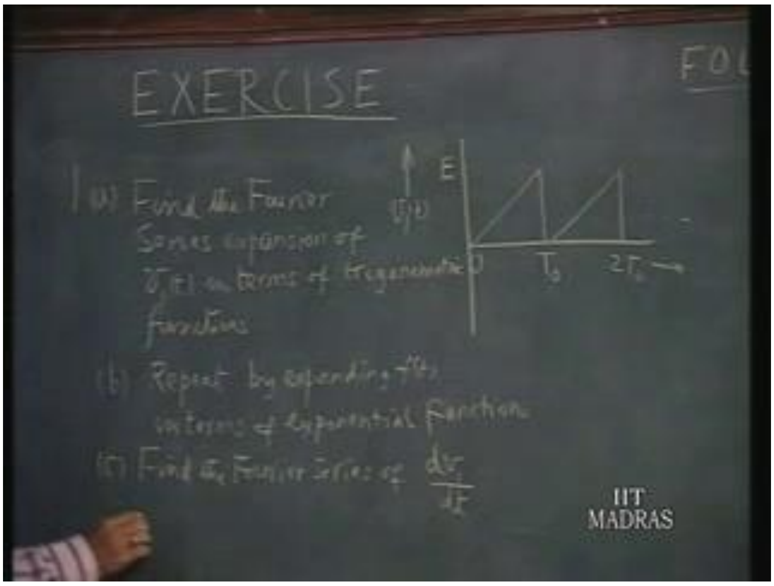
Networks and Systems

Prof V G K Murti
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 12

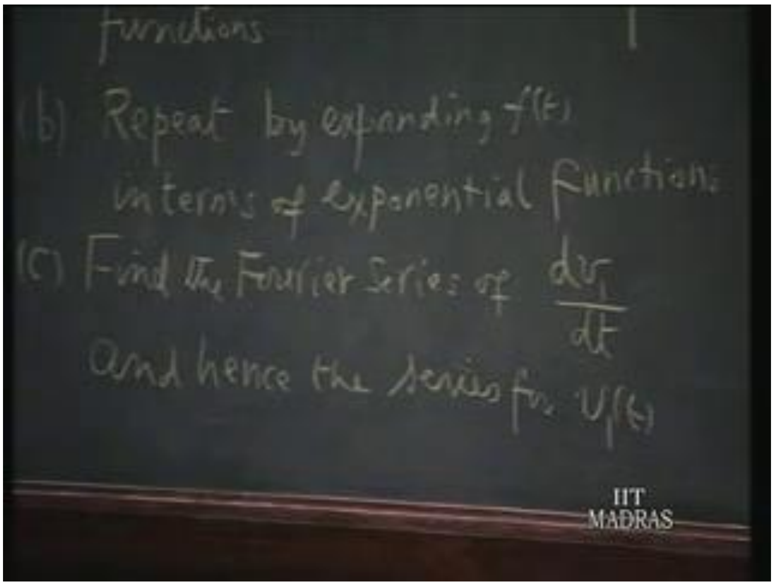
I should now, like to give set of 6 problems to constitute an exercise for you to work out covering the topics that we have discussed so far, on the fourier
series first problem.

(Refer Slide Time: 01:27)



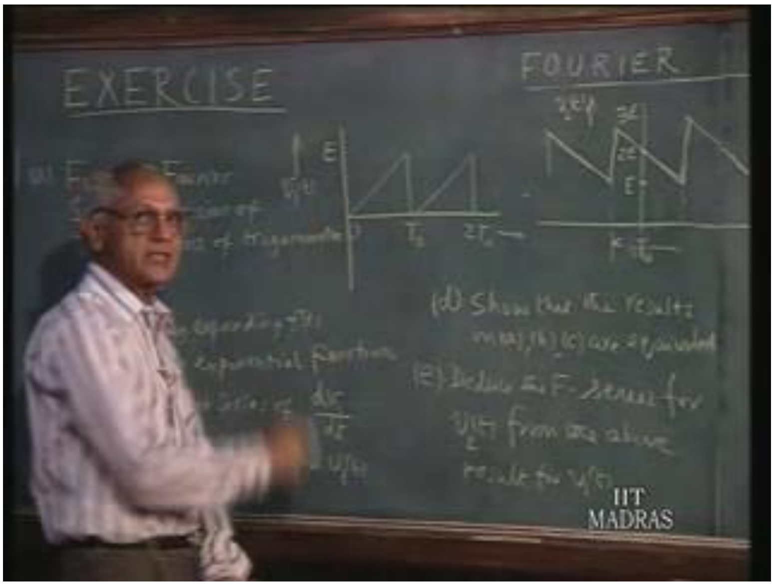
Consider a wave like this triangular wave; this is the voltage wave v1 of t. So, this is the triangular wave saw tooth wave as it is called with an amplitude e and period t not. Find the fourier series expansion of v1 of t in terms of trigonometric functions. That is, in terms of a not a n and bn repeat this by expanding f t in terms of exponential functions, that is; in terms of cn c.

(Refer Slide Time: 01:27)



Find the fourier series of the derivative of this function d v 1 by dt. So, once you take the derivative you should observe the slope is constant here and the slope is constant here. But at this point you have impulses, because there is a certain jump e to 0 e to 0. So, it will have you will have also impulses deal with find the fourier series d v1 by dt and hence the series for v1 of t. So, you do the find the fourier series of v1 t by 3 different methods.

