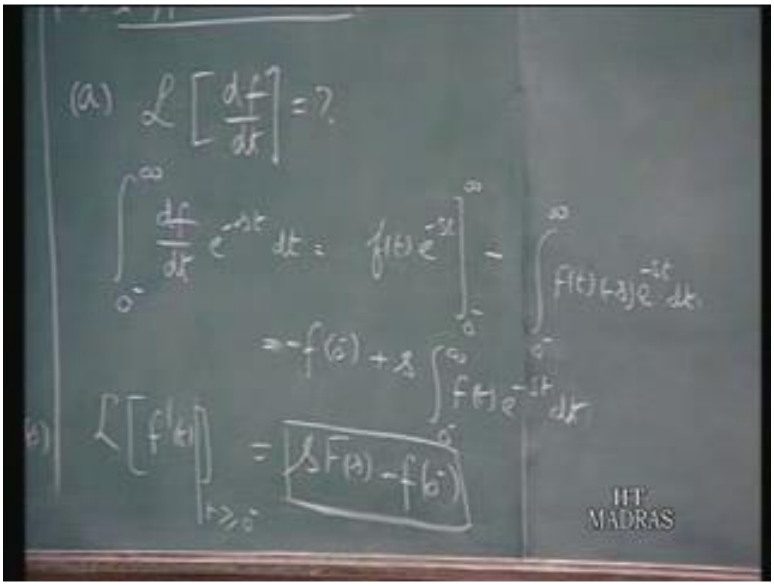

The second property is going to talk about is the differentiation in time domain we ask the question what is the Laplace transform of df by dt? This is the question which we like to also. Given that f off t as the Laplace transform capital f off s you will like to find out what is the differential what is the Laplace transform of df by dt. Assuming that the Laplace transform of the differentiated function exists. Then we set up the differential equation defining the integral 0 minus to infinite of df by dt e to the power of minus st dt could be the Laplace transform of this derivative of this function of time.

Now, this can be integrated by parts can be integrated by parts, you take the integral of df by dt which is f off t e to the power of minus st evaluated with in the limits 0 minus to infinite minus integral from 0 to infinite of f off t and take the derivative of e to the power of st with reference to time.

So, minus s e to the power of minus st dt. So, when we substitute this 2 limits in the first expression in the regional convergence as t goes to infinite limit as s t goes to infinite of f off t e to the power of minus st that goes to 0. That is, the definition of the value of s that we are going to take in the regional converters. So, at the upper limit this is 0 at the lower limit e to the power of minus st becomes 1 and therefore, you have got f off 0 minus. Because it is the lower limit minus of f off 0 minus. And here the integration reference to s; therefore, s times 0 minus to infinite of f off t e to the power of minus st dt. This integral is capital f off s is Laplace transform of f off t itself therefore, this is s times f off s.

So, this will have s times f off s minus f 0 minus. That is, the laplace transform of f off t which is the derivative of f off t. This is s 2 link that f off t talking about we want to take the behavior of the f off t for t greater than are equal to 0 minus; that means, you are taking the derivative of the function of time.

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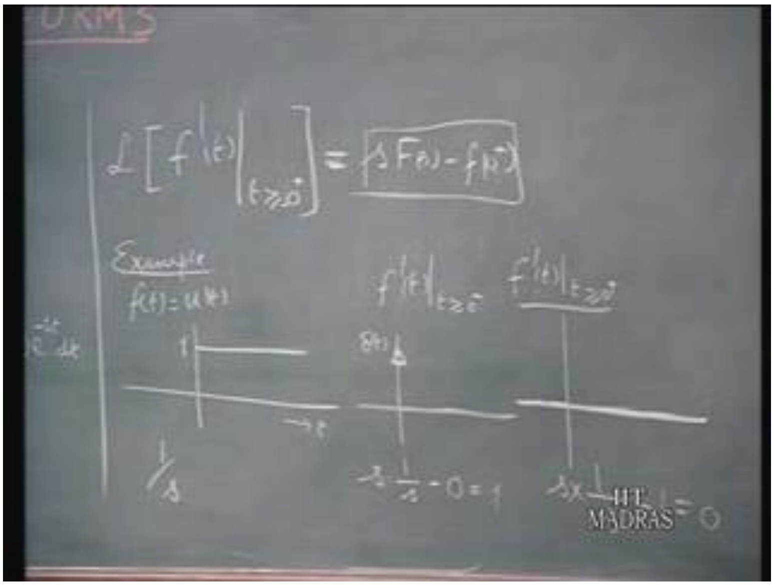

We take into account the variation of f off t from 0 minus onwards often we would like to find out the Laplace transform of f prime t for t greater than or equal to 0 plus; that means, any jump that this function has got in transiting from the 0 minus to 0 plus would like to exclude. That the derivative you want to take only from 0 plus onwards we would like to this will got whatever is happening before that. In such case this will be the laplace transform of that will be s f s minus f 0 plus. That means, you are now taking about the Laplace transformation of the derivative. Starting from 0 plus instead of 0 minus dealings of this will become clear as we take an example.

So, in the literature find the Laplace transform of f prime t is write in this manner or in this manner both are equal both are correct depends up on whether you want to consider the derivative starting from 0 minus or 0 plus. Let be illustrate what we meant by this by example, let us take f off t to be a unit step function of t. So, you have this is f off t. Now let me say, that I want to find out to derivative f prime t for t greater than are equal to 0 minus. So, i would like to take the derivative of this functions starting from 0 minus. So, as take into account the variation of f off t starting from 0 minus at from 0 minus to 0 plus it goes through jump a 1 0 minus to 0 plus.

