Networks and Systems
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Lecture - 14
Fourier transforms (2)

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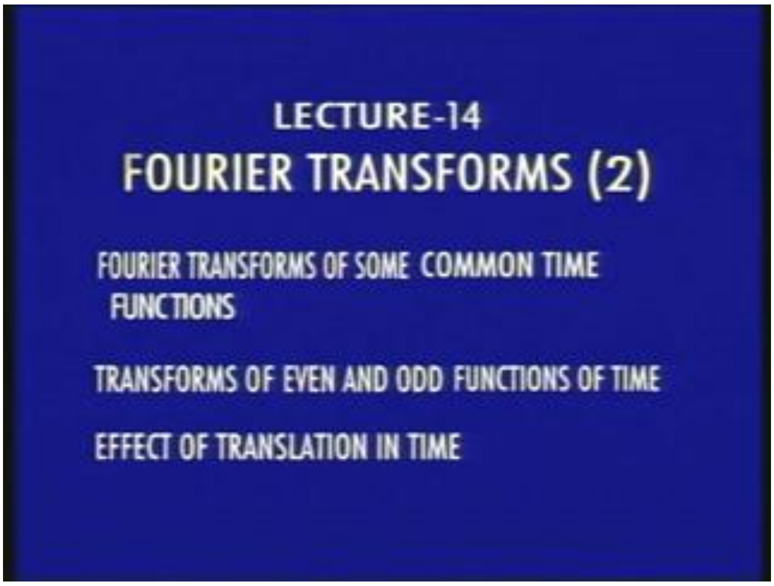

In the last lecture we introduced ourselves to the concepts of the Fourier Integral and the Fourier transform. Let us, quickly recall what we did? We observed that, a periodic function of time can be considered to be a periodic function with period going to infinity and we found that is a consequence the function will have frequencies of all values extending from minus infinity to plus infinity.

But then, the amplitude of the various frequency components will vanishinigly small in fact, tend to 0. So, we will not have any meaningful data if we take up continue to take up the approach of evaluating the coefficient Fourier coefficient as we did in the case of a periodic function. So, we took up an alternative approach we talked in terms of the coefficient density which is the coefficient divided by the base frequency f not.

And the coefficient density turns out to be a meaningful concept in the sense that it does not vanish and it gives the relative idea of the different frequency component magnitudes. Coefficient density we also called the Fourier transform F j omega and Fj omega is in general is a complex number therefore, it has got both magnitude and phase.

So, corresponding to each f of t you have the Fourier transform F of j omega and both f of t and F of j omega can be thought of has 2 different windows through which we can look at a function: 1 is the time description and other description in terms of frequency. Both give equivalent information about the physical phenomenon we are used to observe a function as a sequence of values with respect time. So, a function of time comes more naturally for us to visualize a physical situation of this sort. But imagine that you have the instrument or creature which have got senses receptive different frequencies, different frequency bands.

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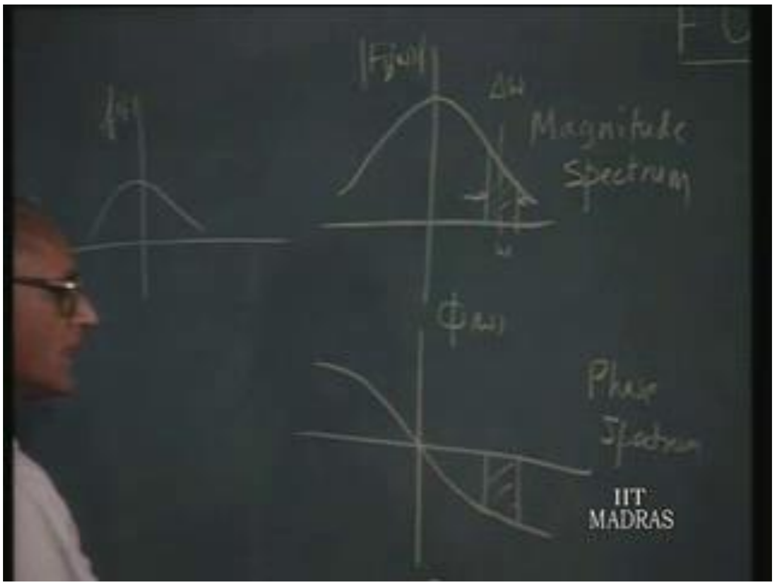

Then, that particular instrument will observe the phenomenon as in terms of the relative different frequencies that will be in terms of the for example: Fourier transform F of j omega. So, let us now look at once again you have a function of time f of t and correspondingly its Fourier transform F of j omega will have magnitude F of j omega magnitude and a phase function phi of omega which is an odd function.

Therefore, this is the magnitude spectrum and this is the phase spectrum. We also observe that is, this both of the magnitude and phase are essentially functions of which are called Fourier coefficient densities if you take a small band of frequencies, delta omega centered around a particular omegathis section of the spectrum both the magnitude and phase together will tell us, about idea of the strength of the signal at this frequency omega.

In fact, these 2 sections represents at function of time which can be written as F of j omega magnitude e to the power of j phi e to the power of j omegat. So, this is the time function which is identified by these 2 sections of the spectrum. In other words , what we are saying is even though there is a small there is small difference in the frequencies in this band if you assume that, entire spectrum represents frequency component at this point omega at the center of this band. The time function corresponding to that is e to the power of j omega t and its magnitude is the Fourier coefficient times of course, delta omega you must also have delta omega.

Because, this is f of j omega i write this again F of j omega e to the power of j phi this is the coefficient density, But since the coefficient density we are talking over a band delta omega is: delta omega over 2 pi because, the density is in terms of the frequency e to the power of j omega t. So, this is the signal that is represented by these 2 sections this is the strength of the signal this is the coefficient and this is the time function.

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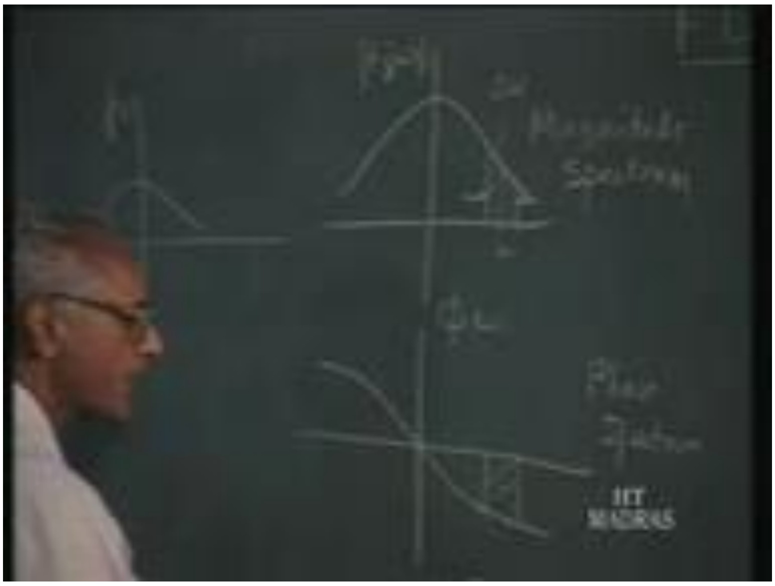

And if you take the limit of all such individual components, over the frequency band extending from minus infinity to plus infinity. This will add up to that is from minus infinity to plus infinity limit as delta omega tends to 0. if you take, such all such function this will be 1 over 2 pi minus infinity to plus infinity F of j omega.

Where F of j omega now, talking about combining F of j omega the magnitude and e to the power of j phi together is a complex number F of j omega e to the power of j omega t that will be your f of t. So, f of t can be thought of as 1 over 2 pi F of j omega e to the power of j omega t d omega. So, this is the Fourier integral and to get F of j omega from f of t this is minus infinity to plus infinity f of t e to the power of minus j omega t dt.

So, these are the 2 relations which are important in the Fourier transform theory. You can get F of j omega from f of t and f of t from F of j omega. We write this relation in more compact fashion in this manner Fourier transform t is indicated in this manner a script f as the function of f of t this will be the Fourier transform this will be F of j omega.