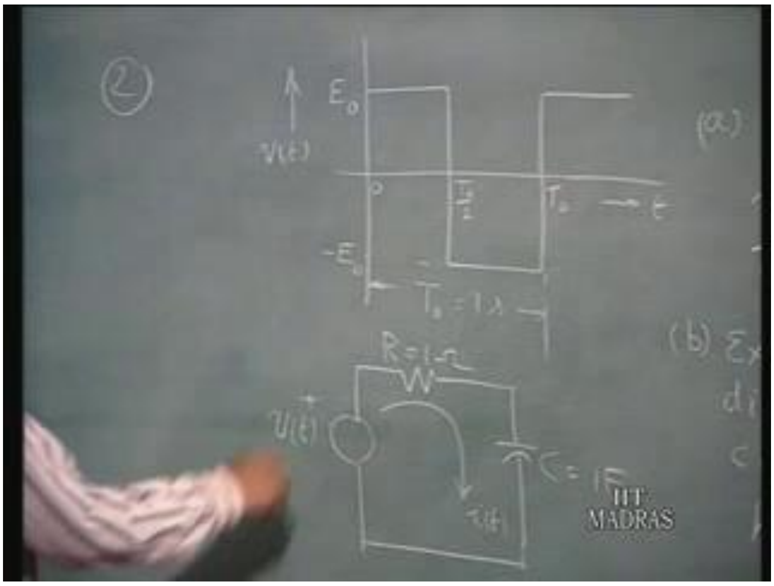
(Refer Slide Time: 04:39)



D: Show that the results obtained in abc are equivalent. Compare them and show that they are all equivalent. E: you take another wave form like this, this is e, this is 3e and this is 2e and the period is t not as before. That means, the wave form is this. This I call v2 of t deduce the fourier series for v2 of t from the above results for v 1t. In other words what we expect here is, you do not find the fourier series for v 2 of t directly. You some where relate v 2 of t with v 1 of t.

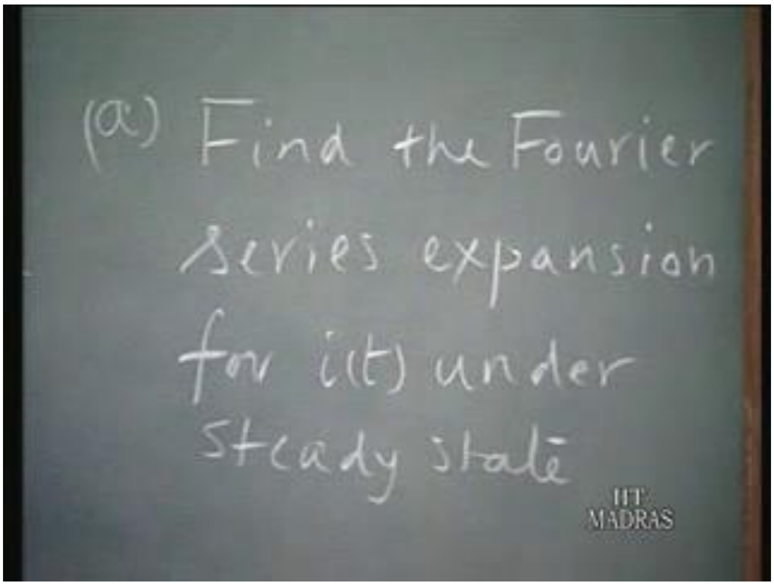
Since we know, the fourier series for v 1 of t you use the information to calculate the fourier series for v2 of t that is the first example, problem number 2.

(Refer Slide Time: 07:06).



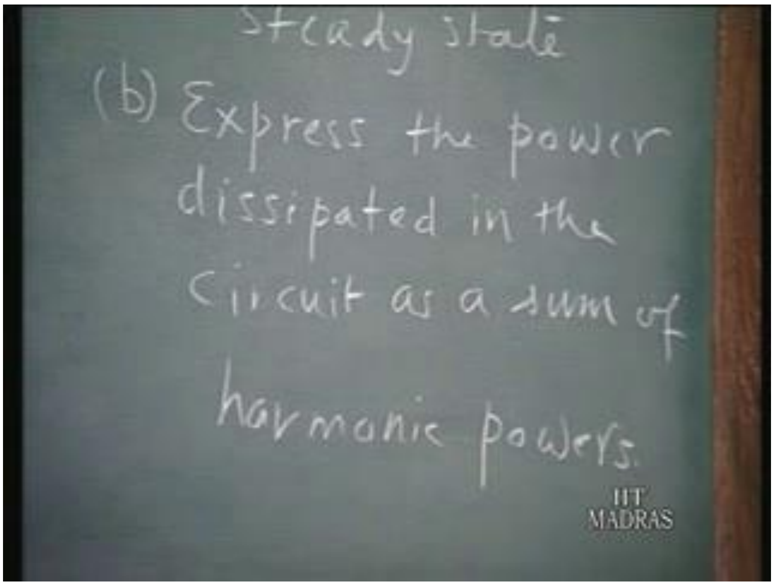
You have wave form like this e not minus e not period t not take it to be 1 second this is of course, t not upon 2 this is t not 0 time t this is voltage v of t. Now, if a voltage adding this type of wave form v of t is given to a rc circuit containing a resistance of 1 ohm and a capacitor of 1 farads. Let under steady state the current i(t) close to the circuit. So, this type of wave form given to this circuit you are asked to find out,

(Refer Slide Time: 08:21)



Find the fourier series expansion for i of t under steady state. That is; the first part of the question.