So, the derivative of this function a t equal to 0 is an impulse. Because we jumping from 0 to 1. So, derivative is infinite and we integrate with impulse we get arise a step 1 that the derivative desired the behavior of the function 0 minus to 0 plus is an impulse. And from t equals 0 plus onwards there is no derivative the 0 because it is a flat function of time. So, f prime of t, t greater than taking into account our variation where t equal 0 minus is in delta function delta t. On the other hand, if you want to consider the derivative only starting from e equals 0 plus onwards. From 0 plus onwards the variation of this function there is no variation; therefore, it is 0 and from all negative values of time we assume the function to a 0 value; therefore, f prime t is identically 0.

So, for this that is the function. Now, let us see the Laplace transform of this we know this 1 over s. The Laplace transform the unit f function is 1 over s. What is the Laplace transform of this if you ask this question f prime of t is greater than equal to 0 minus? You must use this formula s f off s use this formula s f off s minus f 0 minus. This is the formula that we have to use we want to take the derivative that starting from t equal 0 minus; therefore, you write s times 1 over s minus. What is the 0 minus value of this f off t is 0 this is equal to 1 and we know that, the Laplace transform of the unit impulse function which is derivative of this is equal to 1.

On the other hand, if you want to consider the derivative to be 1 which the value from t equal 0 plus onwards. Than we use this form which is s f off s minus f 0 plus. So, that will be s times, the Laplace transform of f off t 1 over s minus f0 plus f0 plus are the unit time function is equal to 1; therefore, we substitute 1 here and that will be 0 and that is should be because our f prime t in this case is identically 0 if Laplace transform as well will be identically 0. So, that is the distinction between this 2 formulas; depending up on how you want to take the derivative whether you want to consider the derivative from t equal 0 minus or 0 plus. So, you can use either of this the difference between this 2 will be the Laplace transform of impulse if there is any existing the jobs. Now, let us now proceed further and consider the second derivative of this.

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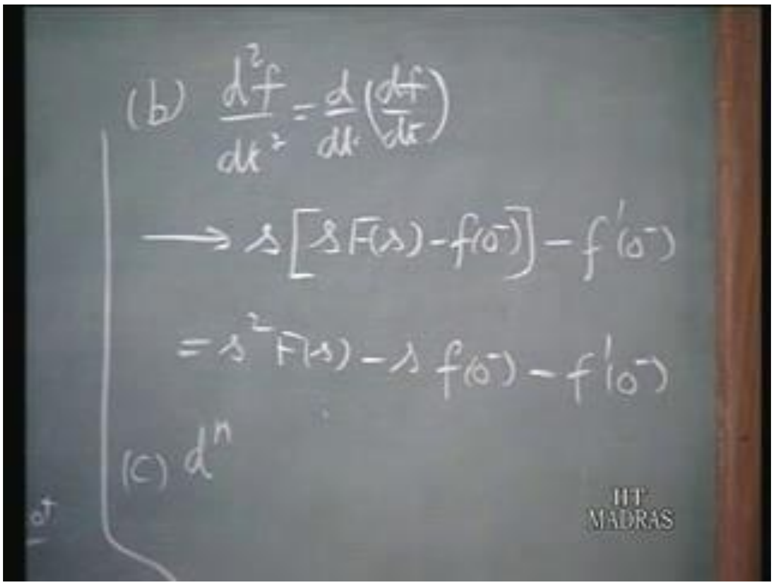

That means; what is the Laplace transform of this d square f dt square. Now, the second derivative d square f dt square required is offteral, the derivative of d by dt off df by dt. Since we know, the Laplace transform of this we have to apply the formula which we already derived. So, this will be s time the Laplace transform of df by dt is s f off s minus f 0 minus i will continue writing 0 minus because that more general, minus the value of the derivative df by dt t equal 0 minus I will write this f prime 0 minus. So, that will be s square f off s minus s f 0 minus f prime 0 minus.

So, the Laplace transform the second derivative of the time function in terms of, the Laplace transform the original time function is given in this and this can be continued.

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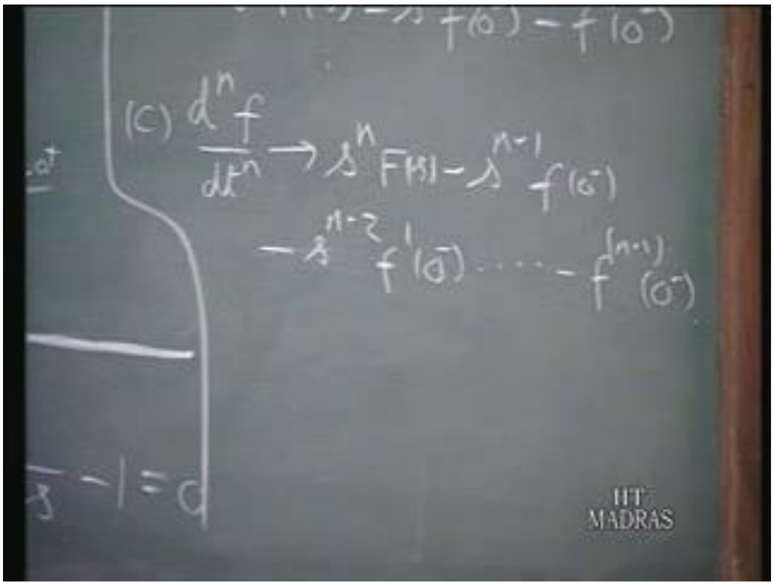

So, a general formula of the Laplace transform of the n’th derivative dn f dt n can be written as, s to the power of n f off s we taking about the n’th derivative of time function. minus s n minus 1 f0 minus like that you take next of the sn minus 2f prime 0 minus down the line to f n minus 1 the derivative evaluated 0 minus that all would be, you don’t have to remember this formula. All we know is, all we should know is the formula for the derivative and then 1 can extrapolate this in this fashion that will be. In fact, you can easily get this by repeating the application of the derivative formula.