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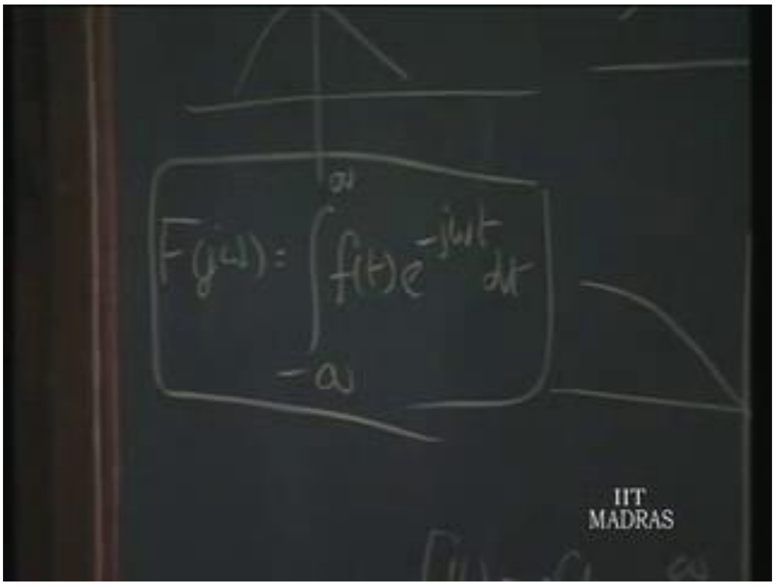

So, to recover f t from F of j omega we write this f of minus 1 the inverse Fourier transform this will give me f of t. So, this is called the Fourier transform that is your transform function of f of t function of ft to get the Fourier transform F j omega and what you have here is called the inverse Fourier transform

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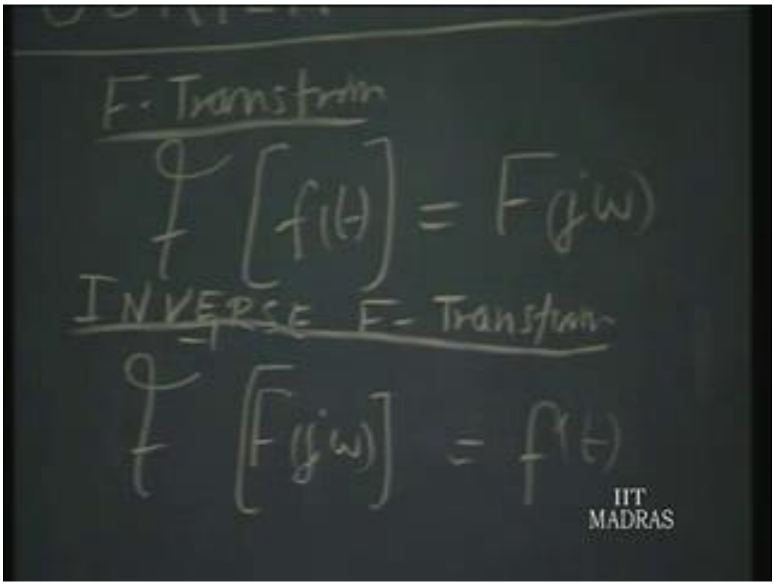

You would also like to indicate this transform relations occasionally, in this fashion f of t and f of j omega from a transform pair. So, we can indicate that functional relationship in this manner f of t arrow F of j omega is there in the forward direction you are doing the Fourier transform and in the reverse direction you are doing the Inverse Fourier transformation.

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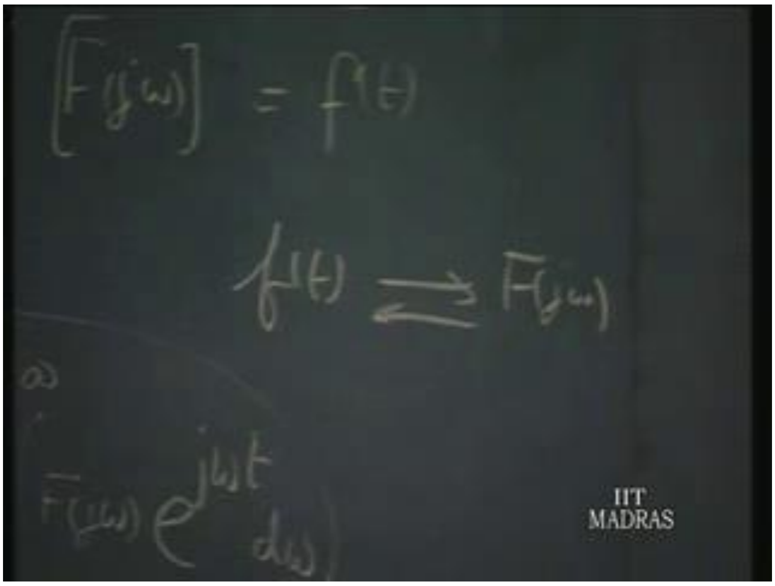

So, as i mentioned f of t and F of j omega are the 2 alternatives descriptions of the same phenomenon and what you like to do now, is to find out the Fourier transforms for a few representative functions of time before we go on to study of the properties of Fourier transform.

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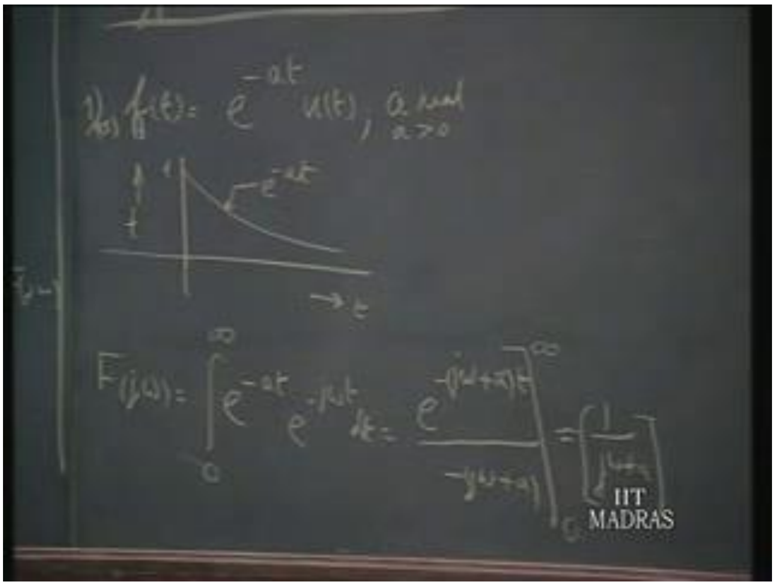

So, to get some physical idea of how this F of j omega comes about common time signals let us, work out a few examples typical Fourier transforms of a few representative time signal: 1 suppose, we have f of t as e to the power of minus at u t i call that 1 a let u, say a is real quantity a is real and a is greater than 0. That is i have taken a exponential starting time t equal to 0 t is the unit step function.

So, it will be 0 up to time t equal to 0 for negative values of t f of t is 0 it starts at 1 and then, decrease exponentially. So, this is e to the power of minus at, but the Fourier transform for this using the formula that we had here will be F of j omega you integrate this from minus infinity to plus infinity of f of t e to the power of minus at u t dt.

But since we know f of t is 0 identically from t minus infinity to 0 i can start the integration 0 go up to infinity strictly speaking we should start from minus infinity to plus infinity, but minus infinity to 0 f is 0. Therefore, i am starting the integration from 0 from 0 to infinity the value this function e to the power of minus at and i have e to the power of minus j omega t dt.

So, this will be e to the power of minus j omega plus at that is what it is to be integrated. So, you have in the denominator minus j omega plus a this should be evaluated between the limit 0 and infinity. Since, we have taken a to be a real number and minus k is the negative real number. When, t goes to infinity e to the power of minus at become 0 and e to the power of j omega t is something which oscillates between 0 and 1 in this magnitude at least therefore, e to the power of minus at become 0 at t equal to infinity.

Therefore, the upper limit is 0 and the lower limit it is 1 because, when t is equal to 0 the exponential become 1 therefore, the result is this will be 1 over j omega plus a. So, e to the power of minus at u t has a Fourier transform which is 1 over j omega plus a.

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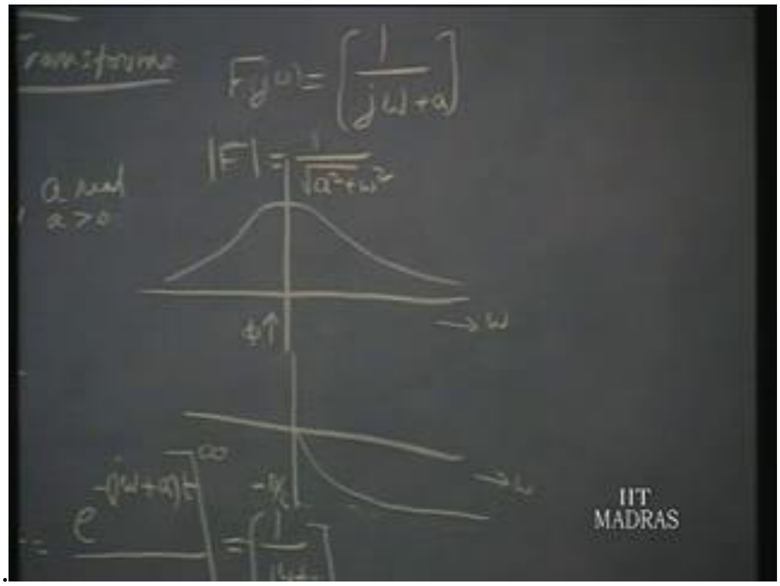

So, how does the spectrum look like? F j omega equals 1 over j omega plus a. So, the magnitude spectrum will be F magnitude 1 over square root of a square plus omega square right. Therefore, it will be something like this and the phase spectrum as a function of this all of the function of frequency when, omega goes to very large positive value the phase of this the angle of this complex number becomes minus 90 Degrees.

