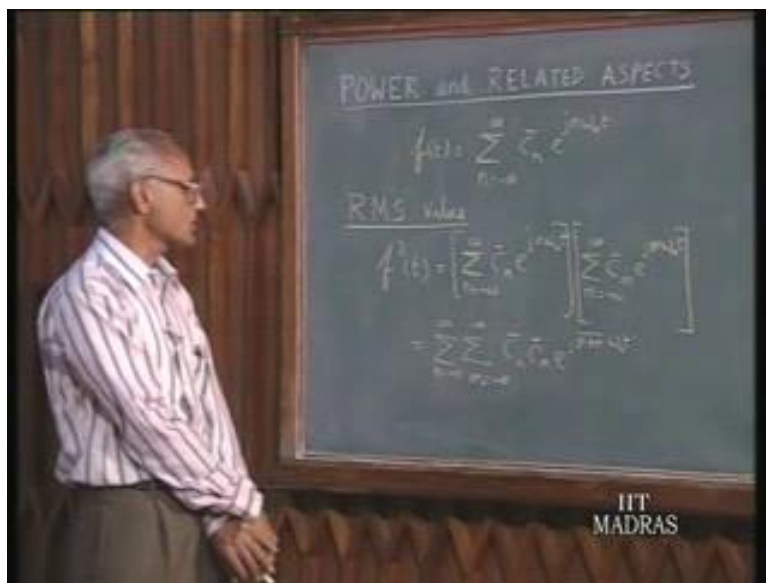


**Networks and Systems**  
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**Indian Institute of Technology, Madras**  
**Lecture - 11**  
**Fourier Series (5)**

Continuing our discussion of Fourier series today, we will take up consideration of power and related aspects associated with periodic signal in its exponential representation. To start with let us, try to calculate the RMS value of this periodic signal in term of Recall that the fourier series expansion of periodic f of t can be put in this form.

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To the fourier co-efficients  $c_n$  in its exponential representation. So, you like to calculate the RMS value of  $f$  of  $t$  you can it in time domain as the average of the mean square of a  $f$   $t$ , but we like to carry out this analysis in terms of the complex coefficient  $c_n$ .

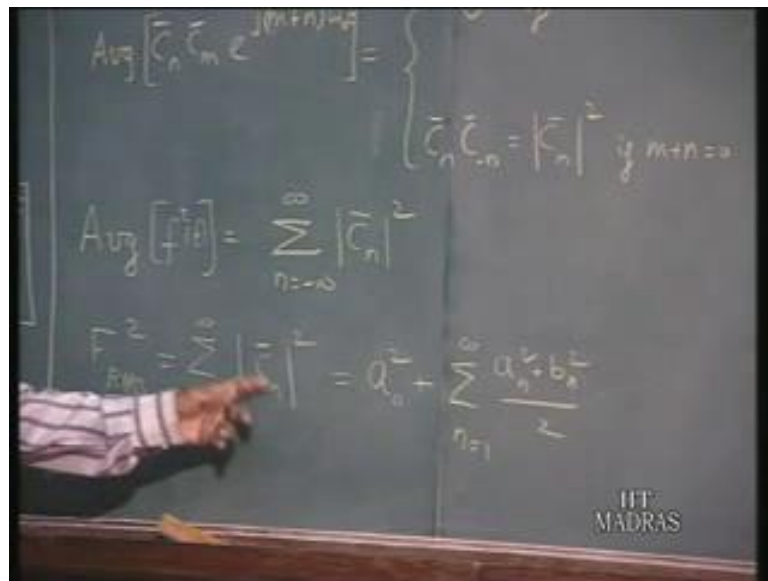
Let us see, how we do it suppose, i like to calculate  $f$  square of  $t$  then this can be written as  $c_n$  into  $e$  to the power of  $j n \omega_0 t$   $n$  of course, ranging from minus infinity to plus infinity multiply by the same series in order to facilitate our taking stock of the various terms, i would like the index here to be  $m$  instead of  $n$   $m$  from 1 to minus infinity to plus infinity.

This of course, can be written as, summed of  $m$  from minus infinity to plus infinity and summation on  $n$  from minus infinity to plus infinity. So, each term here gets multiplied by each 1 of the term of the second summation. So, i have  $c_n$  multiplied by  $c_m^*$   $e$  to the power

$j m$  plus  $n$  omega not  $t$ . Now, in order to find out the mean value of this  $f$  square  $t$  we should find out the average of this whole series. So, the average of the summation is nearly the summation of the averages.

So, we will like to find out the average of each 1 of these terms, if you do that then we can sum up this averages and say that is in the average of  $f$  square  $t$ .

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Because the RMS value of  $f$  of  $t$  is the square root of the mean of the square. Let us see the average value of  $c_n c_m e^{j(m+n)\omega t}$  what it is going to be. When we are talking about the average, we always understand that is the average over a full cycle the complete period of the fundamental. So, this is the value depending term  $c_n$  and  $c_m$  are constant.

So, what do we have if  $m$  plus  $n$  is an integer then  $e$  to the power  $j k \omega t$ , if you integrate over a complete period it is going to vanish because it is after all  $\cos m$  plus  $n$  omega not  $t$   $j \sin$  omega not  $t$  any  $\sin$  term cosine term integrated over a complete period or the integral number of period is going to 0. Therefore the average of this will be 0. if  $m$  plus  $n$  is not equal to 0.

So, if  $m$  plus  $n$  is some non zero constant the integer that is going to be 0. On the other hand suppose, we have a case if  $m$  equal to minus  $n$  or  $m$  plus  $n$  0. If  $m$  plus  $n$  is 0 then this whole term becomes 1 and the average of  $c_n c_m$  is  $c_n c_m$  itself. Therefore the average of

that would be  $c_n$  multiplied by  $c_m$  and what is  $m$  is equals minus  $n$ , if  $m$  plus  $n$  equals to minus  $n$ .

Therefore,  $c_n$  multiplied by  $c_{-n}$  and since, we know that  $c_m$  and  $c_{-m}$  are complex conjugate of each other here multiplying a complex number by its conjugate the result is simply the magnitude  $c_n$  whole square. So, we have that the average of each 1 of the terms here will be either 0 or an expression like  $c_n$  square. Therefore, we can now come to conclusion that the average of  $f^2$  the average of this term is the sum of the averages of these.

Now let us see, what happens if i freeze a particular value of  $m$  and allow the  $m$  to take increments from minus infinity to plus infinity. So,  $m$  goes on changing from minus infinity to plus infinity 1  $m$  takes minus  $n$  then, you have non zero average for all other values of  $m$  the contribution is 0. So, when we take a particular value of  $m$  in the first summation  $m$  equal to the minus  $n$  is the only term which gives the contribution non zero contribution and that we do for all values of  $m$ .

