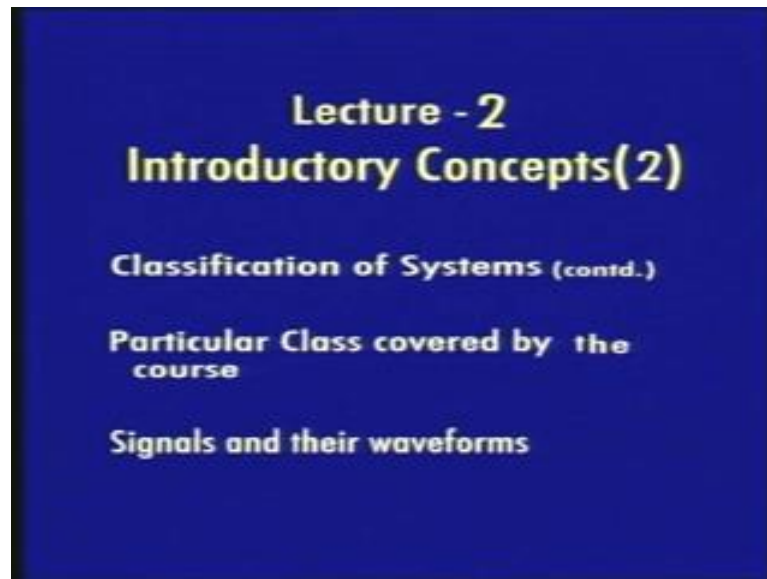


Networks and Systems
Prof V G K Murti
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 2
Introductory Concepts (2)

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In the last lecture we were the considering the classification of systems and networks. We saw the difference between a static systems and a dynamic system the difference between a continuous time system and a discrete time system. We also observed the difference between a linear system and a non-linear. Basically, we said a linear system obeys the principle of super position.

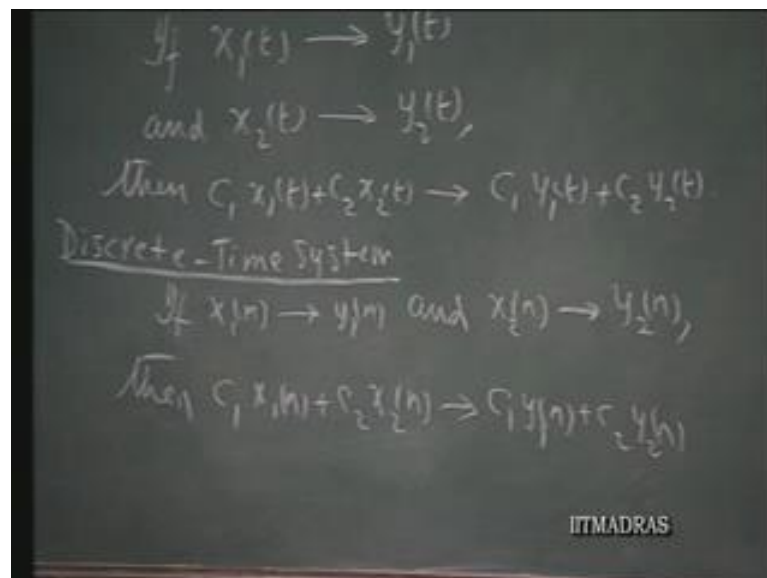
That is a combination of edibility and homogeneity and we promised ourselves at the end of the last lecture. That we look at some examples of the equations governing the performance of a linear system and a non-linear system and this is what we propose to do to start with. So, we recall that we said a linear system satisfies the principle of super position.

If it is a continuous time system then, if x_1 of t as an input gives rise to an output y_1 of t and x_2 of t gives rise to an output y_2 of t . Then, a linear combination of these 2 inputs $c_1 x_1$ of t plus $c_2 x_2$ of t for any arbitrary pair of constants c_1 and c_2 will give rise to an output $c_1 y_1$ of t plus $c_2 y_2$ of t

On the other hand, if we are talking about a discrete time system naturally, now the signals will not be functions of a continuous variable t , but their functions have a discrete variable. Let us, say m ; then the same statement will be carried over in this domain as if $x_1[n]$ gives rise to an output $y_1[n]$ and another arbitrary input $x_2[n]$ gives rise to an output $y_2[n]$.

Then, if the system is linear, if the discrete system is linear; then, c_1 times $x_1[n]$ plus c_2 times $x_2[n]$ will give rise to a similar linear combination of the corresponding outputs. Now, the input, output relations of a continuous time linear system will be described by a differential equation; the linear differential equation and here the corresponding equation will be called a linear difference equation.

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Let us, look at some examples of equations pertaining to linear systems: a 4 d square y dt square plus 3 d y dt plus 3y is 2x plus 5 d x dt. So, x is the input and y is the output all the coefficients of the derivative terms are constant. So, this is certainly a linear differential equation of order 2 with constant coefficients. Therefore, this is an example of an linear the equation corresponding to a linear continuous time system, b y of n plus 2 plus 3y of n plus 1 plus 3y n let us, say is 6x n .

This is an example of the system the equation characterizing a discrete time system which is linear. Again the coefficients or the various output terms and the input terms are constants and therefore, this is again a linear difference equation with constant

coefficients. So, this is a characteristic of a linear discrete time system. Let us, take another example $4t \frac{dy}{dt} + 2t^2 y = 6x + 5 \frac{dx}{dt}$. Now, here we have the coefficients of the various derivative terms as the functions of time. $2t^2$ and so on this also, is an equation pertaining to a linear system. I can easily manipulate these equations find out the y_1 which satisfies this or a given input x_1 , a y_2 which satisfies this for a given input x_2 and combine these 2 to show that if x_1 of t gives rise to y_1 x_2 of t gives rise to y_2 of t .

