Networks and Systems

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Lecture - 18
Fourier Series (6)

In the lecture, we saw how a complicated integral operation like the convolution in time domain is simplified in the transform domain by the pure multiplication of the pertinent transforms.

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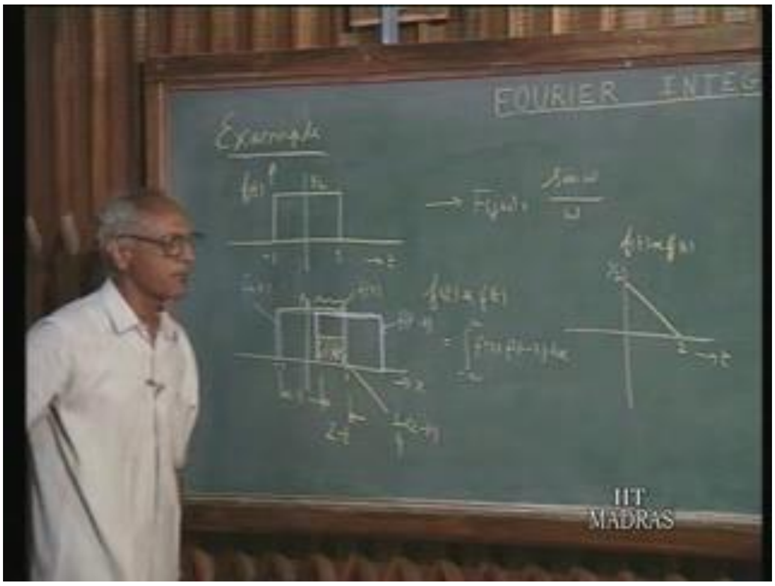

Let me illustrate this, first with an example, before we go on to discussion another properties. Let us consider, here f of t which is a pulse of height half and lasting from minus 1 second to plus 1 second, I will call this f of t it is Fourier Transform f of j omega in our earlier discussion, will be, if the amplitude is a and the pulse duration is d ad that is: half times 2 1 sin omega d up on 2. So, if is 2 d up on 2 is 1 therefore, sin omega divided by omega d up on 2 it is omega.

So in this standard terminology, if we had a height a and duration d it would be a d sin omega d up on 2 by d omega d up on 2 and ad happens to be 1 and d happens to be 2. This is the Fourier Transform of this pulse. Now what I would like to know is: what is the Fourier Transform of f of t star f of t that is: f of t convolve with itself. So, if I want to find out the f of t convolve by with itself then first of all I write this is; let us say, f of x as the function of x. So, in order to find f t convolved with f t I should take the integral of f of x times t minus x dx that is: the integral that, I should workout.

So, this is f of x this also happens to be f of minus x that means; if I take another curve like this, this I can consider to be f of minus x, but I have to multiply f of x by f of t minus x that means; the minus f of x curve I must push forward by an amount equal to t. So that would be obtained by let us say, this curve which I would like to say is f of t minus x. So, originally this was minus 1 this is 1 and I push this forward by an amount t therefore, this intervals equals to t.

So, the overlap interval between f of x and f of t minus x is this is 2 this is t therefore, this is 2 minus t so that is the overlap interval between these 2. And when you multiply these 2 curves each of amplitude half you get the area under the curve after all this is area under this curve and this is the area that, you get amplitude is the height is 1 fourth and the duration is 2 minus t. Therefore, the area of this will be 1 fourth times 2 minus t that would be that, would be the value as you push forward the curve in the positive direction.

If you push it on the negative direction you get a similar expression therefore, the variation of f t convolve with f t would be for positive t.

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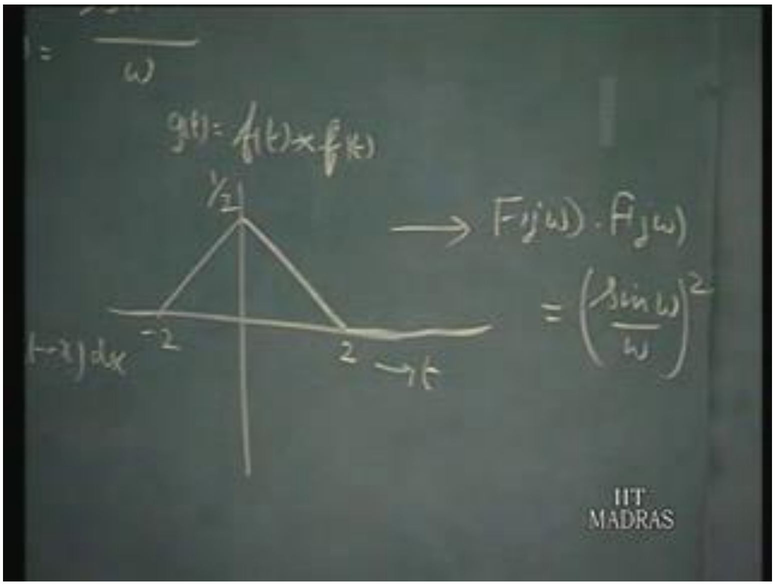

When t equal to 0 we have to take these 2 the f of minus x and f of x multiplied together that, will be 1 fourth lasting for 2 seconds therefore, it will be half but, then as the t progress this reduces in this manner. So, it starts with half and become 0 when t equals to 2. And beyond t equals to 2 this white curves comes away from this therefore, there will be no overlap area therefore, it become 0 from that point onwards and if you proceed in the negative direction you get a symmetrical result it can be easily seen.

So f t convolved with f of t will be having, if you call that, g of t this will be the variation of this. So, to find the Fourier Transform for this, you do not want to spend any additional effort because we know the Fourier Transform of f of t so g of j omega. The Fourier Transform of this is obtained by multiplying f of j omega multiplied by, f j omega which is indeed sin omega by omega whole squared.

So, you can find the Fourier Transform of this; as the Fourier Transform of the convolution of a rectangular pulse by itself. So, you can independently calculate the Fourier Transform of this and we can show that this is equal to this.

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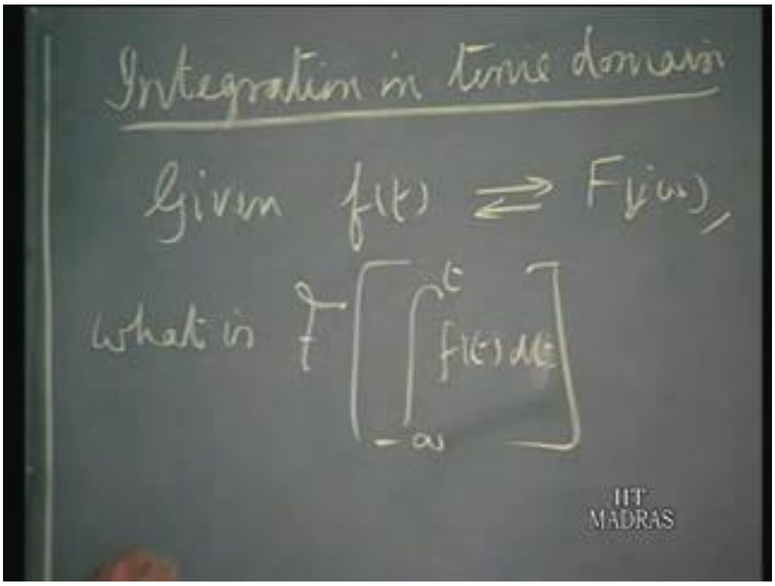

We next study the rule corresponding to integration in time domain, if you recollect that, we said when a function is differentiated in the time domain, in the transform domain it gets multiplied by j omega. And that time I observed that, if you want to integrate a function in the time domain a transform domain it is not nearly division by j omega it may end at something else

Let us see, how that works out now, what we particular, we like to workout is given f of t and f of j omega has the transform pair. What is the Fourier Transform of integral of f of t dt from minus infinity to plus infinity minus infinity to t.