We have seen how differentiation in time domain carries over in the transform domain by multiplication of f off s by s and adding on to get the information according the initial value of the quantities being differentiated.

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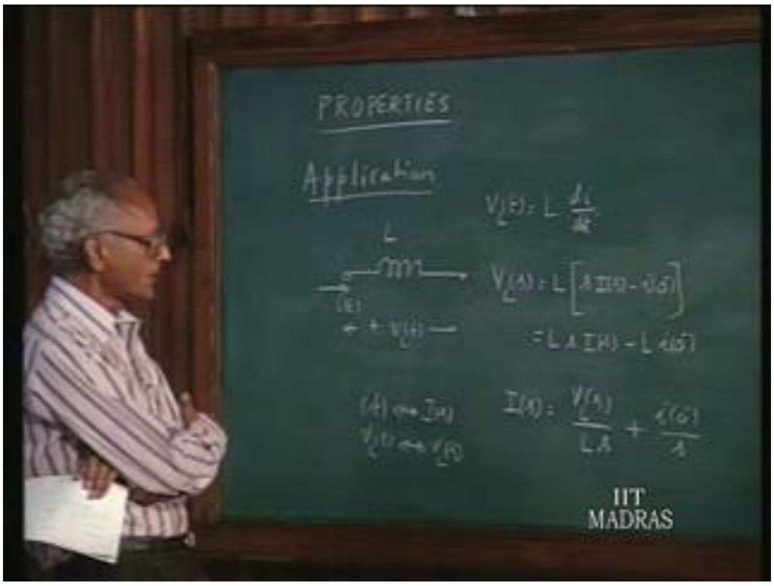

Let us look at the application of this. Application is important in the sense, that the result that are going to get here, will be used r and r again instead froude analysis. Let us consider, an inductor of l henries carrying a current i t that the voltage across the inductance be VL. So, this are the 2 terminals of the inductor and you have the current i t passing through l generating a voltage VL. The fundamental rule relating VL and it domain this is VL off t off course, VL t equals l times di dt.

Now, if you know the Laplace transform for the current i t. Then we should be able to find out the Laplace transform of the voltage VL off t using the differentiation rule that was just now study. So let us, Laplace transform the left hand side and the right hand side assume that i off t has the Laplace transform I off s. So, if you make the Laplace transform for the left hand side and right hand side respectively. And let VL off t have the Laplace transform VL off s then we have VL off s equals l time. The Laplace transform of di up on dt will be s times i off s minus i 0 minus.

We can write this alternately as Ls times i off s minus l i 0 minus. Now, 2 points have emoted. 1 time domain quantities arte given lower case letters VL and i when you make the Laplace transform of that we use the capital letters VL off s and i off s this is the convention. Secondly, we are interesting to finding out VL off t starting from 0 minus onwards. There is the possibility that i becomes to 0 minus to 0 plus. So, if you want to take the derivative of the current including the transition from 0 minus to 0 plus. Then we must take the initial condition here a 0 minus. On the other hand, you are interested in the behavior of the VL off t from 0 plus onwards only and you would like to consider all values of VL from before 0 minus then we have take this for 0 plus. That we are going to illustrate this taking 0 minus values because that is more general situation and optionally you can just take 0 plus as well.

So, this equation tells as that if you know the Laplace transform of current i off s because we calculate VL off s in this manner. But to do this must be the realist value to be current we can inward this expression and find out i off s in terms of VL off s as 1. So, if you do that then I off s can be written as VL off s divided by Ls plus you are going to transfer this to the other side get i 0 minus. So, that gives you an alternative expression relating the terminal current and terminal voltage in the Laplace transform domain i off s is VL off s over Ls plus i 0 minus over s. So, both this equations now are the terminal equations bring that in the transform domain. This the terminal equation bring that in the time domain, this are the equations transform domain both are equal.

Notice 1 thing that, the derivative operator that remove this are purely algebra equations; however, the extra term that you need to have here initial conditions in the invert. Now, provide the initial conditions are 0 suppose i 0 minus is 0. There is the proportional relationship between VL off s and i off s VL off s is Ls i off s i off s is VL off s over Ls; that means, there is a kind of generalized impedance to the current flow in the transform domain. If the current i off s passes through inductor the voltage is Ls times i off s. So, Ls can be regarded at the generalized impedance of the inductor in the transform the way. same thing here also, VL off s over Ls.

So, 1 over ls can be thought of the generalized remittance. So, the induct the impedance concept the proportional relationship between voltage and current whatever domain we are taking about transform domain or j omega domain, sinusoidal circuit take situation for all valid provided you don’t have to the extra term; that means, if there are no initial condition there is a proportional relationship between voltage and current in the transform away. These initial conditions that spoil the correct. But will see how to take care of that relates, but now, this particular equation can be represented by means of circuit representation like this we have an inductance.

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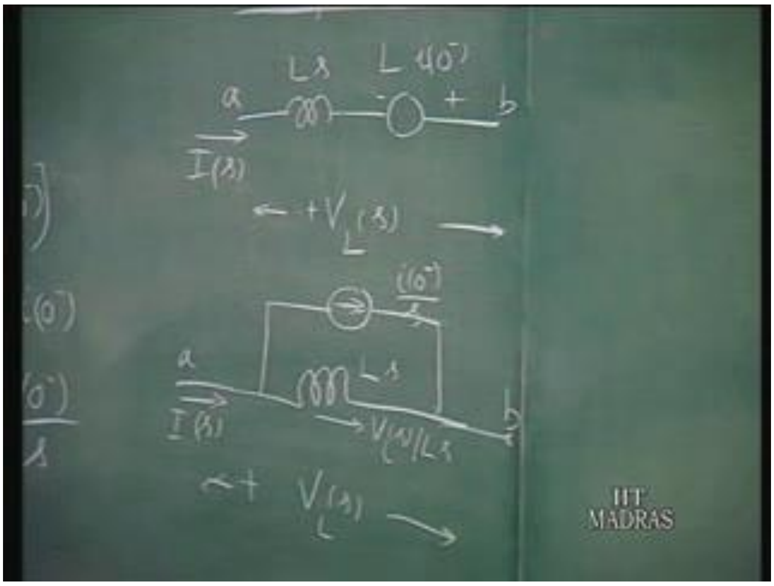