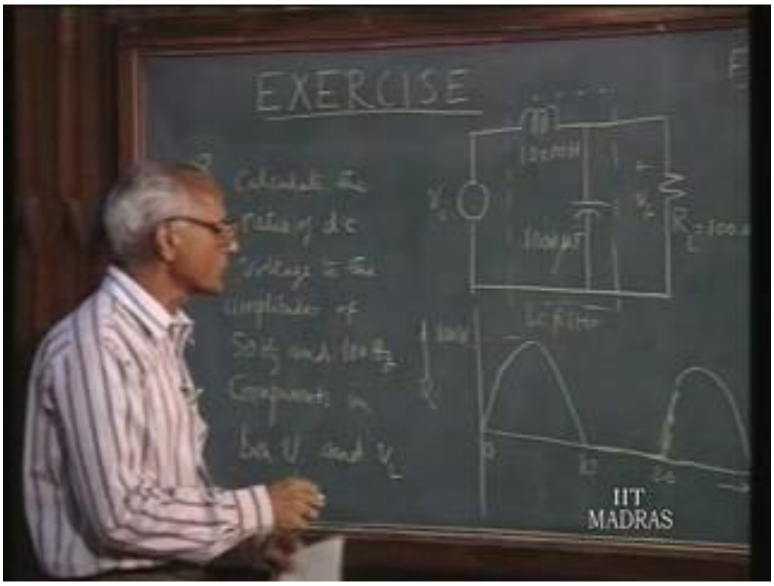
(Refer Slide Time: 08:58)



B. Express the power dissipated in the circuit as a sum of harmonic powers. So, the first problem requires to find out the fourier series expansion for the current i(t) under steady state. You find out the various harmonic components of v of t, find out the component of current at different frequencies given by the source and sum them up and there is fourier series expansion for i of t. In the second part you calculate you are asked to calculate the power as the sum of harmonic powers you have 2 ways of doing this.

You know the various harmonic components of the current; therefore, knowing the value of r and the harmonic component of current you can find out the harmonic power for each harmonic. Alternately you know the fourier series expansion for v of t, you know the fourier series expansion for i of t. So, you can associate the voltage component at particular frequency, the current component at the same frequency find out the angle between these 2 and find out the harmonic power for each 1 of the harmonics. So, find out the summation express this as sum of the various harmonic powers and that will be the total power this is problem number 2.

(Refer Slide Time: 10:40)

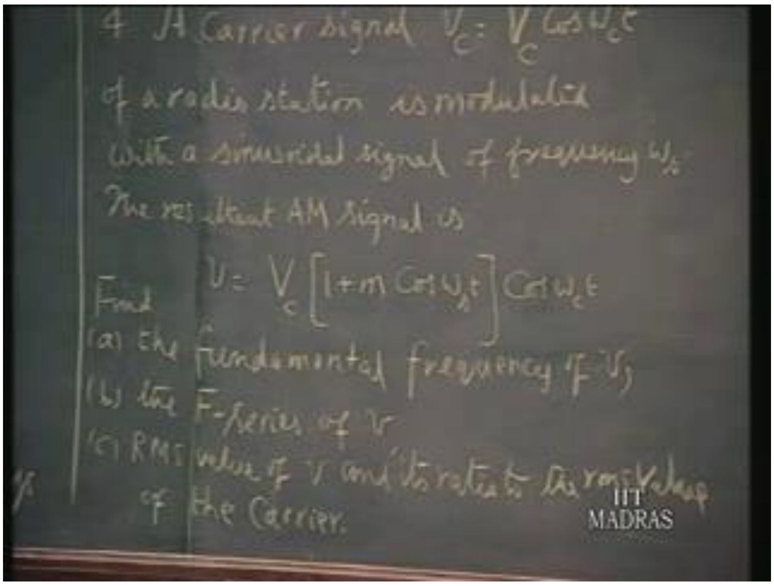


The third example let us consider circuit where we have an input voltage. Which is half wave rectify sine wave like this, with an amplitude of 100 volts this is vi 0, 10, 20 this is t in milli seconds. That means; this 50 cycle's wave from which we generate half wave rectified sine wave 100 volts. This is the input voltage and we like to have a filter l c filter here and try to supply the load resistance rl of 100 ohms as pure as dc is possible.

So, this is the output voltage vl .So, vl should be the dc voltage the extent possible all the ripples should be attenuated. We have a filter which consists of an inductor of 100 millihenries. Let us say capacitor of 1000 micro farads and the input voltage is this. Calculate the ratio of dc voltage to the amplitudes of the 50 cycles and 100 hertz components in both the input voltage vi and the output voltage vl. So, you have the input voltage you have the dc 50 cycles or hundred cycles and so on. We want to calculate dc components of the input voltage and the same quantities in the input voltage and you should show, that the 50 cycles or 100 cycle components are attenuated considerably in relation to the dc when you go to the output.

This is l c filter, we have earlier in the class workout example with an rc filter. So, that this is you will see that this will be more effective filter than the rc filters.

(Refer Slide Time: 13:46)



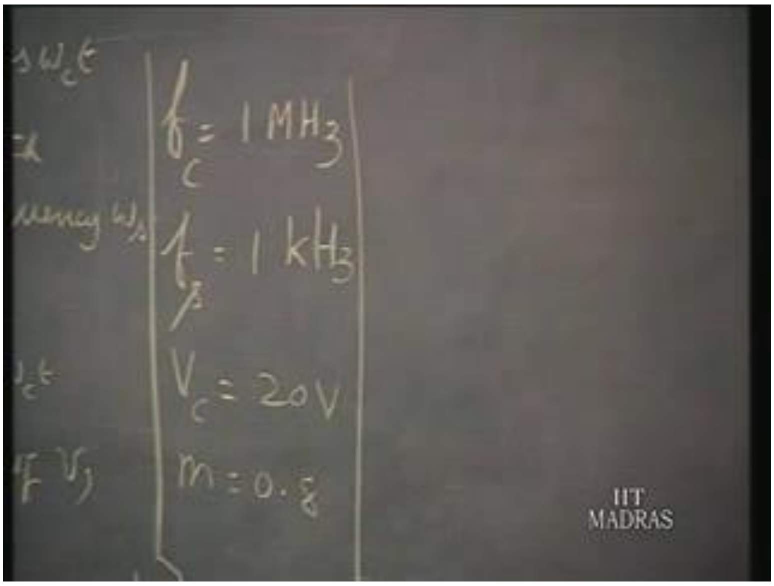
Fourth example:

The carrier signal vc is VC cos omega ct of a radis station is modulated with a sinusoidal signal of frequency omega s radius per second. The resultant amplitude modulator, this is AM means; amplitude modulator signal is v equals to vc 1 plus m cos omega s t. Where omega s is the sinusoidal signal of frequency omega s and this is cos omega ct. So, this is the amplitude modulator signal b. Now, you are asked to find out. a: the fundamental frequency after all this is the periodic wave form because you have got only cosine term with different frequency repeats itself.

