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Fourier Transforms

Lecture No.#16

In this lecture, we will continue our discussion of the various properties of the fourier transform. To start with let us consider aspects related to energy.

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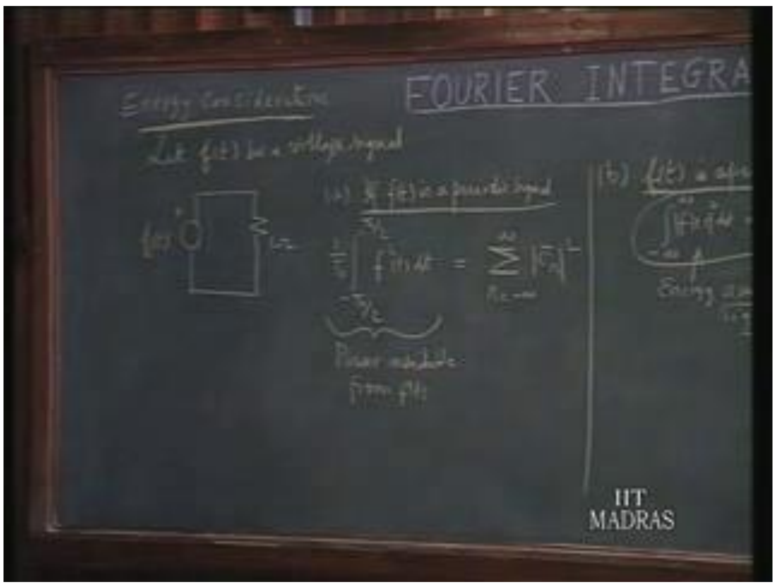

Suppose f of t be a voltage signal. Now, let us imagine that such a voltage is given to 1 ohm resister. Then if f of t is a periodic signal then the power dissipated in the 1 ohm resister is from minus t up on 2 to plus t up on 2 of f square t dt. The average value 1 over t not this is called the power dissipated in the signal or we often refer to power available from f of t. So, irrespective whether the signal is a voltage signal or something else, normally; we refer to this as power available from f of t of course it is quite appropriate if f of t voltage signal.

Even otherwise, we can refer to this as a power available from f of t. And this is a periodic signal and we saw that this quantity power available from the signal can also be obtained from the fourier coefficients as n from minus infinity to plus infinity of cn magnitude square. This is the relation which we already discussed when talked about the fourier series expansion of the periodic signal.

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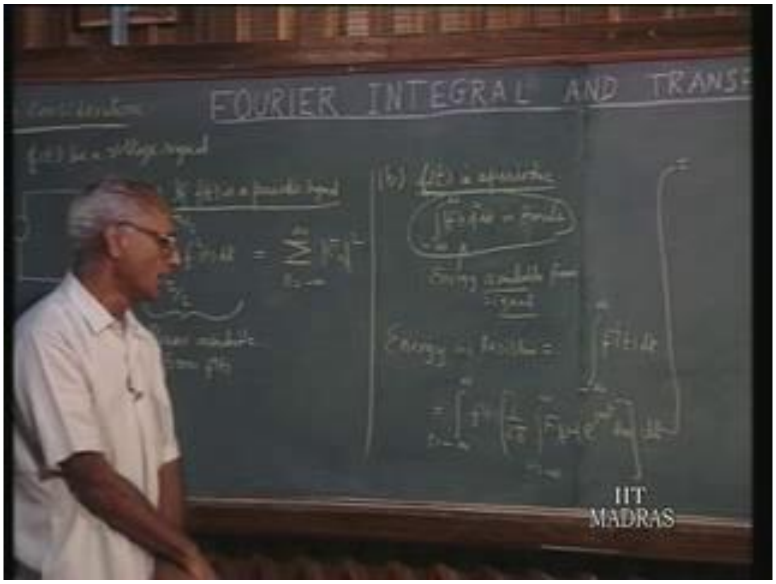

Now, what we like to see is how the concept can be extended in the case of f of t which is not periodic, f of t is a periodic. And we also assume that it has got a finite amount of energy that in the sense that minus infinity to plus infinity of f of t magnitude dt is finite

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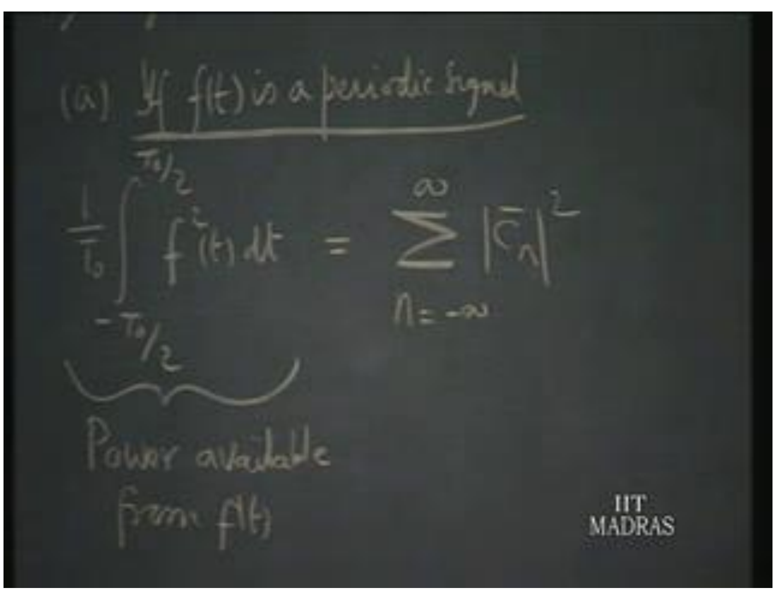

This is the restriction which we already imposed on finding the fourier transform of the various signal that we have considered so far and we also assume that this is also true. So, this is so, then when you divide by t not, when t not goes to infinity then the power become 0. So when we talk about a periodic signals of this type where the square of the integral is over the infinite band of frequencies infinite time is finite then there is no meaning in talking about power because power become 0.

So, all that we can do in such cases is only calculate the value of the integral f square t dt and that has the dimensions of energy because we are not dividing by t not. So, this is energy in the signal is finite is non 0.

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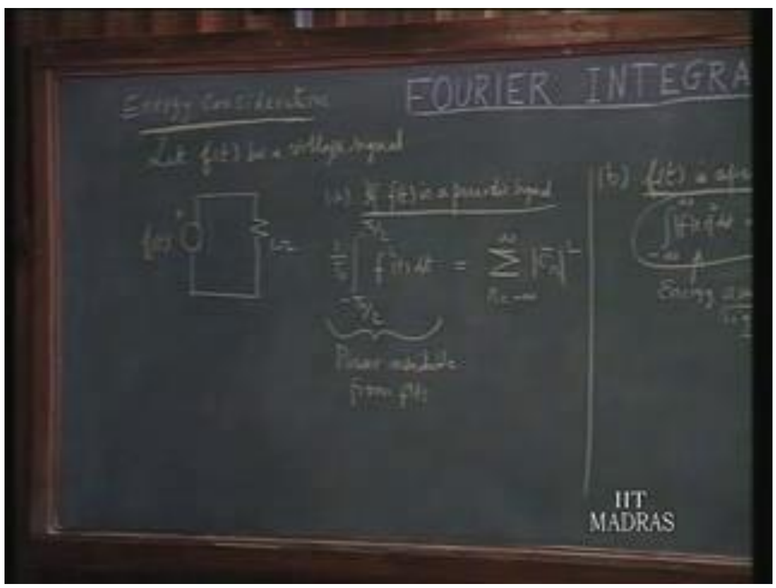

Energy available from the signal so, when we talk about any periodic a signals it would be more convenient for us to talk in terms of energy available from the signal therefore we will have the this particular heading for this topic energy consideration. So, in the case of periodic signals we talk about energy available from the signal where we are talking about periodic signals, we talked about power available from the signal. So, we like to see what is the energy available from the signal in terms of the fourier transform.

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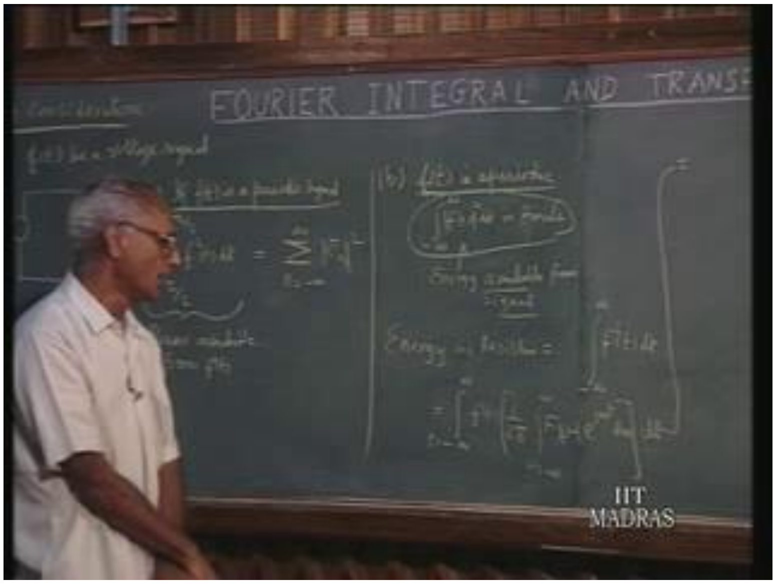

So, the energy that is dissipated in the 1 ohm resister is given by this. So, that energy in 1 ohm resister, energy dissipated in the 1 ohm resister is minus infinity to plus infinity of f square t dt. And this we like to write as minus infinity to plus infinity t is running from minus infinity to plus infinity f of t and other f of t or use the inverse transform formula. Which is 1 over 2 pi omega from minus infinity to plus infinity f of j omega e to the power of j omega t d omega. This is the expression for the f of t so, instead of writing f square t i multiply f of t by the another equivalent expression for f of t times dt.

So, this has got 2 integrals 1 is on time domain and 1 in the frequency domain. What i like to do is interchange the order of integration. That means, instead of integrating the omega domain first and later on the time domain. I like to reverse the role i like to integrate the expression in terms of time domain first and then frequency domain later.

