Networks and Systems
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Lecture - 17
Fourier Transforms (5)

In the last lecture, we were considering the Fourier Transforms of functions of time f of t, which are not absolutely integrable over the infinite range. To do this we lay down certain formulas, which we can assume. These formulas can be justified on the base of distribution theory. And we will just assume these formulas in deriving these Fourier Transforms of the special functions.

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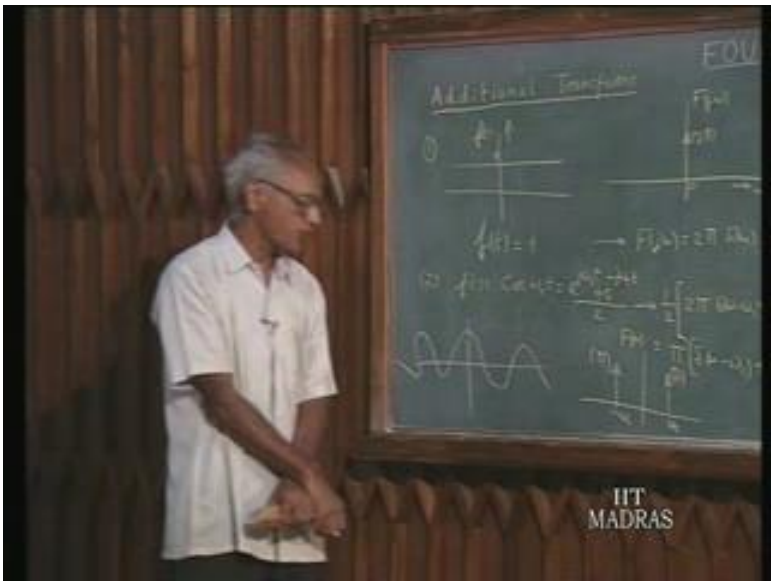

Special in the sense; that f of t is not absolutely integrable over the infinite range. To start with, we took the case of a pure dc. So, if f of t equals 1 then, we said the Fourier Transform for this F of j omega is 2 pi delta omega. The spectrum: Fourier Spectrum then, will comprise just a single impulse at the origin on the omega axis.

Since, this time function now is pure dc, we cannot expect to have frequencies at any value other than, omega equal to 0 and get that point you have a definite non 0 coefficients, in the Fourier Series expansion for f of t which is equal to 1. And consequently you have a density Fourier coefficient density, which is infinitely large and since the Fourier coefficient density is defined in terms of the coefficient per cycles per second. Since, we are plotting on the basis of omega, it turns out to be 2 pi delta omega, you concentration is 2 pi times delta omega.

Now, let us extend this to a sinusoid. Suppose, I have f of t as cos omega c t, a sinusoid of frequency is angular frequency omega c. This of course, can be written as e to the power of j omega c t plus e to the power of minus j omega c t divided by 2. So, in other words f of t consists of 2 exponential functions; half multiplied by e to the power of j omega c t and another half multiplied by e to the power of minus j omega c t.

Now, the difference between half e to the power of j omega c t and f of t equals to 1 is that, it is no doubt scale down by the factor 2. But, the time function is multiplied by the exponent e to the power of minus j omega c t. We had earlier discussed the property of the Fourier Transform. If f of t and F of j omega form a transform pair, if f of t is multiplied e to the power of j omega c t it means: in the transform domain the frequency spectrum is shifted. e to the power of j omega c t times f of t has a Fourier Transform which is F of j omega minus omega c; a property which we have studied earlier.

So, we make use the property and therefore, the Fourier Transform for this will turn out to be, first of all scale factor half is there. So, if f of t has the transform 1, e to the power of j omega ct has the transform 2 pi delta omega minus omega c. Likewise for the other term we have 2 pi delta omega plus omega c which of course, is pi times delta omega minus omega c plus delta omega plus omega c.

So, what do we have? A pure sinusoid here a cosine function goes like this, this is f of t. As far the transform is concerned you have F of j omega. We have 1 impulse here, another impulse here at omega c of the positive side and minus omega c, on the negative side each of strength pi. So, what we have observe here is, once again this is a pure frequency terms that is, only 1 single frequency is present, the pure sinusoid cosine omega c t. And therefore, the energy is concentrated only at 2 discrete frequencies as far as in the exponential representation. e to the power of j omega c t and e to the power of minus j omega c t. There cannot be energy at any other frequency. And these 2 terms together constitute 1 sinusoid frequency omega c.

So, this is a quite revealing that, even a pure sinusoid or a pure dc can be represented by means of Fourier integrals in this, Fourier transform in this manner

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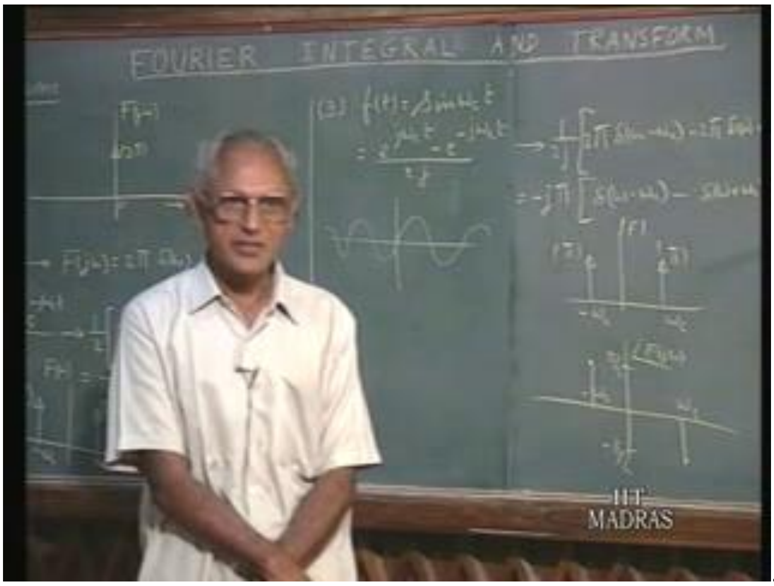

Let us extend this idea further. It can be shown that, if you have sin omega c t which of course, can be written as e to the power of j omega c t minus e to the power of minus j omega c t by 2j. Obviously, the Fourier Transform by the same argument as before will be; pi times 2 pi times delta omega minus omega c plus minus corresponding to second term, you have 2 pi times delta omega plus omega c.