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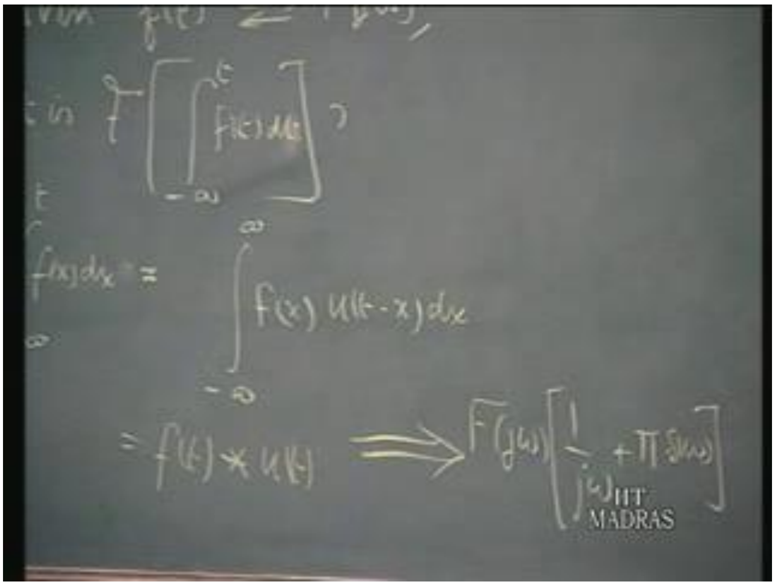

So, this is the function gt which is obtained by integrating f of t from minus infinity to t you can see that; minus infinity to t of f of t dt can be written as minus infinity to infinity of f of x, I can write this also as f of x no problem this can be written as after all this is dummy variable, we can write this as f of x dx. We can write this as f of x times u of t minus x dx because what happens when x goes beyond t when x is more than t t minus is the negative quantity therefore, this become 0 u of t minus is 0 for x greater than t.

Therefore, in this range of integration whenever, x exceeds t it is the integral is 0 therefore, instead of confining our integration from minus infinity to t, we are normally taking the upper limit of integration plus infinity. But we know the value of this integral integrand from x equal to t from x equals to infinity, it is going to 0 so these 2 are equivalent that means; to this, I am adding something which is equal to 0.

So, both are equal and you can see that this is: immediately equal to f of t convolve with u of t after all that is: the formula for a convolution 2 time functions f of t convolve with u of t will give me this. Therefore we observe that, integral of ft from minus infinity to t is equivalent to convolving f of t with u of t and we know from the convolution rule that the Fourier Transform of this is obtained by multiplication of these 2 Fourier Transforms.

The Fourier Transform of this will be f of j omega 1 over j omega pi delta omega that is being the Fourier Transform of unit step function. So, the Fourier Transform of this is given by this you observe now, that f of j omega is not merely divided by j omega but, you have an additional component f of j omega times pi delta omega. So this is the important result that, we have when you want to integrate a function of time from minus infinity to infinity you have to multiply by 1 over j omega plus pi delta omega.

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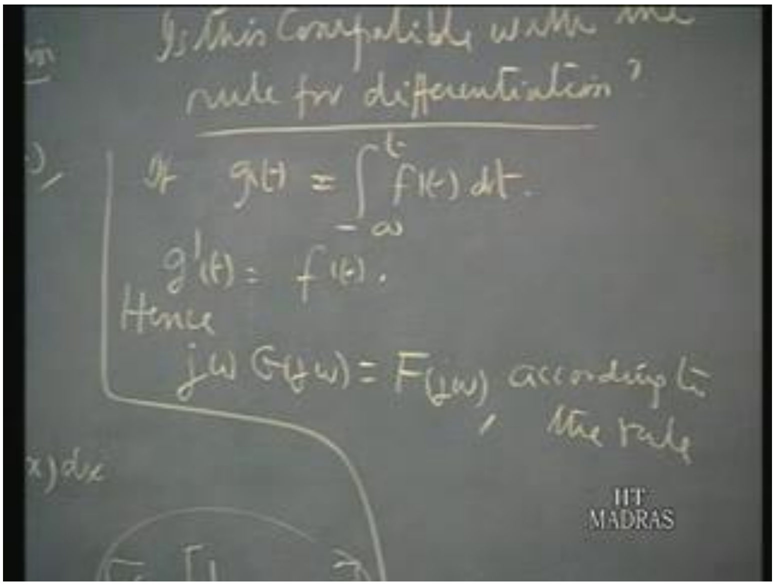

Let us see, the implications of this is this compatible with the rule for differentiation, which we have already talked about that; question we like to settle first. What we are saying is if g of j omega is from minus infinity to t I am sorry g of t suppose, I write g of t I define this as g of t f of t dt then we know the, derivative of g t with respect to t is f of t.

Therefore, if the Fourier Transform of g of t is g of j omega then j omega is multiplied by that; must be the Fourier Transform of f of t hence, j omega times g of j omega must be equal to f of j omega according to the derivative rule according to the rule for differentiation. Now, it is indeed so, we know the g of j omega has got the Fourier Transform therefore.

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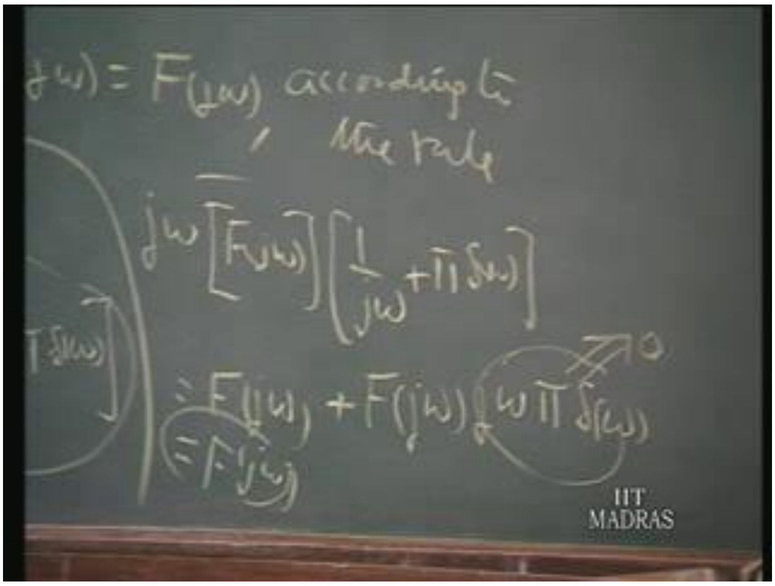

Let us illustrate this; J omega is multiplied by f of j omega multiplied by 1 over j omega plus pi delta omega, if you take that the product of f of j omega 1 over j omega and 1 over j omega gives me f of j omega plus I have additional term which is f of j omega j omega pi delta omega. Now what we would expect is this should result in f of j omega but, we have an additional terms but, we observe that omega times delta omega is 0 because any function delta omega multiplying delta omega will be equal to that function evaluated at omega is equal to 0 multiplied by delta omega.

Therefore since, this quantity goes to 0 this is indeed to f of j omega therefore, there is no incompatibility this is quite alright, if it is consisted with the rule for differentiation that we have already derived. So, in the summary the integration in time domain from minus infinity to t of f of t dt is translated into the transform domain as multiplication of f of j omega by 1 over j omega plus pi delta omega.