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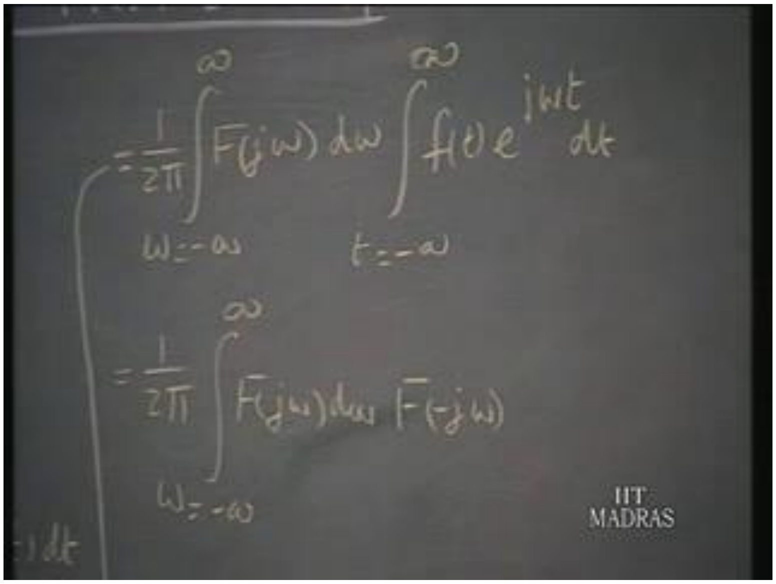

So, if I do that then i want to integrate in the frequency domain at the second stage. So, I put outside and all terms involving omega which do not involve time can be brought outside 1 over 2 pi come out of course, f of j omega will come and the rest of the terms are, f of t e to the power of j omega t and of course d omega is also outside. And you have dt f of t e to the power of j omega t these are the terms that are left. So, in the time integration t minus infinity to plus infinity I have f of t e to the power of j omega t dt. Now let us, look at this if it had been f of t e to the power of minus j omega t dt it would have been straightaway f of j omega.

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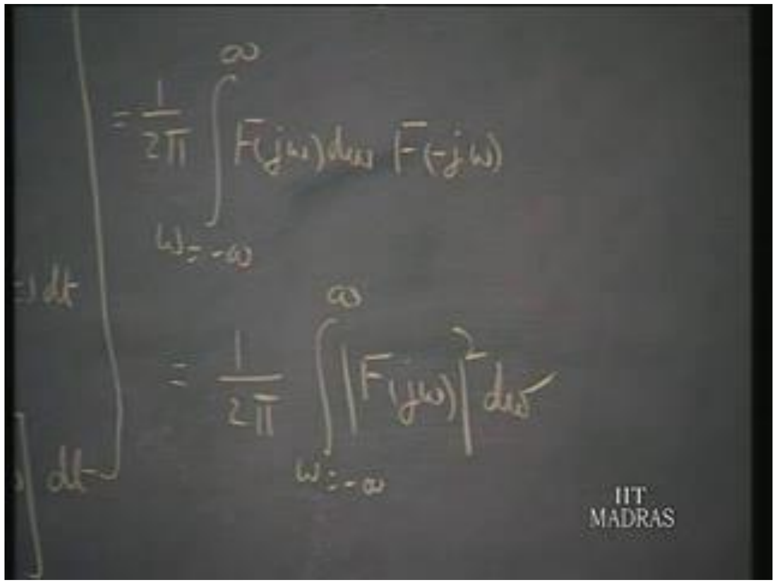

But, instead of minus j omega t we have plus j omega t. Therefore, this whole expression now become f of minus j omega so this can be written as 1 over 2 pi omega from minus infinity to plus infinity f of j omega d omega and this quantity is f of minus j omega and we already observed that for real functions of time your f of t is a real function of time, f of minus j omega is the conjugate of f of j omega. So therefore, this will become 1 over 2 pi omega from minus infinity to plus infinity of f of j omega magnitude square d omega.

So, the power available, energy available from the signal is given in time domain in this fashion and frequency domain in this fashion.

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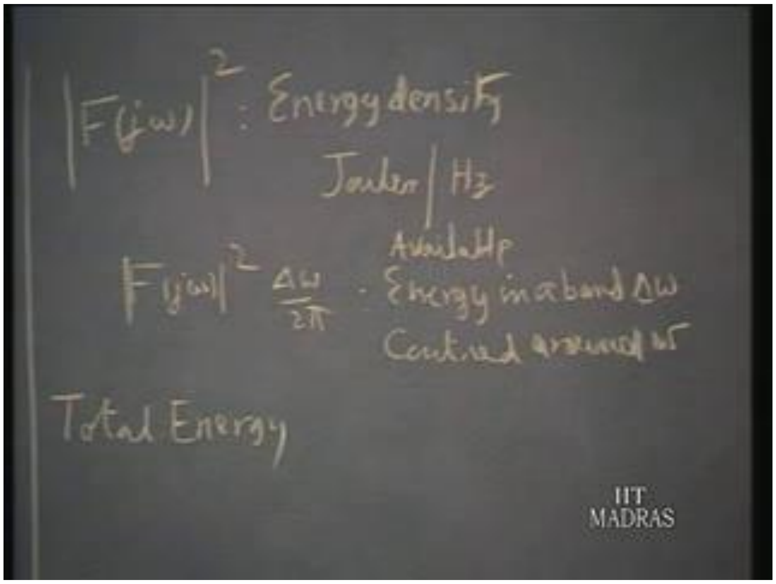

Now, f of j omega squared magnitude squared can be thought of as a energy density. What it means is; around the band, around omega. If we consider a small band of frequency delta f, f j omega squared time delta f is the energy concentrated in that band centered around omega. So, this energy density is expressed in terms of joule per hz. So, in a small band of frequency centered around omega this will be f j omega squared magnitude squared df which is delta omega by 2 pi is the energy available. available energy in a band delta omega centered around omega.

So, if you take a small band of frequencies centered around omega, small delta omega this is the energy that is available all we have to do is; therefore, to find the total energy. We have to sum up all such energy say quanta in different bands and this become this integral,

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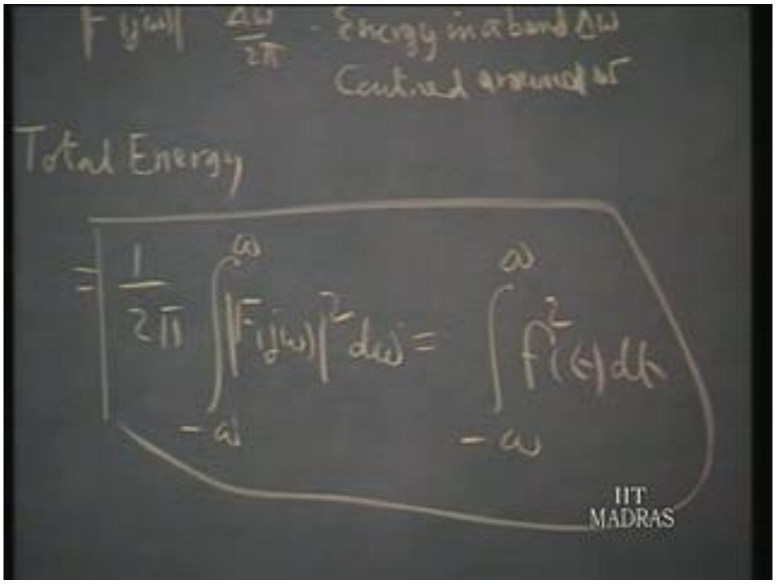

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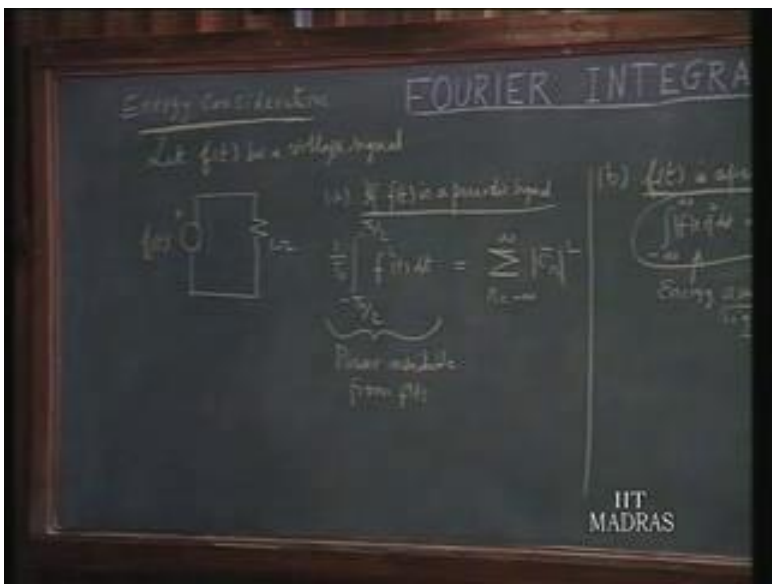

that becomes 1 over 2 pi minus infinity to plus infinity of f of j omega magnitude square. So, this is an interesting relation that f of t this of course is, minus infinity to plus infinity of f square t dt. So, you can calculate the energy in time domain minus infinity to plus infinity f square t dt. That is; called energy available from the signal f of t. You also calculate the energy available from the signal frequency domain in this fashion. f j omega squared is the energy density, energy available per cycle per second.

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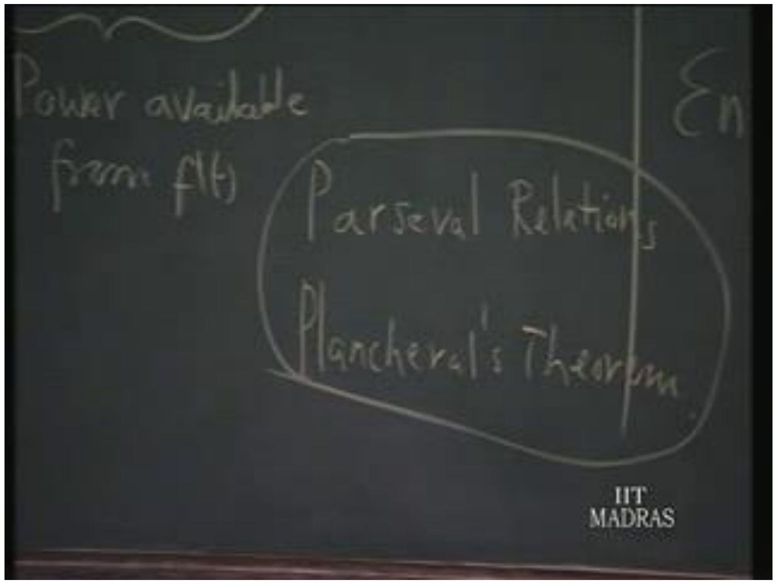

So, you have to multiply by frequency d omega by 2 pi integrate over them whole range of frequencies you get the total energy available from the signal. These relations, relation of the periodic case and this relation for the periodic case are usually referred to Parseval's Relation. It is also case refer to Plancheral's theorem particularly in the case of periodic case. So, Parseval's Relation in the case of, a periodic signal is in terms of the fourier coefficients in the case of a periodic signals in terms of integral; involving the energy density f of j omega magnitude squared d omega.