That means, it goes to asymptotically minus pi upon 2 and because, of the phase spectrum is the r'th function of omega. So, it reaches plus pi up on 2 into positive axis that is so, this is the angle spectrum or phase spectrum, this is the magnitude spectrum. Now, even though we said that in a periodic function has 0 amplitude signals at all frequencies we still from the coefficient density we can see that, the component of the signals at dc are stronger than the component of the signals at some other frequencies.

So, this spectrum gives us an idea of the frequencies at which the densities concentrated in this signal. The energy density is concentrated in the signal because, the components are more done here than here. So, you can see relative proportions of the different frequency component that go to build up the signal in terms of the Fourier transform magnitude which is really the coefficient density.

So, even though coefficients are all 0 the coefficient density gives us as the measure of the strength of the signal strength of the signal at different frequencies. So, let us now continue this now i purposely put this 1 a e to the power of minus at a is real and a is greater than 0. Because, i wanted to extend this idea and say that this particular formula that we had f of t as a Fourier transform 1 over j omega plus a will be valid even if a is complex number.

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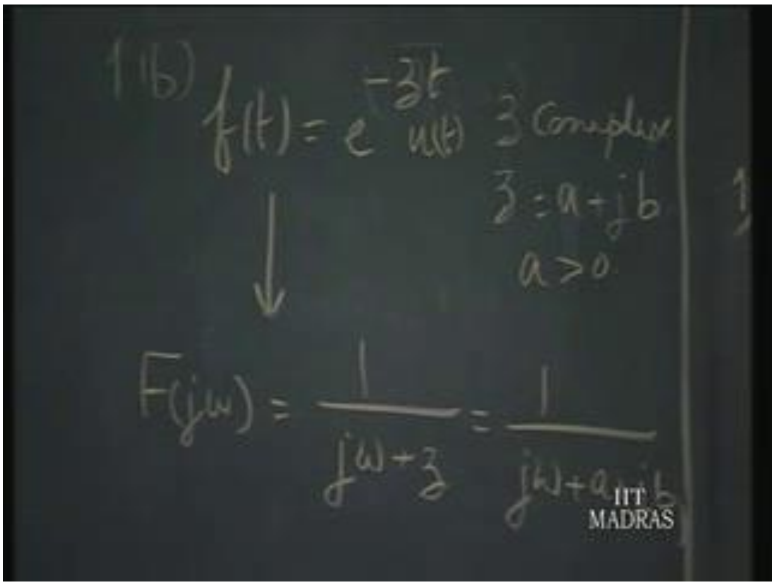

So, f of t suppose is e to the power of minus z t where z is complex z. Let us, say a plus i b and a is the real number greater than 0. Then, the Fourier transform for that can be shown to be 1 over j omega which is 1 over j omega plus a plus i b. So, this formula that e to the power of minus at ut here also, you must have ut as this Fourier transform 1 over j omega plus a will be valid even if instead of a you have complex number. The only requirement is that, the real part of the complex number must be negative because minus z that is minus z is the coefficient of t. Then, the real part of the minus z must be greater than real part of minus must be negative or the real part of the z must be greater than 0.

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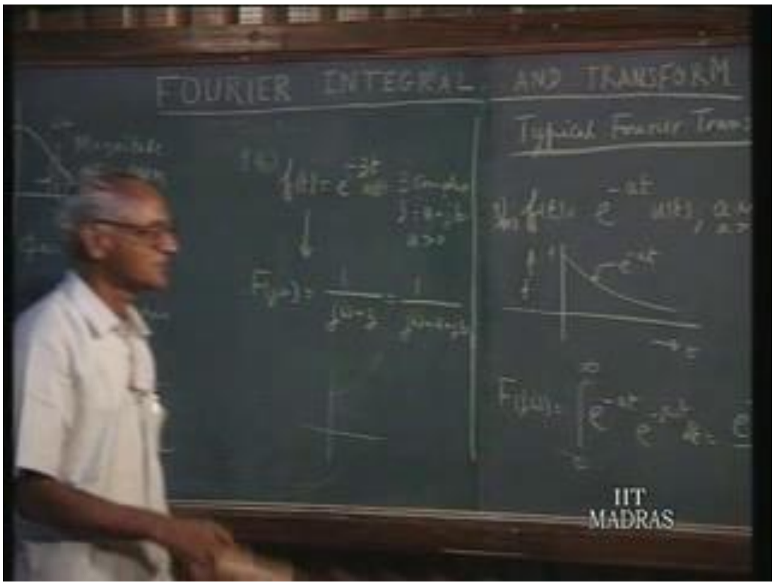

The reason is if you had exponentially rising like this. For example: if you have instead of this being negative suppose it is exponentially rising signal then, this integral will not converge. This integral when, t becomes larger will not converge therefore, you must have an exponential decaying signal then only it converges at t equals to that is the reason why, we had.

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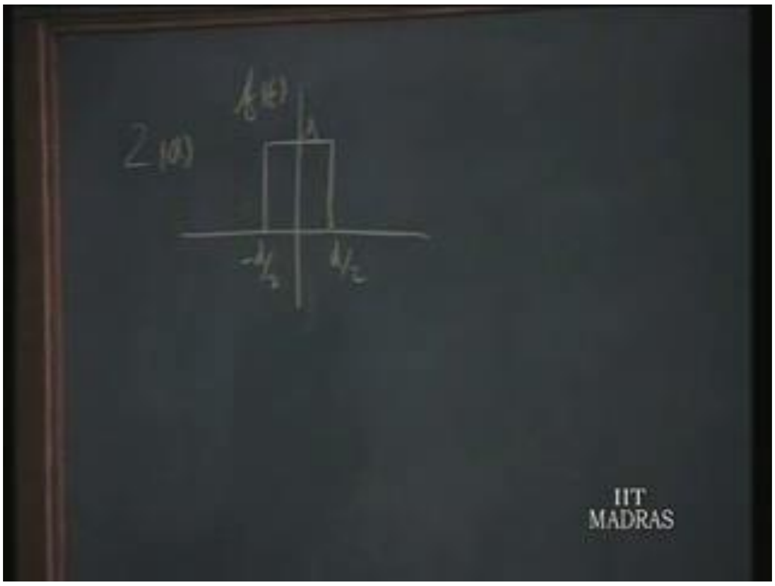

Now, let us work out another example now let us, take a second function of time a pulse function which is quite common and appears quite frequently in Fourier transform theory.

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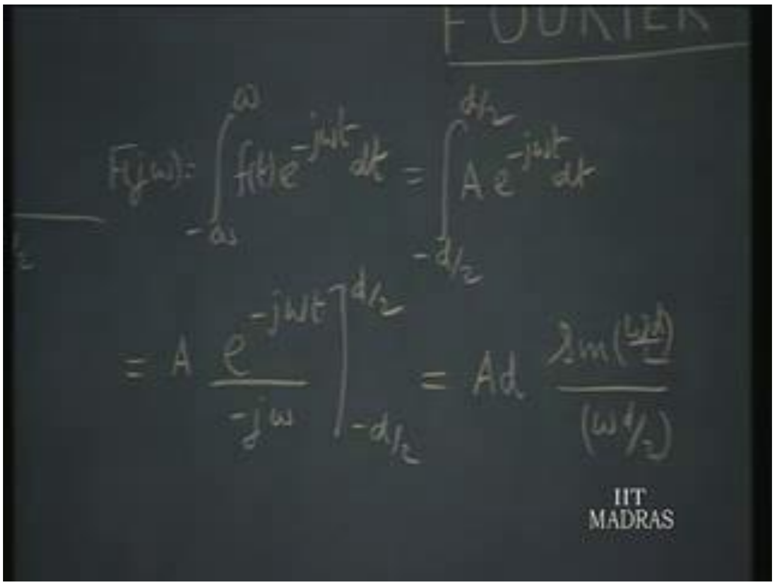

So, we have a pulse of width half d units amplitude a. So, this is f of t so, f j omega will be from minus infinity to plus infinity f of t e to the power of minus j omega t dt this is the standard form in our case, this function of time only from minus d upon 2 to plus d up on 2. Therefore, this can be written as minus d up on 2 to plus d up on 2 and in this interval f of t equals a. So, a e to the power of j omega t dt.And that will be a e to the power of minus j omega t divided by j omega evaluated between the 2 limits minus d up on 2 to plus d up on 2. And this can be shown you can work this out and this can be shown to be Ad sin omega d up on 2 divided by omega d up on 2 that is the Fourier transform for that.