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The image shows a chalkboard with the word "LINEAR" written at the top. Below it are three equations labeled (a), (b), and (c):

$$(a) \quad 4 \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 3y = 2x + 5 \frac{dx}{dt}$$

$$(b) \quad y(n+2) + 3y(n+1) + 3y(n) = 6x(n)$$

$$(c) \quad 4t \frac{dy}{dt} + 2t^2 y = 6x + 5 \frac{dx}{dt}$$

The IITMADRAS logo is visible in the bottom right corner of the chalkboard image.

This condition is satisfied even here in other words, if you have a linear system it does not mean; that the differential equations should have only constant coefficients even, if the coefficients are functions of the independent variables. Which is t in this case even then it corresponds to a linear system. So, this is also an example of a linear system. a corresponding equation for a discrete time system would be something like this: $n y_{n+1} + 2 y_n = 5x_n + 1 - 6x_{n+1}$.

So, you see here the coefficients are not constants, but functions of n . In this location you have got n times y_{n+1} . So, this corresponds to the independent variable here is t the independent variable here is n . Therefore, if the coefficients are functions of the independent variable still it is a linear system; it satisfies this kind of relationship. And therefore, these are examples of system equations which pertain to linear systems.

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(a) $4 \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 3y = 2x + 5 \frac{dx}{dt}$

(b) $y(n+2) + 3y(n+1) + 3y(n) = 6x(n)$

(c) $4t \frac{dy}{dt} + 2t^2 y = 6x + 5 \frac{dx}{dt}$

(d) $n y(n+1) + 2y(n) = 5x(n+1) - 6n x(n)$

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Suppose, I take a non-linear system then, the coefficients are not necessarily constants or functions of the independent variable the characteristic of the differential equation corresponding to a non-linear system is that, the coefficients are functions of the dependent variable. Examples: $4y \, dy \, by \, dt + 6y = 2x + 3x^2$. So, you see that y is multiplying $dy \, by \, dt$ and also you have a x square term; here, and these are the 2 factors which destroy the linearity of this equation.

Similarly, now $y^2 \, n + 2$ suppose I have $y \, n + 1 + 3y \, n = 6x \, of \, n$; $y \, n + 2$ is multiplied by $y \, n + 2$. So, you can think of this the coefficient being $y \, n + 2$ multiplying $y \, n + 2$. So, this is a square term that is involved here. So, this fails to be a linear difference equation.

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Non linear

$$4y \frac{dy}{dx} + 6y = 2x + 3x^2$$
$$y^2(n+2) + y(n+1) + 3y(n) = 6x(n)$$

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So, essentially to summarize what we have is in the case of a linear system whether, it is the differential equation or the difference equation the coefficients of the various terms, the dependent variable or its derivative or its incremented terms like this, should be either constants or functions of the independent variable t or n as the case may be. But if the coefficients turn out to be functions of the dependent variable. Then, it fails to be linear it belongs to the category of non-linear difference or differential equations.

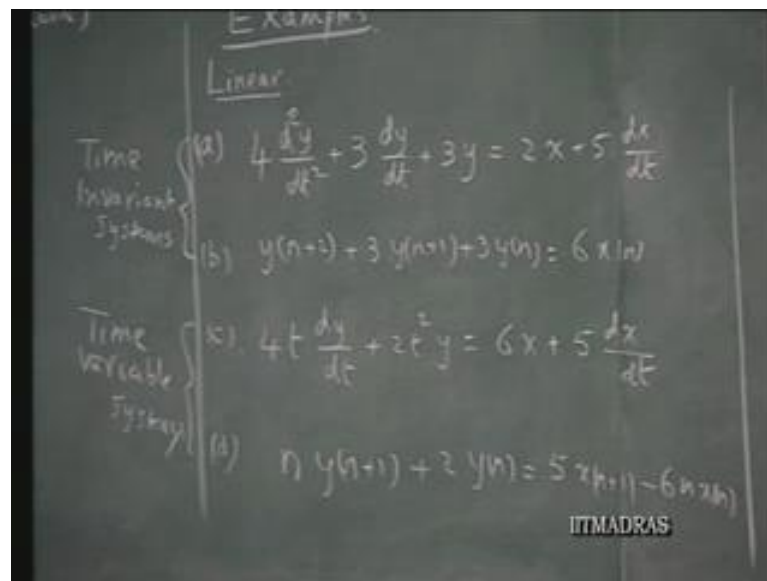
So, we will leave it at that. The next category of the classification that we will talk about is the difference between time invariant systems and time variable systems. So, we have the classification constant parameter versus variable parameter Systems. Another name for constant parameter system is time invariant system. These 2 are equivalent in other words, the parameters which characterize the system the parameters and the various components which constitute the system are constant with respect to time.

Take the case of an electrical network, if r and c are fixed with respect to time; with respect to time it is called a constant parameter system and on the other hand, suppose you have a device in which an inductance and resistance is continuously changing with respect to time ; then, it becomes a time variable system. For example: if i have a carbon microphone and depending upon the input signal that you are having the resistance of the microphone may be changing.

So, that becomes a variable parameter system or a time variable system. So, variable parameter system is also sometimes referred to as time variable system. Generally, we will be interested in talking about time variable system or time invariant system with respect to linear systems. That is because, that is the main focus of our work. So, if you look at these 2 equations here the 4 equations here a and b have the derivative terms with constant coefficients and this is characteristic of a time invariant system or constant parameter system.