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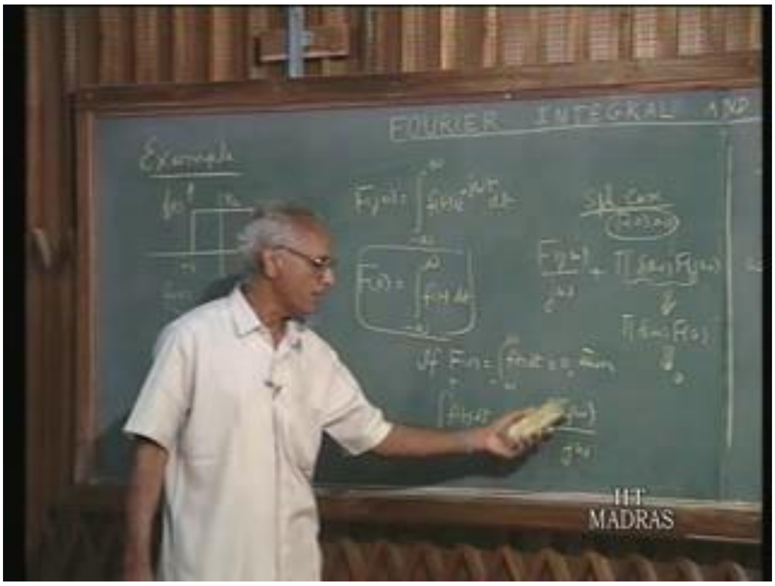

However, we observe that, for a special case, if this g of j omega which is the Fourier Transform of this quantity is f of j omega divided by j omega plus pi delta omega f of j omega. So special case of f 0 is being equal to 0 suppose, we have a special case where Fourier Transform evaluated at omega is equal to 0 is equal to 0 then this will be 0 because this is equal to pi delta omega f of 0 and in this case it equal to 0.

Therefore f of 0, if it is equal to 0 then integration will result in f of j omega you do not have the extra term and when does f of 0 become 0 then you recall once again F of j omega equals f of t e to the power of minus j omega t dt minus infinity to plus infinity. So, f of 0 minus infinity to plus infinity f of t and omega equals 0 this is 1 f t dt. So, the summary of this is. If f of 0 which is equal to minus infinity to plus infinity of f t dt is 0 then the Fourier Transform of f t dt is simply f j omega over j omega the second term drops out.

So, summarize in general whenever, you have an integral minus infinity to t f t dt it is Fourier Transform is obtained by multiplying f of j omega by 1 over j omega plus pi delta omega on the other hand for the special case where the integral of f t dt t from minus infinity to plus infinity is 0 which is f 0. If that happens to be 0 then the Fourier Transform of this integral will be multiplied by f will be obtained by multiplying f of j omega by 1 over j omega.

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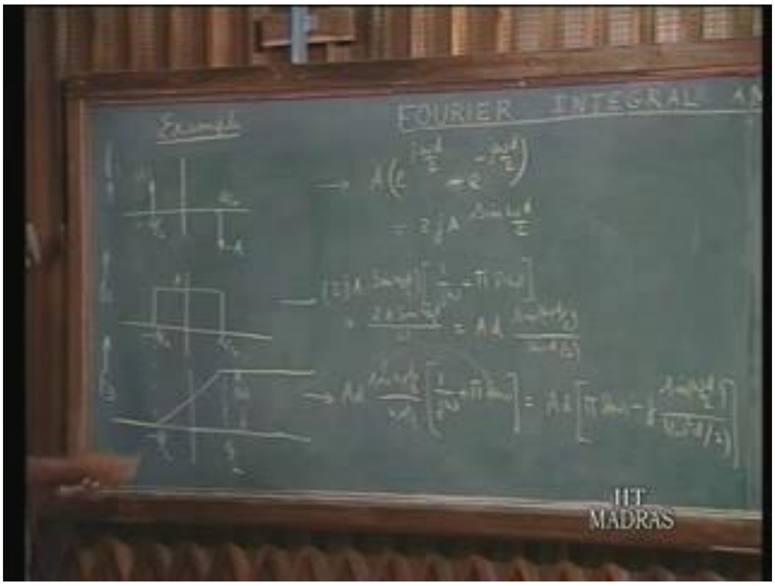

Let us work out an example, illustrating the integration. Let us, consider an example to illustrate this, I will start with a f1 of t which consists of a pair of impulses of strength a and minus a at minus d up on 2 and plus d up on 2 this is f1. I obtain from f1 a function f2, which is the integral of f1 from minus infinity to t as, I integrate from minus infinity to t become 0 area.

At this point I have a jump and the jump is equal to a and that continues up to this point and beyond that, you have another jump of minus a because you are integrating with an impulse and therefore, this becomes 0. So, this will be f2 is obtained by integrating f1 from minus infinity to t therefore, what we have is this kind of f2 derivative of f2 of course, f1.

Now, if I integrate this once more I get a function f3 which is minus d up on up to minus d up on 2 this is 0 and then gradually increases. Because you are picking more and more area under this pulse therefore, it increases like this up to d up on 2 and beyond that it; flat because there is no further area is being added to this curve that will be the nature of f3. And the total area picked up from this point to this point is a times d base area therefore, this is equal to ad.

Now, our interest is to find out the Fourier Transform of such a curve like this f3 now, we can put the problem in this way suppose, you want to find the Fourier Transform of such a curve like this then you know that this is the integral of this which in turn the integral of this. So, if you can find the Fourier Transform of that. By integration rule you can find the Fourier Transform of this and by the integration rule again find.

The Fourier Transform of this, this is what we want to do our motivation for doing this is the Fourier Transform for this is very simple because, it consists of pair of impulses we know the Fourier Transform of this is a 2 pi a the strength is I am sorry the impulse of strength is therefore, e to the power of j omega d up on 2 representing this pulse and this is; minus a e to the power of j minus omega d by 2. Because it is shifted in time by d up on 2 seconds and this is negative therefore, minus omega

So, this is the Fourier Transform of this corresponding to this impulse of strength a you have a Fourier Transform of a since this is advanced in time you have to multiply j e to the power of minus j omega d up on 2. Since. this is delayed in time you have to multiply e to the power of minus j omega d up on 2 and since, the negative impulse you have minus sign here and this of course, equal to 2 a sin omega d up on 2.

Now when you come to this, this curve is obtained by integrating this therefore, you have to multiply this by 2 j a sin omega d up on 2 multiplied by 1 over j omega times pi delta omega, by the integration rule that, we have just now observed and here is a case where the integral from minus infinity to plus infinity of f 1 t is 0 in other words f 0 the value of this evaluated at omega equals to 0 is therefore, this term drops out.

We can also verify this from substituting omega equal to 0 this quantity is 0 that multiplied by pi delta omega leads to 0 therefore, you get contribution only from the product of this term and 1 over j omega. Therefore, that will be 2 a sin omega d up on 2 by omega 2 a sin omega d up on 2 by omega, which I can write this further as ad sin omega d up on 2 divided by omega d up on 2 that is: the form which we are familiar just wanted to put it in that forms so, that once we have this pulse function we know immediately it is Fourier Transform of this of course, the same thing is obtained through finding out the Fourier Transform at the derivative to start with.