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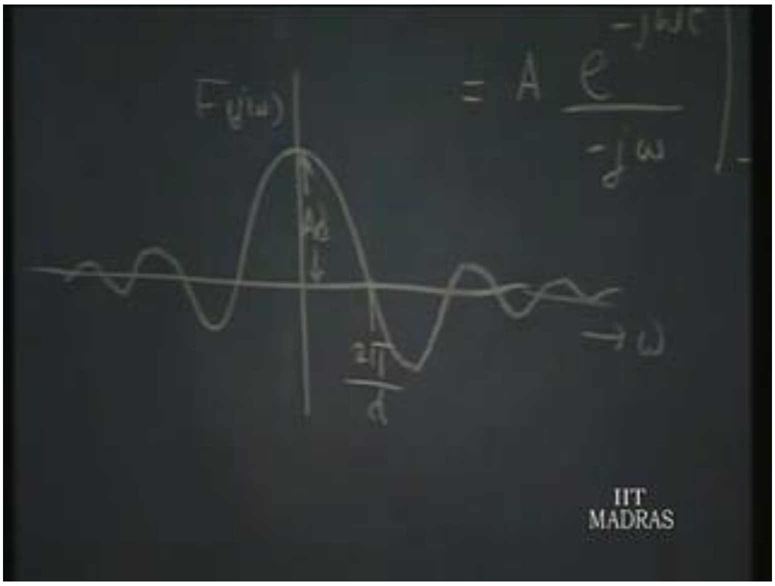

So, this is of the form sin theta by theta type of variation. So, the spectrum for that F of j omega can be plotted in this fashion like this, where at the dc the Fourier transform will be having the value Ad. Because, you recall sin theta by theta will have a value equal to 1 when theta equal to 0. So, this will be Ad And then, it oscillates, but with diminishing amplitude.

Now, when does the first 0 occur first 0 occurs when sin omega d up on 2 is 0 that omega d up on 2 equals pi. Therefore, the first 0 occurs when omega equals 2 pi up on d. In fact, this is the spectrum which we plotted in the last class when, we talked about the evaluation of the Fourier series coefficient density starting from periodic pulse strain this is the exactly the type of spectrum that we plotted.

Now, since F of j omega happens to be real i do not plot the magnitude spectrum and phase spectrum separately. Because, F of j omega is real it is either positive or negative. So, i can combine both the phase and magnitude information in 1 plot like this.

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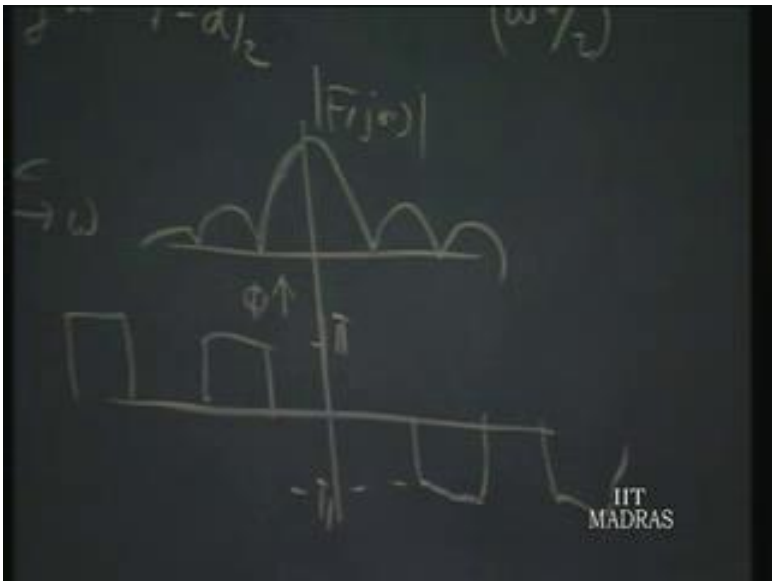

However, if you wish you can plot the magnitude spectrum separately like this, this, is F of j omega magnitude and the phase spectrum will be whenever, this is negative you can say that is minus pi up on 2 minus pi this minus phi and then, you can write this as plus pi.

So, this can be considered to be the phase and then could be the magnitude. So, this is the alternative way of this repeating the spectrum was placed in magnitude. But when the Fourier transform is real there is no point doing this separately as well exhibit the entire f of j omega being the real function of time real function of omega in 1 plot like this.

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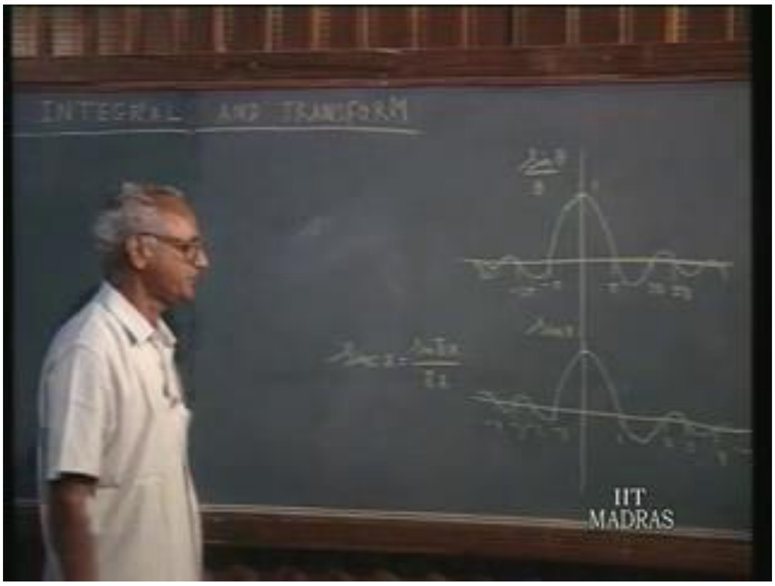

Now let us, have little bit of diversion here we know the sin theta by theta curve which occurs frequently in Fourier transform theory will be like this, this is 1 this is pi, 2pi, 3pi this is minus pi, minus 2 pi etc. Now, there is another function which is called Sinc theta or Sinc x equals defined as sin pi x over pi x. This is in the literature Sinc x is the term the function that is defined as sin pi x over pi x.

Therefore, if you plot Sinc x as versus x it will be similar to this because, when x is 0 both the numerator and denominator is 0 sin 0 by 0 type of thing. So, it will be 1 and it will have oscillations, but the 0 occurs whenever sin pi x is 0 that means, for integral values of when x is equals to 1 2 3 4 the sin pi x become 0 that means, the curve will be something like this.

Similar to that, but what we have now here is this will be 1, this will be 2, this will be 3, this will be 4, this is the minus 1, minus 2 ,minus 3 like that. So, occasionally people prefer to use the Sinc function instead of sin theta over theta compact at special Sinc x, which is will be the value of the function of x will be vary in this fashion.

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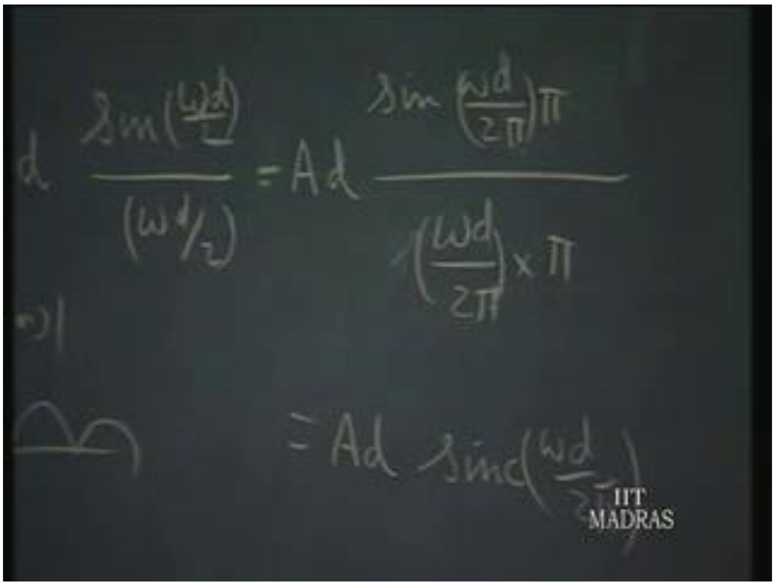

So, we can write this if you like as Ad sin omega d over 2pi times 2pi over i am sorry sin omega d up on 2 pi times pi right divided by omega d up on 2 pi times pi. So, we can write this as Ad times Sinc omega d up on 2 pi. Ad sin omega d up on 2 by omega d up on 2 can be written as Ad Sinc omega d up on 2 pi and the Sinc function vanishes at integral values of the argument. When, x equal to 1 2 3 4 so, what values of omega will become 0 2pi upon d, 4pi upon d and so on and so forth. 2pi upon d, 4pi upon d, 6pi upon d.