So, the result is this will be  $m$  from minus infinity to plus infinity for each value of  $m$  the contribution can come only when  $m$  equals to minus  $n$  and that, condition the contribution is  $c_n$  squared. Therefore this will be  $c_n$  magnitude square and what is the average of  $f^2$  it is the RMS value of the periodic function square square of the RMS value mean square value.

Therefore if i write as Frms of the effective value of this 1 that is equal to  $c_n$  magnitude square  $n$  ranging from minus infinity to plus infinity you can see this also, can be written as  $c$  not squared or i put it a not square because i like to put in the terms of the coefficient of the trigonometric series. So, when  $n$  equal to 0  $c_n$  equal to  $c$  not that is a not square and  $c_n$  square and  $c_{-n}$  square together are both equal to each other.

Therefore, this will be 2 terms  $c_n$  square in terms of turned out to be an squared plus  $b_n$  squared over 2  $n$  ranging from 1 to infinity because an squared plus  $b_n$  squared is 4  $c_n$  square because square root of an square plus  $b_n$  square is 2  $c_n$  as you recall. Therefore, an square plus  $b_n$  square is 4  $c_n$  square up on  $c_n$  squared and  $m$  equals  $k$  and  $m$  equals to minus  $k$  together contribute 1  $c_n$  term will come for positive  $n$  another for negative  $n$  together they constitute an squared plus  $b_n$  squared over 2.

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Handwritten notes on a chalkboard:

- Top line:  $C_{-n} = |C_n|$  if  $m+n=0$
- Middle line (circled):  $\sqrt{a_n^2 + b_n^2} \cos(n\omega_0 t + \phi_n)$
- Below the circled line:  $n^{\text{th}} \text{ harmonic}$
- Bottom left:  $\sqrt{\frac{1}{2}(a_n^2 + b_n^2)}$
- Bottom right: IIT MADRAS logo

So, what we find that the RMS value of a composite wave periodic wave this square of that equals, the square of the square of the dc term and an square plus bn square root of an square plus bn square cos n omega not t plus ph i n as you call is the n'th harmonic. So, what we have here is the peak value square by 2 which means; the RMS value is the n'th harmonic squared root of an squared plus bn squared over 2 is the RMS value the n'th harmonic therefore, the RMS value the n'th harmonic squared an squared is the this c squared.

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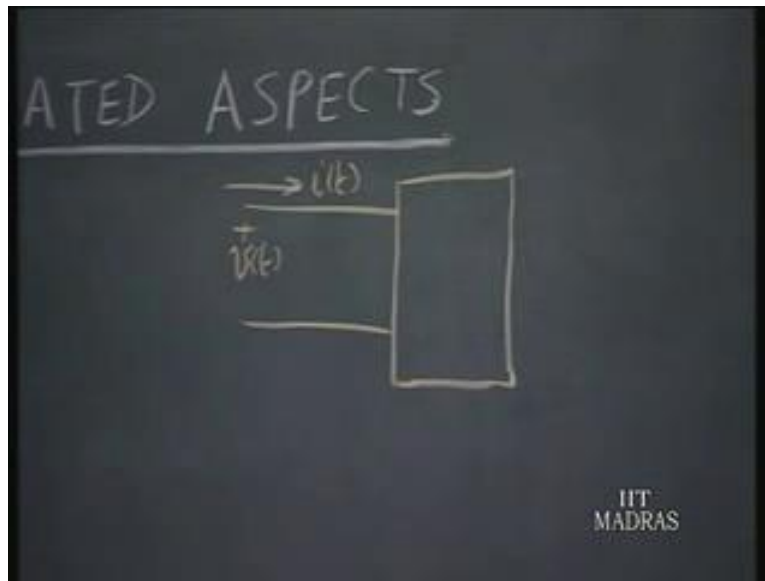
Handwritten notes on a chalkboard:

- Top line:  $\left[ \text{RMS value of a periodic wave} \right]^2 = \text{Sum of squares of RMS values of harmonic components}$
- Middle line:  $= \frac{1}{T_0} \int_0^{T_0} f^2(t) dt$
- Bottom right: IIT MADRAS logo

So, the final conclusion is the RMS value of a periodic wave is square of that, equals the summation of or i can say the sum of the RMS value squared of the harmonic components sum of squares of RMS values of harmonic components including the dc. So, that is easy way remembering how we can calculate the RMS value of periodic wave once we have got the Fourier series analysis for that, this of course, this is also equal to  $\frac{1}{T} \int_0^T i^2 dt$  not this is also equal in time domain  $\frac{1}{T} \int_0^T i^2 dt$ .

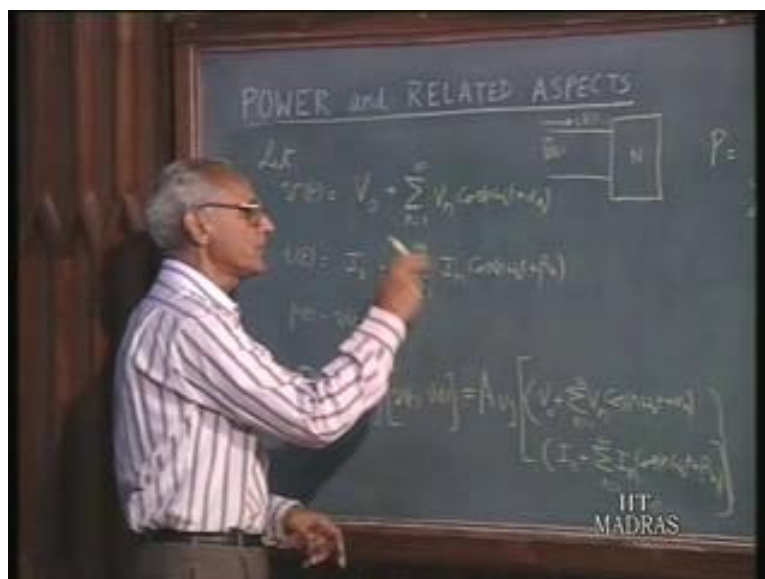
So, you calculate the RMS value either in time domain or in terms of the harmonic components, which are convenient to you at a particular contest a particular contest. Now, let us continuous this and see as far as f of t is concerned, it could be voltage signal, it could be current signal.

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So, can calculate the RMS value of voltage signal or current signal using this kind of formulation. Now let us see, how we calculate power when we are given 2 periodic signals the 1 is being voltage the other is the current. Let us assume that, we have a network or a network element which has the periodic voltage upward cross it and periodic current  $I(t)$  going to the terminals.