Now, the Fourier Transform of this is once again obtained multiplying a d sin omega d up on 2 divided by mega d up 2 by 1 over omega plus pi delta omega and now, f of 0 of f2 of 0 capital f2 of 0 that means Fourier Transform of this evaluated at omega equal to 0 is not 0. Therefore, the second term of also has it place, it is not going to vanish therefore, if you write this, we can have first of all the product of these 2 delta omega multiplied by a certain function is equal to the function evaluated omega equals to 0 multiplied by delta omega equals 0 this becomes ad times delta omega.

Because sin omega d up on 2 by d omega up on 2 evaluated at omega equal to 0 equals 1 and then you have 1 over j put this as minus j ad is already there sin omega d up on 2 divided by omega because this omega is there another d omega up on 2 there omega squared d up on 2. So, that will be the Fourier Transform of this so, this example illustrates how 1 can repeatedly differentiate a function or integrate a function and then get 1 transform of the other using the rule for differentiation and integration of the case may be earlier we had workout some examples, where the function was continuously differentiated.

Now, we have taken a reverse order we started with a simple function whose Fourier Transform, can be easily be obtained and integrate this at 2 successive steps and therefore, from the Fourier Transform of this we are able to find the Fourier Transform of this. After having study the various properties of the Fourier Transforms. Let us now, discuss how this theory can be applied to the analysis of systems.

So, we like to see how the Fourier Transform theory can be used to find out the output of a system corresponding to various type of inputs.

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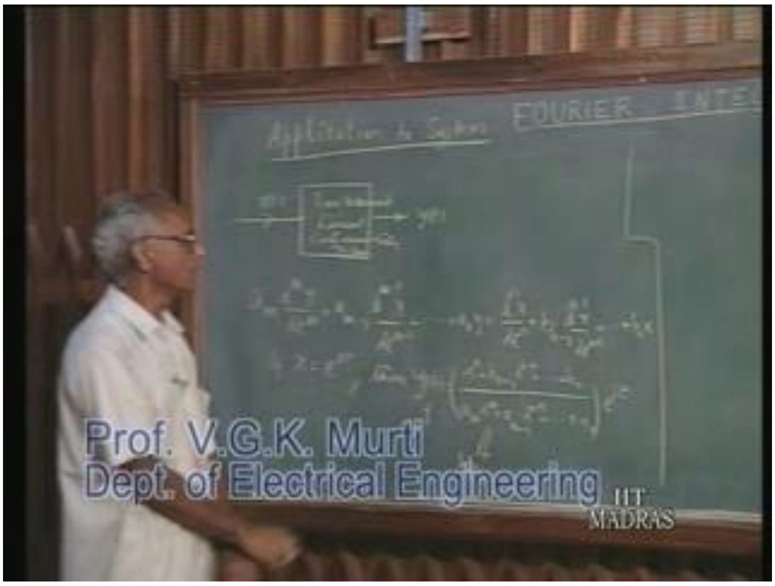

So, in the system terminology, if we represent the input and the output as x of t and y of t respectively, this is the linear continuous constant parameter system. In general the input and output are related by the differential equations of this type dm y by d tm plus a m minus 1 d minus 1 y dt m minus 1 etc plus a not y that is: equal to d n x dt n plus b n minus 1 d n minus 1 x dt n minus 1 etc plus b not x.

This is the type of differential equations that, in general you have for a single input and single out put constant parameter time invariant, linear, continuous time system, in such a system if the input x is the e to the power of say st. Then the force responds will be obtained by taking the instead of d operator you multiply equivalent of s operator that means; is equal to s to the power of n b n minus 1 s to the power of n minus 1 etc plus b not divided by a m sm plus a m minus 1 s m minus 1 plus a n multiplied by e to the power of st.

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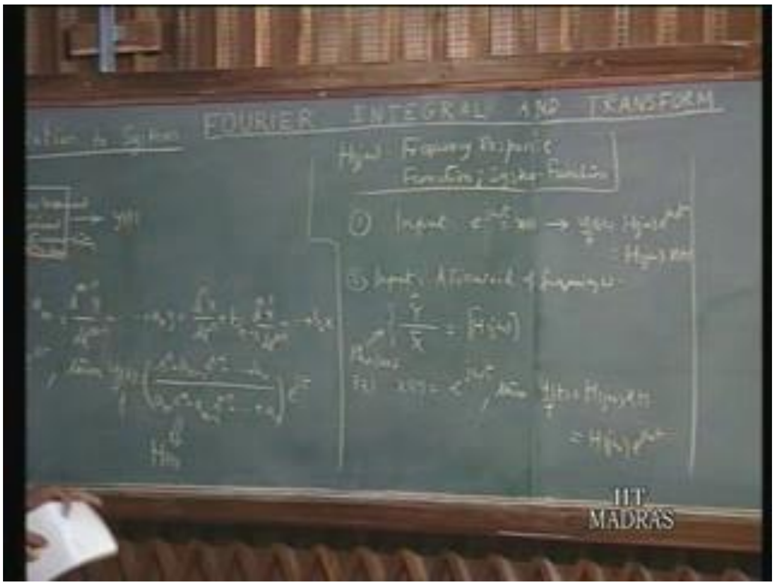

Where this is the input and this is the corresponding output this is the force responds the particular integral solution of the differential equation and this we observed is called the system function of h of s. We are in particular interested in the system function evaluated when s equal to j omega. So, this is h of s h of j omega is the frequency response function also called system function so, often this is: itself called system function and this particular function this will make use of in the fourier transforms applications to systems.

Now, let us see the implications of the system functions 1 suppose, the input is e to the power of j omega t then this is xt it means the force response y f of t the particular integral is the solution of this differential equation the force response will be h of j omega multiplied by e to the power of j omega t that means; h of j omega multiplied x of t. So, the force response and the input are related by multiplicative constant h of j omega which is independent of time that is: why I am calling this as a constant.

A particular case a input is the sinusoid then the output phasor and the input phasor these are phasors whether the voltages are currents may be in the case of electrical network or in terms of general systems you can define phasors in the same manner both are sinusoid function of time this is given by h of j omega once again where, the sinusoid of frequency omega so, under steady state the output sinusoid and the input sinusoid their phasors are related by this h of j omega.

So, that is the significance of the system function h of j omega, if you have an input xt which is an exponential function e to the power of j omega t then the force response y of t is h of j omega multiplied by e to the power of j omega t these after all both are related to each other. And it is this property, which we are going to make use of in our Fourier Transform theory because we are picking up any given x of t as the summation infinite summation of time functions of this type. So, this is what we really want make use of.

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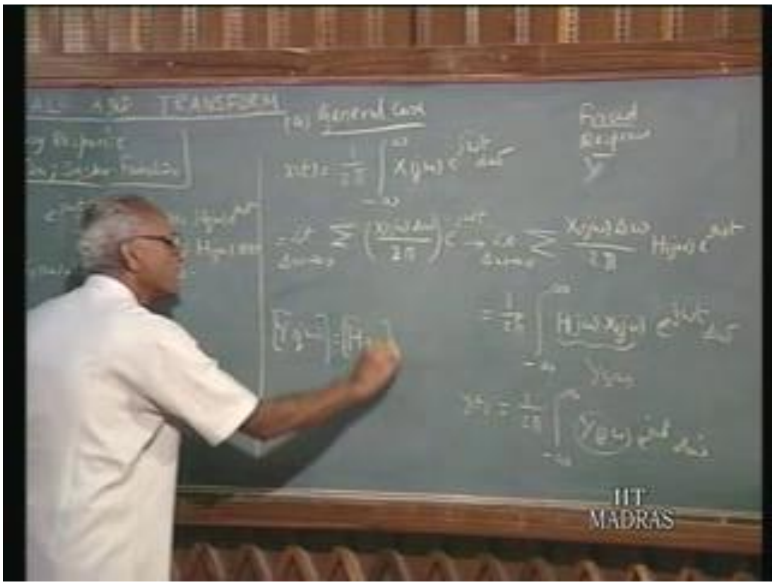

So, in a general case we have x of t is related to its Fourier Transform as 1 over 2 pi minus infinity to plus infinity of x j omega e to the power of j omega t d omega you recall that, this integral arose from considering a number of elementary signals of coefficient density f j omega lasting from minus infinity to plus infinity.