What is the fundamental frequency of the amplitude modulated wave, find the fundamental frequency the amplitude of the am wave v. b: the fourier series expansion of v. Now, if you are clever enough you could see that you do not have to do any major integrations etc here. You can simply your work by combining the cos term suitably and you can do this. c: rms values of v and its ratio to the rms value, rms value of v and its ratio to the rms value of the carrier rms, value of the carrier is vc up on root 2.

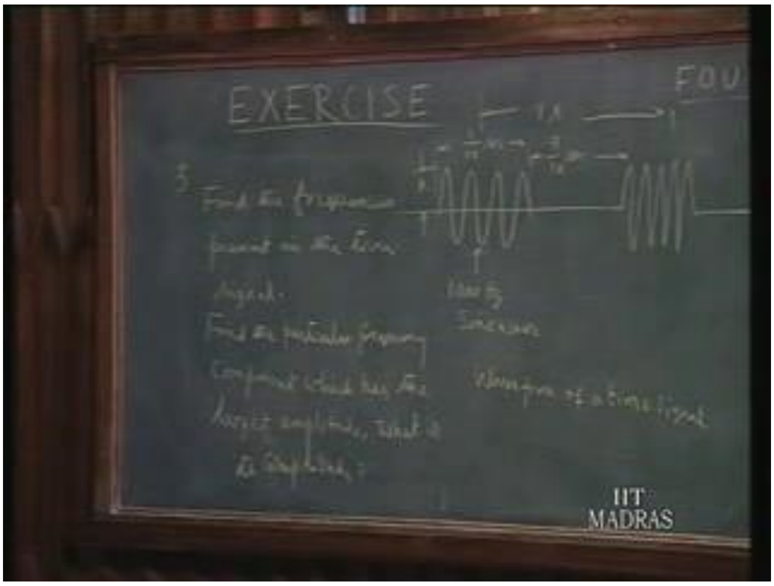
So, you find the rms value of the amplitude modulated wave and see what proportion it does to the rms value of the carrier these are the 3 things you have to find out in this particular problem. Here, I have to give you some additional data for problem 4 fc.

(Refer Slide Time: 17:52)



You can calculate numerically 1 megahertz, that is; the carrier the signal frequency you take it as 1 kilo hertz vc to be 20 volts that is this and m as point 8. So, this is the numerical data which you can use to work out the details of problem 4 .You often heard, the time signal that is broadcast by a radio station immediately before the news pip pip that type of thing. So, the wave form the signal will have something like this like this.

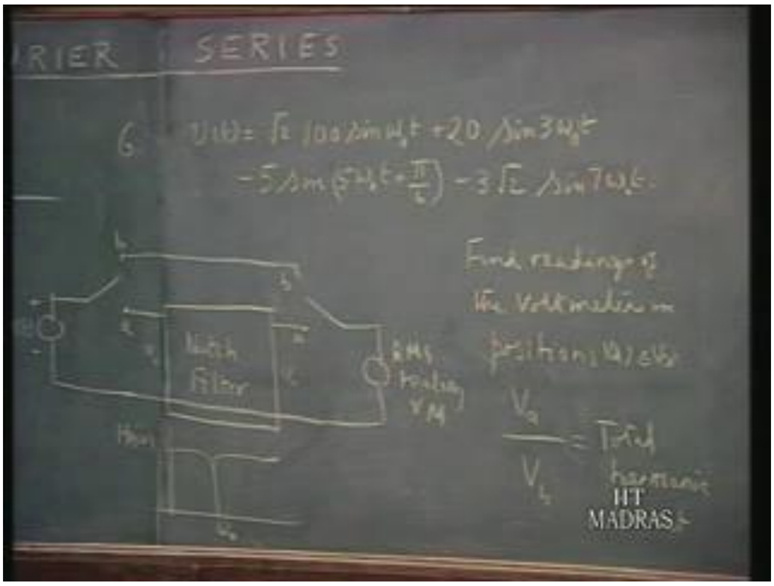
(Refer Slide Time: 18:52)



So, you have the burst of 1000 hertz signals you take just a data a typically. Suppose you have thousand hertz sine wave which is maintained for 1 tenth of the second. And this is repeated every 1 second; that means, this is blank period 9 by 10 second and this is repeated and this is 1000 cycle pure sine wave. This is the wave form of a time signal. So, you are required to find the fourier series analysis of this. So, in particular you are asked to find the frequencies present in the time signal with this wave form.

Find the frequency component, particular frequency component which has the largest amplitude and what is its amplitude? So, if you take amplitude of the sine wave as a. What is the amplitude of the component which has the largest amplitude? Now, in doing these analyses just give a hint you can put your origin, at convenient location and assume that you have the continuous sine wave. If you multiply this by the periodic pulse style you can get this. So, sinusoidal multiplied by periodic pulse style will give this. So, you use that information to find out the fourier series for this.