We can think of that having a generalization impedance Ls and we have a current passing through it not the raw current, but the transformed current Laplace transform variable and across the terminals of inductor we must away among voltage of VL off s. So, this is the voltage of the terminals of that we know, that VL off s is the Ls times i off s minus L i 0 minus. So, this current is passing through Ls will developing voltage which is Ls times i off s in transform domain. But to complete the picture we must have traditional term here, minus Li 0 minus and this quantity is independent of i off s.

So, Li 0 minus is independent of i off s therefore, there must be extra voltage here which is independent of i off s that is equivalent to voltage source. So, you must have a voltage source in the transform domain which is L times i 0 minus like this. So, this description on such the sequence that is this circuit VL off s can be thought of as i off s passing through ls developing a voltage Ls times i off s to as minus Li 0 minus because this is the drop followed by arise. Similarly, if you want to model this equation by means; of equivalent circuit representation. We have a element having a generalized impedance Ls with the terminal voltage VL off s.

So, naturally the current here will be VL off s over Ls. But the terminal current is is larger than this by additional value; therefore, you should have a current source which i s i 0 minus over s. So, the sum of this 2 currents will be your terminal current I off s.

So, this than is another equivalent representation of an inductor in the transform domain. We have, a inductor having generalize impedance L s in parallel with a current source of tend i 0 minus or s in the transform domain this the terminal relation. So, you have 2 equivalent circuits for an inductor, which incorporates initial value in the transform domain.

Both this are called transform diagrams, transform diagram for a particular inductor. One incorporates the voltage source the other incorporates a current source. So, depending up on your convenience when a circuit analysis problem. You favor the useable voltage source we use yes and the other hand, if you are a loud with the current source you can use this in particular if you have a network in which the load equations are preferred in whether of solution may be you need to have like have a current source. But both are equivalent the important thing to keep in mind is that if the terminals of the inductor in the actual circuits are a and b here, the equivalent the terminals in the transform diagram are of this a and b.

We should not thing that the terminals of the induct the transform diagram are this 2. Because the totality of the structure here represents the inductor carrying a initial current. So, that is be kept in mind what we have done here is representation of an inductor in the transform domain. Where the variables in the circuit are the transformed variables are the functions of time where this is the Laplace transform of the voltage this is the Laplace transform the current. So, the variables are the transformed variables and the element tend the Ls this is called generalized impedance of an inductor. In the transform domain the generalized impedance of inductor transform domain is Ls.

In a similar fashion, we can give a similar representation of the capacitor with initial charge on that we will postpone a latter time we will not do this in greater detail at this point of time. Just as we have generalized impedance Ls for a inductor for a capacitance out of the generalized impedance is 1 of our cs. We will do that later, but our point in presenting this at this stage is to show an application of the derivative rule in the Laplace transform theory. How you can apply that to evolve circuit models of this time this are called transform diagrams. So, we have a transform diagram of a whole circuit we have similarly generalized impedance for inductance capacitors so on. A whole assembly is for the transform diagram this is the transform diagram, but 1 element which is an inductor.

So, after having consider this who will now go on to the study of the third property of the Laplace transformation multiplication by t.

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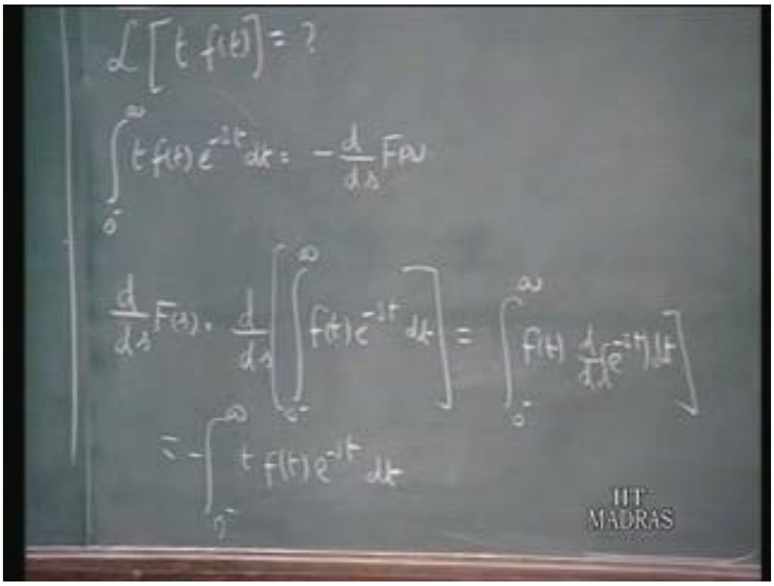

So, we will like ti ask the question what the Laplace transforms of t ft. This is the question that we like to ask. So, the Laplace transform of t ft 0 minus to infinite t ft e to the power of minus st dt, this is what we like to ask. Now, to do this let me take d by ds of f off s. Suppose, I want to take the derivative of f off s which is Laplace transform of f off t this is d by ds off 0 minus to infinite of f off t e to the power of minus st dt this is f. The integration is to the reference to t and we are taking the derivative reference to another variable.