So, in fact we said this is: limit as delta omega goes to 0 a huge a lot of summation of huge a lot of terms x j omega delta omega over 2 pi this is being the coefficient density and delta omega over 2 pi is being width of the elementary slice of the spectrum that, we are taking this is: the coefficient and this is the function of time which takes. And if you take all such signals add them up you get this x of t that is, the general input.

So, general input xt it can be a thought of as the summation of elementary signals of this type each with an amplitude like this and this omega is running from plus infinity to minus infinity. Now from this you conclude since, if it is e to the power of j omega t the force response is h of j omega times e to the power of j omega t all we are doing is e to the power of omega t you have an particular amplitude certain coefficient.

So, the force response will be x j omega delta omega over 2 pi times h of j omega e to the power of j omega and once again here also, we have that limit this is the response for the single term but, we have whole a lot such terms limit as delta omega goes to 0. Since, this integral is viewed as the limit of such quantities the limit of such quantity can also viewed as the limit of such quantities the limit of such quantities can also viewed as the integral.

This, will be equal to 1 over 2 pi from minus infinity plus infinity of h j omega times x j omega e to the power of j omega t and we observed that this is: in the where you form that, we have for its inverse fourier transformation. So, if the response of this is y of t then this can be written as this is your j omega therefore, y of j omega will be this is y of j omega then this will be equal to y of t therefore, y of t which is given by this quantity can be a thought of as 1 over 2 pi minus infinity to plus infinity this is y j omega e to the power of j omega t d omega.

So, that means y j omega so, if this is the inverse Fourier Transform of y of t y j omega must be equal to this. So, the conclusion is that the Fourier Transform of the output quantity equals the Fourier Transform of the input multiplied by the system. So, this is the very important rule the Fourier Transform of the output system here y of j omega will be equal to the Fourier Transform of the input multiplied, by the system function with h of j omega.

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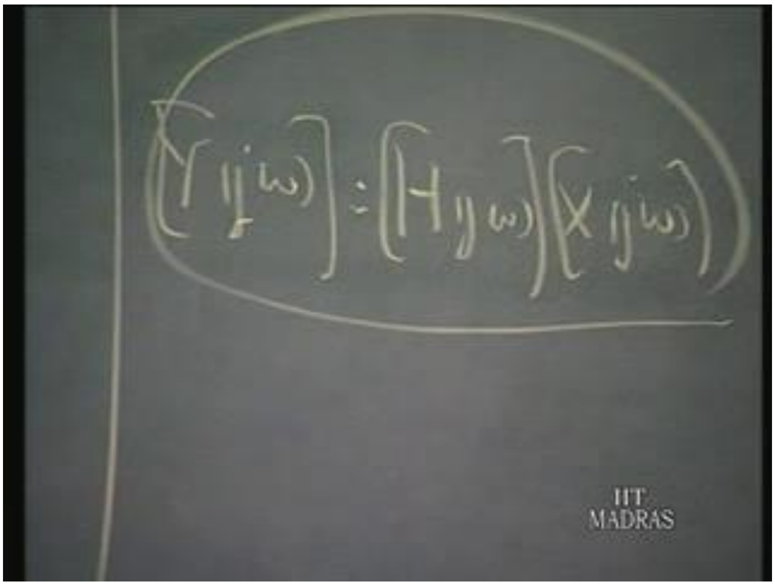

Now, 1 point is we have been taking about the force response particular integral solution and we explain later that, this is indeed will be the total solution because including the transience for the time being assume that, this is.

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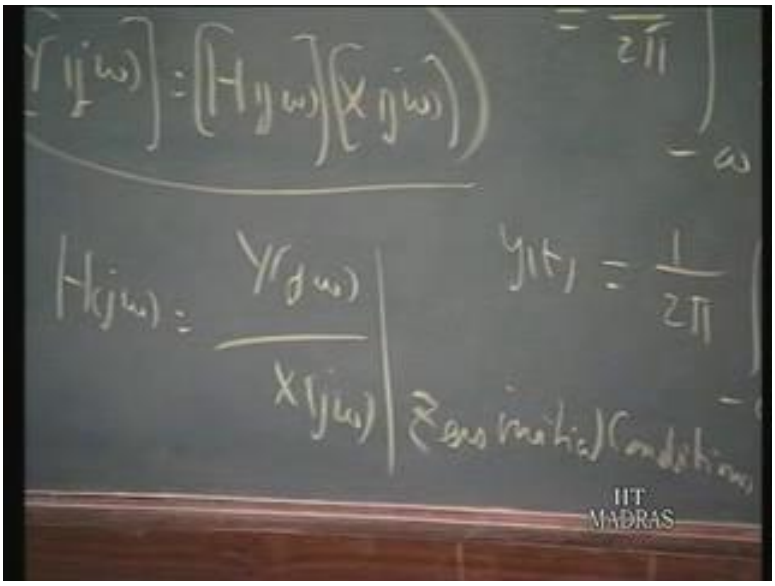

So, I will explain it later point of time we can say the output will be equal to input fourier out put transform will be input Fourier Transform with h of j omega of h of j omega can be thought of as the Fourier Transform out put, the Fourier Transform of the input with 0 initial condition I will talk about that later with 0 initial condition. So this is another way in which we can find out the response of a linear time invariant continuous system for a given excitation of the given input all you have to do is from the given input find it is Fourier Transform.

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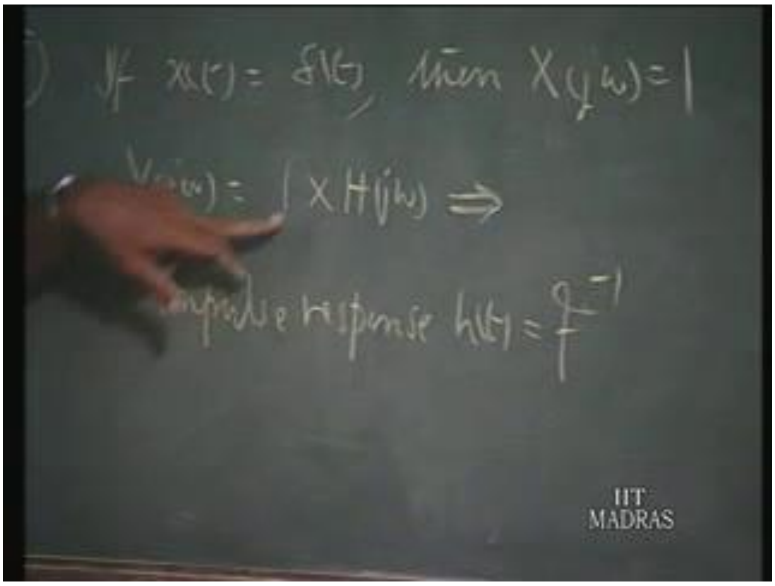

You know the calculate the system function and determine whatever, manner you have at your disposal multiply these 2 that will give you the Fourier Transform of the output. Find out the inverse Fourier Transform that, will give the response quantity that is how it goes 1 more property of the this system function is: if the input happens to be a delta function a unit impulse then, we know that x j omega equals 1.