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Now, let the voltage  $v$  of  $t$  be  $v_0$  plus  $v_n \cos n \omega t + \alpha_n$   $n$  ranging from 1 to infinity. So, that is the fourier series expansion for the voltage wave, which is periodic the

fundamental frequency  $\omega$  not this is the amplitude  $n$ 'th harmonic and this is the dc terms and likewise the current wave form be,  $i$  not plus  $n$  from 1 to infinity of  $I_n \cos(\omega t + \beta_n)$ .

Now, the instantaneous part  $p$  of  $t$  is of course, the product of  $v$  of  $t$  and  $i$  of  $t$  and when you talk about power associated with any periodic phenomenon, we always imply the average power over a fundamental period or integrate multiple of such periods therefore, when we say power associated with this voltage and current that is the power import into this network  $N$ . Simply, call this  $P$  it is the average of the product of  $v$  of  $t$  times  $i$  of  $t$  always the average over a complete period.

So, we take these 2 series and multiple them out. So, you have the average of  $v$  not plus  $v_n \cos(n\omega t + \alpha_n)$  multiplied by  $i$  not plus  $i_n \cos(n\omega t + \beta_n)$ . Now, when you look at this you have whole a lot of terms here, but luckily for us we are interested only the average of the these terms the sum of these terms.

So, you notice that  $v$  not gets multiplied with  $i$  not get multiplied with  $I_n \cos(\omega t + \beta_n)$  plus  $\beta_n$  1  $i^2 \cos^2(\omega t + \beta_n)$  and. So, on, but when we are talking about average, it is only this  $v$  not and  $i$  not which results in nonzero average  $v$  not multiplied by  $i$  1  $\cos(n\omega t + \beta_n)$  is 0  $v$  not multiplied by  $i^2 \cos^2(\omega t + \beta_n)$  is 0 and so, on and so, forth. So, as far the  $v$  not is concerned the contribution to this average to come from, the term  $v$  not  $i$  not.

Similarly, we will take the fundamental here  $v_1 \cos(\omega t + \alpha_1)$  that multiplied by  $i$  not will be the average the 0 average  $v_1 \cos(\omega t + \alpha_n)$  multiplied by  $I_1 \cos(\omega t + \beta_1)$  we have an average after all, it is the voltage and current of the same frequency that, the average will be of course, the product of  $v_1$  and  $I_1$  divided by 2 the product of the 2 RMS values times cosine of the phase angle between  $\alpha$  and  $\beta$   $\alpha_1 - \beta_1$  all other terms the fundamental here multiplied by second harmonic, third harmonic, fourth harmonic we have 0 average.

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$$P = \underbrace{V_0 I_0}_{\text{dc power}} + \sum_{n=1}^{\infty} \underbrace{\frac{V_n I_n}{2} \cos(\alpha_n - \beta_n)}_{n^{\text{th}} \text{ harmonic power}}$$

$$v = V_a \cos \omega_a t + V_b \sin \omega_b t$$

$$i = I_a \cos \omega_a t + I_b \sin \omega_b t$$

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So, consequently P will turn out to be v not i not that is the product of the dc terms the n'th harmonic term will be  $v_n I_n$  divided by 2  $\cos \alpha_n - \beta_n$  this is the power factor associated with the n'th harmonic component n ranging from 1 to infinity. So, this is the dc power, this is the power associated with n'th harmonic.

So, when you have non periodic when you have periodic wave forms which are non sinusoidal and if we make the Fourier series expansion of vt and it as far the power is concerned you can calculate the power for each frequency separately. The product of v not and i not is the dc power the n'th harmonic power is calculated taking the n'th harmonic voltage component, n'th harmonic current component multiplying their RMS values times cosine of the phase angle difference between them.

As a rule power is a non-linear quantity associated with voltage and current non-linear function of voltage and current. However, in the particular case where the components are different frequencies you can calculate superpose, the power for each component separately as we have seen here. Normally we have a voltage v as the sum of  $v_1$  plus  $v_2$  and the currents  $i_1$  plus  $i_2$  you cannot say  $v_1$  plus  $i_1$  plus  $v_2$   $i_2$  is the power because you cannot it does not work out that way.

But, as long as you are calculating the power associated with a pair of current and voltage components of different frequencies than this; summation will be valid and this is; what we are having here you superpose power therefore,if you calculate the power associated



with each set of pair of frequency components current and voltage you can do that also, if you have the same frequency.

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The image shows handwritten mathematical expressions on a chalkboard. The first equation is  $v = V_a \cos \omega_0 t + V_b \sin \omega_0 t$ . The second equation is  $i = I_a \cos \omega_0 t + I_b \sin \omega_0 t$ . The third equation is  $P = \frac{V_a I_a}{2} + \frac{V_b I_b}{2}$ . The IIT MADRAS logo is visible in the bottom right corner of the chalkboard image.

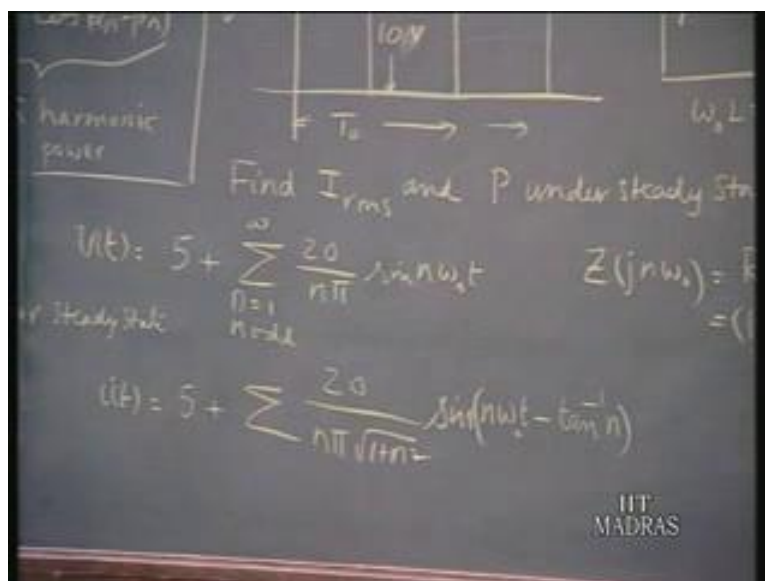
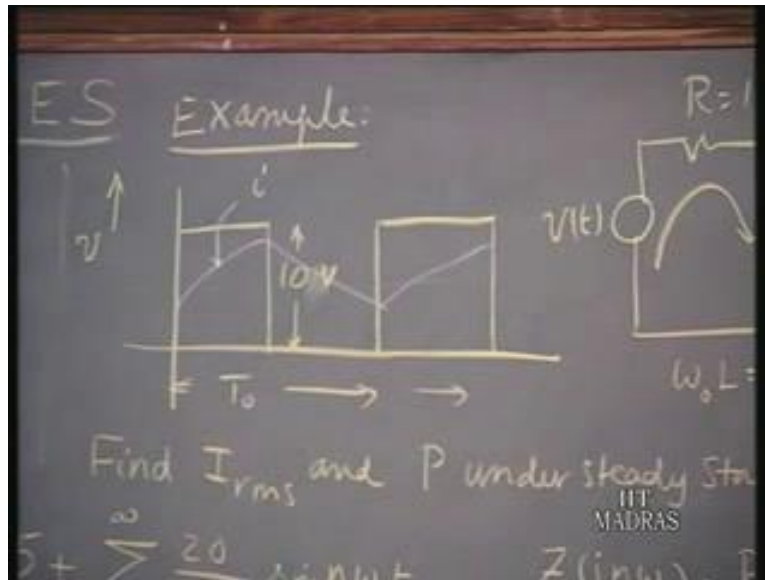
Suppose, you have  $v$  let us say  $v_a \cos \omega_0 t + v_b \sin \omega_0 t$  and current let us say, is  $I_a \cos \omega_0 t + I_b \sin \omega_0 t$  then associated with this, you can say power is  $v_a I_a$  divided by 2 plus  $v_b I_b$  divided by 2 here also you will have some superposition principle will be working out will work out that is; because there is phase difference between this component and voltage and the other component and voltage here also there is 90 degree difference between this current and this current.