On the other hand, in these 2 equations we have coefficients functions of time or n as the case may be, these are time variable systems. These describe the operations of sometime variable systems. So, both are linear, but 1 these first 2 equations belong to the category of time invariant systems. The latter 2 belong to the category of time variable systems. The importance of distinguishing between these 2 classes of systems 1st of all the solutions of these equations is much more simpler than the solution of these 2 equations

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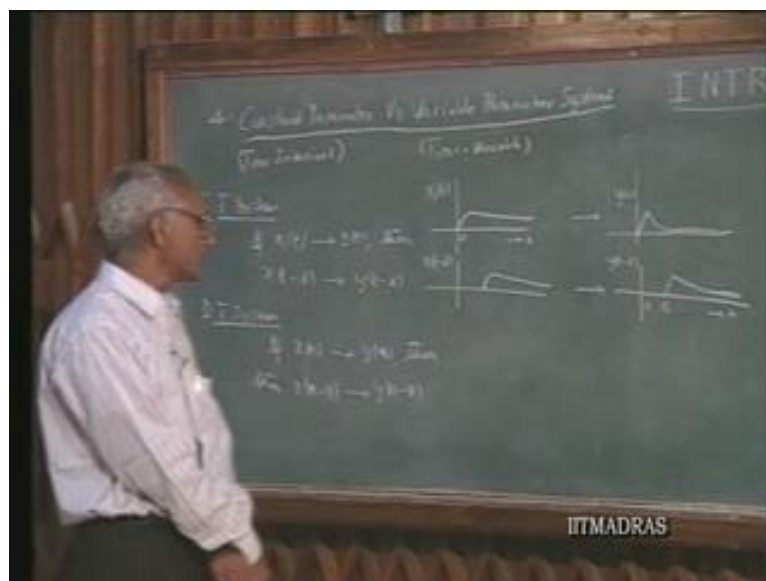
Because, the time factor t is involved here and further there is a very useful property of time invariant systems. Which i will discuss presently and that makes you that makes analysis of our systems far simpler than, what it would be in the case of time variable systems; it goes like this. If you take a continuous time system, if you have an input say x t which gives rise to an output y t.

Then, suppose this x of t is delayed by or translated in time by a certain interval say x of t minus d then, such an input will give a response which is similarly delayed in time by d units of time, no matter what t you take? For discrete time system you have if x of n gives rise to an output y of n and if you consider another input signal which is the same as x of n , but delayed by d or k instance or n instance n minus N , where n is a fixed number.

So, whatever is occurring at n equals 1 now is occurring at n plus 1 units then, the corresponding output will be y of n minus N . So, that means; the response will be shifted by the same amount, but the shape of the response will not be changing at all. Let me, illustrate this by means of some figures; suppose I have x of t and this gives rise to a response y of t like this. Then, suppose I delay this signal by d units. So, instead of 0 it starts here.

So, this is x of t minus d then if it is a time invariant system, we are sure that the output will be the same as this, but starts a little later this is y of t minus d . The same response, same wave shape except that it is delayed by the same amount as the input is delayed. So, this is a feature of time invariant systems and this is a very useful feature as we would see later on. When, we talk about how you use impulse response to characterize the find out the input for find out the response for general input.

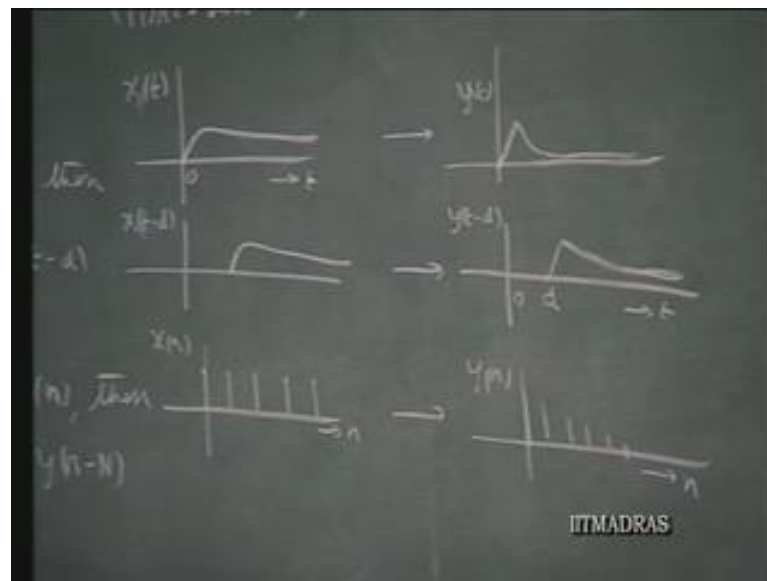
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So, in the case of a discrete time system let us, say we have a x of n a signal like this; x of n is a function of discrete variable n . And let us, say this gives rise to a response y of n like

this. Of course, discrete time signals will have values only at discrete points along the time axis and so, the interval between 2 points may be any arbitrary value depending upon the system. So, let us say for integral values of n you have sequence of values x_n and the corresponding values of y_n . Now, suppose i delay this signal by units. So, that means; it starts here. So, instead of 0 it starts at n that is your x_n otherwise, it is the same.

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Then, the corresponding y of n will be starts not at 0, but at point n here and then, it is as if this whole output is translated along the x axis by n units. So, that is how the response will be occurring for a shifted input signal. This particularly, as you can see this particular property will be very useful in the case of when you want to superpose the responses due to different inputs as we will see later, this is all for the time invariant.

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In the time variable case we cannot have this kind of property. Because, as you can appreciate when this x of t is applied to the system the parameters have certain values and therefore, there is certain response. But if you accept t has been delayed by this amount and we are talking about the application of these at time t equals d . At this time the parameters of the system might have undergone some change.

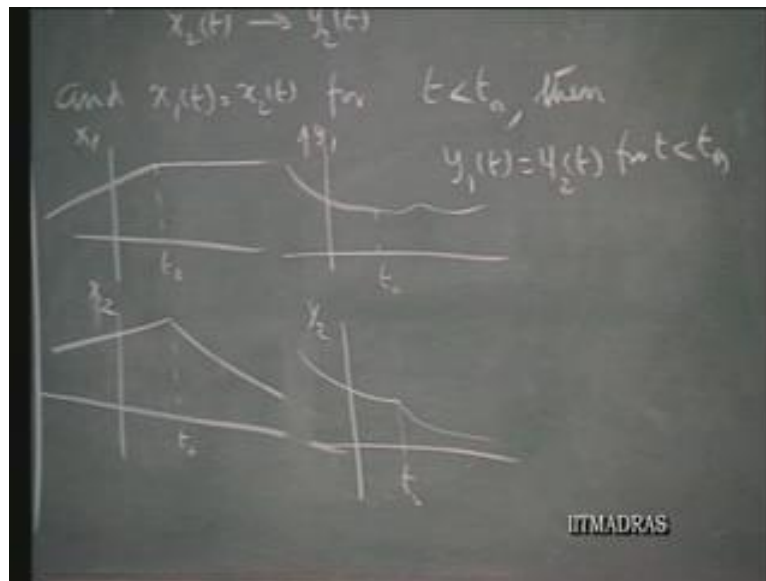
They are no longer the same values that are existing at this point. Therefore, even though the input has the same shape; the output need not follow the same shape. And that is the reason why this such property will not be valid for a time invariant system; time variable system. Time variable system, if X t gives rise to y t then, X t minus d not necessarily gives rise to y of t minus d .