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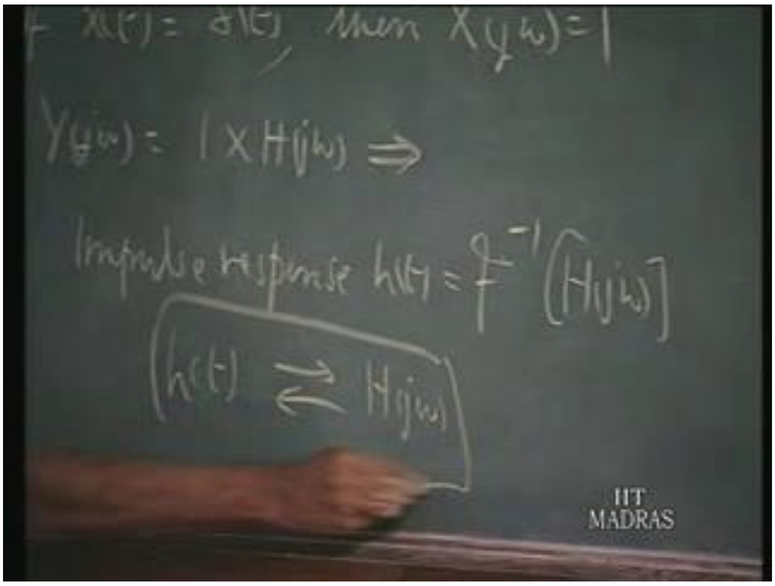

Therefore, the output Fourier Transform y j omega equals 1 multiplied by h of j omega that means; the impulse response, if call impulse response h of t that is the inverse Fourier Transform because under the impulse excitation the output has the Fourier Transform h of j omega therefore, the time function corresponding to this will be inverse Fourier Transform amplitude or to put in other words the impulse response h of t and the system function form a transform pair this is indeed a very useful result. System function is the Fourier Transform of the impulse response or the impulse response is the inverse Fourier Transform of the system function.

Now, this idea plies in nicely with our convolution property you recall that earlier, I mention that in our introductory lecture, I mention that the output can be thought of as the convolution of the impulse response on the input.

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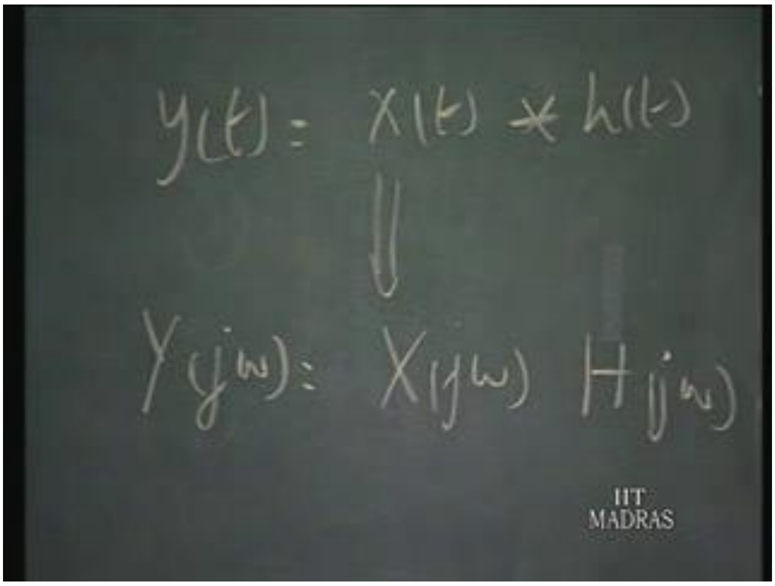

So, you recall that out put y of t is the input xt convolved with impulses and from the Fourier Transform theory we know that, when you have 2 type of functions convolved the Fourier Transform is obtained by taking the product of the Fourier Transform of the 2 functions which are convolved. So, that is exactly what we had here y j omega h j omega times h j omega equals to of this so, these 2 going to right.

Now, with this basic concepts relating to general system let us, apply these techniques let us apply this to a electrical network problem, where we like to calculate the output for different type of excitations. Let us now, apply these concepts to evaluation of the transience in a particular network through an example, now what we are really doing is the given excitation input function is thought of as a number of elementary sinusoids starting from minus infinity and extending to whatever, point of time at which evaluate the response.

Now, if a sinusoid is given as the input network the force response is also a sinusoid and we are trying to find out the summation of the force response to the individual sinusoids. Now let us see, if the system does not have any initial energy to start we assume all capacitors are uncharged all injects do not have any 0 current to start with any non zero current to start with then all these sinusoidal inputs will give corresponding sinusoid responses and we like to add them up.

That is: what we get by finding out y j omega as h of j omega times x of j omega where x of j omega h of omega is the system function which is obtained under steady state sinusoidal conditions.

Now, in a differential equation we have both the complementary solution and the steady state solution which is steady state solution is being the particular integral solution. Now by adding up all these elementary responses, we are adding up all the force response of the particular integrals solutions that we have from these various input functions. Now, what about complementary function since, we are starting our time the application of these inputs and t equals minus infinity and if the system has got a natural response which dies out with time which has got the natural response boards of negative values for the real parts.

That is: natural frequencies have negative real parts then what happens is starting from minus infinity they decay down to 0 when we are talking about the response calculating the response at finite time particularly from t equal to 0 and beyond and therefore, all the natural responses would have died out. And what we are now dealing with is only the force response, that is: why total response is obtained by adding up the force responses of all these elementary sinusoids which are started minus infinity and continued maintaining their amplitude ride through the period up to plus infinity the natural responses would have died down.

This is true for all system whose natural response decay with time there may be certain difficulties where the natural response do not died out with time natural responses have a system natural frequencies have 0 real part that means; they have sinusoids of dc responses as natural responses then there will be some problems but, we will not be dealing with such cases as far as this course is concerned we assume that this system has a stable system with all the natural responses are being decaying with time in exponential manner. And then because they have started at minus infinity by the time we come to 0 in the decay down and.

It also can be shown that the inverse Fourier Transform of the response function through the contour integration on the complex plane it can be shown that suppose, you want to apply the excitation at time t equals to 0 and then want to know the response from t equals onwards for positive t, that means; the function has the excitation is 0 for negative t and we would also like to have the response also 0 for negative t because it is a causal system and if you take the inverse Fourier Transform of y of j omega it trans out to for such systems as we have studied the response for negative t is going to be 0.

Therefore, this is quite in accordance with the causality principle of the system and therefore, there is no necessity for a natural response terms to appear in the final solution. So, the overall to sum up them if you apply the Fourier Transform theory and then apply the steady state methods after all f j omega is the response is the ratio of the response to the input phasor under steady state conditions.

So, we are applying the steady state condition theory and I apply and obtain the natural the total transients response even though, we are applying steady state theory because all the natural response add up to 0 for negative values of time and then decay by the time we start counting our response taking account of the responses for positive t and.