So, the associated the components associated with  $\cos \omega_0 t$  terms can be multiplied together and the component associated with  $\sin \omega_0 t$  will be multiplied together and they can be superposed. So, power can be superposed under these conditions. If we have components at this same frequency, but with 90 degree phase difference you can superpose the 2 components of power or as in this case.

If we have got different frequency components you can calculate power separately for each 1 of these component, that is why we can always terms these quantities as a fundamental dc power. Fundamental power second harmonic, power third harmonic, power and so, on and each harmonic power can be calculated independently of the values of voltage and currents at the other harmonics.

Let us, take an example suppose, i have a 10 volt signal the voltage an half set square wave with the period T not and this voltage signal is given to RN circuit. R is equal to 1 Ohm and L is such that; omega not L is 1 Ohm we have naturally a current i of t.

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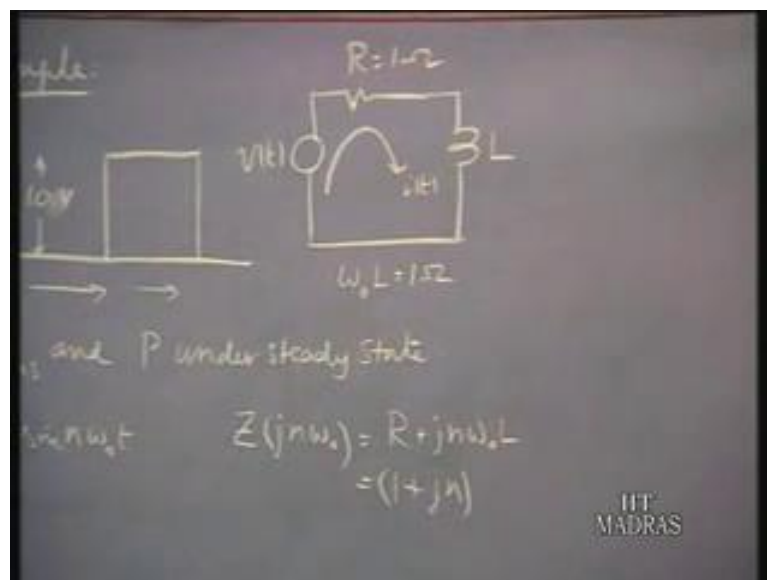
The question that will be asked is: find  $I_{rms}$  and the power  $P$  under steady state. So, this is the question that has been asked, what we proposed to do is we split of the given input voltage  $v$  of  $t$  into its various harmonic components under the influence of each of the harmonic component, we will calculate the current under steady state.

So, we get expressions for  $v$  of  $t$  and  $i$  of  $t$  in the form of Fourier series and once we have them it is easy to calculate  $I_{rms}$  and the overall power the manner we have illustrated earlier. So,  $v$  of  $t$  contains the dc components the average value of this periodic wave is 5 volts therefore, that is the dc value and once we remove the dc value we have the familiar square wave that, have been talking about. So, often in the past.

So, it will have Fourier series expansion of 4 times this amplitude divided by  $n\pi$  and you have only odd harmonic terms are presented and sin terms only will be presented because once, we remove the dc term this will turn out to be an odd function of time. Anyway the final Fourier series would be  $\sin n\omega t$  and  $n$  from 1 to infinity and  $n$  is odd because this as once, we remove the dc terms this will have odd half wave.

So, we have  $v$  of  $t$  and now we do the steady state analysis circuits and to do that; we need to find out the impedance of the circuit for different frequencies. So,  $Z(j\omega)$  not so, for the  $n$ 'th harmonic frequency the impedance of the circuit is  $R + j\omega L$  and since  $R$  is equal to 1 Ohm and  $\omega L$  is 1 Ohm this is  $jn$  so, many Ohms.

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So, we have the impedance for different frequencies we have the voltage under steady conditions, under steady state  $i$  of  $t$  will be for each component separately we find the steady state current. If the dc voltage is 5 volt in a circuit consisting of  $R$  and  $L$  the current will be  $B$  divided by  $R$ . So, 5 up on 1 that is 5 and the  $n$ 'th harmonic component the voltage is 20 up on

n. So, the peak value of the current will be divided by this voltage divided by the value of the impedance which is square root of 1 plus n squared sin n omega not t.

Since, the impedance such an angle tan inverse of n therefore, there will be a phase difference between the current between the voltage and the current this will be tan inverse of arc tangent of n. So, we have the complete description of the voltage and current in terms of the Fourier components. So, we can calculate the whatever, results that are required Irms and P under steady state conditions.

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$$\begin{aligned}
 V_{rms}^2 &= 5^2 + \left(\frac{20}{\sqrt{2}\pi}\right)^2 + \left(\frac{20}{3\sqrt{2}\pi}\right)^2 + \dots \\
 &= 25 + \frac{200}{\pi^2} \left[1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right] \\
 &= 25 + \frac{200}{\pi^2} \times \frac{\pi^2}{8} = 50 \\
 \boxed{V_{rms} = 7.07V}
 \end{aligned}$$

First of all let us, also calculate the Vrms just for sake of interest. So, Vrms squared will be the square of the dc term 5 squared plus the fundamental amplitude square divided by 2 because, we have the RMS value of the fundamental RMS value of the fundamental is 20 upon root 2 pi squared. The third harmonic component 20 upon 3 root pi whole square and etc etc and if you calculate this this will turn out to be 25 200 upon pi squared times 1 plus 1 by 3 squared plus 1 by pi squared etc etc and this series is going to have summation equal to pi square upon 8.

