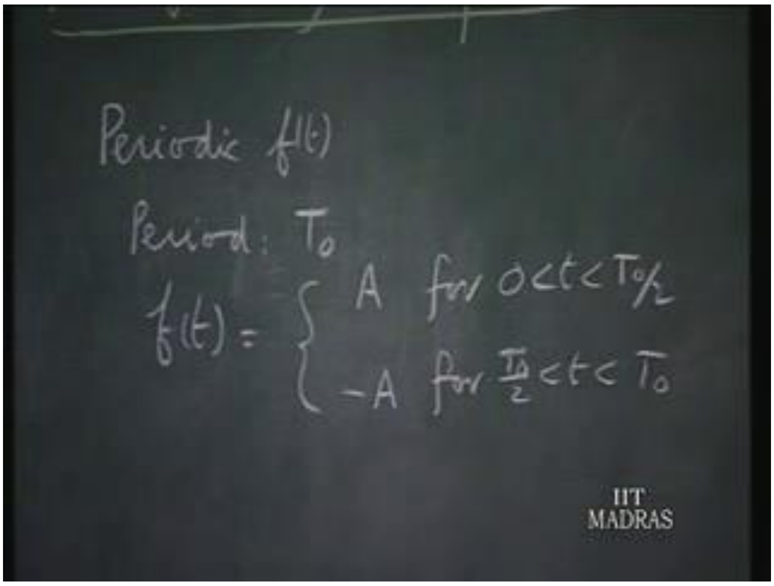
Networks and Systems
Prof V.G K. Murti
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture - 10
Fourier Series (10)

What we have seen in the previous lectures, is first how to calculate the Fourier coefficients of a given periodic waveform and through an example. We saw, how this information could be utilized to access the steady state performance of an electrical network, excited by periodic non sinusoidal sources.

Today, we will start with; we will take up the concept of frequency spectrum. To introduce this let us, consider the statement of a periodic function of time.

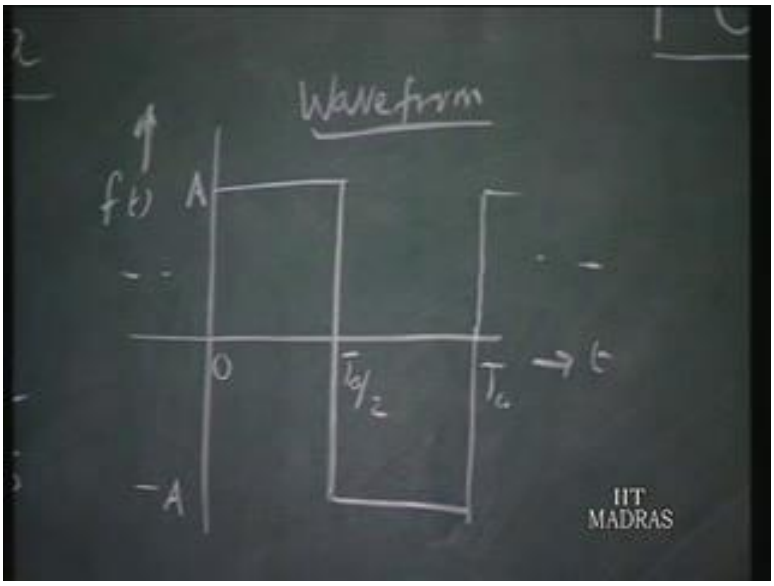
(Refer Slide Time: 01:49)



Suppose, I describe the periodic function of time as follows: period is T nought and f t equals a for t ranging from 0 to T nought upon 2 and minus a for t ranging from T nought by 2 to T nought. This is the complete description of the periodic function. But this is just a cold statement of facts.

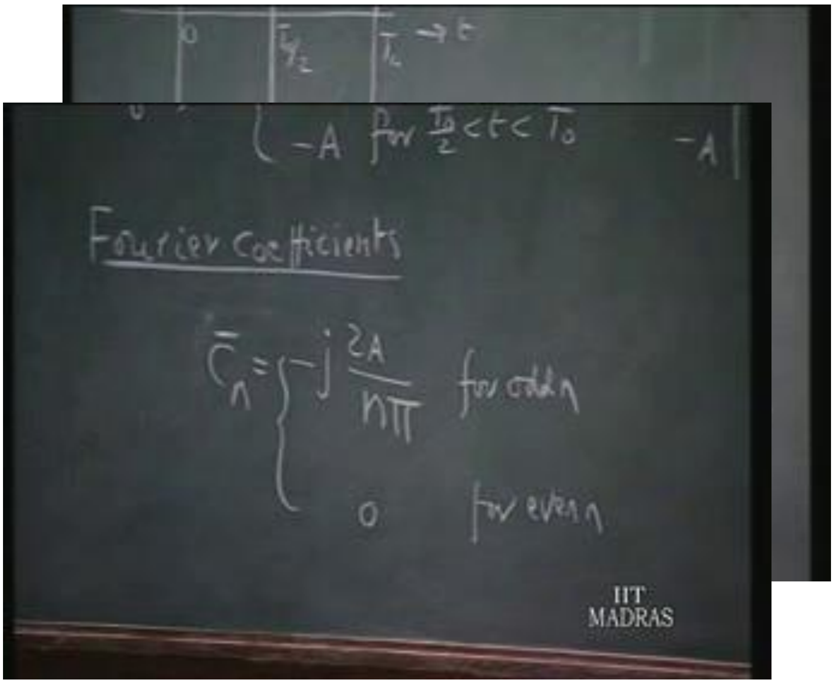
On the other hand, if I picture this information in the form of a waveform then, the whole situation comes alive.