(Refer Slide Time: 22:14)



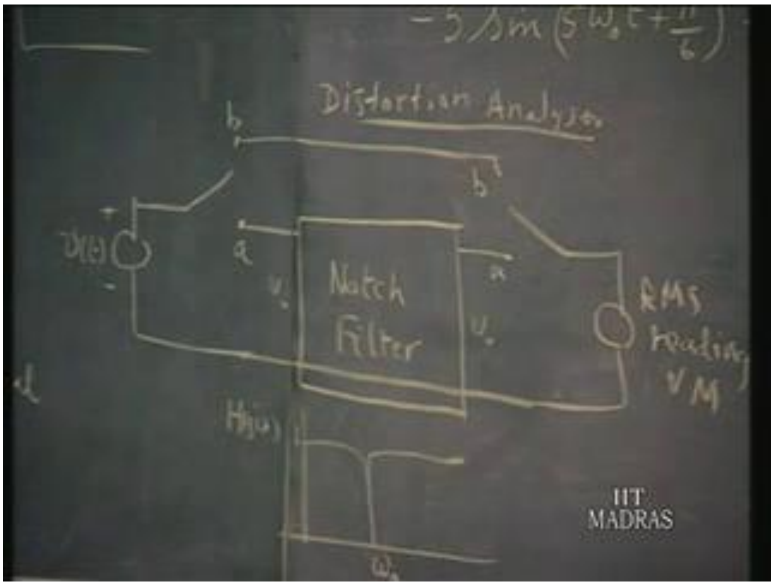
Lastly it is often interest for us, to know the rms value of the various harmonics. As proportion of the total rms value. This example, illustrates an instrument. Which measures this? Suppose, we have v of t which is root 2 times 100 sin omega not t plus 20 times sin 3 omega not t minus 5 times sin 5 omega not t plus 5 up on 6 minus 3 root 2 sin 7 omega not t. So, we will stop with that up to the 7th harmonic. Now, we have an instrument which incorporates. What is called a notch filter will explain the characteristic of filter in a moment.

So, this periodic non sinusoidal signal is given to this notch filter it can be connected to the notch filter. So, have the switch here which can be thrown to either a or b. So, likewise here is switch can be thrown either b or a. And this is connected to an rms reading volt meter. What this notch filter does is? This is the output v not and this input vi. Suppose, it has a frequency response function h of j omega of this notch filter is it allows all frequencies, but cuts off any desired frequency. So, if this amplitude is 1. If this frequency is omega x or omega not in our case that particular frequency omega not is out changed fully.

If you introduce a sinusoid with that frequency you get 0 outputs. All other frequencies come out unscapped, without any amplification or attenuation. So, that is the characteristic of the notch filter this appears to be a nock that is why it is called notch field. So, we have notch filter like this. So, what happens is if you put the switch on b then the meter here volt meter reads the rms value of the input signal of v of t. On the other hand, if you put switches on a both these on a the fundamental is cut out. And the meter reads the rms value of all the other harmonic components right. So, you have 2 readings in a and b. Find readings of the volt meter in positions a and b. In position b it is vb is suppose the rms value that is the total in position a. Suppose, you put this a va corresponds the rms value of the various harmonics, vb will be the reading corresponds the rms value of the input voltage.

the ratio va to vb is said to be the total harmonic content of this voltage. And suppose, this assumed to be sinusoidal this total harmonic content is said to be the total harmonic distortion of the sinusoidal. So, this meter as such is called distortion analyzer and this is used to find out. How wave form which is supposed to be sinusoidal how it gets started by contamination with various harmonics. So, you have suppose an amplifier system you feed input sine wave the output also the amplified version must be a pure sine wave, but sometime the amplifier into this distortion and you like to measure. What is the amount of distortion and that is caused by a device like amplifier which may have some non-linearities.

(Refer Slide Time: 27:18)



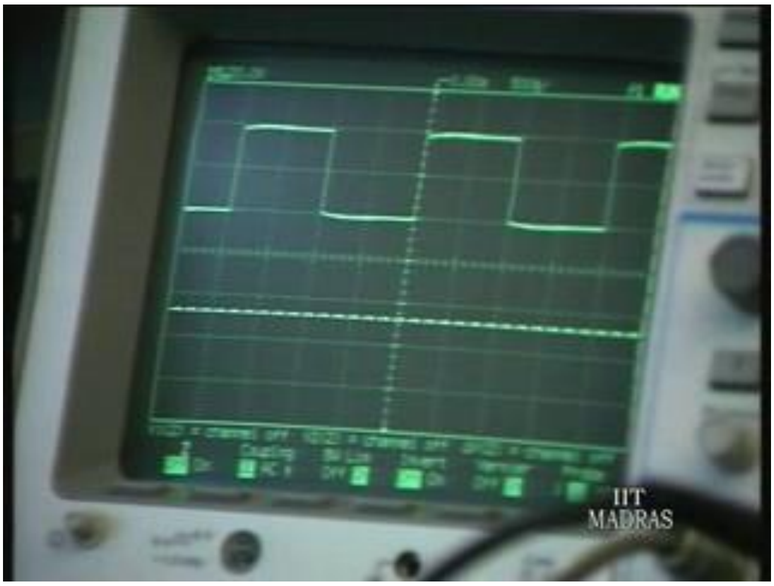
So, this particular instrument which is called harmonic analyzer a distortion. Analyzer measure the harmonic distortion this is called distortion analyzer. So, this distortion analyzer is essentially an instrument which measures the rms value of the various harmonics. As the ratio of the rms value of the composite wave or usually in practical cases this harmonic this rms value of the total wave very nearly the rms value the fundamental. So, we can say it measures the rms value of the various harmonics, as a fraction of the rms value of the fundamental as well.

So, at end of this exercise; let us go to the laboratory to where we have the demonstration experiments, illustrating some of the concepts on fourier series that we have been discussing. so far, What we have here is an experimental,

(Refer Slide Time: 28:23).

Set up to demonstrate the harmonic analysis of a square wave we have a function generator here, which generates the square wave. And that is how displayed on the oscilloscope this is the square wave of 2 units pit to peak value or 1 unit peak value. Now, what we are going to do is; put this square wave through low pass filter which is this. So that, we can allow only harmonic up to certain order to go through and come out of the output. And the output voltage will be also displayed on the oscilloscope in the bottom trace.