Therefore this will turn out 25 plus 200 upon pi squared into pi squared of up on 8. So, this is total 50 watts. So, Vrms equals 7.07 that is: 50 volts the results which we have obtained straight way from here working out in time domain quite simple once, the peak values turn hold you integrate you get this result straight away. But just wanted to demonstrate the use of the formula for RMS value in terms of v t that is; how we got this we could have got this

same results working out time domain straight away, but it is not so, you could not have got this time domain.

So, easily for  $i$  of  $t$   $I_{rms}$  because the once we have this type of voltage the current under steady state would have a characteristic like this: under steady state that would be the value of the current will vary. So, you must, if you want to find the rms value the current you would have to find out the expression for this  $i$  of  $t$  in this manner. So, that the starting point is equal to the same at the end of the period to fit the initial conditions suitably and then do the integration.

Now we have got the term expression for  $i$  of  $t$  in the form of Fourier series we can calculate  $I_{rms}$  in terms its various harmonic components.

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$$V_{rms} = 7.07V$$

$$I_{rms}^2 = 5^2 + \sum_{n=1, \text{ odd}}^{\infty} \left( \frac{20}{\sqrt{2} n \pi \sqrt{1-n^2}} \right)^2 \quad \text{Under}$$

So, 5 squared the dc term plus summation 20 up on root 2 n pi root over 1 minus n squared whole squared because this is the peak amplitude the RMS value will be obtained by dividing by root 2 and then n from 1 to infinity n is odd. Calculate the first a few terms you will get 25 from the fundamental from the dc from the fundamental you will get 10.13 for the third harmonic you get 0.23 for the third harmonic you get 0.03.

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$$I_{rms}^2 = 5^2 + \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \left( \frac{20}{\sqrt{2} n \pi \sqrt{1+n^2}} \right)^2 \quad \text{Under Stea}$$
$$= 25 + 10.13 + 0.23 + 0.03 + \dots$$
$$= 35.39$$
$$I_{rms} = 5.95 \text{ A}$$

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For the fifth harmonic you get 0.03 and therefore, afterwards it is negligible. So, this will be equal to 35.39 and  $I_{rms}$  will be 5.95 Amperes. What about power we have  $v$  of  $t$  and  $i$  of  $t$  here. So, power we can calculate taking the particular term here and their associated current terminal, current term here and finding of the phase angle. So, let us see the dc power  $P$  not product of these 2 the dc voltage and the dc component of the current 25 watts.

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The image shows a chalkboard with handwritten mathematical derivations. At the top, it states  $P_0 = 25 \text{ W}$ . Below that, the calculation for  $P_1$  is shown:  $P_1 = \frac{1}{2} \left[ \frac{20}{\pi} \times \frac{20}{\pi\sqrt{2}} \right] \cos \frac{\pi}{4}$ , which simplifies to  $= 10.13 \text{ W}$ . To the left of this equation, there is a vertical line and the expression  $\frac{1}{\sin n}$ . Below the  $P_1$  calculation, the calculation for  $P_3$  is shown:  $P_3 = \frac{1}{2} \left[ \frac{20}{3\pi} \times \frac{20}{3\pi\sqrt{10}} \right] \cos(\tan^{-1} 3)$ , which simplifies to  $= 0.23 \text{ W}$ . In the bottom right corner of the chalkboard, the text "IIT MADRAS" is visible.

Fundamental power, fundamental power we have the amplitude the fundamental voltage is 20 up on pi the amplitude of the fundamental component of the current is 20 up on pi n equals 1 square root of 2 and of course, n equals to 1 these are the product of the amplitudes. So, multiplied by half because we wanted to take the product of the RMS value and cosine of the phase angle difference is tan inverse of 1 that is equal to pi by 4. So, cos pi by four.

And this turns out to be 10.13 watts similarly  $P_3$  will be given by half of 20 up on 3 pi multiplied by 20 up on 3 pi and when you substitute n equal to 3 this root 10 cosine of whatever, angle you get tan inverse of 3 and this will turn out to be 0.23 watts.

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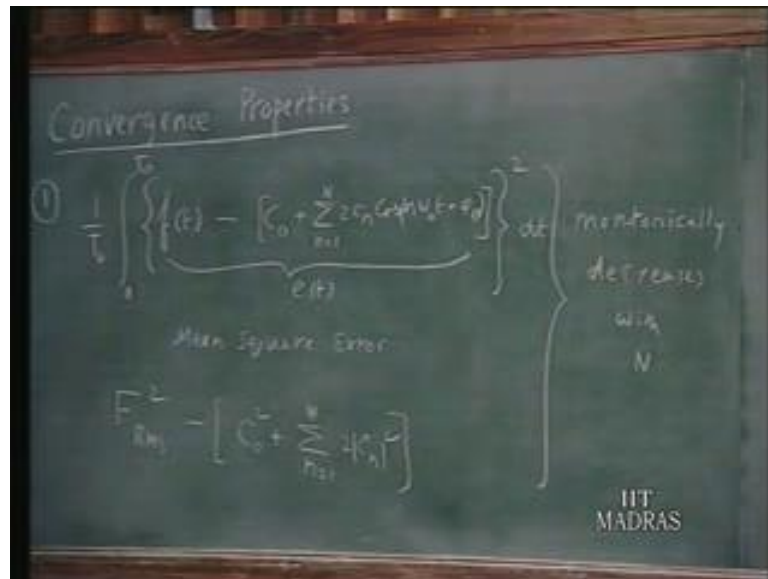
The image shows a chalkboard with handwritten mathematical work. At the top, it says  $L = 1 \Omega$ . Below that, there is a calculation for total power:  $P_0 + P_1 + P_3 = 35.4 \text{ W}$ . The result  $35.4 \text{ W}$  is enclosed in a hand-drawn box. To the left of this, there is another value  $25 \text{ W}$ . At the bottom, there are some partially visible formulas involving  $\frac{1}{2} \left[ \frac{Z_0}{T} \times \frac{Z_0}{T} \right] \cos \theta$ . In the bottom right corner, there is a small logo for IIT Madras.

The total power, if you assume all other harmonic component powers are negligible  $P_0$  plus  $P_1$  plus  $P_3$  will be 35.4 watts. So, that will be the total power that, will be delivered by the source into the circuit. If you look at closely you will find that once you have got RMS value you could have calculated power without going to this analysis after all you know the RMS value of the current in this circuit.

So, if you know the RMS value this kind of the circuit the RMS value squared multiplied by, the resistance will be the total power in the circuit. So, indeed it turns out  $I_{\text{RMS}}^2$  squared 35.39 and the power that, we got also the same thing 35.4 watts. But it only illustrates the procedure the principle that is: involved that is you can calculate the power for different harmonic components separately and add them up to get the total power in this circuit.