That means; it is not necessary that this gives rise to that particular response. So, you cannot say anything about it unless you know the actual behavior of the variables with respect to time. The next property that we will talk about next classification of systems is: causal systems and those which are not causal called non causal systems. A causal system is 1 which is defined as follows:

If you have 2 inputs x_1 t which gives rise to a response y_1 t and x_2 t which gives rise to a response y_2 t and further. Let us, say x_1 t equals x_2 t for t less than some point t naught. That means: up to some point t equals t naught both our inputs are the same. So, you may have x_1 of t up to point t naught and after wards, may be it goes like this. And you have

another x_2 which also follows the same waveform up to point t_{naught} , but afterwards may be deviates from the values which x_1 had taken. Then, if this is so, then for a causal system we can expect that $y_1(t) = y_2(t)$ for $t < t_{naught}$. So, the corresponding outputs here, if y_1 had some output like these up to t_{naught} and afterwards may be it goes like this. We can say that y_2 also we have the same response may be from here onwards. That mean, up to the point t_{naught} the same input the inputs are the same, the outputs will be the same.

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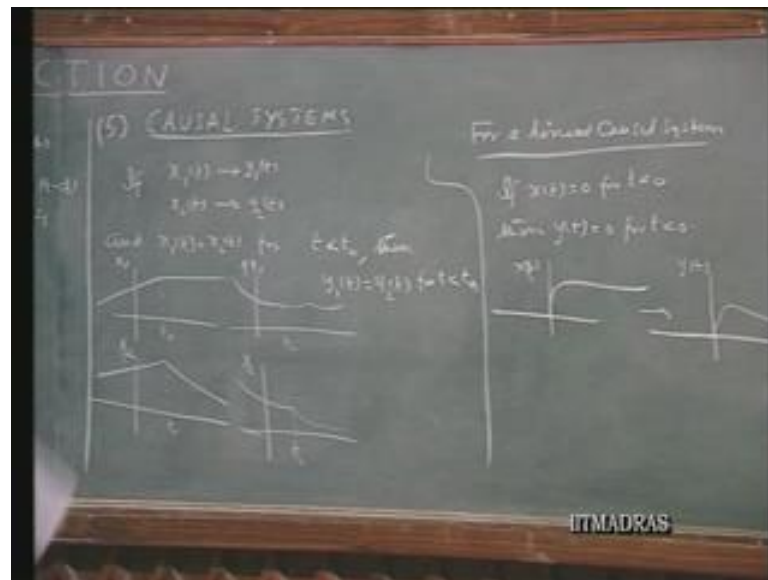


Such systems are said to be causal systems now, if this is a general definition of a causal system. In particular for a linear causal system if the systems is not only causal, but also linear. Then, you can easily see if i have an input which is x_1 minus x_2 which means up to time t_{naught} x_1 minus x_2 is 0. Then, the output must also be 0 because, i superpose these 2 this also will be 0. Or we can take t_{naught} as equal to 0 as a general case we can say if $x(t) = 0$ for $t < 0$ then the output $y(t)$ is 0.

That's the consequence of a linear causal system. That means, whatever input you had if it is 0 up to this point and this is you $x(t)$ your output must be 0 up to this point. And later on whatever, output you get you have it, but it must be less than 0 for $t < 0$. That means, before you apply the input you cannot get an output. Which is seems to be quite reasonable for physical arguments and therefore, we believe that all physical systems follow this causality principle. And a system which is not causal is referred to as non

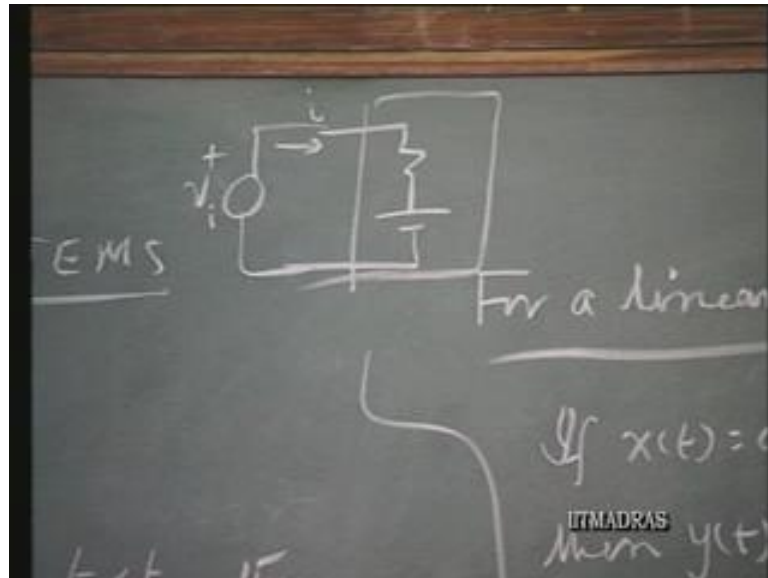
causal or anticipatory systems. This is the opposite of the causal system; we believe as i said all physical systems that we can build follow the principle of causality. Very often 1 describes causality principle in these terms. But that is not quite correct in the sense, if i have a system like this and i is the response and your v i is the input.