So, this will be minus j times pi, minus j pi i can take out as a common factor delta omega minus omega c and minus. So, it is pure sinusoid therefore, we have sine wave extending from minus infinity to plus infinity and for the Fourier spectrum you have magnitude spectrum f magnitude, we have as before 2 impulses at omega c and minus omega c.

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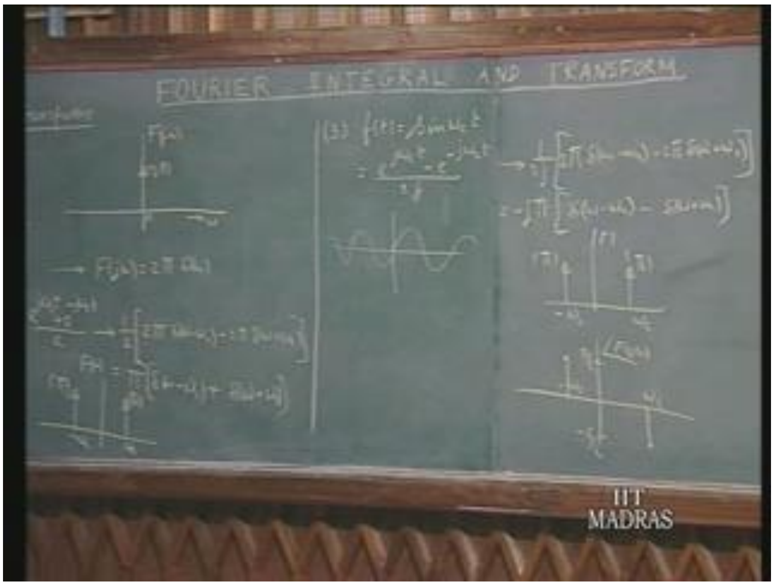

each of strength pi and the angle of F of j omega will be for the frequency omega c and minus omega c, these are the components are present only at these 2 frequencies when the impulse at omega c is minus j pi. Therefore it is minus 90 degree just plus 90 degree. Therefore, this is pi up on 2 and this is minus pi up on 2.

So, this is the angle spectrum phase spectrum and this is the magnitude spectrum. Now, why do we stop at this point, when after all now you are able to find out the Fourier Transform of d c a pure sinusoid? 1 can as well go to the extreme case, of a pure of non sinusoid periodic functions. And indeed it can be shown that, the non sinusoidal periodic function which admits the Fourier Series can also, have a corresponding Fourier Transform. Let us see how it goes.

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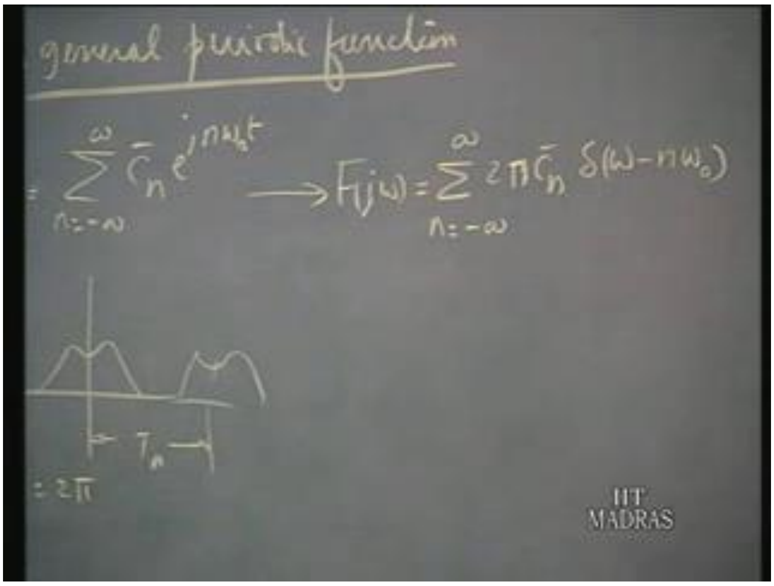

A general periodic function: Suppose, f of t is a periodic function which admits the Fourier Series representation in this manner; n from minus infinity to plus infinity. So, this periodic function is something which repeats itself at regular intervals, with a time period T n and omega not be is 2 pi over T not. So, that is the fundamental frequency. Now, we have seen that a constant will have a Fourier Transform which is 2 pi times delta omega times constants. This is 1 therefore; this is 2 pi times delta omega.

Now, each of 1 these terms in the Fourier Series expansion will therefore, have a corresponding Fourier Transform. And the summation of all those transforms of individual components will, be the Fourier Transform of the entire function. So, it is from the discussion, it is quite obvious F of j omega will be equal to. For this term C n of course, you have 2 pi times C n. If, it is C n alone, it would have been 2 pi times C n times delta omega, but you have e to the power of j n omega not t. Therefore, this becomes delta omega minus n omega not. And you have all such terms starting from, n from minus infinity to plus infinity. So, that is its Fourier Transform.

So, a periodic function of time, which has the Fourier Series, also has the corresponding Fourier Transform. Now, the Fourier Transform of this would have a magnitude spectrum like this; F of j omega. Let us see, a dc it will have 2 pi times C not, at omega not it has 2 pi times C 1 and so on and so forth. At minus omega not, it will have 2 pi times magnitude spectrum is even spectrum 2 pi C 1 and you may have a 2 omega not 2 pi C 2. These are the strength of the impulses etcetera.

Now, if you plot the Fourier spectrum of the Fourier Series in the convention of sense, what you would have done is the spectrum of the Fourier Series. What we talked about the spectrum when we were discussing the Fourier Series, we would have plotted a line spectrum like this C 1 C not C 1 C 2.