To summarize discussion here, when you talk about a periodic signal with finite amount of energy, then you cannot talk about the power available from the signal which is going to 0. Because, the base is t not is going to be infinity the average power over a complete time period is 0 we talk about only energy and the total energy available from the signal can be computed in 2 ways;

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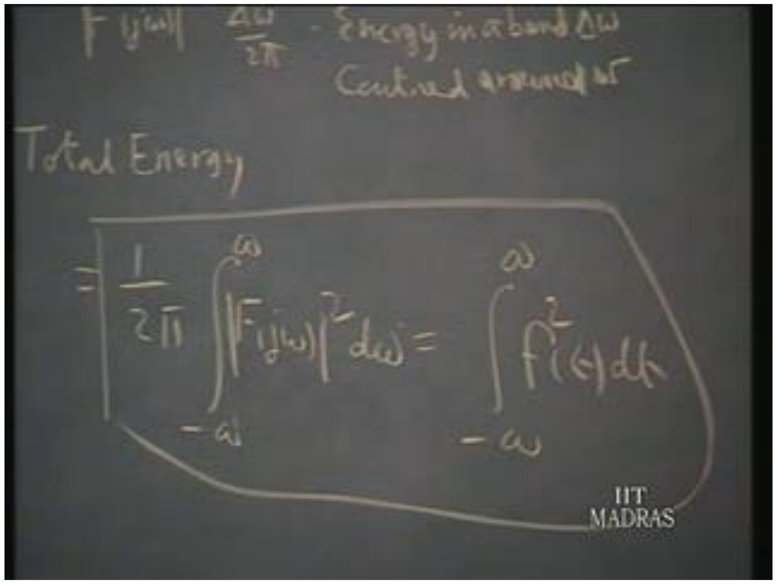

one in time domain this fashion and 1 in the frequency domain. The meaning of this is, if this is a voltage signal this is given to 1 ohm resister. This is the energy dissipated in that 1 ohm resister. You can calculate it either in time domain, frequency domain the case may be. Now, let us work out an example to illustrate the particular concept.

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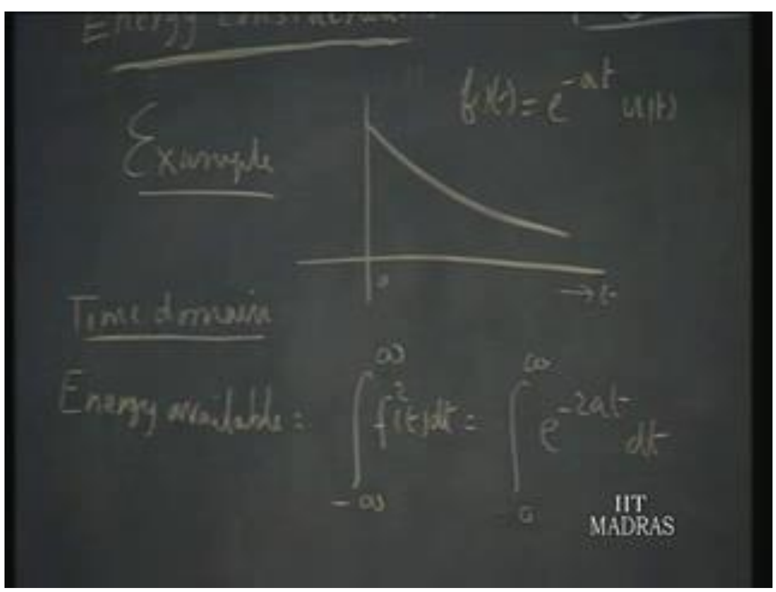

Take this example; we have f of t e to the power of minus a t u t. Now, we like to calculate the energy content in the signal energy available from the signal in 2 ways. Time domain doing with the time domain energy available integral from minus infinity to plus infinity of f square t dt. That is; minus infinity to plus infinity of e to the power of minus 2 a t dt. But this lower limit is not correct because f square t exists only from t equal to 0 to infinity; therefore, we have to write substitute 0 to infinity e to the power of minus 2 a t dt.

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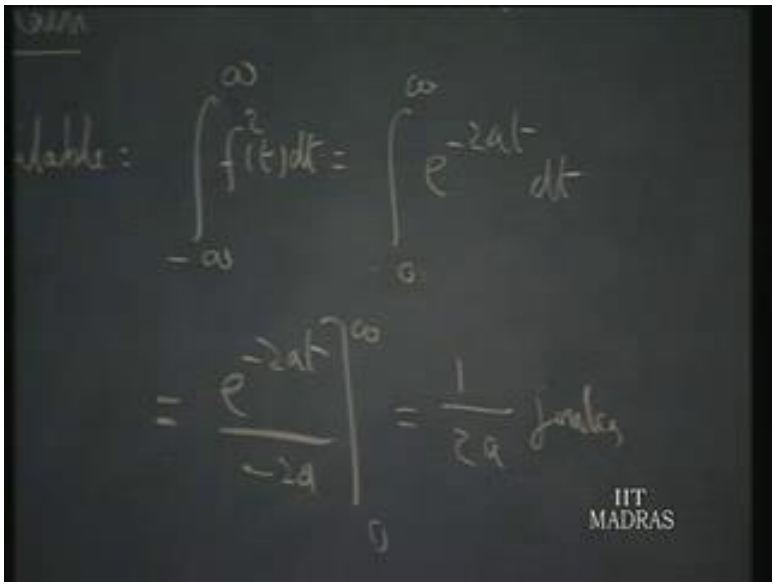

Therefore, this will become e to the power of minus 2 a t divided by minus 2 a evaluated between 0 and infinity, but the upper limit this is going to be 0 because are taking to be a the positive quantity and the lower limit of this is 1 this will become 1 over 2 a so 1 over 2 a joules is the total energy available from the signal.

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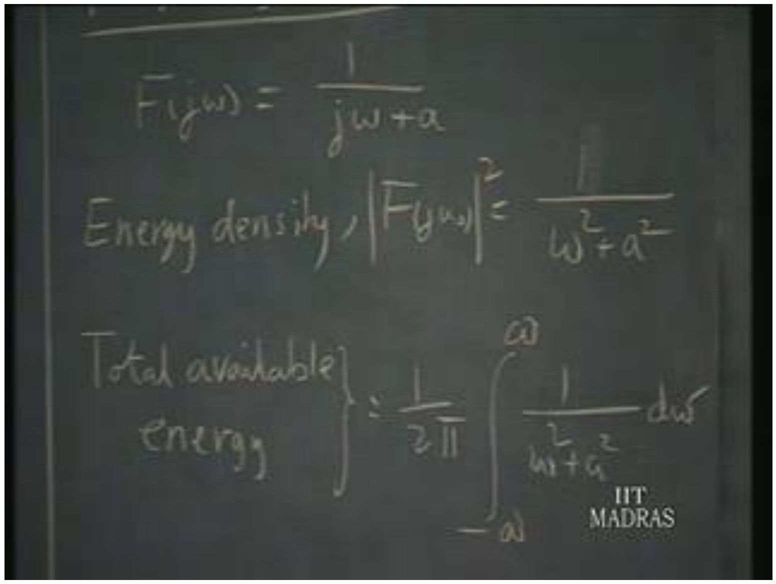

Let us work out, in frequency domain f of j omega is 1 over j omega plus a, but we are talking in terms of energy therefore energy density is f of j omega magnitude squared is 1 over omega square plus a squared so many joules cycles per second. So, the total energy available from the signal is the 1 over 2 pi times the integral minus infinity to plus infinity 1 over omega squared a squared d omega.

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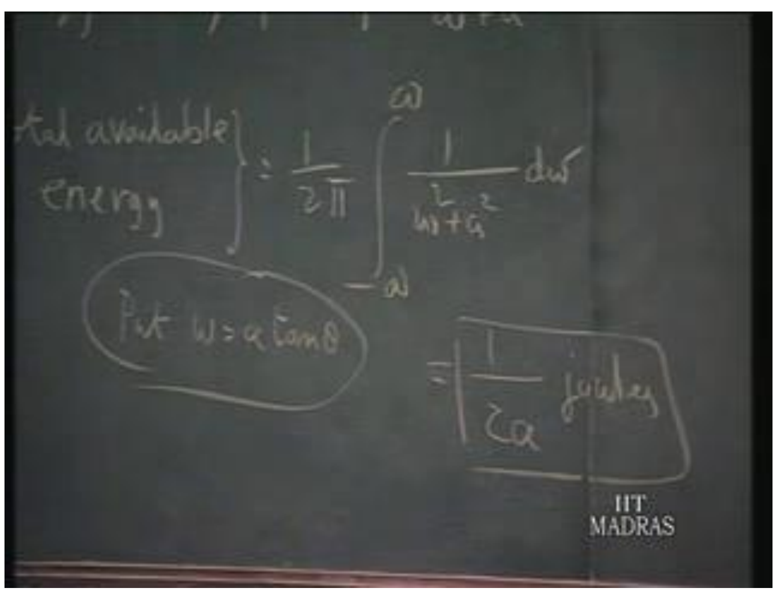

and this can be shown by substituting omega put omega as a time tan theta. And carry out the integration and you can show this is indeed 1 over 2 a.