(Refer Slide Time: 29:06)



So, let us see what we have here, is a 500 square wave. And now, I am putting a low pass filter cut off frequencies. So, that only the fundamental pass through and when you do that.

(Refer Slide Time: 29:23)

You, observe that the output of the low pass filter corresponds only, the fundamental a pure sine wave. And this has the amplitude which is about 4 upon 5 times the amplitude and square wave as we should expect. Now this is the, that is the only the fundamental is pass through. Now, if you allow other frequencies for example, if increase the cut off frequency at the low pass filter. So, that I allow up to the third harmonic. Now you can see, the third harmonic frequency also allow; Therefore, not only the fundamental the third harmonic.

(Refer Slide Time: 29:47)

Is also coming through and we can increase the cut off frequency and allow more and more harmonic come through. And as you can see, as you pick up more and more harmonics the output becomes closers to the square wave. In fact, if you go to very cut off frequency.

(Refer Slide Time: 30:10)

In fact, if you go to very cut off frequency, almost a square wave. A point which we should notice here is, that just at the pointer jump just immediately after the jump there is a overshoot as we expected from the discussion in the earlier lecture classes. This is due to gibbs phenomenon and immediately after the discontinuity the series fourier series will not converge. At the point of discontinuity it will not converge uniformly; therefore, we expect lower shoot. The surprise thing is that, in strict mathematical analysis you should have lower shoot both immediately after the jump and immediately before the jump. So, strictly speaking mathematically you should have over shoot here also.

But, in practice experimentally you never get that the reason is that the our low pass filters are not ideal low pass filter in that all the frequency component that they allow will not be faithfully passed out, but there will be phase distortion there will be phase delay. Because of this reason you will not get this kind of overshoot just, before the jump. This is all also in accordance what is called principle of causality because no physical device can anti speed the jump that is going to take place latter. So, no physical device can respond to jump that is going to occur later and start ringing here.

Therefore, it is physically is not possible for us to construct any experimental set up where you can demonstrate this kind of ringing even before the discontinuity. So, this just demonstration of how the various harmonics is going to built up the square wave.

Let us go back, once again this is 1 all harmonic have to let us say 30 third harmonic pass through and you go back as you allow small and smaller only up to smaller order harmonics the wave form is no longer is square wave. And in fact, when you have the cut off frequency just above the fundamental only the fundamental is pass through. As mentioned in the lecture you can perform harmonic analysis.

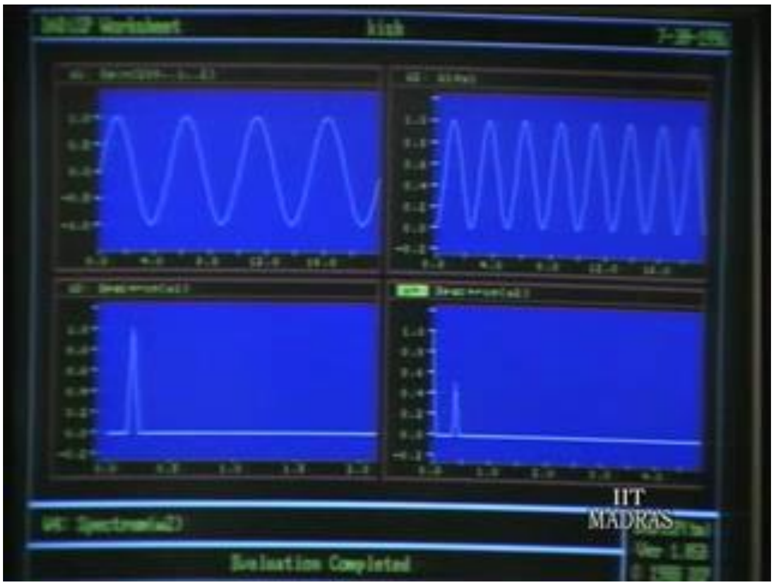
(Refer Slide Time: 32:16)

Using instruments known as harmonic analyzer using instruments known as harmonic analyzer spectrum analyzer. It is also possible nowadays to use the computers to affect the harmonic analysis of any given periodic wave form through software. And here is the demonstration 1 such situation we have square wave here and computer has calculated the various harmonics and displayed them in the windows that are shown here. This is the fundamental, this is the third harmonic, this is the fifth harmonic and sum of the fundamentals third and fifth harmonics are displayed here and we also observe this is the spectrum of this.

The spectrum shows peaks at the fundamentals third and fifth and so on. and we also observe the peak progressively go down you would expect that the amplitude of the various harmonic orders will go in the manner 1 over n where n is the order of the harmonic. Now, strictly ideally speaking the spectrum should be line spectrum there should be lines at the discrete frequencies. But, because of the various round of computational errors and inter phase on the system. You will only get the peaks it will not be a pure line spectrum in the sense that you have a complete blank between 2 lines. Rather you have some kind of continuous spectrum, but at the frequencies which are presented in the composite wave. The computer programs will also calculate the various amplitudes various amplitude components and displays them with suitable scale factor on the screen.

So, this is very convenient way of analyzing the any given periodic wave form and finding out the various harmonic components. Now, let us use the computer program to find out the harmonics of another wave form rather than the square wave.