So, we can introduce this inside the integration time and this can be written as 0 minus to infinite of f off t d by ds of e to the power of minus st dt. And this when you take the derivative of this with reference to s that is minus t is sin outside. So, 0 minus t infinite of a input a minus sign because of that, t ft e to the power of minus st dt. And that is exactly what we are having here you wrote minus infinite t ft e to the power of minus st dt. And that integral is equal to minus d by ds of f off s as we can see. So, this integral is equal to minus d by ds of f off t because is a negative sign; therefore, this will be minus d by ds of f off s.

So, when you multiply a function of time by t as for the transform is concerned. It is equivalent to the negative of the derivative of the f off s to s. So, you can see that there is a nice arrangement here, you multiply by t you get minus d by ds this. And the other hand, when you take the derivative of f off t df by dt you are multiplying that s f off s. In the derivative formula you get d by dt of f off t has the Laplace transform s times of s. On the other hand, when you multiply t off t is d by ds of f off s except for the negative sign is somewhat a dual relationship exists.

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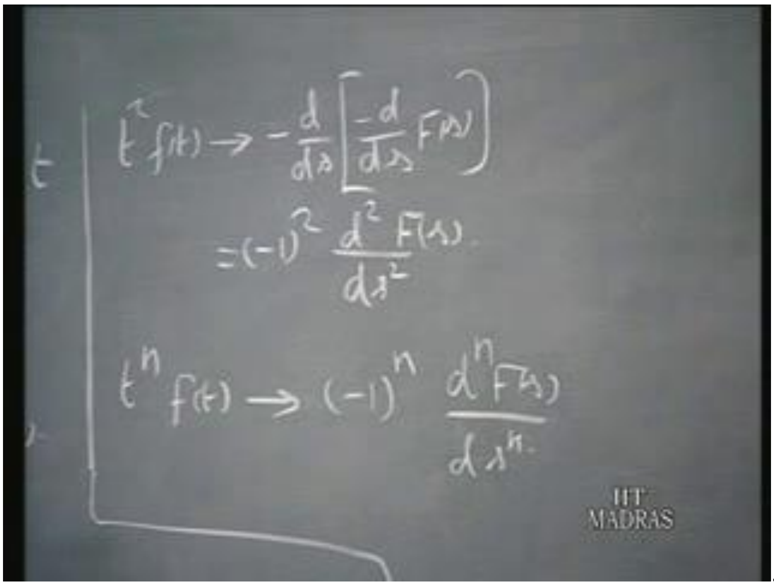

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Now, based upon this let me extent this result. Suppose, I take t square f off t then t f off t is minus d by ds of f off s; therefore, I am straight minus off d by ds off the Laplace transform of t ft which is minus d by ds of f off s. So, this will be minus 1 square d squared by ds square f off s that is all. So, you have tn f off t the terms to be if you take tn you have minus n to the power of n n'th derivative of f off s. All the time we are assuming that the causal functions 1 a verity an f off t we assume that tn f off t will be 0 for negative values of n this is understood.

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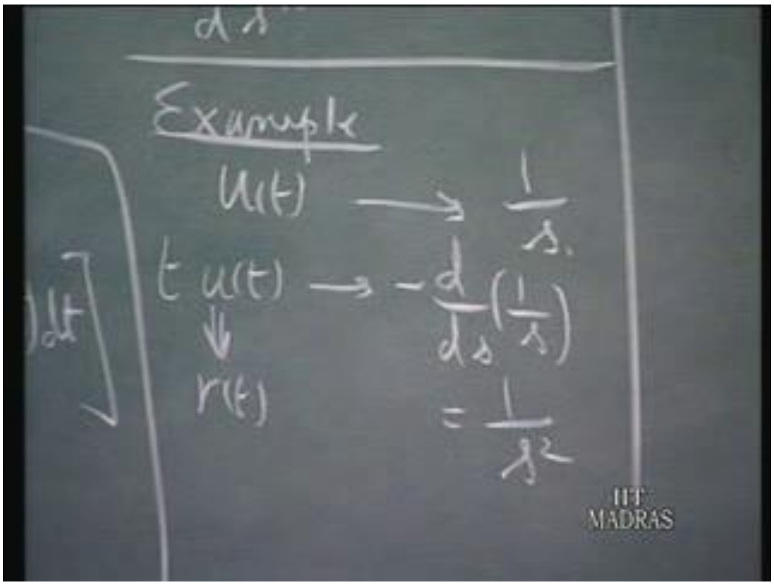

So, let me take simple example. We know that, u off t as the Laplace transform 1 over s. Now what happens, if I take the Laplace transform of t ut. I multiply this by t, then you must take the negative of the derivative of this with reference to s minus d by ds off 1 over s that is 1 over s square and t ut is indeed rt. So, that Laplace transform of a ram function is 1 over s square.

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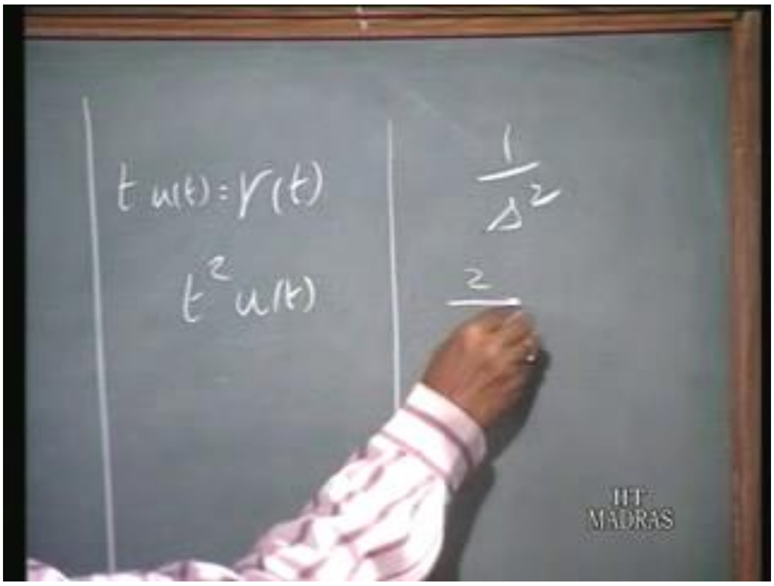

So, we can write the extend over Laplace transform implies the table, we have already consider u off t delta t sin omega not t cos omega not t we can also say Rt Laplace transform is 1 over s square.