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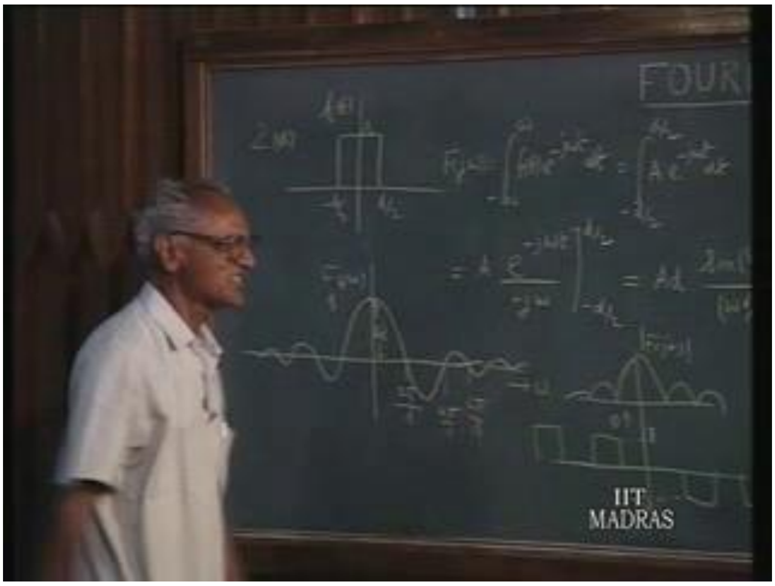

So, this is an alternative way of writing this So, this function this pulse function sometimes called gate functions. It has the non-zero value only between these limits rectangular pulse function as the Fourier transform Ad Sinc omega d upon 2pi. The Sinc function tries to normalize the values in a nice way because, you are now having integral values of x and this will become evident. Now, if i take the second example we normalize the pulse so that you have unit amplitude and unit duration that means, this is minus half, this is half, this is t and A equals 1.

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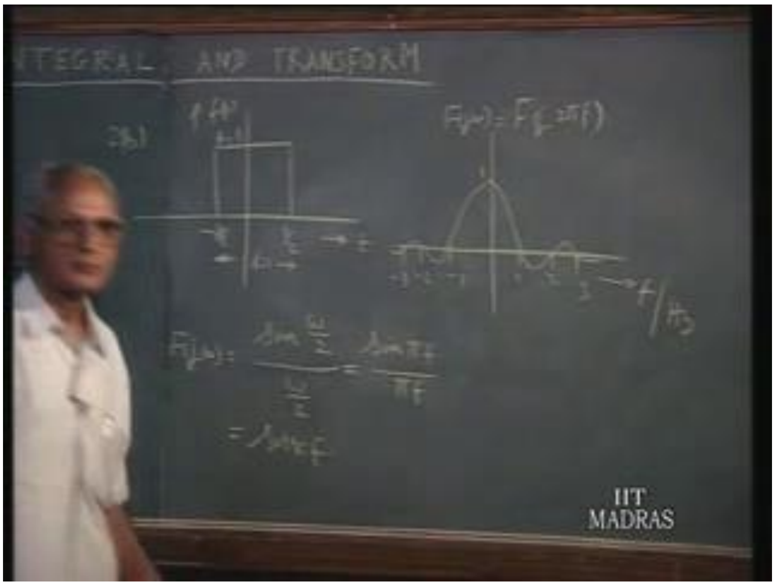

Example where you normalize the pulse so, that you have unit amplitude and unit. This is f of t. So, the Fourier transform for that if you go back to our old formula, sin omega d upon 2 by omega d upon 2 A happens to be 1 d happens to be 1. So, Ad is 1 sin omega d is 1 this is A is 1. So, omega d upon 2 d is 1. Therefore, this is omega upon 2 divide by omega upon 2 which if you like to put this in terms of frequency, this is sin pi f omega being 2 pi f this is sin pi f.

therefore, and this is indeeds Sinc f so, consequently the pulse normalize to a unit amplitude and unit width we have a Fourier spectrum which is given by Sinc f that means, if you plot the frequency F of j omega, but it turns now F of the frequency you calculate the x axis in terms of frequency you will have this is 1 f is 1 2 3 4 minus 1.

So, what does Sinc function only tries to normalize the things if you have normalize pulse unit amplitude unit width then, the Fourier spectrum will have will be like this unit height and going to 0 1 2 3 cycles per second. So, this is just a especial case of this because, once we normalize we have the Sinc function which surprisingly very simple function. Let us, now work out the third example you recall that when, you had periodic impulse strain in the case of Fourier series we found that all Fourier coefficients have the same magnitude.

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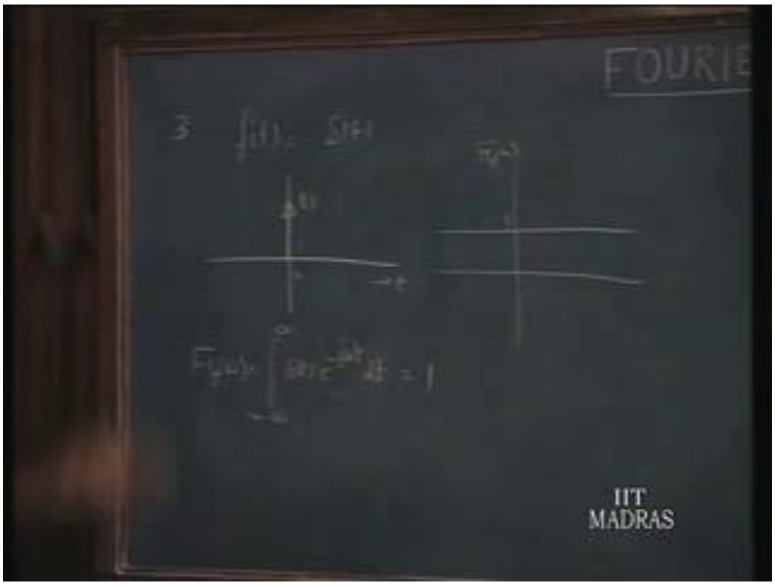

So let us, now take an impulse and see just 1 single impulse this is of course,, a periodic and what its Fourier transform would like. So, f of t is taken to be a delta function. So, you have a unit delta sitting at t equal to 0 that is a time function. The Fourier transform for that f of j omega minus infinity to plus infinity the function of time is delta t e to the power of minus omega t dt.

And since, we have the delta function in the integrand naturally the value will be the value of e to the power of minus j omega t t equals to 0 that is equal to 1. So, we have very nice and compact result F of j omega equal to 1.Which means the Fourier spectrum so, you have impulse you have Fourier spectrum which is plot. So, we can say now this particular pair of transforms delta t as the Fourier transform which is equal to 1.

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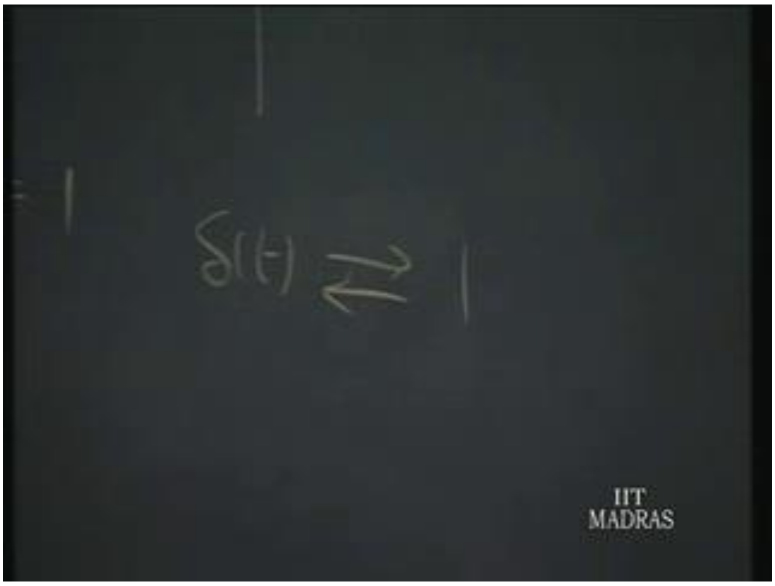

That means, the coefficient density at all frequencies ranging from minus infinity to plus infinity equals 1 very nice and useful result. Let us, take now a more complicated example e to the power of at cos omega ct u t that means, the time function would be something like this which is oscillating sinusoid, but with decaying amplitude.