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So, if v i is the input and i is the response even if v i is 0 you are having a kind of current there i in that circuit. Therefore, we cannot say that this particular system follows this: if x of t is the input in vi voltage and the current is the y t i will exist even if y v i t is 0. Therefore, if you start a signal v i of t from t equals 0 onwards, it does not mean that the current will be 0.

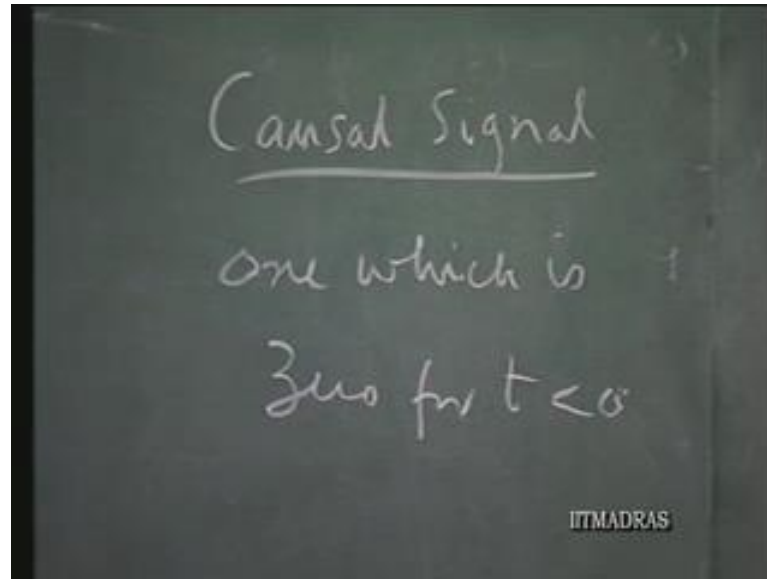
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Because, the current will be driven by the belt, but this is not a linear system. Because of the presence of the source you do not have a proportional relation between b and i . That is not a linear system the systems part is linear, but the source destroys the linearity of this. So, this particular property is not valid for this. But on the other hand, this property will be valid for this system. If you have 2 voltages v_{i1} and v_{i2} therefore, it gives the same response as long as: v_{i1} is equal to v_{i2} up to t equals t_{naught} .

So, this is a more general definition of causality and for a particular special case of linear causal system we can simply say if the input is 0 for time t less than 0 the output will likewise be 0 for time t less than 0. Now, another terminology that we can introduce at this stage it is convenient to use such a term causal signal is 1 which is 0 for t less than 0.

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So, a causal signal is 1 which is normally defined as 1 which is 0 for t less than 0. That means: this x of t is a causal signal it is 0 for t less than 0. So, 1 can say that the property is that if a linear causal system as a causal signal as the input, the output will also be a causal signal. Because, when we talk about transients and linear systems and so on. It would be nice to have a term which describes all those signals which are 0 for t less than 0.

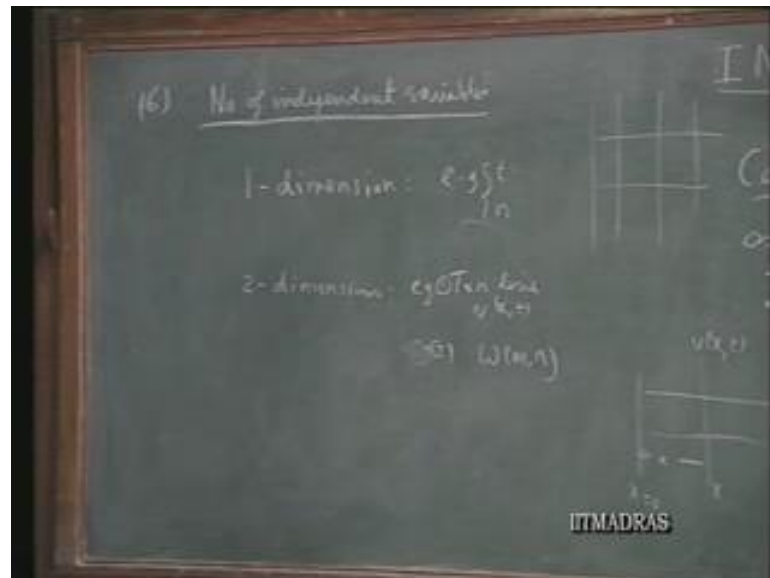
We could call such signals causal signal. So, we can say a causal signal given as an input to linear causal system will produce a response which is also a causal signal. We will have 1 more property classification which we will discuss that is, the number of independent variable systems can be classified depending upon the number of independent variables that we have to reckon with in the system. And this number of independent variables is also referred to often as the dimension of the system.

So, 1 dimensional systems the transients in electrical circuit are 1 dimensional systems because, t is the independent variable. For example: t or n in the case of a discrete time system. Let us, on the other hand think of the transient that arrives at transmission line: we have a transmission line and therefore, at any point x you have the voltage between these 2 is a function of x and the time t .

So, x is 0 at starting let us say therefore, the voltage here depends upon 2 independent quantities x and t . So, along the line the voltage will change and not only with distance,

but also with respect to time. So, you may have 2 dimensions example: transmission line where the currents and voltages are functions of x and t . Similarly, you may have a grid like, structure and you are describing some parameter here as a function of the x coordinate and the y coordinate. Therefore, some quantity w which is a function of the m and n 2 dimensions.

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These are called 2 dimensions next course, we can continue with other dimensions also. For example: you have a field problem electromagnetic field or electro static field. So, it is a function of x y z and perhaps it is a dynamic you also have a function of time. So, it may be 4 dimensions and when you deal with such multi-dimensional problems in the case of a continuous system.

You have partial differential equations to contend with and in the case of a discrete time system you have a multi-dimensional discrete difference equations that you have to deal with. So, after having looked at the classification of various systems who would now like to say, as far as this course is concerned we will be talking about linear time invariant systems which are causal of course, and which are 1 dimensional.