(Refer Slide Time: 34:24)



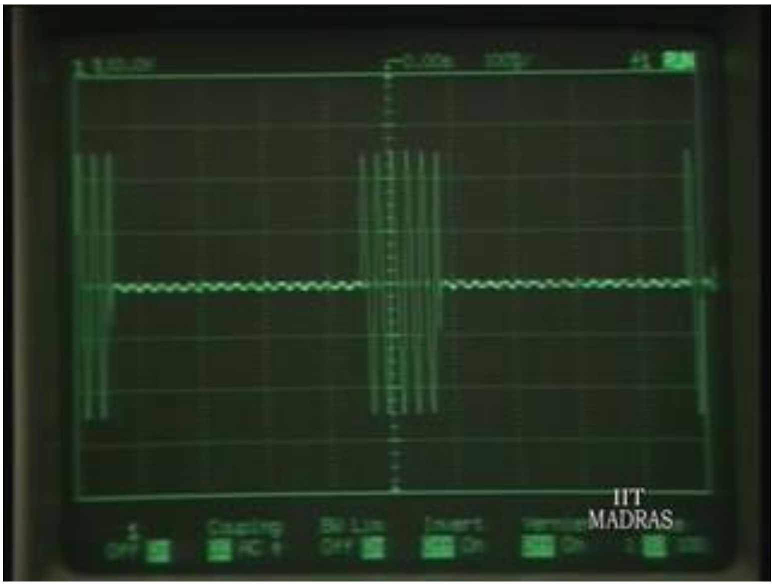
As a second example, let us take a pure sine wave and perform its harmonic analysis so; obviously, this spectrum consists of only 1 line at 1 frequency which is the frequency of the sinusoid; therefore, this is the spectrum of the sine wave. You have 1 spike at the fundamental frequency and no other harmonics are present.

Now, if i multiply sine wave by itself, get sin square theta this can be expressed as sum of a dc component plus second harmonic component. And that is what is shown here, this is the function which is obtained by squaring this. And the spectrum of that; obviously, consists of a dc term plus sinusoidal which is not the twice the frequency of the fundamental and that is these are the 2 lines corresponding to that.

These are dc components and this is the sinusoidal and all other harmonics are absent. You observe that the both these wave form are very smooth. In the sense that they are infinitely differentiable. So, all the derivatives whatever order you take are continuous and therefore, the spectrum comes down. The amplitude of the spectrum very past larger than n to the power of k .Where k is any large that as you wish and therefore, you observe the spectrum lies down 0 after few harmonics. Because the 2 functions very smooth functions.

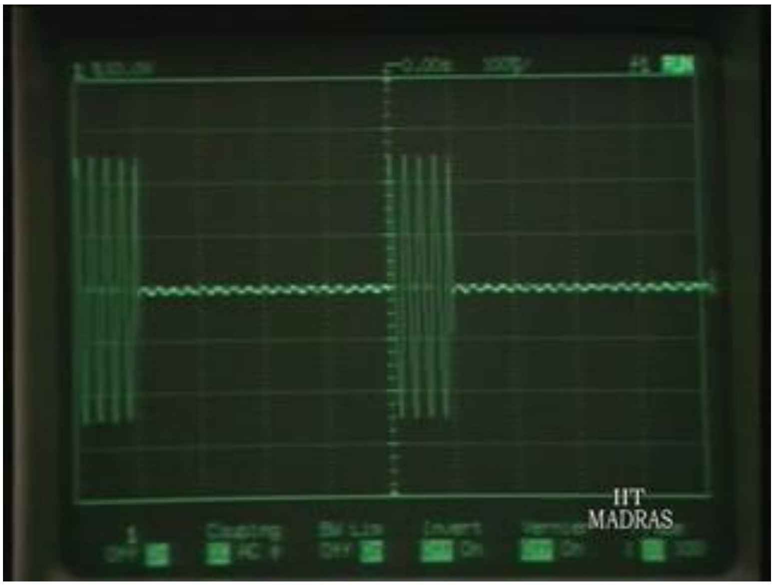
What you are hearing now, is the sound of the time signal which we get before the beginning of the news broadcast on the all India radio. This is the type of wave form which has been given to you as the part of the problem in the last exercise. These are 1000 cycles sinusoidal which is the blank per 9 second of the period and then it is kept on the onto off ratio is 1 to 9, every 900 million is off. So, these such time signals produce this kind of sound. Now, let us see the wave form of this on the screen.

(Refer Slide Time: 36:42)



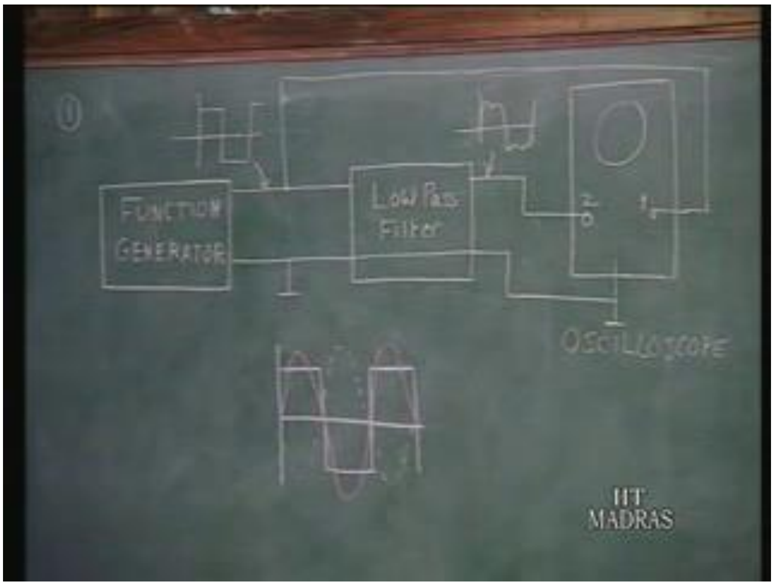
The sound you just now, heard corresponds a wave form like this.