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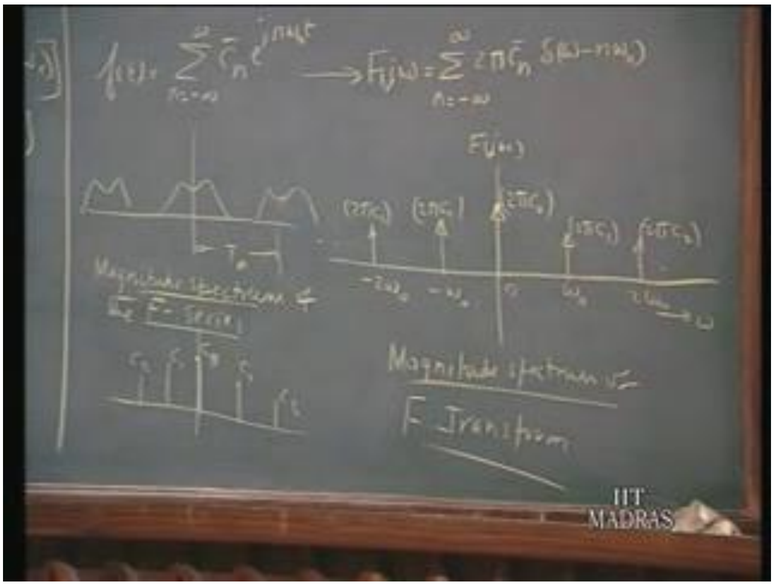

C 1 C 2 etcetera. This is the magnitude spectrum of the Fourier Series, magnitude spectrum of the Fourier Series. Now, what we have in the case of Fourier Transform is; instead of this being the coefficients we have impulses. So, instead of finite constants here, we have impulses. The strength of each impulse is 2 pi times the corresponding coefficients here. Therefore, this is the Fourier magnitude spectrum of the Fourier integral, in the sense; this is the magnitude spectrum of the Fourier Transform.

So, we have now seen that a Fourier integral gives more generalized concepts of, finding out the various harmonic components of a time functional function of time. It can be used not only for a periodic functions, but a periodic functions as well. Because, once we have the Fourier Series for a periodic functions in the Fourier

Transform domain all it means the same, line spectrum is carried out carried over into the transform domain as well. But, instead of coefficient finite values of coefficients, we have impulses each of which has strength which is proportional to the Fourier coefficients.

Since, this particular wave form has frequency component adjusting at 0 omega not 2 omega not and nowhere else, here also the coefficient density, we have concentration of energy at these particular frequencies and nowhere else. So, it is not a continues spectrum because, f of theta periodic not only here, here here. And even in the case of dc can be considered to be a periodic functions. So, the conclusion that, we get is when we have a periodic function of time then, the spectrum will turn out to be a line spectrum.

Let us now continue with our discussion and derive some additional Fourier Transforms of functions of time, which are not once again absolutely integrable over the infinite range. But at the same time they are not periodic as well, like the cases we just now considered.

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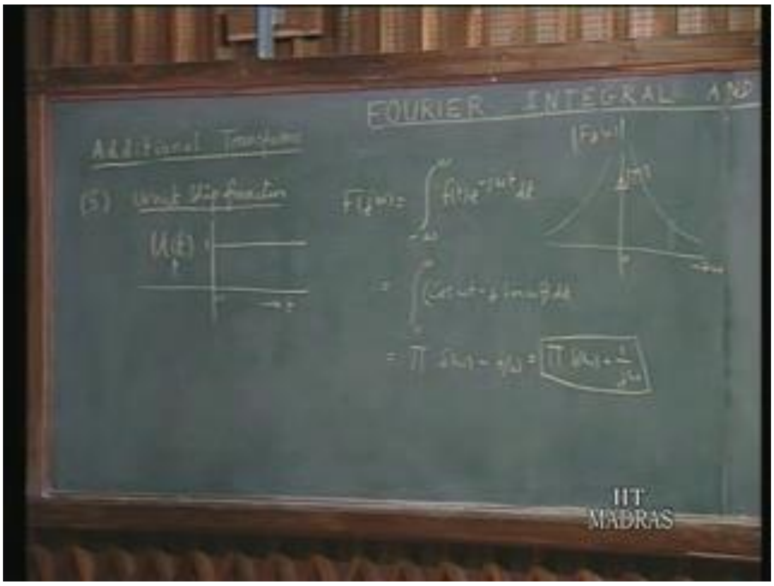

To start with, let us take the unit step function u of t. This of course, has a value 1 for positive t and 0 for negative t u of t is the unit step function in the sense. Now, 1 we like to find out the Fourier Transform for this. So, F of j omega 1 over 2 pi integral from minus infinity to plus infinity of f of t e to the power of minus j omega t. And since, f of t is 0 for negative values of time, you can start integration from 0 to infinity of f of t is 1 in that range. So, you have cos omega t minus j sin omega t minus j sin omega t dt. And from the formulas which we had developed earlier, this is seem to be pi delta omega, the formula which we noted down in the last lecture. And this will be minus j this will be pi delta omega plus 1 over j omega. This is the Fourier Transform of unit step function.

So, if you plot this spectrum, there will be an impulse of magnitude pi sitting at the origin and you have a continuous spectrum of decreasing amplitude as omega increases corresponding to 1 over omega. So, that will be the magnitude spectrum that we have. Now, look at these, this particular function of time u of t is either even or odd. Therefore, we cannot expect the Fourier either real purely real or purely imaginary.

Indeed it has got a real component and purely imaginary component.