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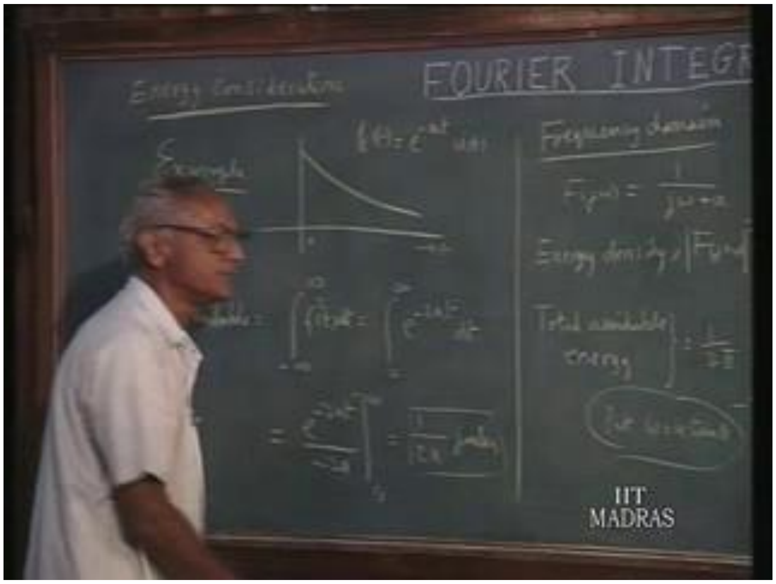

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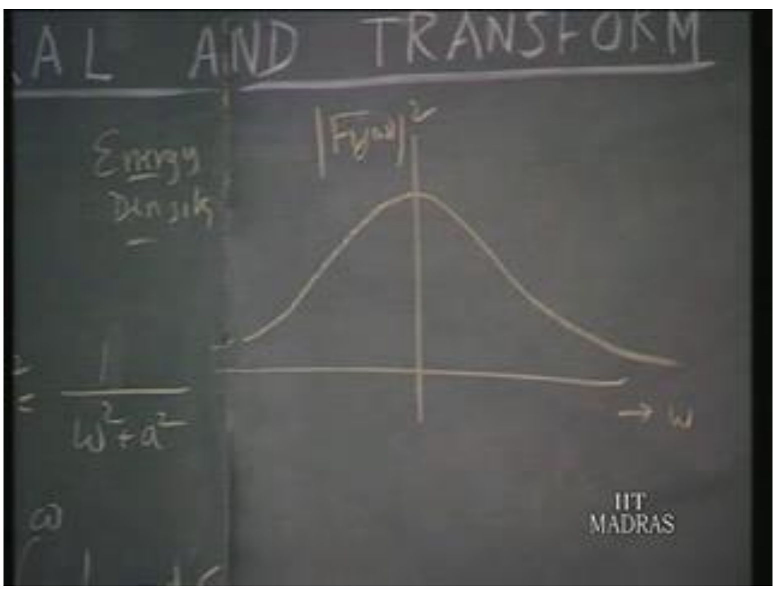

Which is the exactly the result that we have obtained earlier, if you plot f of j omega squared versus omega, the energy density is a function of frequency omega. You will get a curve like this. So, this is the energy density. So, this gives us an idea where the energy is concentrated at what frequency band there is more energy. Now you can see, there is a lot of energy in the low frequency band centered around the D. C. But higher frequency the energy is somewhat less. This additional information is obtained through the fourier transform analysis as compared with time domain analysis.

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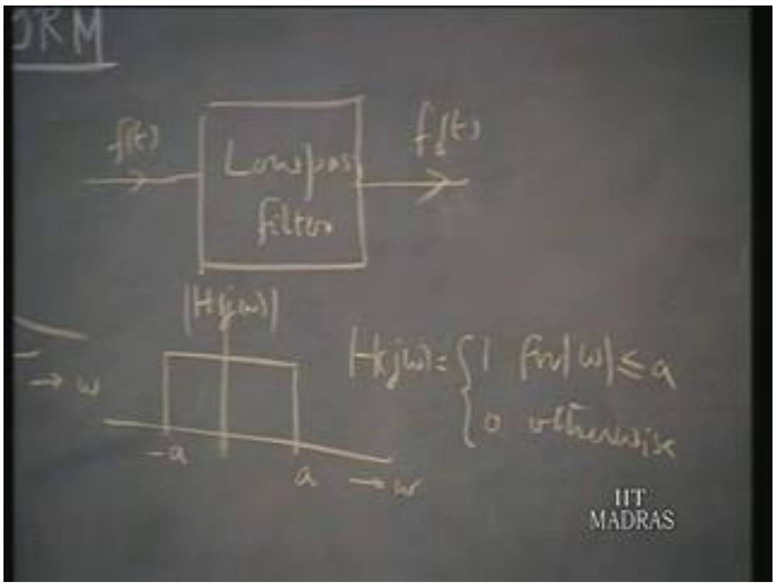

Let us see, how this such a concept can be made use of in practice. Let us assume that, this particular f of t which we had the exponential signal is put through a low pass filter. And we get output signal f not of t this low pass filter has a characteristic like this. H of j omega magnitude a low pass filter essentially allows all frequencies up to a certain point then shuts off all other frequency components.

So, let us imagine that a low pass filter here has a cutoff frequency equal to a and minus a. In other words in the spectrum that is available for the given f of t all frequency components having a magnitude omega greater than a become 0. That means; H of j omega we are talking about the transfer function or the system function relating f not of t and f not f t is equal to 1 for omega less than or equal to a and 0 otherwise. So, essentially what we are doing is, this low pass filter is such cuts off all frequency components beyond certain value a.

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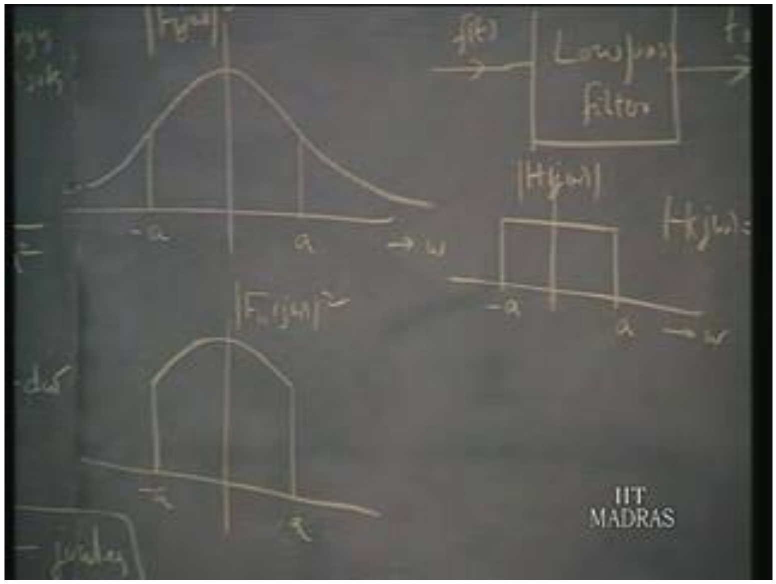

So, if this is a, this is minus a, as far the output is concerned f not of t. The corresponding energy density f not of j omega squared would be the 1 which contains frequncey components within this band only. That means, it will have only this minus a plus a the rest of the terms are being cut off. And so, suppose I want to know; what is the energy available from f not of t? Then you have to take the same function but integrate between minus a to plus a rather than from minus infinity to plus infinity.

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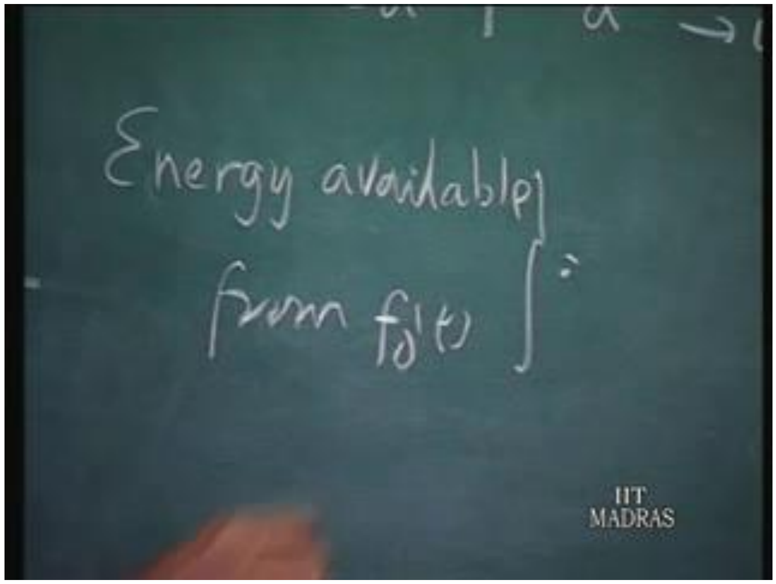

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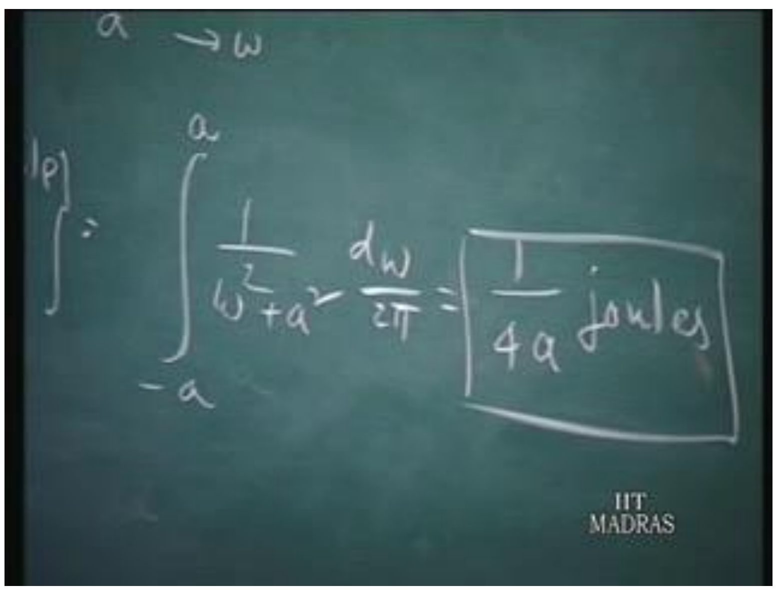

So, we have the energy available from f not of t equals the integral of 1 over omega square plus a square. That is the energy density into d omega over 2 pi the integral now, from minus a to plus a instead from minus infinity to plus infinity. And then, making use of the standard substitution that omega equals to a time theta this can be shown to be 1 over 4 a joule. It means, that out of the total energy available from the input signal ft which is 1 over 2 a joules 1 half of that energy resides in the band, frequency band omega from minus a to plus a.

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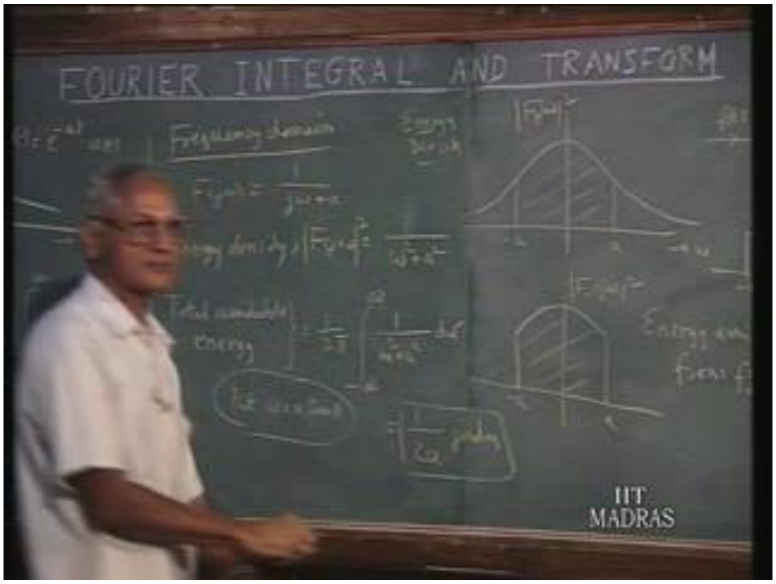

So, since when signals are transmitted through linear system. Like a low pass filter and these systems are characterized quite nicely in terms of, the system function H of j omega. And if you like to find out, the properties of the signals that come out the system or the response of the system then, frequency analysis in this manner trans out to be very convenient tool because filters are like this characterized in terms of the frequency response. Not in terms of their time response and therefore, the energy available from the f not t can be more conveniently to handle in terms of frequency characteristic of the signal in terms of the spectrum as shown here.