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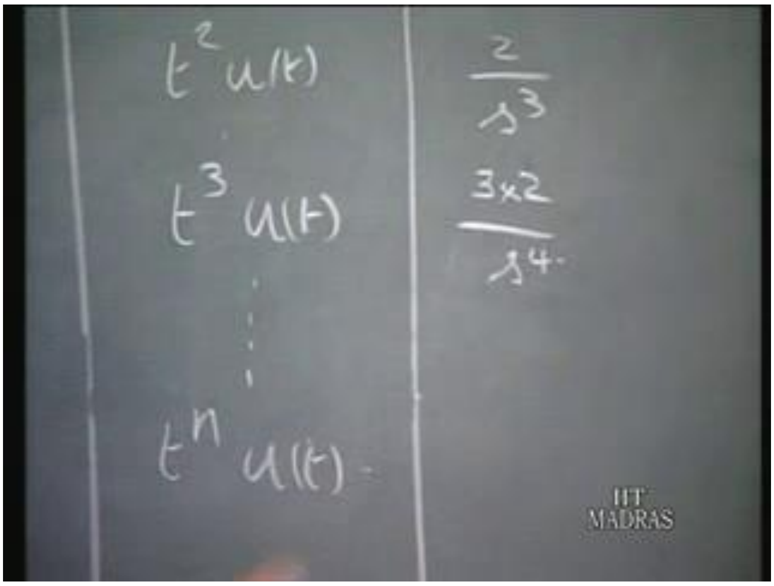

Now, suppose i take t square ut this is t ut off course, t square ut; that means, state this derivative of these ones again and data's are negative sign. So, if you do that it is minus 2 over s cube will be derivative and you attach the negative sign it is a 2 over s cube. Continuing in this fashion suppose i have 2 cube ut then it will be again you take the derivative of this is 3 times 2 divided by s to the power of 4. And if you proceed further in this fashion, if you have tn ut then you have n factorial over s to the power of n plus 1.

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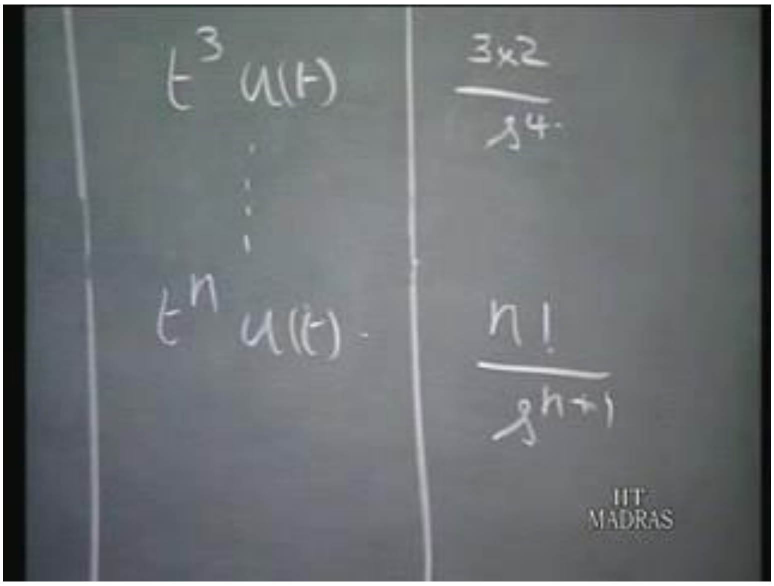

That’s how it goes as we take higher and higher powers of t the Laplace transform goes in this manner. Let us now consider another example, suppose I have e to the power

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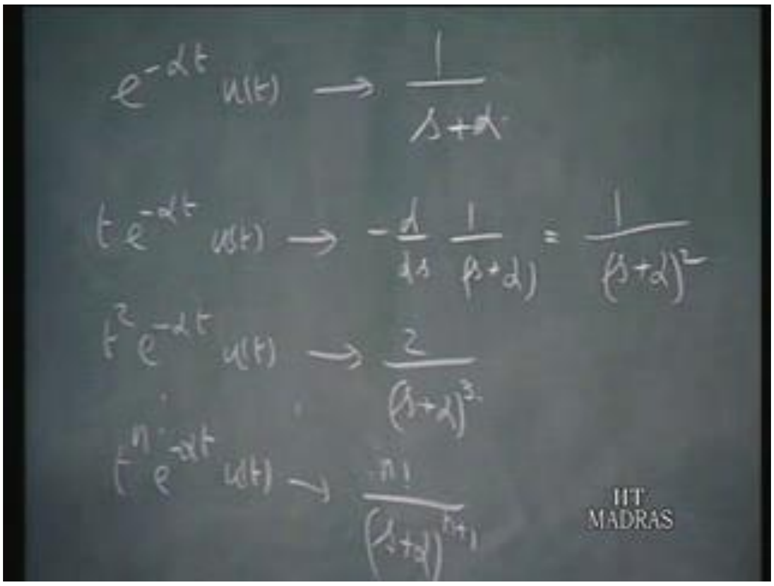

of minus alpha t ut, then we know the Laplace transform is 1 over s plus alpha. Now, i suppose multiply this like t e to the power of minus alpha t ut then its Laplace transform is minus d by ds off 1 over s plus alpha. And that is indeed 1 over s plus alpha Whole Square 1 over s plus alpha Whole Square. And suppose I have t square e to the power of minus alpha t ut than it can be shown this is indeed 2 over s plus alpha to the power of 3.