(Refer Slide Time: 36:45)



Where you have thousand hertz sinusoidal which is kept on the certain small amount of time. One- tenth of the period and then it switched off for the remaining nine-tenth of the period. So, you have burst of sine wave and it itself periodically. So, this is the wave form that you get. And if you increase this, this is how it looks you have actually it can be produced by multiplying a sine wave with periodic pulse strain of width corresponding the interval of time. And in the exercise last exercise you are given this kind of wave form and you asked to find out the various harmonic components given the data into the frequency it is on and also the blank period and the mark period of the pulse strain. The few commons on the,

(Refer Slide Time: 37:38)

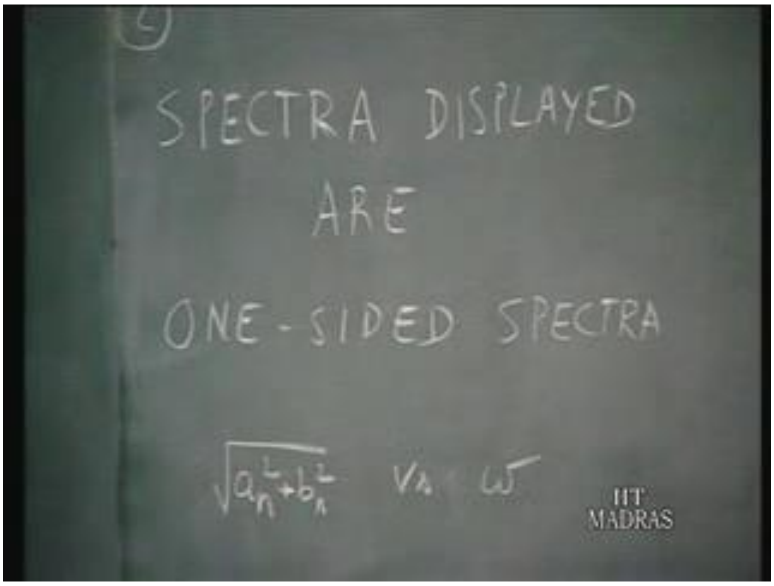


Demonstration we have just now witnessed in the first part of the experiment. You had a function generator which produces the square wave. This is put through low pass filter which allows frequencies up to certain order to pass through. And both these square wave and output of the low pass filter are displayed on the oscilloscope. oscilloscope has 2 channel 1 channel you have the square wave and the channel below we displayed this type of wave form.

When frequency only up to a point with just above the fundamental frequency of the function generator is put, ideally this is the square wave you should get sine wave which is the fundamental which is 4 by 5 times of the amplitude of the square wave. Some of you might have observed that the square wave. The this is the square wave the sine wave is not sitting here at this point, but slightly shifted like this as shown by the dotted lines here. This is because the low pass filter gives the phase shift when it is allowing the signals to go through and because of the phase shift produced by the low pass filter, but particularly frequencies very close to the cut off frequency you would observe the phase shift.

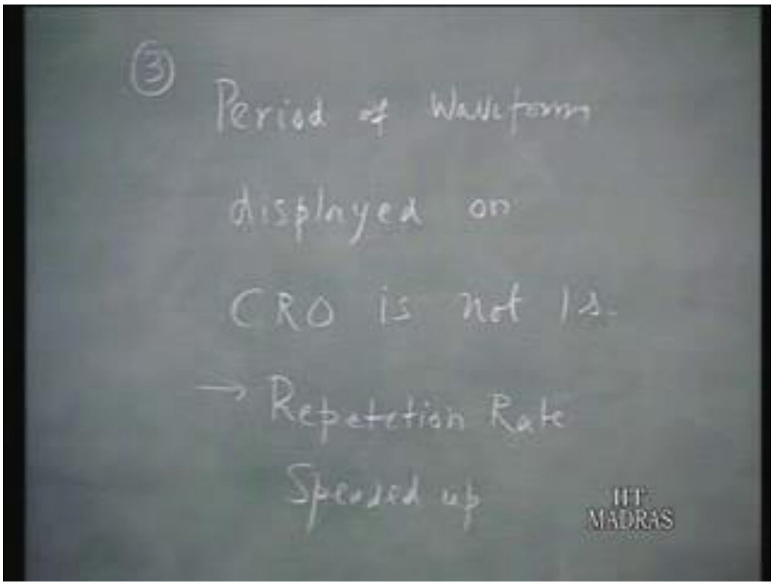
Those of a few might have noticed it will know that the reason is the phase shift produced by the low pass filter. It is because of the phase shifted produced by the low pass filter for different frequencies, that 1 of the gibbs phenomenon is lost finally, we allowed a lot of frequencies you find the gibbs phenomenon overshoot occurs at this point not at this point. That is again because of the phase shift produced by a practical low pass filter this is the reason for this. Now, let us move onto the second part of the experiment where on the computers screen you have seen.

(Refer Slide Time: 39:29)



The spectra that are displayed for different types of wave forms; the spectra that are displayed there are 1 sided spectra not the 2 sided spectra not cn versus omega for both positive and negative values of omega. But only for positive omega we are talking about the amplitude of the sinusoidal versus omega. So, that is why the dc is that at 1 end of the displayed wave spectrum. So, it starts from and dc and goes up to higher frequencies. So, you recall that is 1 sided spectrum whatever is displayed not the 2 sided spectra that we have been talking about the lectures. Let us go to the third part of the demonstration where we had a time signal.

(Refer Slide Time: 40:09)



That is; we had the time signal and then we displayed the burst of sine wave on the screen. Now, we played a little trick there they actually sound of course, 1000 cycle signal which is repeated at 1 second, but then if you repetition rate of 1 second is too slow to enable us to capture on the cro. Because the writing speed is too slow; therefore, what we did was the repetition rate speeded up it is not 1 second it is something much less.

So, that is the reason why in the burst of the each sine wave you will not see hundreds of cycles, but may be 4 or 5 cycles only you have seen. That is because the while the general shape of the wave form is the same, but the repetition rate has been increased and that is why you would not see in the single burst of the sine wave a number of cycles. Actually the sound we heard is of course, is corresponds to 1000 hertz repeated at once per second. That is; the reason for this. So, these are common you can now go back and look at the demonstration once again please go ahead.