If we wanted to the same analysis in term of time domain, then it would be much more difficult because you have to characterize the filter. In terms, of the time response which can be done but it much more complicated. So, these examples illustrate how we can utilize the energy concepts of also in finding out the characteristic of various signals. So far, we have been talking about signals with a finite energy content and signals which are absolutely integrable over the infinite range of time from minus infinity to plus infinity.

You might recall, that I mentioned earlier that we can extend the concept of the fourier transform to signals which are not absolutely integrable over the infinite range and in particular i mentioned that signals like u of t a dc term, sin omega c t cos omega c t. Such functions also admit fourier transforms provided we use impulse functions. Which are justified from the theory of distributions in mathematics? We would now, like to familiarize ourselves with some of the operational rules involving such functions. We will not go in deep mathematical details of this we will just assume some results and use them to derive the fourier transform of signals, whose energy content is not lesserly infinite or whose is not finite.

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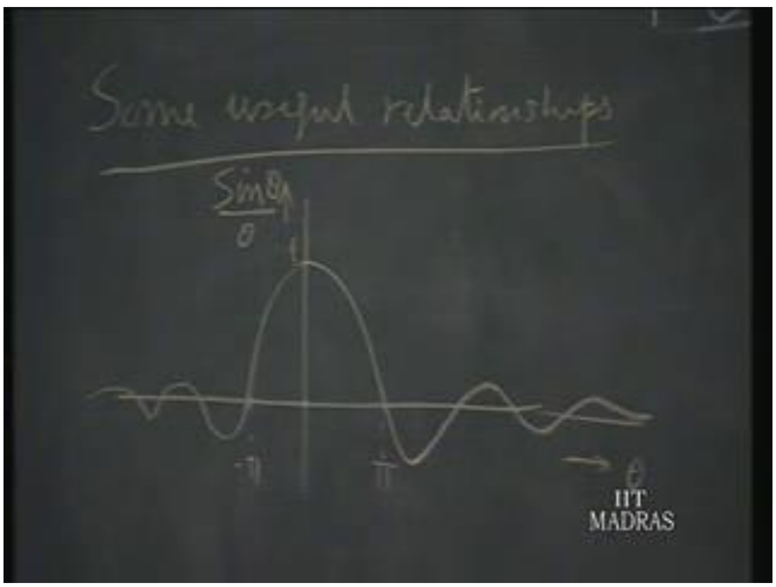

So, to start with we like to have some useful relations. We have discussed earlier sin theta by theta. This will have a variation like this, with an amplitude 1 and this will be pi and this is minus pi.

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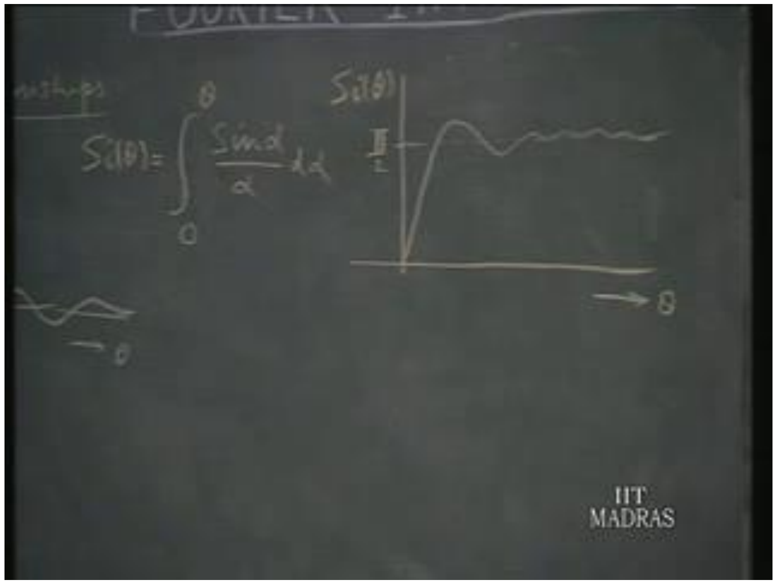

We are interested in the integral such a quantity. Suppose we say 0 to theta of say sin alpha by alpha d alpha, such an integral is referred to a sine integral of theta. This is given in the symbol si of theta; sin integral of theta sin alpha by alpha from 0 to theta. And the variation of this is pronounced as sine integral of theta versus theta would be something like this; asymptotically it reaches the value which is equal to pi by 2.

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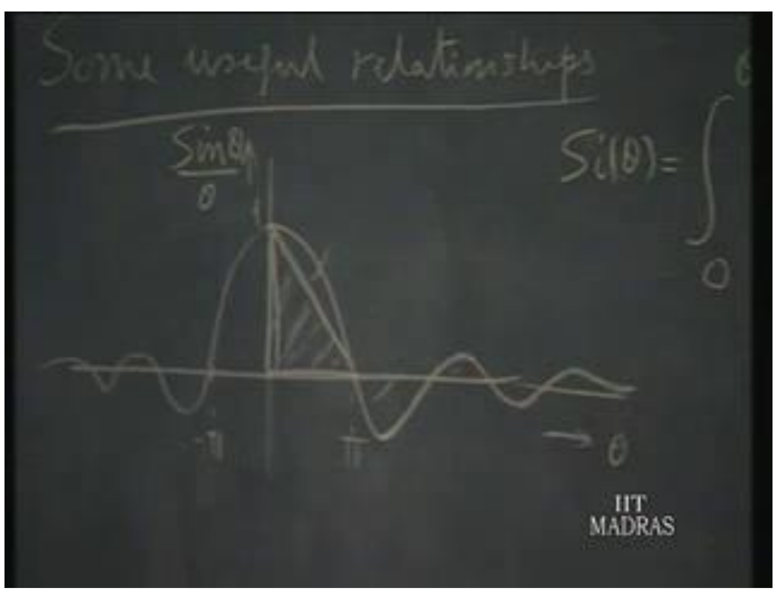

So, this is the relationship which we assume, that means if you start from this point and go on integrating it reaches a value pi by 2. That means, the integral the area under this curve starting from theta equal to 0 to theta equal to infinity will be pi by 2. It is easy to remember this particular value because if you take this triangle, this is pi this is 1, the area of the triangle is pi by 2. So, the area of the triangle is pi by 2 and that is also the area under the curve from 0 to infinity. Which means that, the rest of the area; this area, this 1 and this 1 all add up to 0.The positive areas are cancelled out by the negative areas and therefore, the areas the curves sin theta by theta from 0 to infinity becomes pi by 2.

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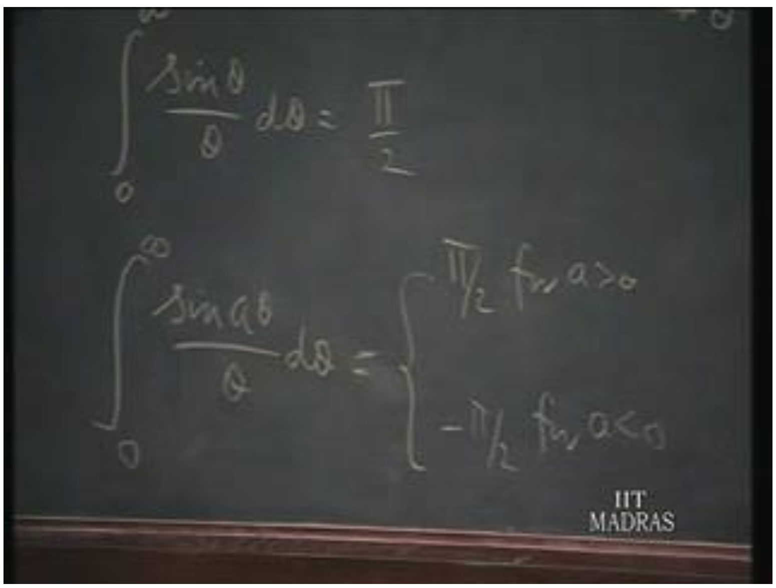

So, we have the relation; therefore, 0 to infinity of sin theta by theta d theta equals to pi by 2. Now, sometimes we like to have instead of theta we have sin a theta by theta d theta 0 to infinity. Now, if a is positive then certainly we can say d of a theta by a theta then this same relation as this. And the limits are not interchanged therefore this will also be equal to pi by 2. Because a theta can be replaced by another variable x, but if a is negative then no doubt d a theta by a theta a can be cancelled out. But if a is negative instead of plus infinity you have minus infinity. And you know, sin theta by theta is a even function of time, but you are integrating the negative direction; therefore, it will be minus pi by two.

So, the result is this value pi by 2 for a greater than 0 equals minus pi by 2 for a less than 0. Or in other words, if you extend this sin theta sin integral theta in the negative it will be like this. And it reaches minus pi by 2 eventually.

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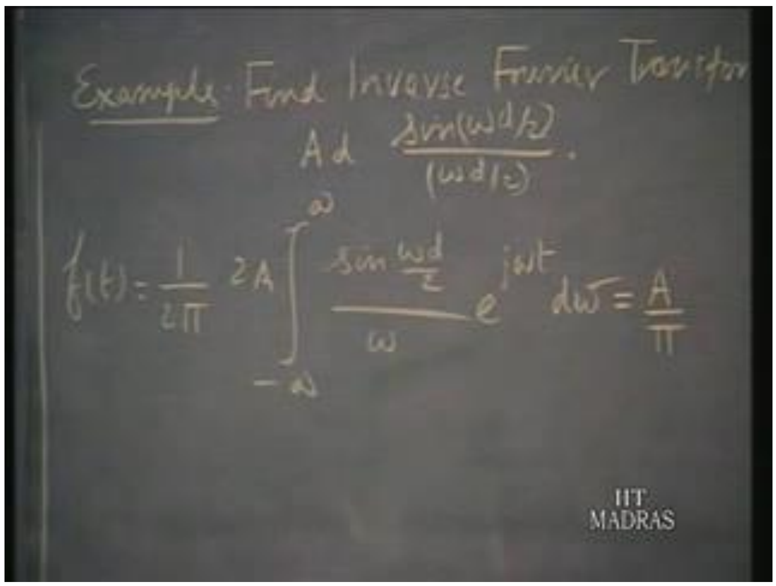

This is the relationship which is useful for us in our work. Just illustrate this particular rule. Let us consider a example once again. Find inverse fourier transform of A d sin omega d upon 2 by omega d up on 2. We know this results, we know that sin omega d up on 2 by omega d up on 2 times A d is the fourier transform of pulse function of amplitude a and width d centered around the origin. But we like to find out this independently. So let us do this, so f of t would be 1 over 2 pi and i take out this d by 2 outside; therefore, it will be 2 A by t 2 A by 2 A. Integral from minus infinity to plus infinity of f of j omega which is sin omega upon 2 divided by omega e to the power of j omega t d omega.