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The last example, showed that the size of the various harmonic components and the powers associated with the harmonics go down as the other harmonic increases you should like to see, how fast these various co-efficient go down and that is; related to what is meant by, convergence of the Fourier series you should look at the convergence like to look at the convergent properties of the fourier series.

So, first let us consider this particular point suppose,  $f$  of  $t$  is approximated by the Fourier series up to order capital  $N$  that means; we have  $c$  not plus  $2 c_n \cos n \omega t \phi_n$  if you take  $n$  from 1 to infinity that constitute the entire fourier series now suppose, i calculate the series at capital  $N$ . So, this is the approximated Fourier series we take only finite number of terms and the difference between the actual  $f$  of  $t$  and the truncated fourier series will call that the error  $e$  of  $t$ .

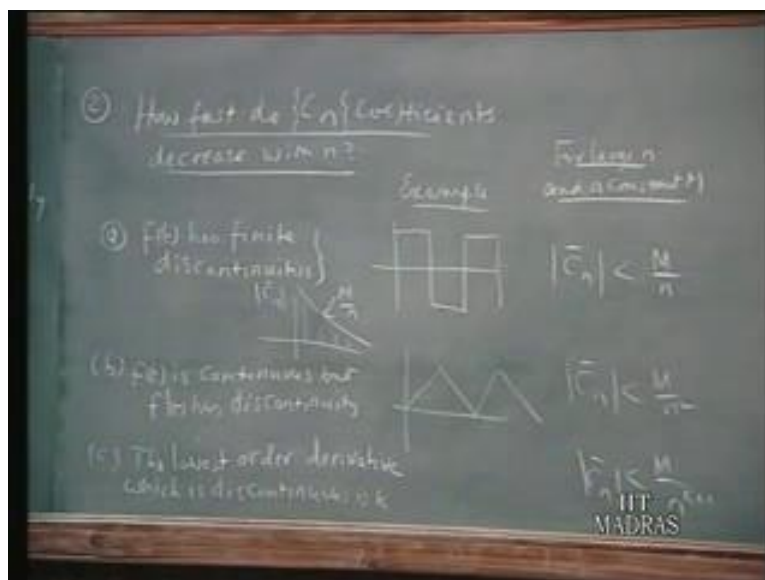
So, this error now depends up on the value  $N$  that we take, now if you take square of the error integrate this from 0 to  $T$  not and take its average then this is referred to as the mean square error, mean square error of the square of the mean the mean of the square of the error is called the mean square error. So, we see that this decreases the capital  $N$  the value of  $N$  that you take close to related to this and actually this expression equivalent to another term like this.

Suppose, you take the RMS value of the given wave form and it is square FRMS square assuming that  $f$  of  $t$  is a voltage signal this represents square of the RMS value the voltage signal and this voltage is applied to 1 Ohm resister, this is the power dissipated in the 1

Ohm resistor therefore, this is the voltage signal FRMS square can also be thought of the power available from the signal.

However, if you take only the sum of the RMS values of the various harmonic components up to  $n$ ; that means, you take  $c$  not square plus  $n$  from 1 to capital  $M$   $2$   $c_n$  magnitude square  $2$   $c_n$  magnitude square is the sum of the square of the RMS values of the harmonic components 1 to  $N$  because after all  $2$   $c_n$  equals square root of an square plus  $b_n$  square therefore,  $4$   $c_n$  square is an square plus  $b_n$  square and RMS value is an square plus  $b_n$  square up on  $2$  therefore,  $2$   $c_n$  magnitude squared is the square of the RMS value of the  $n$ 'th harmonic component.

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Therefore, the difference this is the power available from the signal this is the sum of the power available from the various harmonic components of order  $N$ . So, this quantity can be shown to be equal to the other quantity which we have written earlier both these will monotonically decrease with  $N$ . So, the larger the value of  $N$  you take the less the discrepancy will be the error and the decrease is monotonic and as you take larger and larger values of  $n$  these terms go down.

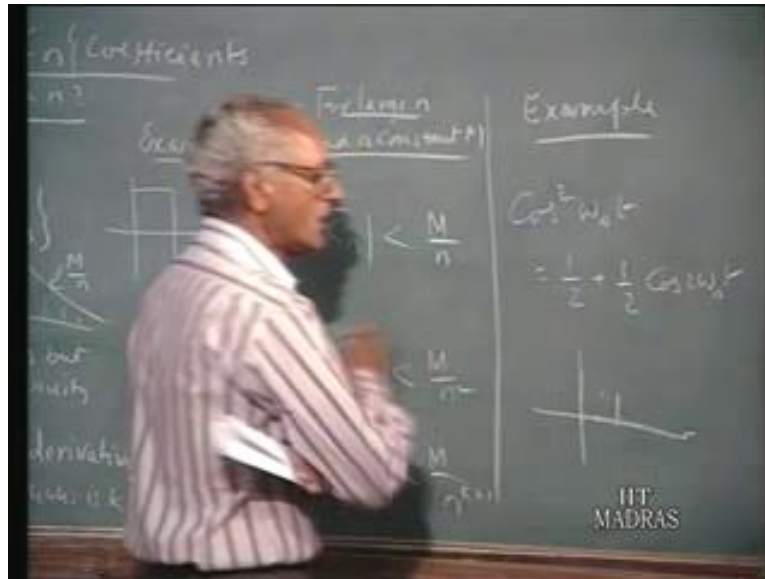
So, the next question we like to ask is how fast do the  $c_n$  terms decrease with  $N$ . So, that is: the next property of interest that we like to know. So, the question that we ask is: how fast do the  $c_n$  terms the  $c_n$  coefficients decrease with  $n$  we like to give the answer in the following manner suppose, we take  $f$  of  $t$  has finite discontinuities example. Suppose, my

wave form like this square wave familiar, square wave there is a jump here there is a finite discontinuities.

For such wave forms it can be shown that for large  $n$  and a constant  $M$  the Fourier coefficients for large  $n$  are constrained by a factor like this.