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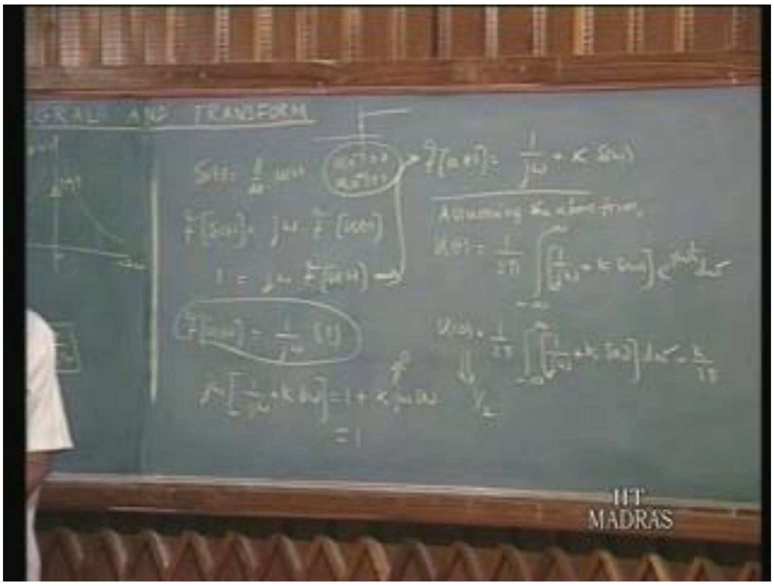

So, this unit step function therefore, has a Fourier Transform which is pi delta omega 1 over j omega. And let us see how we can justify this or we know that, unit step function is obtained by integrating a delta function or in other words, a unit impulse function is the derivative of the unit step function. Since, we know the rule for derivative of f of t how it gets transform in the transform domain. Let us see how both these relationships are related to each other. We know that delta t is d by dt of u t.

Therefore, we would expect that the Fourier Transform of delta t will be obtained by multiplying the Fourier Transform of u of t by j omega j omega Fourier Transform. So, you take the Fourier Transform of u of t multiply by j omega, you must get the Fourier Transform of delta t. On this basis we conclude, the Fourier Transform of unit step function delta function is 1. And this is j omega times Fourier Transform of u of t. If this is so, the Fourier Transform of u of t can we conclude from this; divide everything by j omega can we say this is j omega. If so, then Fourier Transform of u of t is 1 over j omega that is not in accordance what now just derived.

So; obviously, there is something inconsistent here. The steps here also seem to be harder, but the catch here is whenever you are dividing by j omega, you must also take possibility of a delta function is being presented. In fact, if you have j omega times 1 over j omega plus some K times delta omega. Suppose, I have an impulse of magnitude K sitting at the origin, if you multiply right through you have 1 plus K j omega time delta omega. And whenever a function f of omega is multiplied by delta omega, that f of omega can be replaced by a f of 0. Because, after all this delta function here has value at omega equal to 0 only and nowhere else and therefore, the value of this function at omega equals 0 is only 1 that matters. And in this case K j omega equals 0. Therefore, this is equals to 1 this is equal to 0.

So, in other words j omega multiplied by this is equal to 1. Therefore, if you take 1 over j omega it could be 1 over j omega plus K omega delta omega as well. So, that is where, we have a discrepancy here. This is of course, 2, but it is not a general expression. The general expression for this would be F of j omega from this it follows that, the general expression for this, you want to solve for this f of u t must be Fourier Transform of u of t. The most general way in which this equation can be solved is if you write 1 over j omega plus K delta omega.

So, this is whenever you are dealing with impulses 1 has to be careful in this manner, in the algebraic solutions of equations, you divide by j omega you must also take into account the possibilities that it is the delta function is there. Because, if you multiply by this j omega it is equal to 1, the second term dropped out. Having agreed to this, now what is the value of K? How do we get the value of K? In the earlier derivation we saw that K equals to pi. So, how do we justify this pi from these reasoning?

Now, let us see assuming that Fourier Transform of assuming this relation assuming the above relation let us, calculate u of t u of t therefore, must be. So, assuming the above, assuming the above form. We can obtain u of t through the Inverse Fourier Transform formula which is 1 over 2 pi minus infinity to plus infinity of F of j omega which is 1 over j omega plus K delta omega e to the power of j omega t d omega. That is the Inverse Fourier Transform formula.

Now, in this formula let me substitute the value after all, this is the function of t, this is the function of t. Let me substitute the value of t equal to 0. Now, what is the value of u of 0? I ask this question. And on the other side, I substitute t equals to 0. Therefore, 1 over 2 pi minus infinity to plus infinity of 1 over j omega plus K delta omega e to the power of j omega t now, becomes 1 d omega.

Now, we observe carefully the expression on the right hand side d omega by omega is an odd function of omega, 1 over omega is an odd function of omega and integrating between the symmetrical limits minus infinity to plus infinity. So, the positive and negative area cancels themselves out and therefore, the contribution of this term 1 over omega d omega whatever it might be will, be 0 whatever it might be on the positive side and whatever might be on the negative side it is going to be 0.

So, the only contribution that comes from this is from the second term. And when you are integrating from K delta omega over the infinite range, that infinite range covers the where the impulse function stands. And therefore, this will be K over 2 pi because, integral from minus infinity to plus infinity of delta omega d omega equals 1. Therefore, K by 2 pi. But, what is the value of the step function at the origin? You recall from the Fourier Series theory, we said whenever there is a discontinuity of any functions, the Fourier Series converges to the average value of the limit you get from the starting from the right side and the left side.

So, in other words here also u of 0 u of 0 minus is 0 u of 0 plus is 1. Therefore, u of 0 and the point t equals to 0, this will turn out to be half. This is how the Fourier Transform and Fourier Series both of them converge. And since u of 0 is half and that must be equal to K upon 2 pi K equals pi and that indeed is the values that we obtained here. So, this is how tied up the formula that we obtained, starting from the Fourier Transform of the impulse that we know already.