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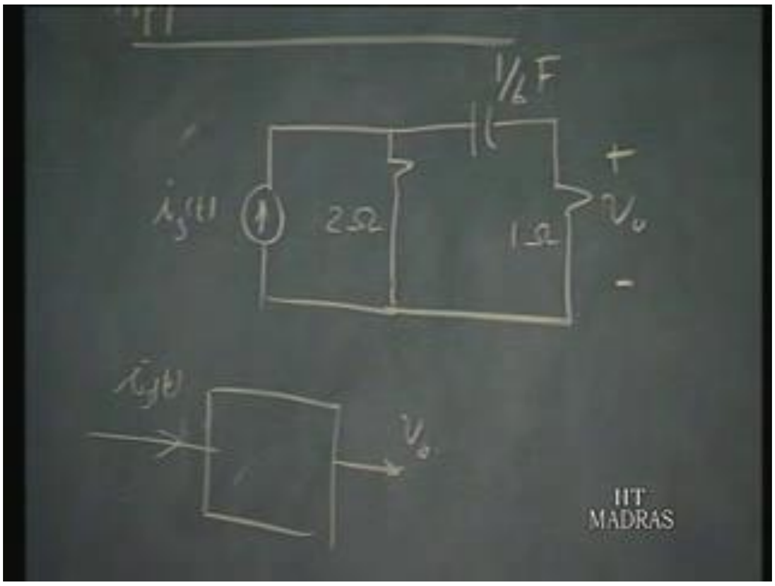

So, let us take an example, to illustrate this suppose, I have a current source I s of t which is given to a circuit like this, where this is 2 Ohm resister 1 by 6 Faradays capacitor and 1 Ohm resister and this is my out put that I desire. And this can be viewed as the system where I s of t is the input and v not is the output so, from steady state theory, we would like to find out what is h of j omega for this what is the ratio for the phasor v not to phasor I s

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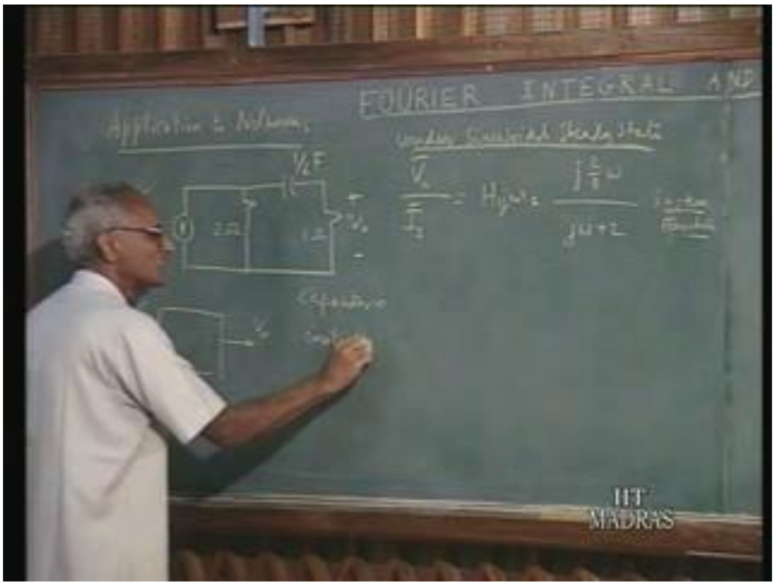

So, v not phasor over I s phasor under steady state under sinusoidal steady state this is our h of j omega once, you identify this as the output and this as the input we have the output of the input the system can be defined v not over I s you can work this out and so this is equal to j 2 by 3 omega divided by j omega plus 2.

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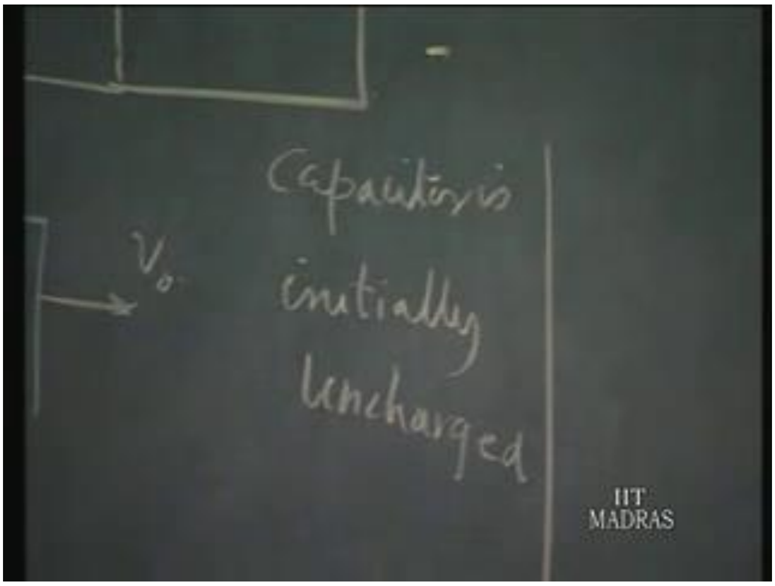

Impedance and impedances you calculate the ratio this phasor the input phasor the response to any kind of excitation I s of t that, we have we assume that the capacitor is initially uncharged. So, let us take first a case where I s of t is the delta function then the input is the impulse function therefore, the input the Fourier Transform of the input equals to 1.

Therefore, the output Fourier Transform v not of j omega equals the system function which is equals to j 2 by 3 omega over j omega plus 2 times the input Fourier Transform which is equal to 1 and this I can write this as now, I have a function like this, I must find out the inverse Fourier Transform I must put this in a recognizable form so, this I can write this as 2 up on 3 minus 4 up on 3 over j omega plus 2 this is what we call a partial fraction expansion, we talk about this in greater detail, when we go to Laplace Transform but, we can easily see that this is equal to this.

I try to break up this into 2 parts each of which can be recognized to be the Fourier Transform of known functions that is the general idea. Therefore, v not t is obtained as two-thirds, two-thirds in the transform domain therefore, in time domain it two-third of delta t two-third of delta t and the Fourier Transform of that is 4 up on 3 e to the power of minus 2 t ut, because 1 over j omega plus 2 corresponds to e to the power of minus 2 t ut and you have minus 4 up on 3 as the coefficient.

So, this is the total response of the system, when the input is a unit impulse a second case suppose, I s of t is the ut then I s j omega the input Fourier Transform is 1 over j omega plus pi delta omega.

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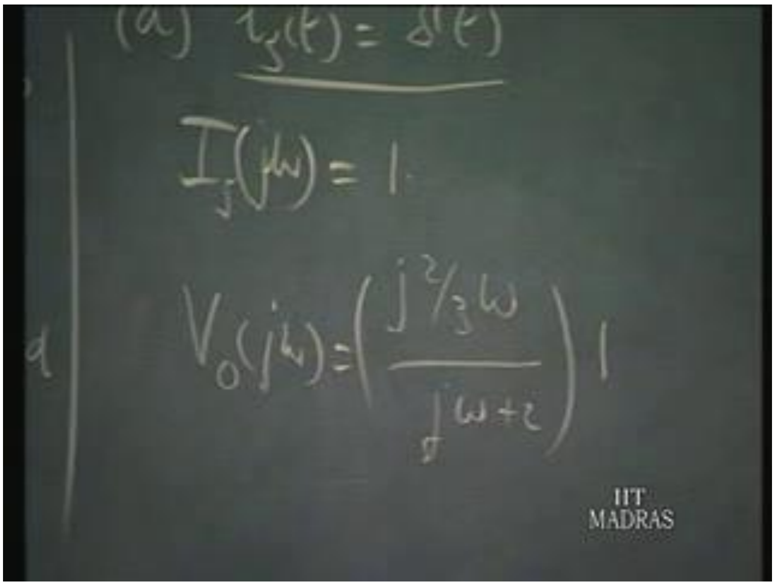

The output the v not j omega is 1 over j omega plus pi delta omega multiplied by this system function, which is j two-thirds of omega divided by j omega plus 2 and you can simplify this and show this to be equal to you see pi delta omega is multiplied by this become 0 because the whole function is 0 at omega equal to 0 therefore, this does not contribute to any term in the final result this will turn out to be 2 by 3 j omega plus 2 or v not of t equals two-thirds e to the power of minus 2 t ut.