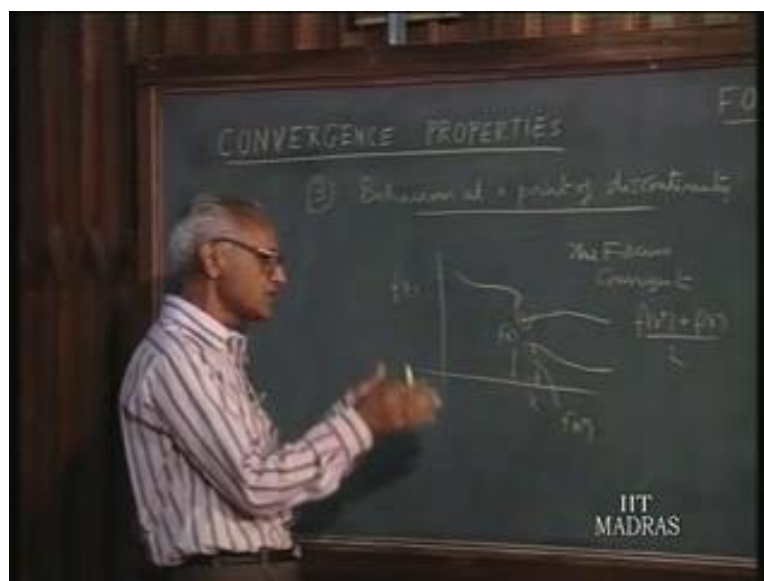
So, as you take  $n$  to be the large more and more then the coefficients go down as  $1/n$ , that means: the spectrum for this suppose it is like this the values of various coefficients are constrained by a line like this  $M/n$  and the other hand suppose,  $f$  of  $t$  is continuous, but the derivative  $f'$  has discontinuity; that means, the function itself is continuous, but then there is a discontinuity its derivative and example function like this will be like this.

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Suppose, I have sample like this, this is the periodic function it is continuous there is no jump anywhere, but you take its derivative, the derivative here will be like this the derivative here will be negative; that means, the derivative function of this will be something like this. So, there is the derivative of the discontinuity, but the function itself is smooth in this event the various Fourier coefficients go down as  $m$  by  $n$  square; that means, they decrease as  $1$  over  $n$  square; that means, the decrease is faster in this case because this is smoother function than this you can continue like this.

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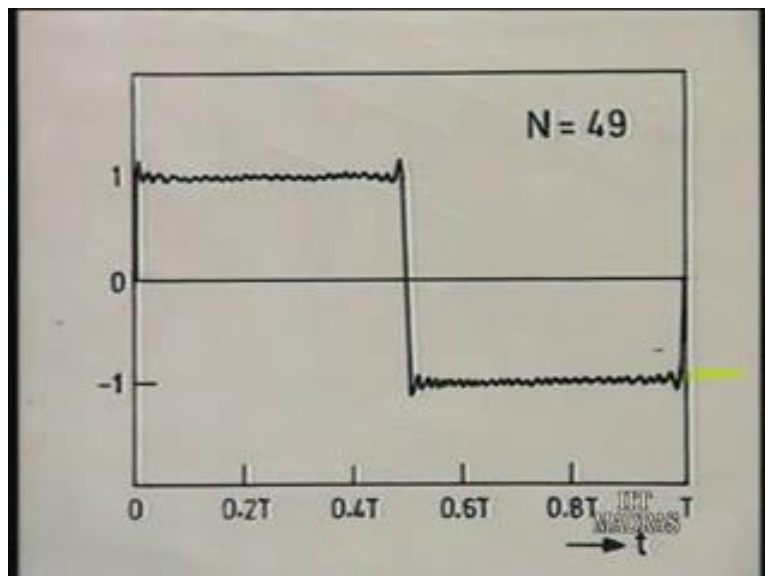


Suppose, you say the lowest order of the derivative which is discontinuous is  $k$ ; that means, first  $k$  minus 1 derivatives are continuous the first time going to be discontinuous is order  $k$  then it can be shown that  $c_n$  goes down as  $m$  over  $n$  times  $k$  plus  $n$  to the power of  $k$  plus 1 this is much smoother function than the earlier 1. Therefore, the decrease of  $c_n$  coefficient will much faster as an example suppose, you take  $\cos^2 \omega t$  this is the very smooth function all its derivatives are continuous and the Fourier series expansion really half plus 1 half of  $\cos 2 \omega t$ .

So, you have the dc term the second harmonic term all the other terms have 0. So, you have really the dc term and the second harmonic term here. Further terms are all 0 because this particular function of time is very smooth all its derivatives are continuous therefore, the fourier series go down very fast and in fact, they become 0 after  $n$  equals 2. So, this is the example of the very smooth function; that means, in other words if the function is smoother than the faster will be the rate of decrease of the  $c_n$  coefficients that is the summary of what we have discussed under this header.

The third we would like to ask is: what is the behavior of the function at the point of discontinuity. Let us say, this  $f$  of  $t$  has the discontinuity at the point  $x$ .

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Now, if you take the Fourier series and evaluate the value of  $f$  of  $t$  at  $f$  of  $x$  in turns out that, the series the Fourier series converges to  $f$  of  $x$  plus  $f$  of  $x$  minus divided by 2 the limit of  $f$  of  $t$  as you approach  $x$  from this direction this value is  $f$  of  $x$  minus the value here

is  $f(x)$  plus. So, depending up on how you approach  $x$  from the right or left you get  $x$  minus or  $x$  plus, but the series will converge to mid point between this 2 this is the point to which the series will converge irrespective of how you define  $f(x)$ .

So, you may define  $f(x)$  some value not necessarily this, but as far the series is concerned it converges to this.

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The image shows a chalkboard with the following handwritten text:

Time Shift

$$f_1(t) = \sum \bar{c}_{n_1} e^{jn\omega_0 t}$$
$$f_1(t-\tau) = \sum \bar{c}_{n_2} e^{jn\omega_0 t}$$
$$\bar{c}_{n_2} = \bar{c}_{n_1} e^{-jn\omega_0 \tau}$$

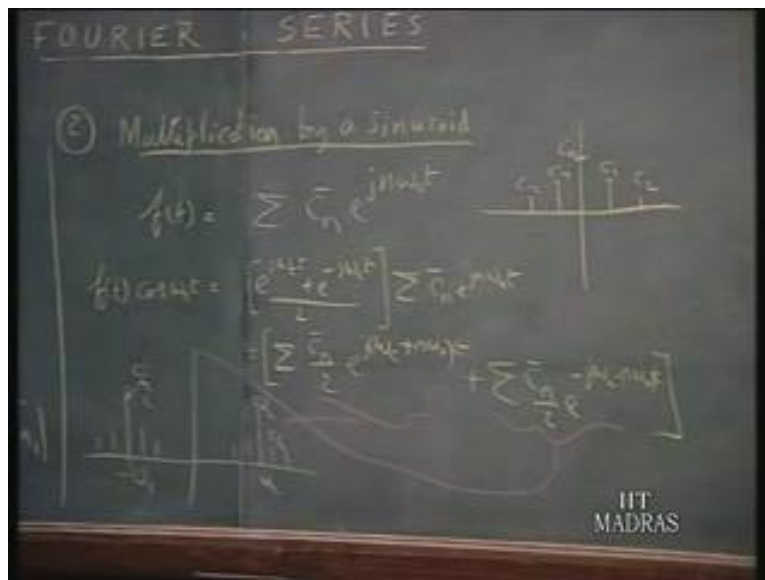
Below the equations, there is a note:  $|\bar{c}_{n_2}| = |\bar{c}_{n_1}|$

In the bottom right corner of the chalkboard, the text "IIT MADRAS" is visible.