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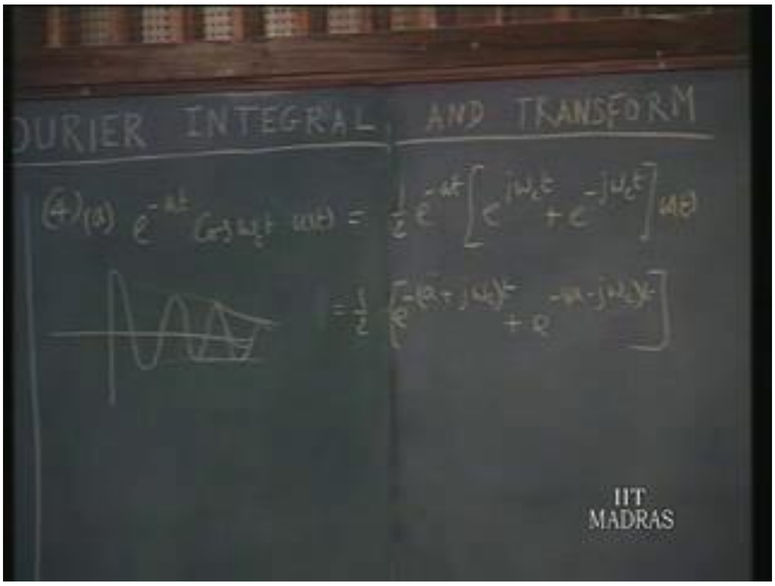

Now, this can be written as 1 half of e to the power of minus at e to the power of minus j omega c t plus e to the power of minus j omega c t which is 1 half of e to the power of minus a plus j omega c t plus e to the power of minus a minus j omega c t.

So, this decaying exponentially decaying sinusoid can be a thought of as 2 exponential functions of this type. And very first example we observed that e to the power of minus a plus j bt can also, have the same type of Fourier transform as when you have e to the power minus j 1 over plus a we have 1 over j omega plus a plus j omega in this case.

So, the Fourier transform for this will be half of 1 over Fourier transform for this particular function for this i also i have ut that continuous. J omega plus a plus j omega c that is for the first function, second function we have 1 over j omega plus a minus j omega c.

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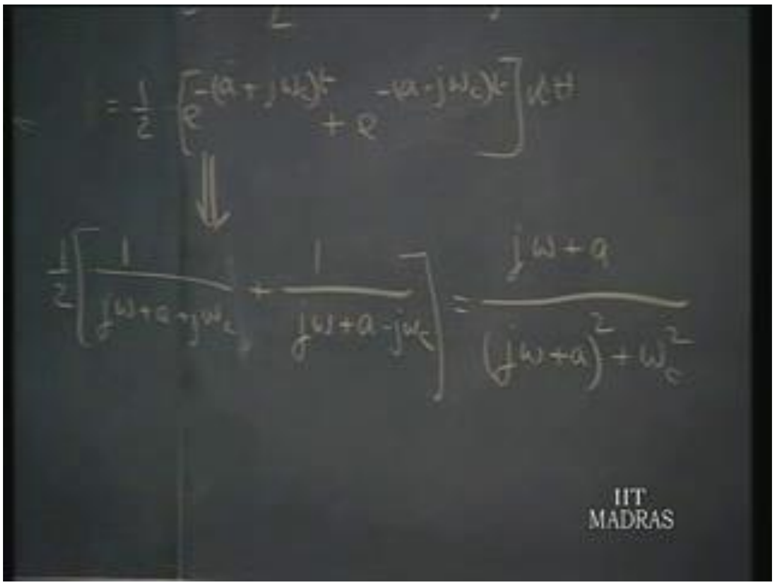

And these 2 can be combined and together we have an expression which looks like this j omega plus a over j omega plus a whole square plus omega c square.

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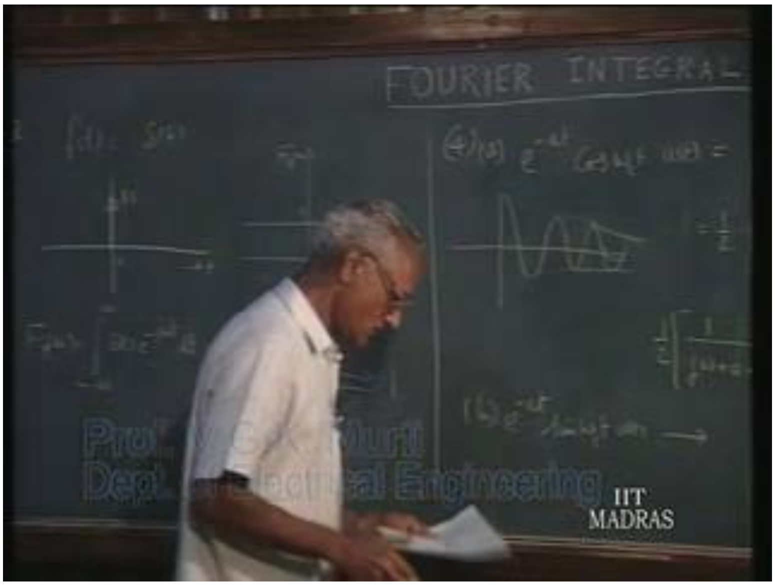

Likewise, you can plot this spectrum then, you will find that around omega c you have some kind of peaks, but we do not do that Similarly, e to the power of minus at sin omega c t ut can be shown to have a Fourier transform which is equal to omega c over j omega plus c whole squared plus omega c square right. And the last example if i have a function like this, A d upon 2 minus d upon 2 this is f of t.

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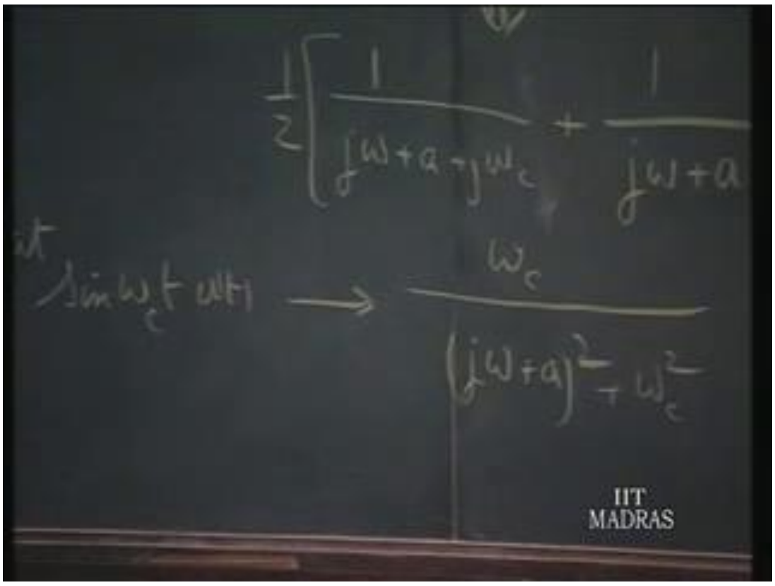

That is a pulse the form of a triangle lasting from minus d upon 2 to plus d upon 2 it can be shown that this will have a Fourier transform which is equal to A d upon 2 sin omega d upon 4 by d omega d upon 4.

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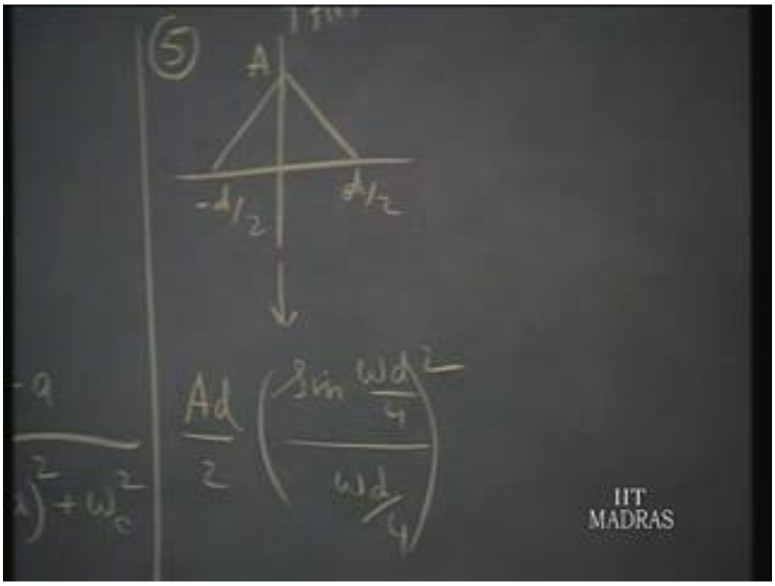

So, this is something like sin theta by theta type of arrangement, but squared that means, the decay is faster. The spectrum for this would be like this and this i leave to you as an exercise to show.