Because, we are talking about only 1 variable at a time t or n as the case may be and however, we will- and also dynamic, linear time invariant, causal 1 dimensional systems.

This is the main focus of our course. We will also have we will be talking both about continuous time systems as well as discrete time systems. Perhaps for 70 percent of the course material we have to do with continuous time systems and 30 percent with the discrete time systems which we will take up at the end of this course. Even though, we are talking about systems in general.

We would be taking specific examples from electrical networks, as examples of systems and the variables therefore, will be voltages and currents and the independent variable will be time. In the matter of analysis of dynamic systems electrical engineers have all had always head start over others because, they developed powerful tool for the analysis of linear systems under various excitation patterns.

The impedance concept, the Phasor notation and the early application of operational methods for dealing with dynamic systems are all due to electrical engineers. And further, more electrical engineers had also an access to very sophisticated experimental techniques or the measurements of the various parameters under dynamic conditions, high speed recording and wave form observation, not only for verifying the results from theory, but also to gather data which will be useful to supplement the theory.

With the result that the methods developed by for the solution of electrical networks can be profitably employed by for the solution of other class of networks as well. So, we often find that systems and networks of other kinds like, mechanical systems or mechanical networks, acoustical networks or acoustical systems, hydraulic systems and so on.

Or usually, are sometimes modeled in terms of electrical circuits the analogous electrical circuit for a given mechanical system set up and you analyze this electrical system with all the gamut or the techniques that are available for the electrical circuit analysis. And once, you have that you translate the results back to the original domain interpret the results suitably and then, you get the solution for the non electrical system.

For experimentation also this is convenient so, instead of having to do with large masses, springs and dash box and so on. You can set up a r l c circuit which is a replica which simulates the actual mechanical system and carry out all those experimentation in terms of voltages and currents which are easy to measure in the lab. And then, interpret the results suitably with the result domain.

So, when we electrical networks and the dynamic performance of the electrical networks in this course. We have the assurance that whatever, techniques we employ and whatever methods we use here can also, be profitably employed for other kinds of networks and other kinds of systems. With this we close our discussion of the introductory remarks for this course. Next, we will take up the consideration of the different signals that we come across in our discussion of linear systems and networks.

Literally, a signal is a means of conveying some information, but in the context of systems. We take the meaning of a signal to collectively, indicate the various variables, which describe the status of the system at any particular point in the system or at any particular point of time. As far as, electrical networks are concerned the variables or the signals that we deal with are the voltages and currents. It is the signals as, i mentioned earlier which show to say give the breadth of life to a system or a network.

Because, in the absence of signals, the network or system is completely lifeless. And therefore, an analysis of signals and knowledge of the various kinds of signals that 1 comes across is important for us. And when, we are dealing with dynamic sub systems and networks the signals are functions of time. We would have naturally, an infinite variety of signals possible which are functions of time; however, a few special kinds of signals are important for us because, of their simplicity for 1 thing

Later on, we will also argue that any composite signal, any general signal can be decomposed in terms of this elementary simple signals. And therefore, a study of these of these signals is: of the simple signals in important for us and we will see we will 1st of all review some of the signals which are familiar to us already and introduce new types of signals, which will be found useful in our study. Let us, now talk about some various types of signals, elementary signals that we are interested in: 1 d c signal.

So, f of t is a constant a is independent of time. So, nothing further needs to be said about this so, we will leave it. Like that, then we have a c signal, what we mean by that is a sinusoidal signal. So, a general form of the sinusoid f of t root $a \cos \omega t$ plus θ and the whole a c circuit analysis is based upon signals of this type and you know, the importance of sinusoids in circuit analysis. Because, this sinusoid has got a very distinctive property.

This is the only periodic signal which retains its shape, which retains its general wave form under the linear operations of addition of 2 sine waves of the same frequency subtraction of 1 sine wave from another the same frequency of course, differentiation of a sine wave will lead to another sine wave of the same frequency. Integration of a sinusoid will lead to another sinusoid of the same frequency.

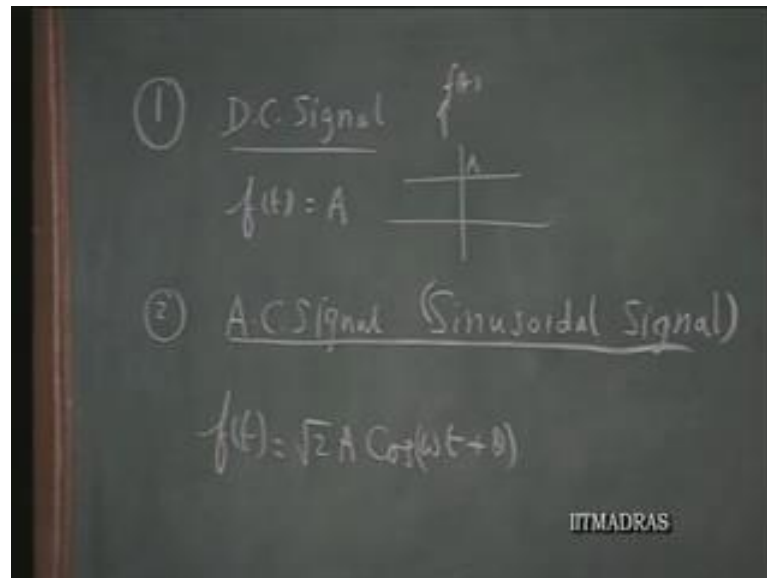
So, under the linear operations of addition and subtraction, differentiation and integration the sinusoid retains its character, the same wave shaped tails is retained. So, other periodic function has got this wonderful property. The result therefore is that, if you have a whole system with sinusoidal sources of the same frequency distributed all over. Then, when a sinusoidal current passes through an inductor it produces a voltage which is sinusoid of the same frequency.

When, a sinusoidal voltage is applied across a capacitor it produces a current a sinusoidal same frequency. Therefore all these currents and voltages in the entire system as for example: in the power system where we have got different sources at the same frequency all currents and voltages ideally in every point of the system are sinusoidal at the same frequency.