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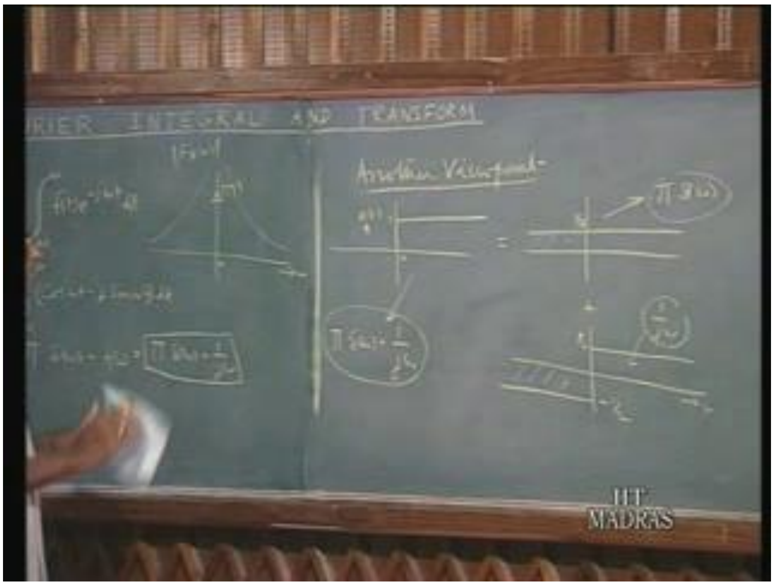

Now, let us look at some from another angle. Let us say another view point. We can always think of u of t to be the sum of; suppose, there is a pure dc term half, what else should be added to the pure dc term to get this? I must add to this a function which has the value half for positive t and minus half for negative t. If, I add these 2 functions of time, I would indeed to get unit step functions because, on the negative side this area is cancelled by this area and the positive side both of them both these values add. So, it will be plus 1 for positive t and 0 for negative t.

Now, we know that the Fourier Transform for this a pure dc if, is 1 it would have been 2 pi delta omega, but this is half therefore, this pi delta omega. And we know the Fourier Transform for this, we just now derived is pi delta omega plus 1 over j omega. So, it would appear that the Fourier Transform of this should have been 1 over j omega.

So, we have a unit step function can be resolved into 2 functions 1 is pure dc and another which has the odd function of time, which has the value half for positive t and minus half for negative t. And this is an odd function and therefore, its Fourier Transform is purely imaginary and everything ties nicely and therefore, it appears as if, that a function like this which has the negative value for negative t. And equal and opposite value, which is the positive value over positive t has the Fourier Transform of this type 1 over j omega. And this is the type important function that appears again and again.

So, we will try to derive this, a Fourier Transform for such functions independently that is, the what we will do next, but now we can see that a unit step function consists partly of a pure dc and partly of the function like this.

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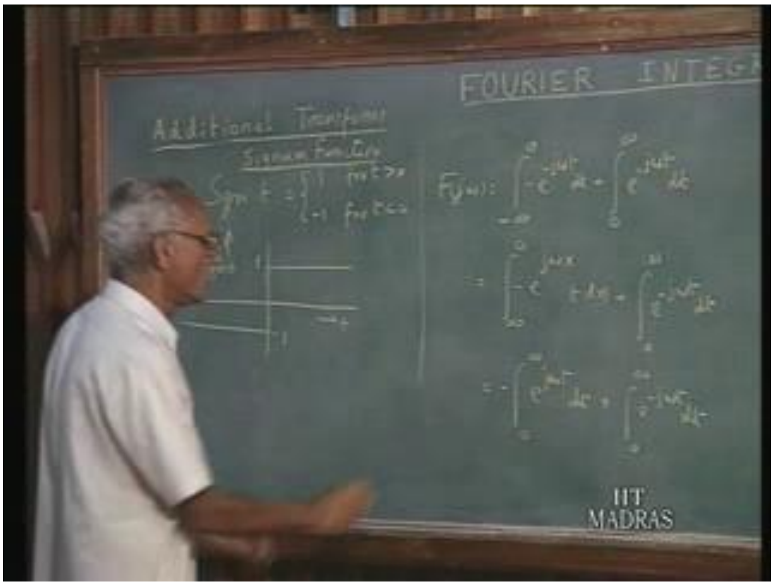

We just now saw that, an interesting function which has the constant for positive t and constant for negative t, but the 2 constants are negative of each other. A function like this is defined as a signum function and usually symbol like this. This is signum function of t denoted as sgn t has the value 1 for t greater than 0 and minus 1 for t less than 0. And such a function is will have a time variation like this. This is called signum function so; obviously, its Fourier Transform can be derived in the usual way; from minus infinity to 0 its value equal to minus 1.

Therefore, I will write this as minus e to the power of minus j omega t dt and I split it up this integration into 2 parts: minus infinity to 0 and 0 to infinity 0 to infinity its value equal to 1.

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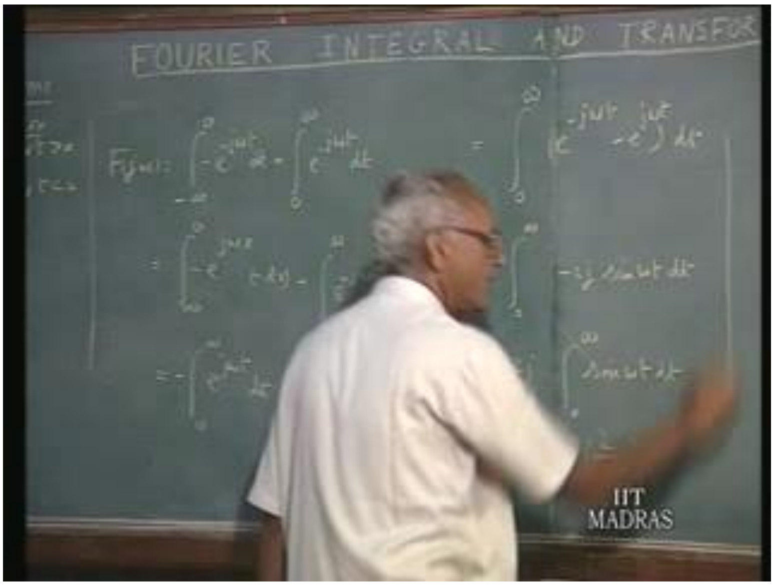

Therefore, 1 times e to the power of minus j omega t. Now, I can put t equals minus x if I do that, it will be minus of e to the power of minus j omega t equals minus x therefore, I can write this as plus x j omega x. And dt equals minus dx and the integrals limits will become infinity to 0. My idea is to combine these 2 terms in some fashion and put them between the same limits of integration 0 to infinity of e to the power of minus j omega dt.