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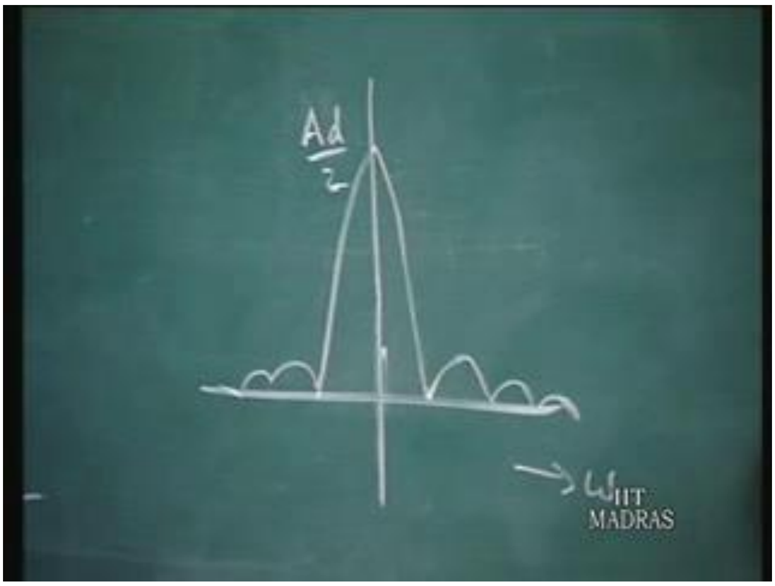

Now, all the function that we have taken so far are nice well behaved functions which have the property that, all functions considered so far considered so far satisfying the relation from minus infinity to plus infinity f of t dt converge. That means, the magnitude of the function is integrable from minus infinity to plus infinity that is why, we have exponentially decaying function, exponentially decaying function sinusoid here. A pulse with last for a finite amount of time either in a rectangular pulse or a triangular pulse and impulse can be integrated that is equal to 1.

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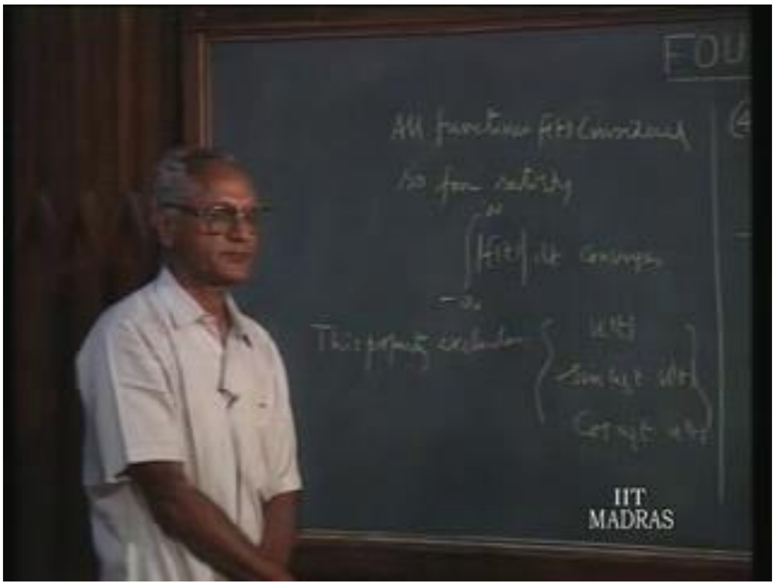

So, all such functions have we have considered nice properties like this, this excludes this property excludes some desirable function which we should like to find Fourier. For example: u of t last from 0 to infinity sin omega ct u t a sinusoid cos omega ct ut functions like this are important. And we come across them quite commonly, but in the classical sense these functions will not satisfy this property therefore, we do not find the Fourier transform for this not yet anywhere.

In the classical Fourier transform theory these functions were excluded from this code. But that impulse functions have become a matter common usage once, we have impulse functions we should be able to find out the Fourier transform of these functions as well which we do a little later. The mathematical justification for such usage comes from what is called the distribution theory which has Scientified the use of impulse functions and related mathematics we will not however, go into that. But nevertheless, we assume certain relations which are sanctioned and use those results to find out the Fourier transforms of such functions as well a little later. But before that, we like to have a closer look at the some of the properties of the Fourier transforms in order for us to get familiarity with their characteristic before

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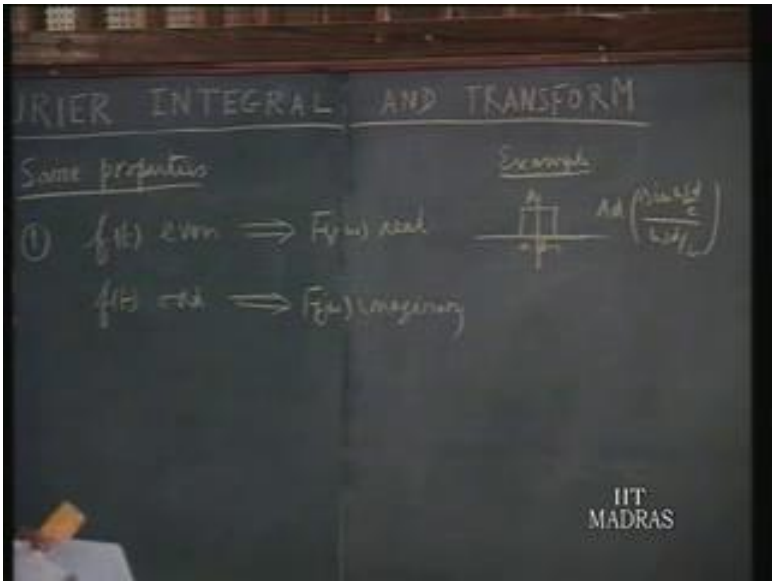

We move on to finding out the Fourier transforms of special functions of this type. So, what we now like to do is discuss the properties of Fourier transforms 1 this property will run somewhat parallel to the properties we discussed when, we talked about the Fourier series after all Fourier integral Fourier series are related to each other.

Therefore, it is natural for us to talk about properties which are similar to what we discussed in the case of the Fourier series. So, if f of t is even then it means that F of j omega is real. So, when you substitute j omega in the Fourier transform it trans out to be real. Example what we have seen this pulse this is an even function and then we found that F of j omega is real there is no j term in F of j omega.

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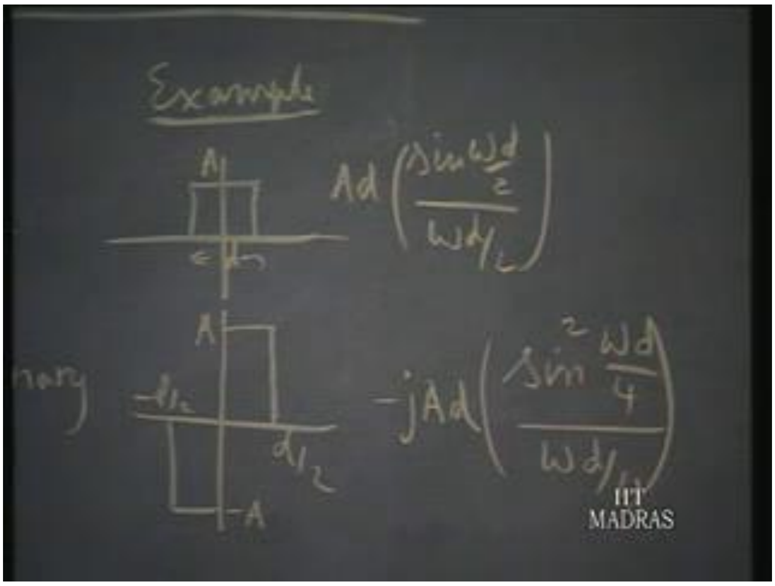

If f of t is odd then, it turns out that F of j omega will be imaginary there is a j term sitting outside will take just give an example which i will not however, derive that. Let me, write this this is A, this is d we know this is Ad sin omega d upon 2 divided by omega d upon 2. Because, i am repeating this again and again because, this come so frequently it is better to remember that.

Now, suppose i have a function like this A d upon 2 minus d u on 2 this is certain a not function. We have pulse of value A is lasting from 0 to d upon 2 and minus A is lasting from minus d upon to A. Its Fourier transform Trans out to be minus j Ad sin squared omega d upon 4 divided by omega d upon 4. So, you observe that this is immediately j times the j sign attached to it is purely imaginary.

The proof of this is straight forward if you write the expression for F of j omega as minus infinity to plus infinity of f of t cos omega dt minus j times minus infinity to plus infinity of f of t sin omega t dt.

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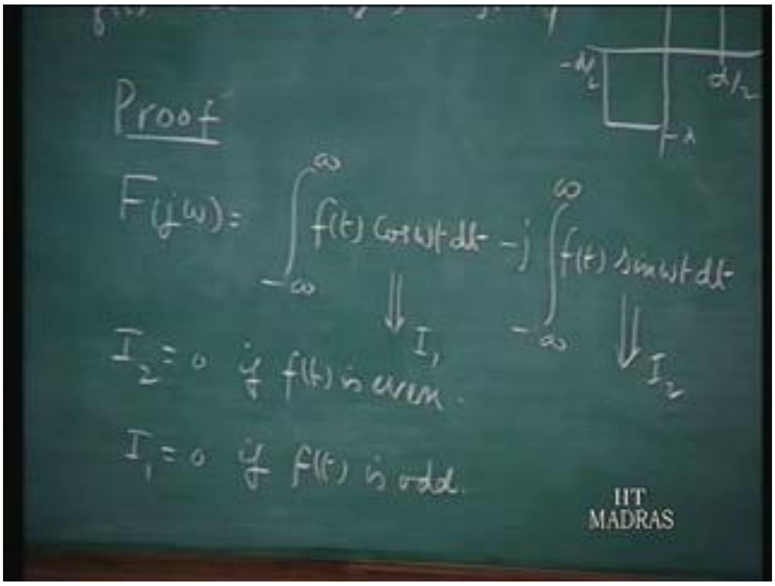

We recognize that e to the power of j omega t is cos omega t minus j sin omega t. So, we split the defining integral for Fourier transform into 2 parts like this now if you call this integral i 1 and call this integral i 2. Then, we notice that f of t is even the product f t cos omega t is also even. Therefore, we are integrating as far i1 is considered between 2 symmetrical limits.