And in fact, as before you have t to the power of n e to the power of minus alpha t ut. You continue in the same fashion you have n factorial over s plus alpha to the power of n plus 1.

So in other words, in we need not remember the Laplace transforms for t multiplied answers like this either e to the power of minus alpha t or in terms of u off t. It's enough you know, the basic formula for ut then when ever is multiplied by t to the power p square t cube extra than you can easily derive them without any difficult. Let me now take the next property which is integration.

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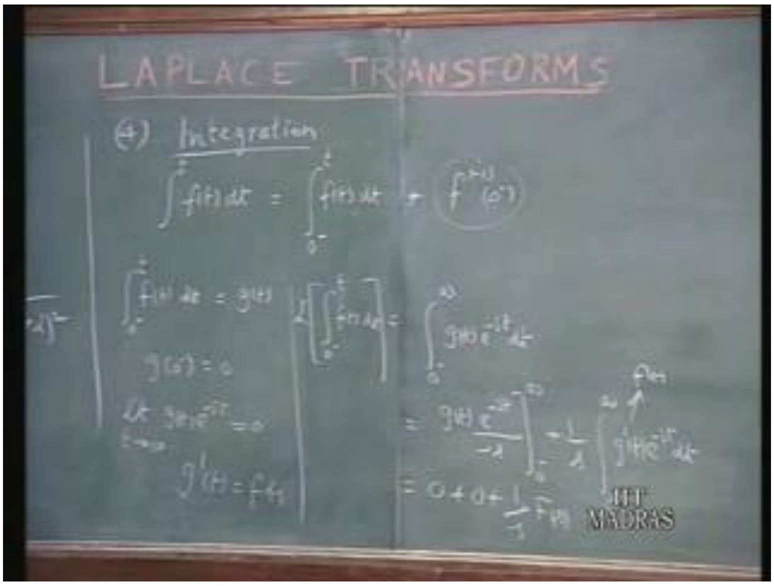

Which is again very important and which commonly occurs. Let me consider, f off t dt in infinite form which we can always regard this as 0 minus to t of f off t dt plus. Suppose, this f off t is a current let us say then what we are doing this by integrating only go for the charge. So, we find it convenient to consider the charge at a function of point at a time t. As the initial charge plus the additional charge that is carried by the current the point 0 minus t the interval 0 minus to t. So, this is the value of integral to limit 0 minus to t to this the initial value of the integral of this write this as f inverse minus 1 with in bacl at 0 minus. That is, this is the initial value of this integral plus the addition to the integral in the value in the interval from 0 minus to t.

So, this is how we can pit the interfered integral in the form of definite integral because they initialize. Now, you not to find out the Laplace transform of that. Let us find out the

Laplace transform of the 2 individual quantities. So, 0 minus to t of f off t dt suppose I called the g off t. Then, we know it straight away that g 0 minus equal to 0 because like in the integral between the limits 0 minus to 0 minus that must specify. And further limit as t has going to infinite of g off t e to the power of minus st equal 0. Because, we are taking the value of g off t to be Laplace transformable and you are taking the value of real part of s is that this tends to 0. That is value of st is the region of convergence of the Laplace transform of g off t.

With these conditions and further we also know that g prime t the derivative of g off t g prime t is f off t because this the integral of this the fundamental role of the integration tells as that the derivative of this is going to be f off t .Using this the Laplace transform ft dt Laplace transform of that equals 0 to infinite of this quantity which is gt e to the power of minus st dt. And that would be obtained as, let us do the integration by the parts. I will take g off t e to the power of st take the integral of this divided by minus s between 0 minus t infinite plus 1 over s 0 minus to infinite of e to the power of minus st g prime t the derivative of g off t times e to the power of minus st dt. And this will be at the upper limit this will be 0 because limit as t tends to infinite of g to the power of minus st this is 0 at the lower limit this is also 0 g or g minus is 0 therefore, this is 0 and you have 1 over s of g prime t is the same as f off t.

Therefore, 1 over s times f off s. So, we have that the Laplace transform of this is a f off s over s and this is a constant f. The initial value of the integral is going to a constant that will be the Laplace transform of this is f initial value divided by s and the Laplace transform of this is f off s power s.

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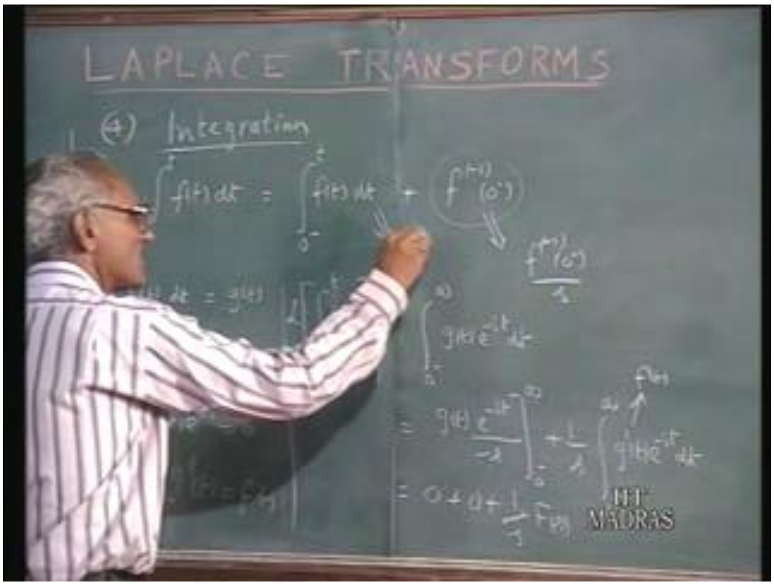