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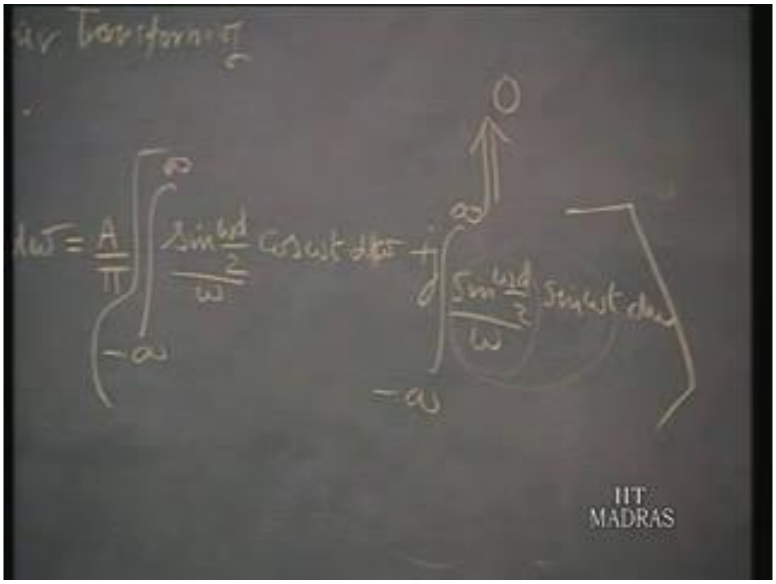

That is, the inverse fourier transform of this function. This i can write as a upon pi minus infinity to plus infinity sin omega d up on 2 cos omega t dt d omega plus minus infinity to plus infinity sin omega d by 2 sin omega t d omega and j is outside (voice is not hearable) now, this also omega by continue to have here. What you observe here is, as far second integral is concerned sin omega d divided by omega this is an even function of omega, this is the odd function of omega. And you are integrating between symmetrical limits from minus infinity to plus infinity.

So, the contribution of this integral from minus infinity to 0 would be exactly equivalent and opposite to the contribution that arises from 0 to infinity. So, since this entire function is an odd function of omega and you are integrating between symmetrical limits this goes to 0. So, what we have left with is only this and this can be written further as you have sin theta and cos theta; therefore, you can combine these 2 as sin alpha plus beta plus sin alpha minus beta type of formulations you can make.

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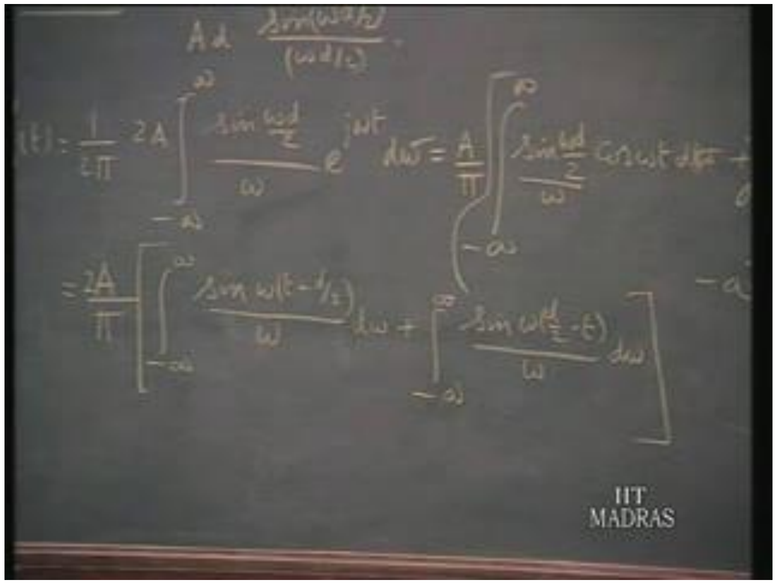

Therefore, I can write this as a by pi and then you have 2 also will come because you are going to express this as sin alpha plus beta and sin alpha minus beta. So, you have sin of omega t plus d by 2 divided by omega, that is; d omega and minus infinity to plus infinity of sin omega d by 2 minus t by omega d omega d. So, you have 2 such terms. Now, we can also observe here first of all before going further. This is an even function of omega, this is an even function of omega; therefore, whatever integral you get minus infinity to plus infinity would be twice the value of the integral which would have got if you extended if you integrate this from 0 to infinity. So, i can write this further as 4 a upon pi then 0 to infinity and 0 to infinity here, also that also, I can write this here;

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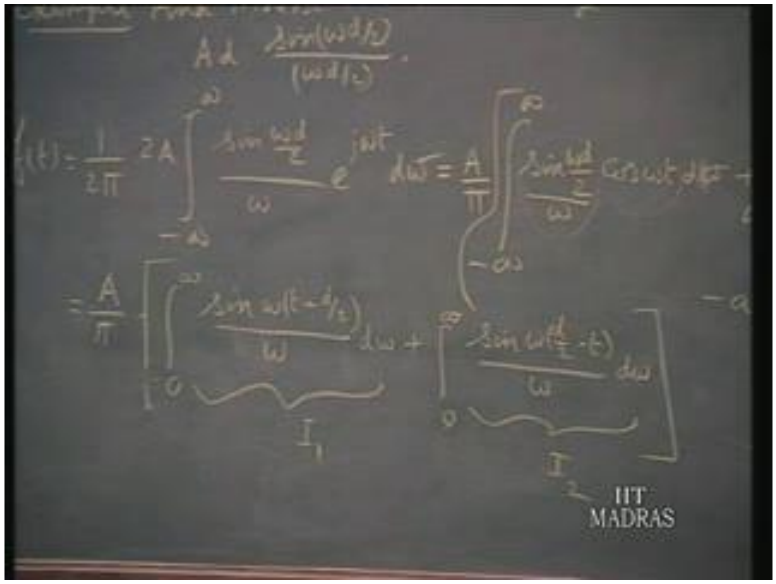

So, I will do that. Now, I think this should be corrected first of all since I am doing the integral 0 to infinity. I should have 2 a upon pi, but sin omega t d by 2 sin omega d by 2 minus t will give me 2 times sin omega d up on 2 cos omega t. therefore, I must divide by 2 further; therefore, finally this will become a upon pi. So, a upon pi this integral from sin omega t plus d up on 2 by omega and sin omega d by 2 minus t omega.

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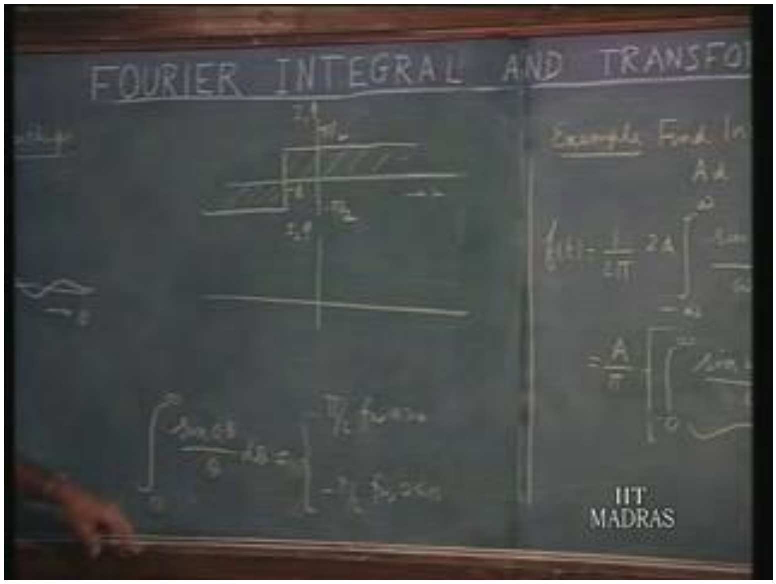

Suppose I call this integral i1 and this integral i2. And let us see, what their values would be for different values of t. For this i make use of this relation here, sin a theta by theta d theta will be pi by 2 for a positive and minus pi by 2 for a negative. Here, if i plot i1, the value of i1 we have sin omega something by omega d omega something very similar to this and instead of a i have t plus d upon 2. So, as long as t plus d upon 2 is positive it is pi by 2 as long as t plus d by 2 is negative it is minus pi by 2. That means this value of this integral will be like this is pi by 2.

This is minus d, this is t and this is minus pi by 2. So, as far i1 is concerned it will have positive value of pi by 2 for t greater than minus d. So, that t plus d by 2 is greater than 0 and as long as t plus d by 2 is negative it has minus pi by 2 that is the value of i1. Value of i2 again the same formulation should be used we have sin omega by omega d omega but omega is multiplied by d by 2 minus t, look at this expression again, sin a theta by theta d theta is pi by 2 for a positive minus pi by 2 for a negative. So, as long as d by 2 minus t is positive then it is pi by 2; otherwise, it is negative that means, as long as t does not exceed d by 2.

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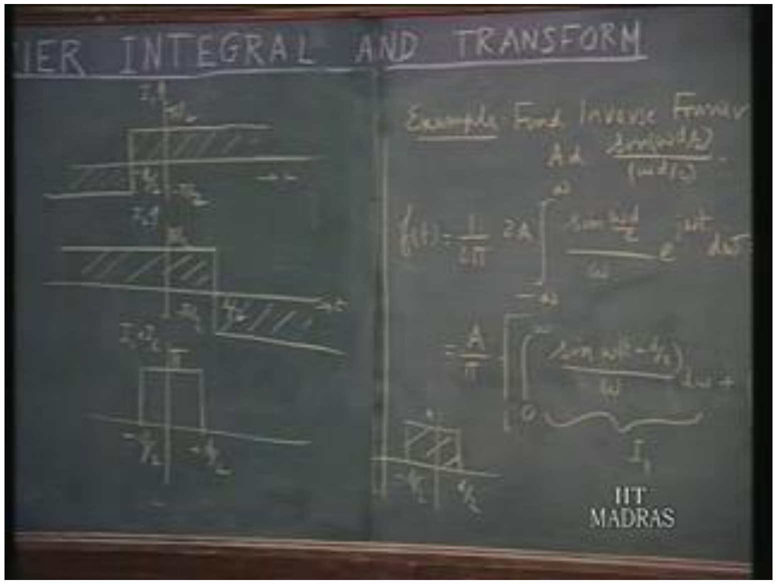

Then it is going to be pi by 2; otherwise, it is minus pi by 2. This will be the relation that you get finally as far as i2 is concerned. So, what you have got for when you add i1 plus i 2 now, no longer need this. So, if you add i1 and i2 add up these 2 things then this positive area will be cancelled by the negative area here and here so we have left with a pulse like quantity which will have a value of pi by 2 and pi by 2 this is pi by 2 this is minus pi by 2; therefore, this is pi and this last from minus d by 2 to plus d by 2. I should have put here, because t plus d by 2 and minus d by 2. I should have written here minus d up on 2 and this also minus d upon 2.