Now, in this integration x is dummy variable of integration. I can reintroduce t and then, I notice as the negative sign here and then negative sign here therefore, they cancel each other out. And interchange the limits of integration therefore, there is a another negative sign is coming. Therefore, this becomes 0 to infinity minus of e to the power of j omega t dt plus 0 to infinity of e to the power of minus j omega t dt. We can combine these 2 as 0 to infinity of e to the power of minus j omega t minus e to the power of j omega t dt. e to the power of minus j omega t minus e to the power of j omega t dt 0 to infinity. This will be minus 2 j sin omega t minus 2 sin omega t dt. And minus 2 j can be pulled out the integral and what is the 0 to infinity of sin omega t dt.

And this integral has a special value, which we integrated in the last lecture that will be 1 over omega.

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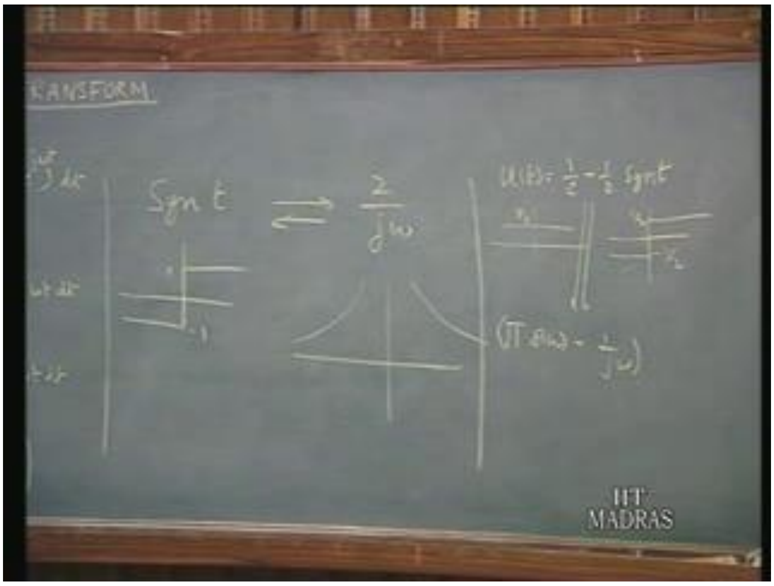

Therefore, this will become minus 2j over omega or I can write this as 2 over j omega. So, the absort of this now, to say that the signum function of t minus 1 plus 1 that is, signum function of t has its Fourier Transform 2 upon j omega. And therefore, when we had in the derivation of the unit step function, you recall that once again that let me redo what we done there.

A Unit step function can always be thought of; as 1 half plus 1 half of the signum function of t because, if you add a pure dc of magnitude half and the signum function which has minus half and plus half for negative and positive sign then, the sum of these will indeed result in u of t. And therefore, the Fourier Transform for this, from the dc part you get phi delta omega. It is the even function therefore, this is real and for the signum of t you get 2 over j omega but, we have a coefficient half here this is 1 over j omega that is the Fourier Transform.

So, a signum function can be represented in the fashion and it has a Fourier Transform which is 2 up on j omega. So, what we earlier said, 1 over j omega corresponds to a function of this type, we wondered if that is so then, we justified now based on this particular analysis. Now, you also observe that is the purely odd function of time therefore, the Fourier Transform turns out to be purely imaginary.

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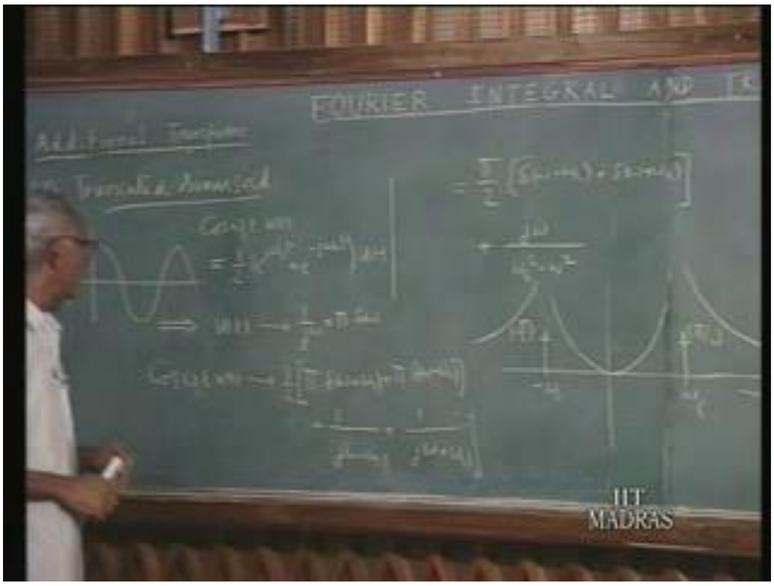

Let us take next, a truncated sinusoid. What we are really mean by this is; suppose, we have a function which is only lasting for positive t. So, we are taking cos omega c t u t; that means, for the negative values of time it is cut off. This we are calling truncated sinusoid. To find out the Fourier Transform for this, we recognized that, this is equal to half of e to the power of j omega c t plus e to the power of minus j omega c t times u of t. Now, that we have the Fourier Transform for u of t, we can find out the Fourier Transform of u of t multiplied by e to the power of j omega c t and e to the power of minus j omega c t.

So, this will be; we know that u of t has for the Fourier Transform 1 over j omega plus phi delta omega. Now, we have u of t multiplied by e to the power of j omega c t which means; instead of omega we have omega minus omega c and delta omega minus omega c. It is also multiplied by half. So, we take that into account. Similarly, u of t is multiplied by e to the power of minus j omega c t. What it means is; the Fourier Transform of e to the power of minus j omega c t times u of t would be 1 over j omega plus omega c plus pi times delta omega plus omega c.