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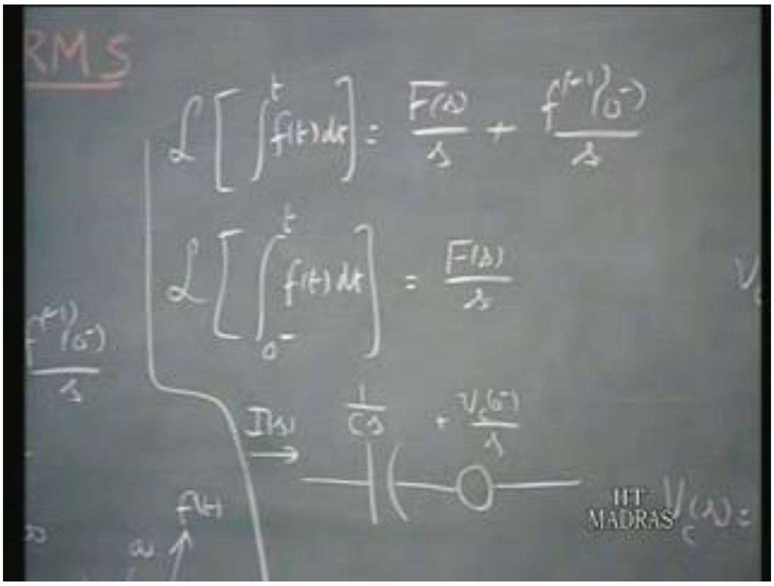

So, we have Laplace transform of the integrate integral f t dt as f off s over s plus the initial value of the integral at 0 minus over s. On the other hand, if you want to take the Laplace transform of the definite integral 0 minus to t of f off t dt; that means, the initial value of the integral is ignored. Where this is simply f off s over s, that this are the 2 important results relating to integral. And we can make use of this, when suppose have a capacitor value c and this is the voltage across the capacitor vct and the current the capacitor is it then we can write vct is 1 over c the integral of the current which is the charge or the capacitor. I can write this further as 1 over c 0 minus to t of i dt plus the initial charge on the capacitor 0 minus.

So, if i take the Laplace transform of the various quantities. I have Vc off s the Laplace transform of the capacitor voltage is 1 over c this is I off s over s plus a constant this is the constant q 0 minus over s. So, i can write this as I as over cs plus q 0 over cs which is I s over cs plus q 0 minus over c is the initial value voltage across the capacitor; therefore, I can write this as Vc 0 minus over s.

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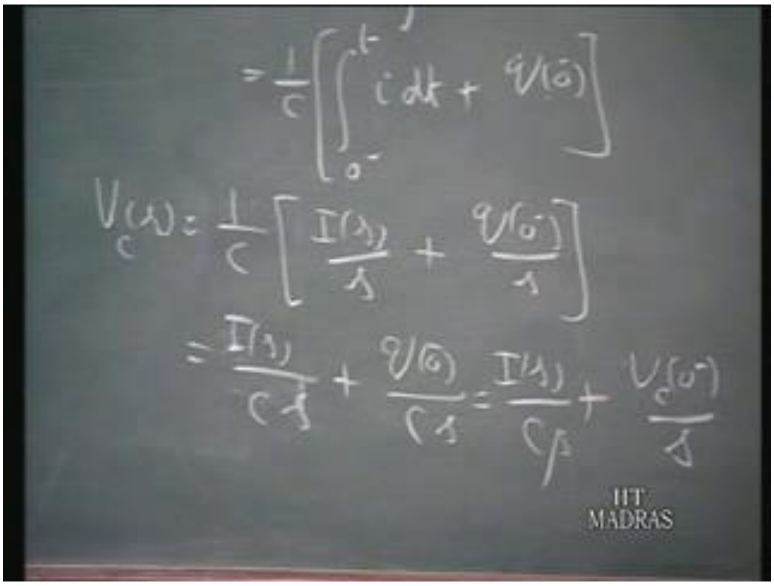

So, this equation can be represented nicely in these fashions. This is the Laplace transform the current this is the laplace transform of the voltage across the capacitor and this is the generalized impedance 1 over cs I s passing through 1 over cs create this voltage in addition, I have the initial value of the capacitor voltage divided by s a voltage source.

So, this is then the transform diagram of the capacitor in the same way as we derived it for the inductor. An equivalent in the term in the parallel form the capacitor parallel with the current source can also be derived, I will not do that it leave it the exercise for you that it can be drawn up in similar fashion as we did in the case of an inductor is very important. Now, if you see that you must distinguish between that 2 formulas and when you have the initial charge in the capacitors we have that is equivalent to a source which is the represents the initial charge in the capacitor or the initial voltage of the capacitor. So, will stop this discussion of the properties of the Laplace transform of this point.

So, in this lecture what we have covered is we started of with revive of the definition of the Laplace transformation and type of functions which are Laplace transformable then we went to discuss the various properties of the Laplace transform in particular what we talked about with linearity property then we talked about the derivative how the derivative time function carry over in the transform domain. Then we discussed how multiplication of f off t by various powers of t how such functions are transformed in the Laplace transform domain.

In particular we said t ft becomes as the Laplace transform minus d by ds of f off s then we went to the integration of the time functions and we distinguish between the definite integral indefinite integral and in both the derivative case and the integral case we saw how they can be applied to the situation of the induct than the capacitor how the relation between the transformed variables of currents and voltages across inductor and capacitors can be represented by means of such a diagrams which we called transform diagrams. We will continue the discussion of the properties of the Laplace transforms in the next lecture.