Therefore, this pulse extends from minus d upon 2 to plus d up on 2 and therefore, this is the value of i1 by i2; therefore, you multiply by a by pi f of t would be a pulse of width a lasting from minus d by 2 plus d by 2. A result which would have expected from this. So, what we have really done is we know, in the forward direction this is the pulse this is the fourier transform. We demonstrate it validity in the reverse direction making use of the relation of this sin integral as the arguments goes to infinity. Illustrated, this because we may like to use the this type of formulation later on in our further work. To discuss cases; where the fourier transforms involve impulses. Let us see the origin of the trouble if we follow the classical rules.

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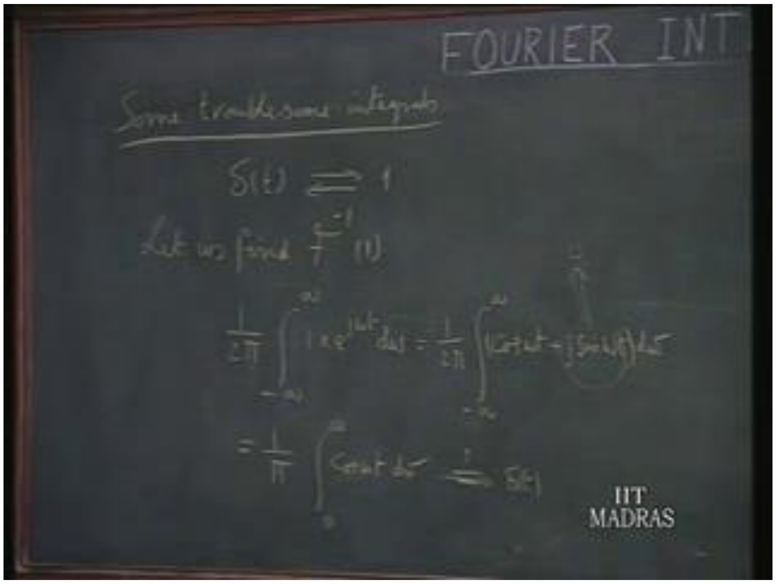

So, let me say some troubles of the integrals. We have just now seen a example, where in the forward direction we have got a pulse and then found out the fourier transform. When you went in the reverse direction we have to work a little hard to get the final pulse function. Let us consider the another example, we know that delta t has the fourier transform equal to 1. Let us find the inverse fourier transform of this 1 we should able to get delta t. How do we do it? then we write for the inverse fourier transform of 1 one over 2 pi minus infinity to plus infinity of 1 e to the power of j omega t d omega

When you try to integrate we have some difficulty because you have got infinite limits. So, in any case let us see, what it really means; so, minus infinity to plus infinity of cos omega t plus j sin omega t d omega. and once again sin omega t is an odd function of omega and you are integrating between the symmetrical limits we know the contribution from this is goes to 0 contribution from this goes to 0. So, we can write this as 1 over 2 pi minus infinity to plus infinity cos omega t d omega and once again, this is cos omega t is an even function of omega. We can write this as 1 over pi. And take the limit from 0 to infinity of cos omega t d omega. now, 1 over pi cos omega t d omega so far so good.

But if you try to integrate, you get minus sin omega t by sin omega t by t and then when we try to substitute for omega infinity then you do not know, what that amount too. So this type of integration does not allow itself lend itself to analysis by the classical methods; however, since we started with inverse fourier transform of 1 the inverse fourier transform must lead to delta t0. We know, delta t has this fourier transform. So, apparently the whole thing must be equal to delta t, this must be equal to delta t.

Now, that is where we have a problem in the classical mathematics. And after the impulses came to be used in analysis of systems then after their usefulness recognized mathematicians have developed a branch of mathematics is called theory of distributions with the help of which the analyses of the mathematical methods using impulses or put in more rigorous basis, but we will not go into that however make use of certain result of that and put them down here. So, that we will use the for our further work. Basically then we have some important results which i will list them out and these results which will make use of in our future work.

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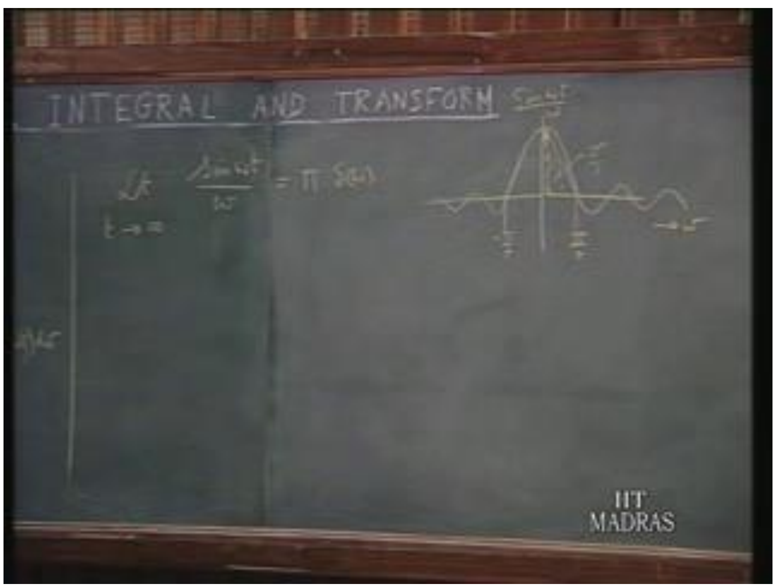

First of all limit as t tends to infinity of sin omega t by omega will be pi delta omega. It can be explained on the following basis; sin omega t by omega how it look like. So, this will be equal to t the height equal to t and the first 0 occurs when omega t equals to pi or t equals pi by or this is sin omega this is with respect to omega or omega equals to pi up on t. Now, enquire as what happens when t becomes larger and larger. When t becomes larger and larger sin omega t by omega the function becomes this pi upon t so the first loop becomes smaller and smaller, but the height goes. And we already seen, that the area under this curve of this is as far as the positive side is concerned area equals to pi pi by t times t into half; therefore, this is pi by 2 and the other side is pi by 2.

That means, the area under this curve is pi area under this curve is pi no matter what t use. So, as the area from minus infinity to plus infinity of this is pi and d becomes larger and larger this loop strings and then you have very large amplitude. And the first loop becomes 0. That is essentially the character of the impulse and since the area under this curve is pi then it is impulse of value pi and the impulse is situated omega equal to 0 that is where we have a spike. So, we can justify this in this fashion that sin omega t by omega t goes to infinity is tends the limit the pi delta omega.

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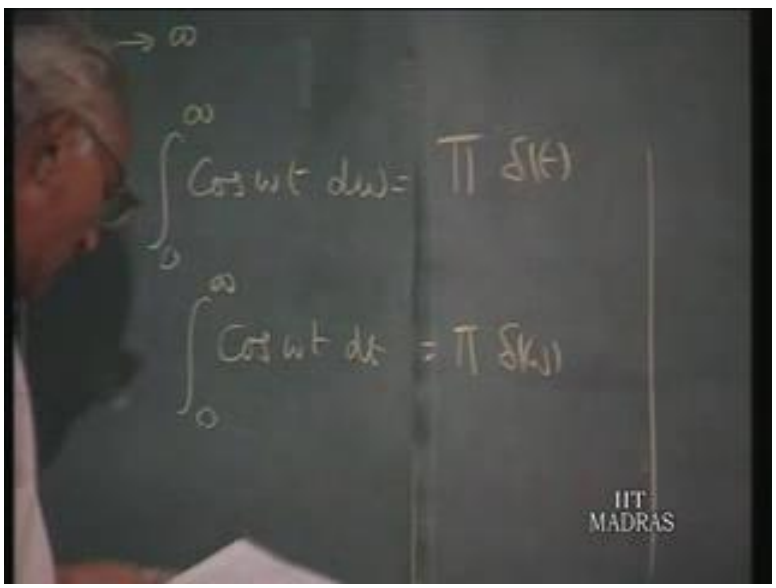

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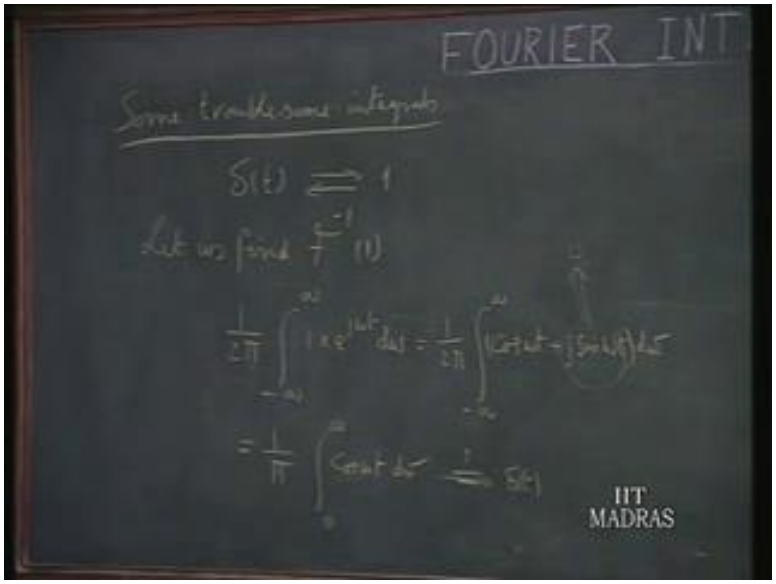

Similarly, we can have 0 to infinity of cos omega t d omega equals pi delta t. That we have seen, from this we have earlier come to this we got into difficulty 0 to t cos omega t d omega. What it means, it is pi times delta t. We did not know how to deriver this but we can assume this relation. Similarly, we have 0 to infinity cos omega t d t after all we change the variables here this must be pi delta omega.