So, on this basis cos omega c t u of t will have the Fourier Transform 1 half of pi delta omega plus omega c plus pi delta omega minus omega c, because of the multiplication by this. In addition you have 1 over j omega minus omega c plus 1 over j omega plus omega c. You put all these terms together, you can show that this will be equal to pi up on 2 delta omega minus omega c plus delta omega plus omega c t, 2 delta functions plus j omega by omega c squared minus omega squared.

So, the spectrum would be like this; at omega and minus omega c you have delta functions of strength pi up on 2. In addition, you have a continuous variation of the magnitude corresponding to omega over omega c squared minus omega squared. So, something like this we observe that. We are not taking a pure cosine wave lasting from minus infinity to plus infinity if e to the power of minus j omega t minus e to the power of j omega t dt. If, it had been a cosine wave lasting from minus infinity to plus infinity, there is only 1 signal frequency term that is present therefore, you would have plus pi omega c and minus omega c. This is not a pure cosine wave; it is cut off for negative values of t.

Therefore, it is not a periodic phenomena lasting from minus infinity to plus infinity.

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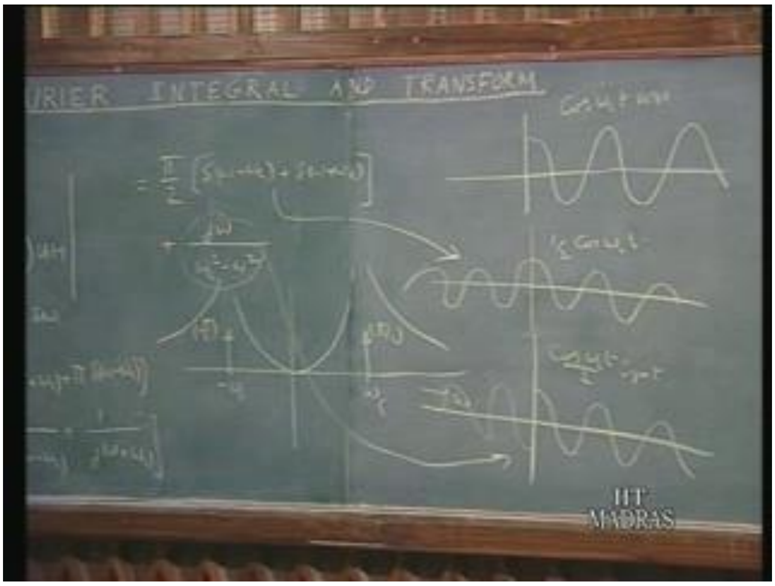

Therefore, you have other frequency components, not only the infinite the coefficient density plus omega c and minus omega c plus the continuous band of frequencies with various magnitudes as shown in this. In fact, you can show that, if I have a cosine omega c t u t, you can think of this as the sum of a cosine function which last forever. This is half cos omega c t plus the same function repeating for positive half under by the negative of this occurring for the negative t. So, this is cos omega c t divided by 2 multiplied by the signum function.

So, it is this part which contributes the delta functions. It is this part which contributes to this spectrum. So, the delta part corresponds to this and the continuous frequency. So, that is how it goes. So, a pure cos omega c t u t if, it is a cos omega c t as such without multiplication by u t, it would have been pure impulses at these 2 frequencies. But, because it is not periodic function in the sense it is lasting from minus infinity to plus infinity, we have the additional portion of the spectrum like this.

Similarly, you can derive the spectrum for sin omega c t u t, but we will not do that in this, leave that as an exercise for you. We will discuss 1 additional property in this lecture, which relates the convolution.

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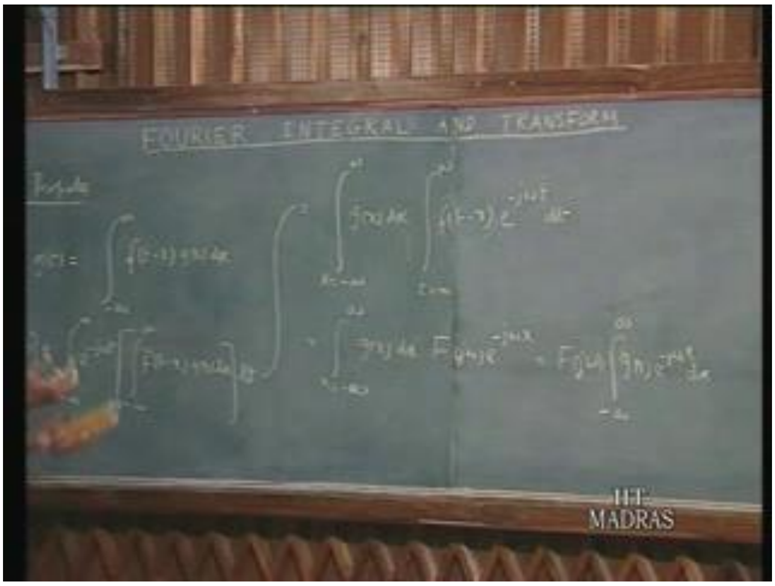

You recall that the convolution of f of t; 2 time functions f of t and g of t have been defined to be f of t minus x g of x dx from minus infinity to plus infinity. So, this is the function of time and this is defined in this manner; f of t minus g of x dx. The physical meaning of convolution etcetera we have discussed earlier. Let us try to find out the Fourier Transform of this, in terms of the Fourier Transform of f of t and g of t.

So, the Fourier Transform of f t g t convolve g t can be written as; minus infinity to plus infinity of e to the power of minus j omega t times this. This is the function of time now minus infinity to plus infinity of f of t minus x g of x dx. This is the function of time, the new function of time dt. This is what the Fourier Transform of this would be. This is integration with respect to time t this is integration with respect to x.