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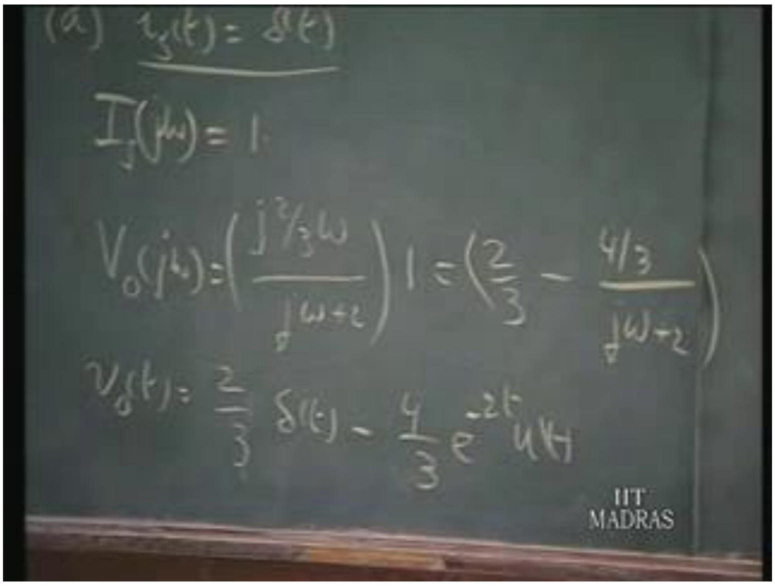

So, that is how you can calculate the transient perform of network using the fourier integral Fourier Transform concepts. To summarize in this particular lecture, what we have done is we started out with the formula for integration and we saw how the integration rule is compatible with the differentiation rule in particular, we said any function f of t integrated from minus infinity to t is equivalent to multiplication in the transform domain by term 1 over j omega plus pi delta omega and in particular when the transform f of j omega is 0, when omega equal to 0 the second term drops out.

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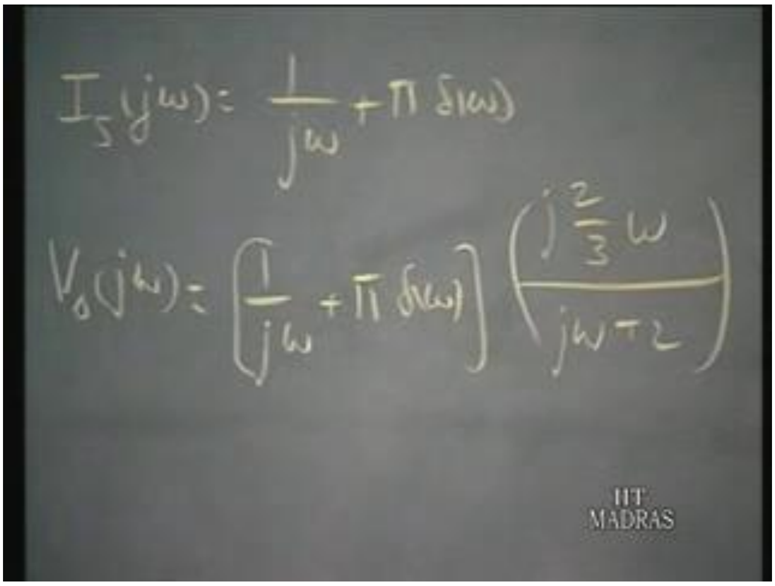

Then, we saw how the Fourier Transform theory can be applied to general system studies and in the general system studies frequency response function h of j omega plays very important role as far the Fourier Transform is concerned. And h of omega is the value of the of the phasor of the output the phasor of the input at the particular frequency. So, under using steady state method you can evaluate the h of j omega or either experimentally or through the analytical calculation.

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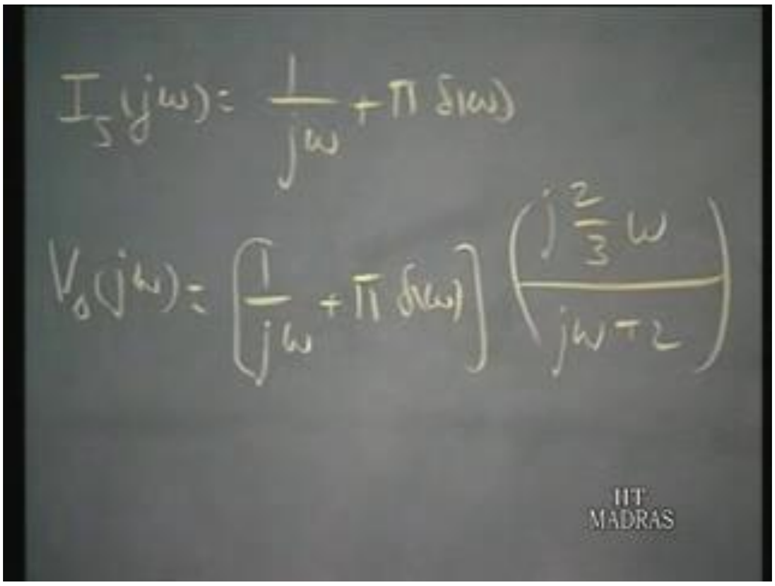

Once, you have h of j omega the output Fourier Transform and the input Fourier Transform are related by the system function h of j omega and I mentioned that, even though this is the ratio of the force response the Fourier Transform of the force response to the Fourier Transform of the input in many particular cases, many cases which are common interest by the natural frequencies decay, natural response decay with time this also gives us the total solution and we took up a specific example to illustrate this idea and we took a r c network.

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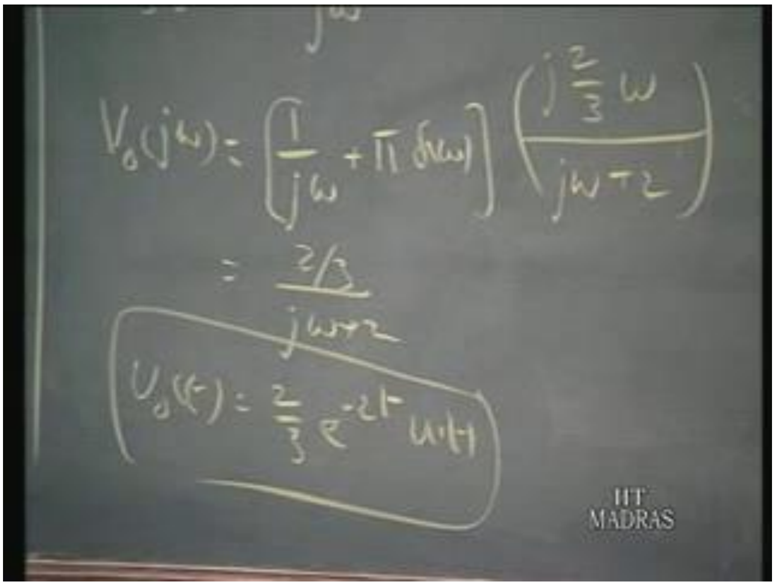

And found out the response of the r c network response in the sense the output occurs 2 terminals calculated for 2 different excitations 1 is an impulse excitation, the other is step excitation and this shows that, the Fourier Transform theory can be applied to a evaluation of the transients and network, even though the Fourier Transform is essentially related to steady state behavior of networks. It can be made use of evaluate transients as well.