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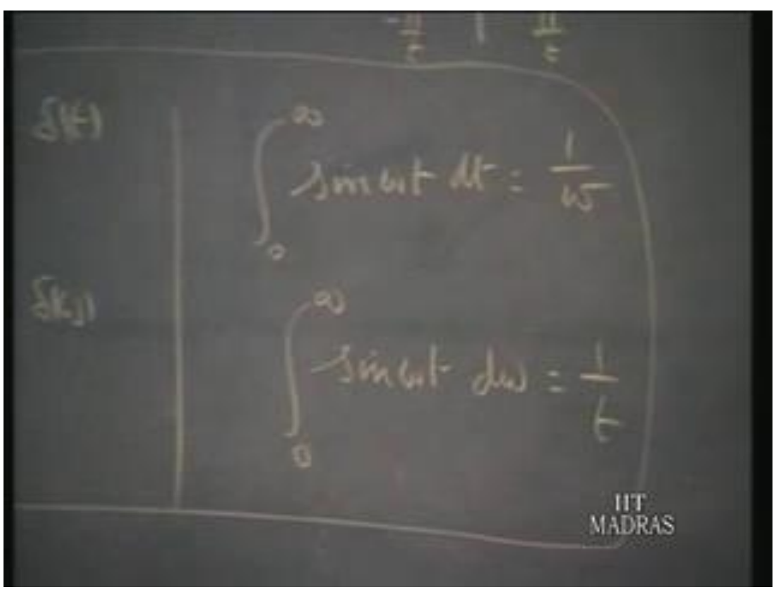

And 2 more relations are useful 0 to infinity sin omega t dt equals 1 over omega and 0 to infinity sin omega t d omega is 1 over t. We assume these relations in our work when we derive the fourier transforms of functions whose energy content is may be infinite may not be finite. So, these 4 relations are important for us we will take this to be true in our further work. We will make use of this as I said in our further work but just as an example we will consider 1 particular case.

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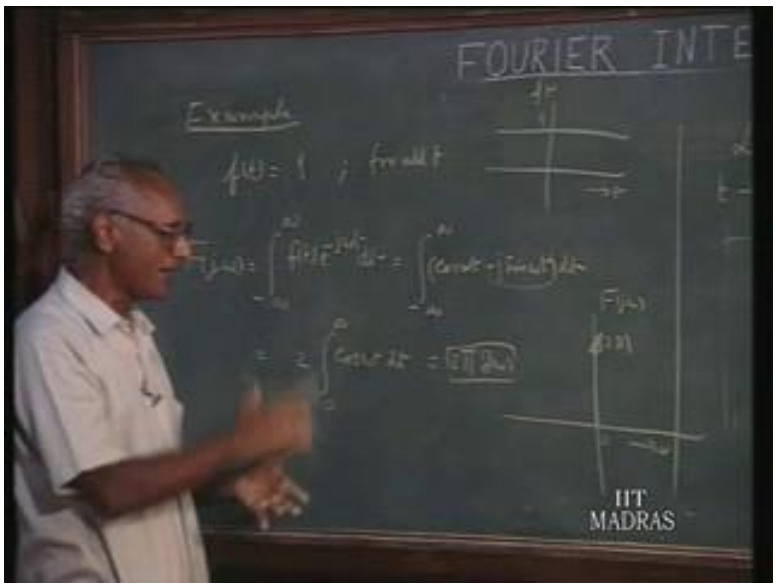

Let us take a simple example now, to illustrate these relations. We will take f of t to be a d c quantity equal to 1 for all t. So, that means f of t is a constant f of omega for this obviously from minus infinity to plus infinity of f of t e to the power of minus j omega t dt. Now, e to the power of j omega t can be written as cos omega t minus j sin omega t. And once again we can make as the symmetry relation and sin omega t term will be this is minus infinity cos omega t minus j sin omega t dt.

Because f of t happens to be equal to 1 and the contribution of this will be 0 because it is an odd function and we have symmetrical limits; Therefore, I can write this as only integral involve cos omega t now i take from 0 to infinity of cos omega t d t and i introduce 2 sin because 2 here because instead of integrating minus infinity to plus infinity i am integrating 0 to infinity. And now, cos omega t dt integral from 0 to infinity is pi delta omega according to what we had already seen therefore this is 2 pi delta omega.

So, the fourier spectrum for this f of j omega. You have an impulse of magnitude 2 pi and d c this is what we should expect this is a pure dc term; therefore, you will have only a dc component here and no other components. And at the dc we have finite amplitude for this voltage; therefore, you have an impulse function that means you no longer even though we are talking about coefficient density. Now, this coefficient density is infinite. Because there is a definite non 0 amplitude at d c; therefore, the coefficient density is going to be infinite. And it has got a magnitude equal to 2 pi we develop this further when we take about other periodic signals.

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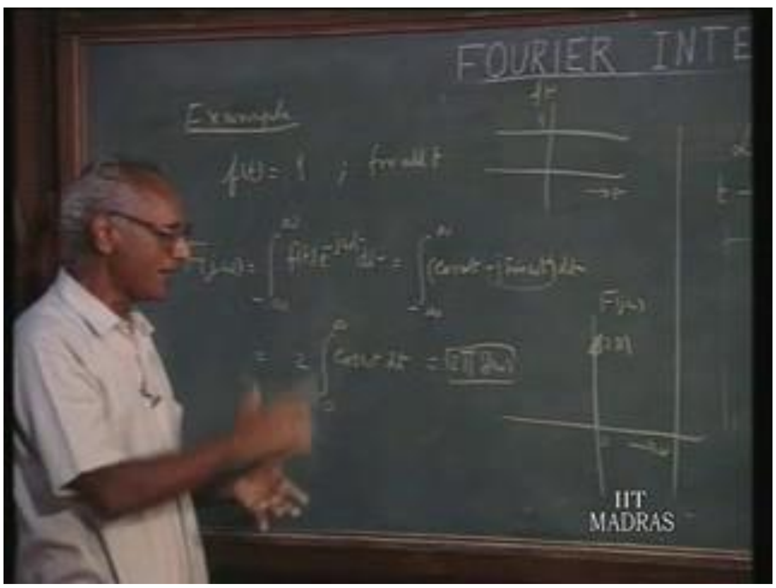

So, in this lecture; therefore, what we have done so far is, we started with the concept of energy in a periodic signals we said when we talk about a periodic signals with finite energy it no longer fruitful for us to talk in terms of power. Because the average over a infinite interval goes to 0; therefore, we talk only in terms of energy available from the signal. And what we mean by energy available from the signal is. If this signal is a voltage signal if it is given to 1 ohm resister. What is the total energy dissipated in that 1 ohm resister from minus infinity to plus infinity is what we call the energy available from the signal.

And this energy available in the signal resides in the different frequency band by different amounts; therefore, we saw that if we integrate from minus infinity to plus infinity the energy density which is given by f j omega squared integrate it with reference to omega and divide by 2 pi because the energy in terms of frequency in hertz cycle per second. That will you give the total energy and this we said answer refer to Parseval's theorem. Then we discussed the possibilities of extending in the fourier analysis to signals whose energy content is not finite and in that contest we started with some interesting relations involving sin theta by theta functions and we familiarize with sin integrals. And then we put down some results here, which are scientified by the distribution theory in mathematics which we cannot prove classical methods we assume this results.

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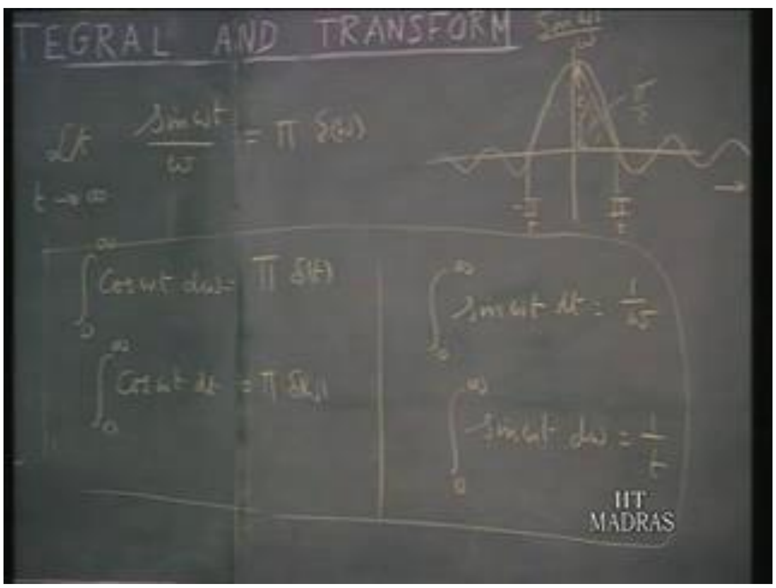

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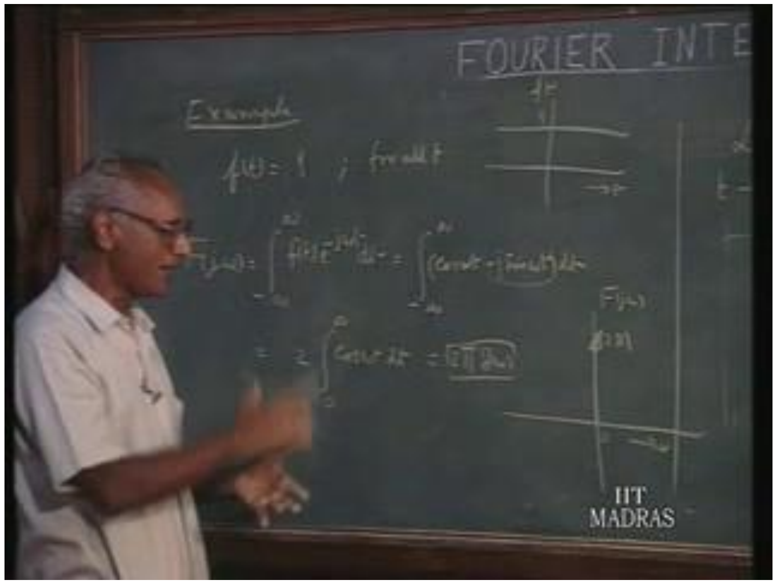

and making use of these results we try to find out the fourier spectrum of a pure d c quantity. And came up with results which is a quite perhaps quite have been obvious this particular signals has only 1 frequency component and that is a d c. And therefore, you have only 1 this becomes line spectrum once again in the fourier transform also but the coefficient there is coefficient which is non 0; therefore, the coefficient density has got to be infinity; therefore, we have a delta function. And the coefficient density is 2 pi omega and the density occurs at 0 and so we can expect that when we talk about periodic functions in the line spectrum the magnitudes are going to be infinite involving delta functions, more about this in the next lecture.