If we did have this wonderful property then, a sinusoidal current in an inductor will produce a non sinusoidal some kind of if you have non sinusoidal current in an inductor you will have a waveform of the voltage which is completely different. And if we have all this different types of waveforms mingling with each other in a complicated system we will certainly it will drive 1 to insanity if 1 has to analyze this because, the waveforms are so unpredictable and so complex.

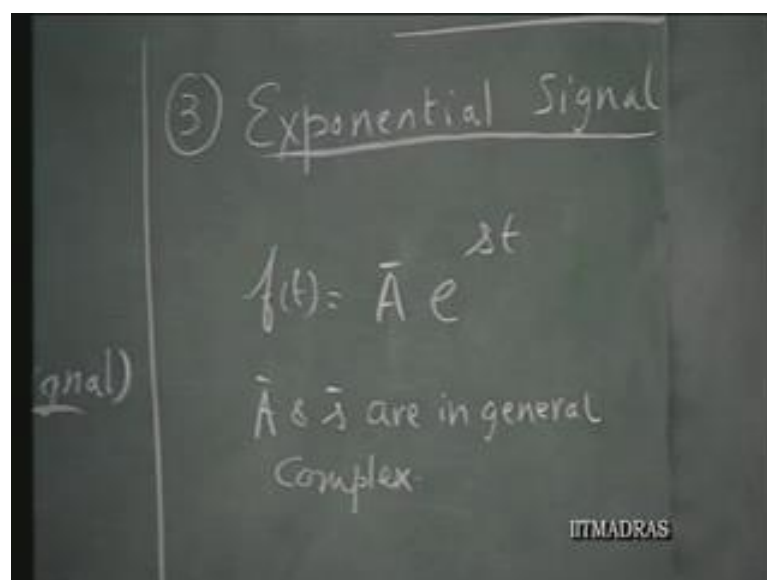
So, sinusoid has got a wonderful property and it stands out as I said this is the only periodic signal which has got this kind of property. And we have Phasor notation, which deal with sinusoidal signals analysis of systems driven by sinusoidal systems. So, we know all about this from the a c circuit analysis. So, we will leave it at that except to remark at this stage, that this is really a very wonderful signal.

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Then, we have the exponential signal. So, the general form of an exponential signal would be $f(t) = A e^{st}$ where A is a complex number and s is also, could be in general complex. Now, mathematically therefore when you substitute a value of t here. Because, s is complex and the coefficient A is complex, you get a complex value for this. So, this is a complex exponential signal.

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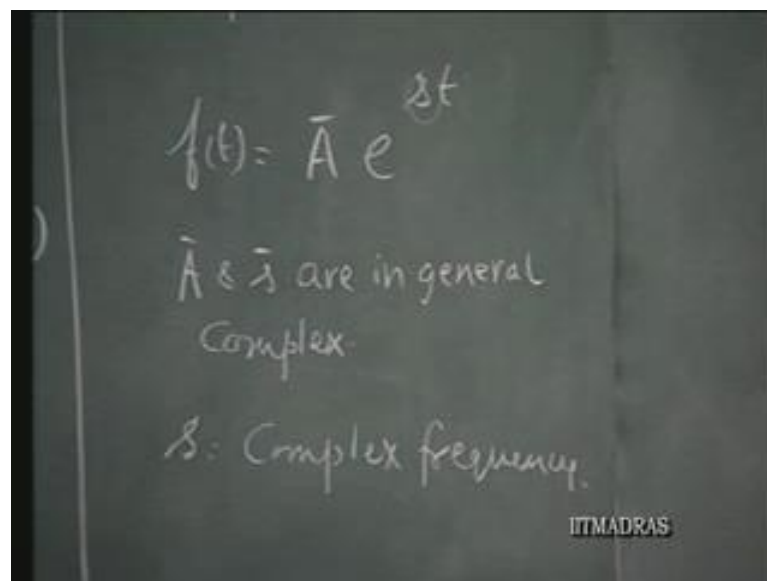
Now, what about the dimensions of s ? You know from your may be from some of your earlier courses that whenever, you have a physical equation in which an exponent

appears the argument of the exponent: the exponential function that is $s t$ must be dimensionless. Similarly, $\sin \omega t$ whatever it is that argument must be dimensionless. So, $s t$ must be dimensionless otherwise, you cannot match the dimensions on both sides of an equation. If $s t$ had a dimension then, e to the power of $s t$ has $1 + s t + s^2 t^2 + \dots$ so on and so forth.

The dimensions of that you do not know what they are because, if it is $s t$ the dimension has some x say m or l or whatever it is your $m^2 l^2$ and so on. So, because of these reasons this must be dimensionless. If this is dimensionless then, the dimension of s must be something per second, $s t$ is dimensionless. Therefore, s must be something per second and what is the quantity which is something per second. It is called frequency therefore, we turn this quantity s as a frequency and because, it is in general complex we call this a complex frequency.

So, s is termed a complex frequency, it is not a frequency in the sense something is repetitive in character like in a periodic function, but because its dimensions are something per second therefore, it is called frequency: this is called complex frequency. So, this is the most general kind of exponential signal. Then, if it is going to yield a complex value for f of t what is the use for this? That is the question that naturally arises.

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As it turns out that whenever, we have to deal with real systems real signals any complex signal of this type will always be matched by its mate such that, the sum of these 2 will

always be yield a real term. For example: if i have a for example, suppose is a complex number a e to the power of j field that is a complex number a. Suppose i put it in this form and e to the power of say s is equal to sigma plus j omega t such a complex signal this is: the same signal i am writing for s sigma plus j omega and for complex number a. I am writing a e to the power of j field i have written this. This will always, be matched by its counterpart which we have the coefficient its conjugate and likewise, the complex frequency also will have its conjugate.