Now, let us interchange the limits of integration. This is t from minus infinity to plus infinity, this is x minus infinity to plus infinity. Let me interchange the limits of integration; let us interchange the order of integration. I would like to do integration first with respect to t and then later with respect to x. So, all terms which are independent of t can be put outside. Therefore, I put g x dx and those which are depended on t, I will put inside the first integral which is to be done; f of t minus x e to the power of minus j omega t.

Now, from the property of the translation in the time domain, if f of t has the Fourier Transform of F of j omega, when you have f of t minus x, the Fourier Transform of this we know is F of j omega multiplied by e to the power of j omega x. So, this turns out to be x from minus infinity to plus infinity of g x dx e to the power of F of j omega multiplied by e to the power of minus j omega. And in this we are integrating with respect to x. So, i can write this as F of j omega can be pulled outside the integration sign and then, you have minus infinity to plus infinity of g of x e to the power of minus j omega x, which is the standard formula for the Fourier Transform of g of t, instead of t, I have put x is the dummy index.

Therefore, this is indeed G of j omega therefore; F of j omega multiplied by G of j omega is the Fourier Transform of this. So, that is the nice result, in the sense that, you have the convolution in the time domain corresponds to multiplication the transform domain.

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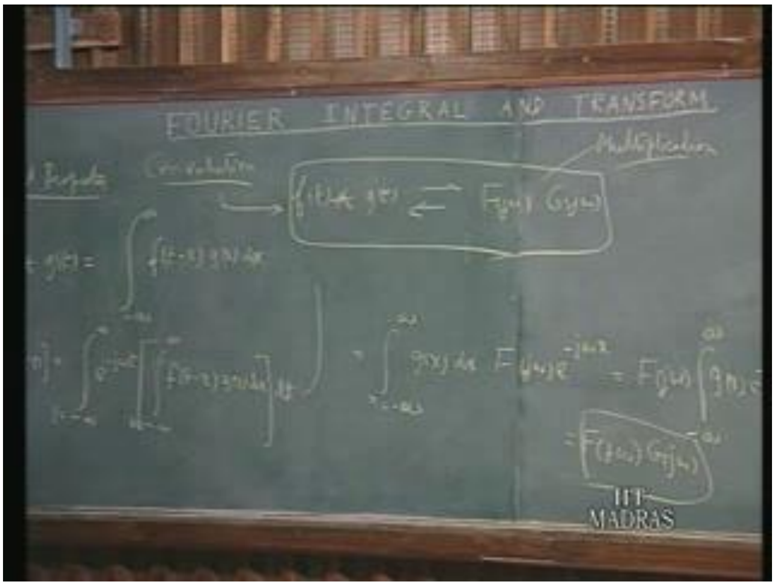

So, f of t convolved with g of t which is complicated integral operation is, carries over in the frequency domain or the transform domain has pure multiplication of the 2 time functions. So, convolution in the time domain corresponds to multiplication in the transform. This has a interesting application in system and network theory which will, come across as go along. And in the convolution formula here, when you take the, apply this Fourier Transforms; Fourier Transform theory, all functions of time for assumed to be lasting from minus infinity to plus infinity.

So, the limits must be taken from minus infinity to plus infinity. This may be different when, you go to Laplace Transform later on, but for the Fourier Transform theory is concerned, the convolution must take minus infinity to plus infinity. And 2 functions which are convolved in time as far the transform domain is concerned, it is equivalent to multiplication in the transform domain.

In this lecture, we first of all looked at the Fourier Transforms of periodic functions of time. To start with a pure dc, will have in the Fourier Transform is spike that is, a impulse at the origin and the Fourier Transform is 0 everywhere else indicating that, concentration of energy is only at 1 frequency that is the, dc. Then, we took up a sinusoidal function of time angular frequency omega not and we found in the Fourier spectrum we have, 2 delta functions at plus omega c and minus omega c.

Therefore, the spectrum consists of 2 impulses sitting at these frequencies; plus omega c and minus omega c. That means: there is no energy at all other frequencies. We extended this concept to a periodic function of time, which admits the Fourier Series representation. In the Fourier Series spectrum consists of lines having a finite amplitudes that is, the various coefficients; C n coefficients and this is a line spectrum in the case of Fourier Series.

But in the case of Fourier integral, it is continued to be a line spectrum because, the energy is only at the discrete frequency. But, now since we are talking in terms of coefficients density, the line spectrum consists of series of impulses sitting at the various discrete frequencies. And each strength of each impulse is related to the Fourier coefficients, in the Fourier Series representation by a factor 2 pi. That means: instead of C not in the Fourier coefficient, we have 2 pi times C not being the strength of impulse in the Fourier spectrum.

After having dealt with, the periodic functions of time, we looked at functions of times at the unit step function or a sinusoidal function, which last only for positive t and then try to look at the Fourier spectrum of this. We notice that, this is not a pure periodic phenomena, this has an a periodic attached to it. Therefore, you have in this spectrum, concentration of energy at 1 or more discrete frequencies corresponding to the basic character of the time function. If, it is unit step function, the concentration of energy is dc. If, is cosine omega c t u t then, the concentration of energy is plus omega c and minus omega c.

But, since these are not periodic functions, their cut off for negative values of time, you have also a portion of the spectrum, which is continuous, which is representative of periodic function. Therefore, we have continuous spectrum as well as 1 or more lines representing the particular frequencies which are present. We saw that, in the analysis of such functions, the signum functions play the important role. Signum function is the 1 which, has got minus 1 for negative t and plus 1 for positive t. And we saw a signum function of the Fourier Transform 2 upon j omega.

Lastly, we look at the convolution property in the time domain and how it is going over in the transform domain, a 2 time functions f of t and g of t. When they are convoluted in the time domain, in the transform domain the result in function of time, will be having a Fourier Transform which is, the product of the Fourier Transform of the individual components, which are convolved together. In particular, f t star g t which is the convolution of f of t and g of t, has the Fourier Transform which is, F j omega times G of j omega. This particular property is very useful in network and system study and we will see later.

In the next lecture, we start with an example illustrating the convolution property and we will go on from there