So, whatever sequence of values this integrand takes from minus infinity to 0 it takes from minus infinity to 0 as well. Therefore, the integral i 1 will be twice the value of the integral from 0 to infinity. On the other hand, i2 involves ft times sin omega t therefore, that is the odd function of time and we are integrating between the symmetrical limits. Therefore, the value of the integral from minus infinity to plus 0 will be exactly the negative of the integral from 0 to infinity.

So, as a consequence i2 is 0 if f of t is even similar arguments we show that if f of t is odd f of t sin omega t is even. Therefore, the second integral will be non-zero, but the first integral which involves f of t cos omega t that turns out to be the integrand is odd. Therefore, if f of t is odd, f of t cos omega t is odd and you are integrating between symmetrical limits.

Therefore, i1 will be 0 if f of t is odd. So, we have the situation that if f of t is even then, the second integral vanishes and the first integral i1 is real. Therefore, f of j omega itself is going to be real. On the other hand, if f of t is odd the first integral vanishes and i2 remains, but i 2 proceeded by j therefore f of j omega will be purely imaginary. Let us, now discuss another property of the Fourier transform translation in time if f t has the Fourier transform F j omega then, F of t minus d that means, you delay the signal to by some amount then this turns out to be F j omega e to the power of minus j omega.

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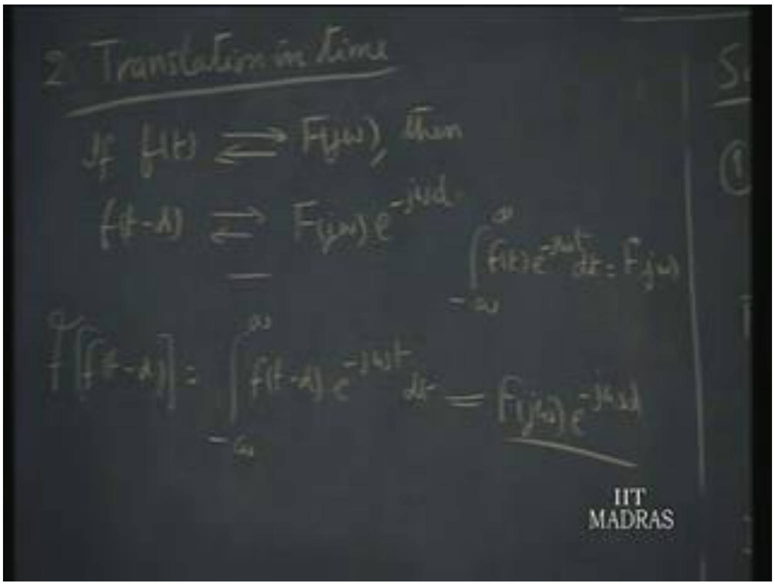

This is the useful result if you delay this time signal by d units, the Fourier transform gets multiplied with by e to the power of minus j omega d. It means, the magnitude of this being equal to 1 the Fourier transform magnitude will remain the same, but the phase will be disturbed by the amount minus omega d.

Very similar, to what we had in the case of cn coefficients in the Fourier series the proof is quite straight forward to find, the Fourier transform of f of t minus d all you have to do is from minus infinity to plus infinity f of t minus d e to the power of minus j omega t dt this defining relationship for the Fourier transform of f of t minus d.

Now, the what we have to do further is quite evident for you all you have to do is bring it we know that minus infinity to plus infinity of f of t e to the power of minus j omega t dt is F of j omega F j omega from this. Therefore, we have to bring it into that form so, all you have to do is put t minus d as x. Then, f of x e to the power of minus j omega t plus instead of t t minus d is x therefore, t equals d plus x and then, dx we get and then you have e to the power of minus j omega t term is coming outside. Then, you have minus infinity to plus infinity of f of x e to the power of minus j omega dx which is indeed the F of j omega.

So, it can be shown that this is equal to F of j omega very useful result extremely useful result.

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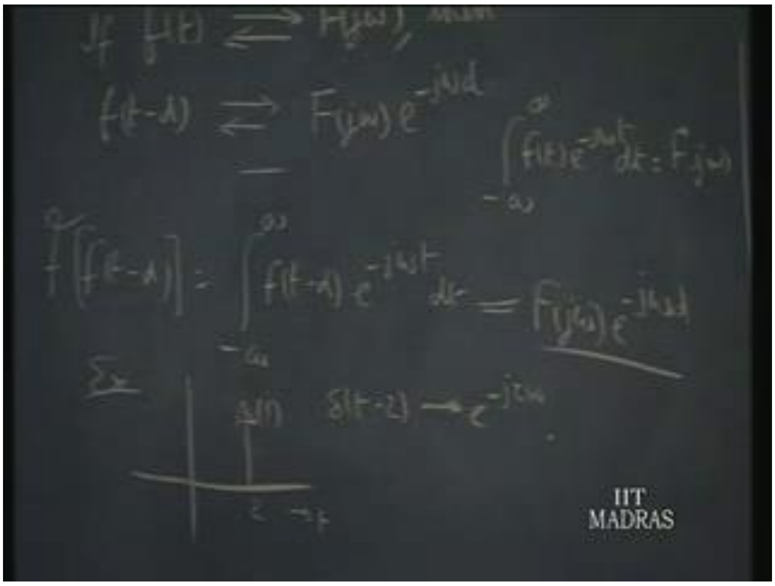

As an example simple example I will work out suppose, I have delta function here of unit magnitude sitting the time t is equal to 2. That means, this delta t minus 2 and we know the Fourier transform of delta t is 1 the Fourier transform of this is e to the power of minus j 2 omega. Because, you have to multiply the Fourier transform of delta t is 1 you are multiplying this by j e to the power of minus j omega d the delay is 2 second.

Therefore, e to the power of j omega 2 omega We will work out, some more examples in the next lectures, but before that let us now summarize what we have done today. So, in this lecture we started with review of the Fourier transform defining integral and the Inverse Fourier transform. Then, we constructed the Fourier transforms of a few typical type functions.

Specifically, these are the impulse functions, the rectangular pulse functions, the exponentially decaying sinusoidal functions, a triangular pulse functions and we observed that, the if the Fourier transform turns out to be either is real turns out to be real. Then, there is no necessity for us to plot the magnitude spectrum and the phase spectrum separately.

Because, the entire function is real we can exhibit this by means of single spectrum which represents the value of F of j omega. We also, introduce ourselves to a new function Sinc x which is defined as sin pi x over pi x: Sinc x has the value 1 at x equal to 0 and vanishes when x takes the integral value like 1 2 3 minus 1, minus 2, minus 3 etc.

So, this new function is helpful in finding debiting the Fourier transforms of pulse functions. Then, we took up for steady some important properties of the Fourier transforms: the first property we studied is that, if f of t is even then F of j omega is real and if f of f is odd F j omega is purely imaginary. So, we also saw that we also mentioned earlier that F of j omega is real.

Then, the magnitude spectrum and the phase spectrum need not be exhibited separately they can be exhibited by 1 spectrum which use both this magnitude and phase of course, will be 0 or 180. Same can be done if the spectrum is purely imaginary, in that case we will plot of F of j omega by j that means, the imaginary part of F of j omega is can be plotted by means of 1 spectrum and because, the real part is 0 we can also do that extend this principle of having only 1 spectrum even F of j omega is imaginary by keeping at the back of mind.

Whatever, values which we plot of the spectrum are purely imaginary quantities. The second important property that, we studied was that if the function f of t is delayed by tau seconds then, the Fourier spectrum gets modified F of j omega multiplied by e to the power of minus j omega tau. That means, the new function will have the same magnitude spectrum as the earlier function f of t, but the phase will be modified.

This is not surprising because, we saw a similar result in the case of the Fourier series where the cn coefficients the magnitude will remain undisturbed if the periodic pulse periodic function is delayed by certain amount of time per tau seconds only the case will get modified.

After all f of j omega is closely related the cn coefficients therefore, we have the similar result here also. We will stop at this point and continue our discussion of the properties of the Fourier transform in our next lecture.