Usually, these 2 come together so, if you have a e to the power of s t you will also have a star e to the power of s conjugate t. These 2 will always occur together. So, when you combine these 2 you observe that you have a e to the power of sigma t and e to the power of j omega t plus phi and e to the power of minus j omega t minus phi. Therefore, you will have 2 a cos omega t plus phi so, this is what it yields.

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The image shows a chalkboard with the following handwritten equations:

$$(A e^{j\phi}) e^{(\sigma + j\omega)t}$$

$$+ (A e^{-j\phi}) e^{(\sigma - j\omega)t}$$

$$\Rightarrow 2A e^{\sigma t} \cos(\omega t + \phi)$$

The IITMADRAS logo is visible in the bottom right corner of the chalkboard image.

So, this is the real signal so, we do not have to be particularly alarmed about the fact that this is going to yield a complex number. Because, we can be sure that in analysis of any system that we are going to take up this will always be matched by a conjugate quantity of mate like this. So, the sum of these 2 will always be of this type.

Now, depending upon the location of the complex frequency here in the complex plane you have different kind of behavior as for the time dependence of this signal is concerned. Let me, illustrate this here so, the complex value of s can be represented in

the complex plane like this with x axis representing σ and the y axis representing ω . So, suppose if the value of s is equal to minus 2. Let us, say a particular location s is equal to minus 2 that means: the value the general value of s . Now, takes a real value this represents a signal which will be $a e$ to the power of minus $2t$. So, you will have something like this. So, the time dependence of that suppose let us, take a to be a real number because, it does not have this is a real quantity.

Therefore, you cannot have a conjugate of this complex frequency therefore, a has to be real So, for example, a signal like this may be $3 e$ the power of minus $2t$ on the other hand if a have sorry s is equal to 2. s is equal to j have made a mistake here if s is equal to 2 here this represents an increase in signal like, this an example like this: $3 e$ to the power of $2t$. That means: it is a n exponentially growing signal. If a take s is equal to minus 2 then, this is an exponentially decreasing signal may be this is $3 e$ to the power of minus $2t$.

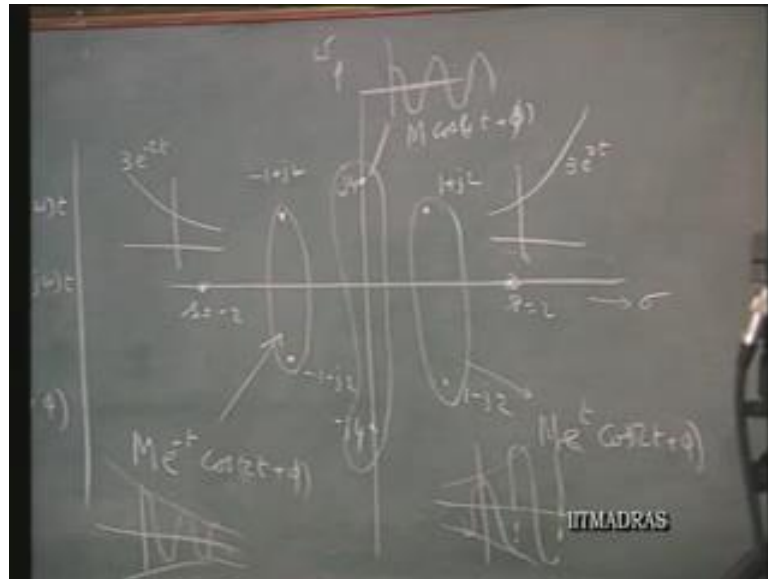
On the other hand, if a have 2 complex frequencies which are conjugates of each other like this say this: value equals minus 1 plus $j2$ and this is minus 1 minus $j2$ these 2 frequencies together. These 2 terms like this will give rise to a response which is of the form some $m e$ to the power of minus t minus σ is now, minus 1 cos $2t$ plus d . So, that means: you have a decaying sinusoid that's how the quantity will vary with respect to time.

On the other hand, if a have 2 conjugate complex frequencies with positive real parts say 1 plus $j2$ and 1 minus $j2$ such 2 terms will yield a function of time which is $m e$ to the power of t cos $2t$ plus p and that would yield a time function which is growing in time something like this. On the border line between the left half plane and the right half plane suppose, a have 2 frequencies.

Let us, say $j4$ and minus $j4$ these 2 terms will yield a pure sinusoid and we have say, a typically $m \cos 4t$ plus θ the m and θ or m and p depending upon the coefficients complex coefficients a that you are having. So; that means: your complex frequency exponential signal like this, Encompasses in its generality the whole lot of time functions of this type. So, it can be a decaying exponential signal where s happens to be real and negative, it can be a growing exponential, where s happens to be real and positive and if s happens to be purely imaginary a pair of such frequencies will give rise to a sinusoidal

signal pure sinusoidal signal. And on the other hand, if s has a negative real part it indicates a decaying oscillations and if real part of s happens to be positive thing it has a increasing oscillations. So, we have this e to the power of $s t$ has as a special case the sinusoidal signals. That means sum of 2 such exponential functions will give rise to a pure sinusoidal.

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So, we have taken now a look at 3 different kinds of signals which are namely the d c signal, the sinusoid and the exponential signal e to the power of $s t$ and I mentioned that even though exponential signal in general gives rise to a complex value for a real time t , but 2 such signals can combine and will give rise to a real function of time. So, the value at any time at any point of time t will turn out to be real. We also, have acquainted ourselves with the meaning of the term complex frequency.

It is mainly because, that particular coefficient of t in the exponential representation of the exponential signal has the dimensions of frequency we call it a frequency. It does not necessarily mean that, it gives rise to a periodic quantity because, none of these signals are purely periodic. The characteristics of all these signals that, we had talked about so far is that they are smooth functions of time. That means, not only they do not have discontinuities, but you can take derivatives of these signals or they continue to have continuous and therefore, this is a characteristic of these signals.

So, in the next lecture we will consider some examples bearing on these concepts particularly, the exponential signal and the complex frequencies. And we will also go on to discuss some signal waveforms which are not continuous, but which are important in our further study.