Now let us look at this, the Fourier series for a square wave and we have taken up to the 49th term. Now, the series converges to 0 at this point which is the proper thing to do which is the average of the left limit and right limit, but you should also observe that, there is small amount of overshoot just before the jump and just after the jump and this overshoot is before to literature, gibb's phenomena and the amount of overshoot is 9 percent of the total jump and this is something which will pursuit no matter to what order harmonic you go to. Just discuss 2 additional properties of the Fourier series suppose, we have a function  $f_1$  of  $t$  and its fourier series is this with  $c_{n_1}$  and the coefficients.

Now, if this function  $f$  of  $t$  is shifted in time translated in time there certain amount. let us say,  $f_1(t - \tau)$  it is the same function, but the origin is shifted it is delayed by an amount it equal to  $\tau$  second.

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Then, it can be shown that the corresponding Fourier series for this will be such that  $c_{n2}$  is  $c_{n1}$  multiplied by  $e$  to the power of minus  $j n \omega_c t$  which immediately, shows that the magnitude of  $c_{n2}$  is same as the magnitude of  $c_{n1}$  how are the phase is different, the angle is different, which means that the magnitude spectrum of  $f_1(t)$  and the magnitude spectrum of  $f_1(t - \tau)$  will be the same.

The phase; however, is decreased by amount proportional to the frequency and example of this we have seen earlier, but when you took a square wave with the origin at 2 different places and you could express square wave in terms of cosine functions and sin functions. So, this is the important property that the constitution of the various harmonic component they proposed at the various components will not depend up on where we place the origin.

The second property relates to suppose, you have a periodic function you multiply by a sinusoid again let,  $f(t)$  be the periodic function and this may be the fourier series for that the spectrum for this this is  $c_0, c_1, c_2, \dots, c_{-1}, c_{-2}, \dots$ . Now, i would like to ask is: if  $f(t)$  is multiplied by  $\cos(\omega_c t)$  then we can say this is: after all  $\cos(\omega_c t) = \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$  and you multiply this by  $c_n e^{jn\omega t}$ .

The result is you get 1 expansion  $c_n$  up on  $2 e^{jn\omega t}$  as i may write  $c_n e^{jn\omega t} + c_n e^{jn\omega t - j\omega_c t}$  plus another group of terms  $c_n e^{jn\omega t + j\omega_c t}$  to the power of minus  $j\omega_c t - jn\omega t$ . So, you have frequencies  $\omega_c + n\omega$

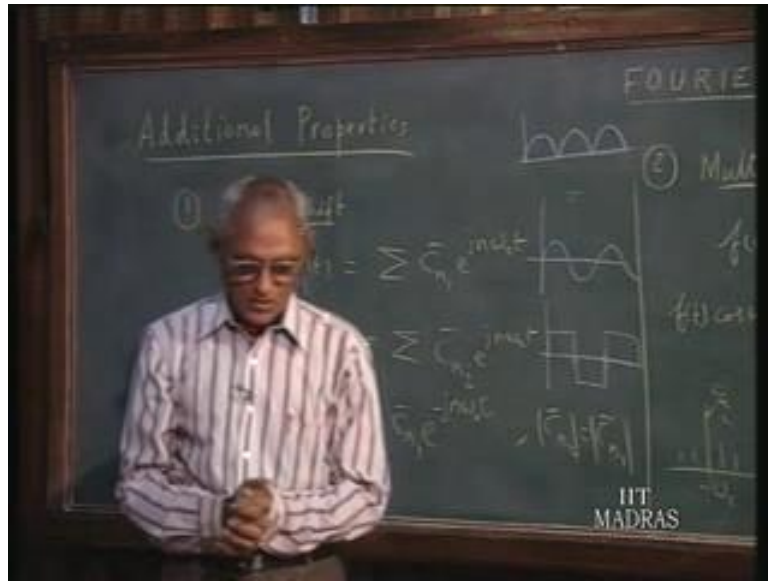


$n\omega$  not  $n$  of course, ranges from minus infinity to plus infinity and another set of terms frequencies  $\omega_c$  minus  $n\omega$  minus  $\omega_c$  minus  $n\omega$ .

So, in other words, if look at the spectrum here, centered around  $\omega_c$  you have various harmonic components this will be  $c$  not by this is  $c_1$  up on 2 etc and centered around minus  $\omega_c$  you have second set of component this is again  $c$  not up on 2. So, this portion corresponds to this and this portion corresponds to this.

So, what it means; is this: spectrum gets divided into 2 parts 1, part is shifted to  $\omega_c$  and other part is shifted to minus  $\omega_c$  it has been centered around the origin centered around plus  $\omega_c$  and minus  $\omega_c$ .

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This is done in communication and instrumentation. When you want the information contained of the signal it has been centered around dc value you like to be shifted to a more convenient frequency for purpose instrumentation or communication and in that contest this is: set to be the carrier frequency because it carries the information content associated with  $f$  of  $t$  you can use this information as an example, you can think of if you have a full wave rectify sine wave you can think of this as the product of a sin term and a product of and the square wave.

Suppose, you multiply these 2 out you can get this therefore, to find the Fourier series for this you can use this property and then find the Fourier series, i will leave this as an exercise to you to sum up we have discussed, at length the various properties of Fourier series. How the series can be obtained, the characteristic of the series and how it may be used to analyze circuits excited by periodic, but not sinusoidal wave forms we derived a little expression for calculating various harmonic components.

This is the process which is refer to harmonic analysis, if the function  $f$  of  $t$  is given in the form of analytical expression we can carry out this harmonic analysis analytically the wave we have done in the various examples. However, if the function is available in graphical form with no analytical expression on hand available with you then we can use numerical methods and computers can also be used software programs can be developed to give the various harmonic components.

Once you have the wave form prescribe or it can be captured by means of discrete data points experimentally. We also have instrument available do this harmonic analysis. The 2 categories of instruments: 1 is called the harmonic analyzer, where you feed the harmonic analyzer and tune it to get the amplitude of magnitude of each harmonic components 1 at a time and read it out and a meter. The other class of instruments are: known as spectral analysis where, if you feed the signal you will find the entire magnitude spectrum displayed on the screen this is called a spectrum analyzer.

The ideas of Fourier series or the harmonic analysis of periodic signals, can be carried over to signals, which are not periodic they are called a periodic signals and this is the subject will take up next, when we talk about the Fourier integral concept with this, we conclude our discussion, on the Fourier series in the next presentation, I will give you the set of examples as an exercise for you and also we include elaborate, it in demonstration and some of the concepts we have discussed.