(Refer Slide Time: 02 :47)



So, if you plot this for a whole period, you have this and repeats itself endlessly in both directions. This is A, this is minus A, 0, T nought upon, 2 T nought and this is f of t. So, information about the periodic waveform presented in this manner, gives you real feel for the variation and it is often useful therefore, for us, to plot these functions of time through waveforms like this. We know this is called a waveform and that is the reason why, an oscilloscope is such a useful instrument in the laboratory.

(Refer Slide Time: 03:54)



As we can, see the variations with respect to time which, are often masked in an electrical expression of this type.

Now, when you perform the Fourier analysis of this, you can describe the Fourier coefficients of this waveform. We know that, C n the Fourier coefficient in the exponential representation is minus j 2 A by n pi. You recall, that in the trigonometric expansion of this function of time, the coefficients of sine n omega nought t is 4 A by n pi and since, C n is a n minus j b n by 2, b n terms only are present 4 a by n pi for odd n. Therefore, C n will be minus j 2 A by n pi for odd n and 0 for even n.

Now, this statement again gives you the complete picture, but there is nothing like representing them in the form of a figure. So, suppose you want to do this a similar operation like this with respect to functions of time. So, we should be able to plot C n for different values of n in the form of a graph or a figure. But then there is a problem. C n is a complex number. So, if you have to indicate a complex number, we have to indicate both its magnitude and phase. They are 2 parameters we associate with each value of C n.

Therefore, a convenient way of doing this would be.

(Refer Slide Time: 05:34)

To plot the magnitudes for C n for different values of n and the angle associated with C n which, we call phi n for different values of n. So, in this case, we can graduate the x axis in terms of n or omega nought; suppose you do it in terms of, n 0 1 2 3 4 5 etcetera and on the negative side minus 1 minus 2 minus 3 minus 4 minus 5 etcetera. So, when you plot the magnitude of C n then, there is no dc term. There is 1 component here, another component here, third component here. This is even harmonics are absent.

Therefore, that will be how it looks like and phase every non vanishing C n will have minus 90 degrees as you can see here. Therefore and since, for minus n it is the conjugate you have: plus phi pi by 2 minus pi by 2. You can graduate this x axis not necessarily in terms of n, we can as well do it in terms of frequency or angular frequency. Now, I can write this as 0 omega nought 2 omega nought 3 omega nought and 4 omega nought 5 omega nought etcetera. The point to notice that, this is what is called a line spectrum. Such a representation is called a spectrum.

This is called the magnitude spectrum and this is called the phase spectrum and together they go by the name spectra: the magnitude and the phase spectrum. 1 point we should notice that, as long as the function is periodic that is the type of functions that, we are dealing with at this stage; the spectrum consists of components only at the d c and integrals multiples of the fundamental frequency. There cannot be any term in between. So, these are called line spectrum. Both these magnitude and phase spectra are called, are now for a periodic situation are, line spectra. And the spacing between 2 adjacent components this is also a component, but 0 magnitude, this spacing between 2 adjacent components is omega nought, if you graduate this in terms of frequency.

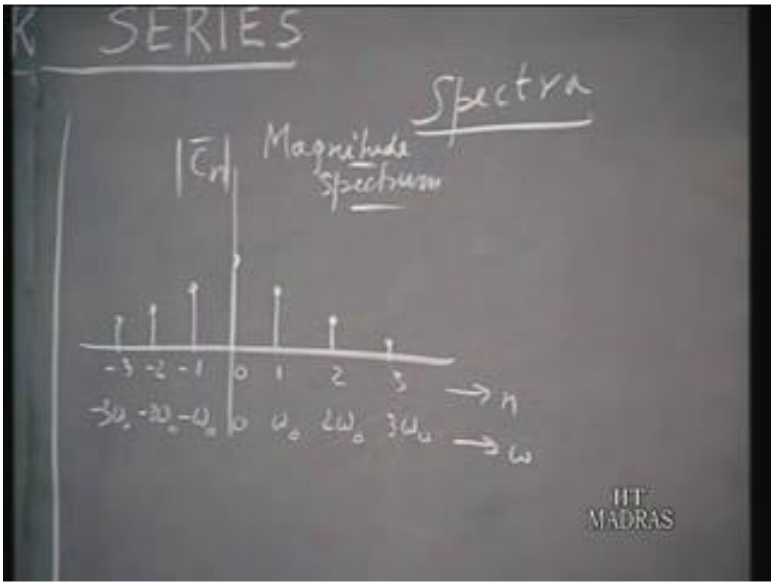
So, the entire information regarding C n can be pictorially represented in this fashion. It gives you in a graphic way: what are the frequency components present and what are the relative stages. All this information is given in a pictorial representation like this. Therefore, it is often convenient and useful for us, to plot this spectra of periodic waveforms in this fashion.

For example, you draw an envelope like this. You can say this is how, the various Fourier coefficients decay for large values of n. So, generalizing this we can say now that.

(Refer Slide Time: 09:50)

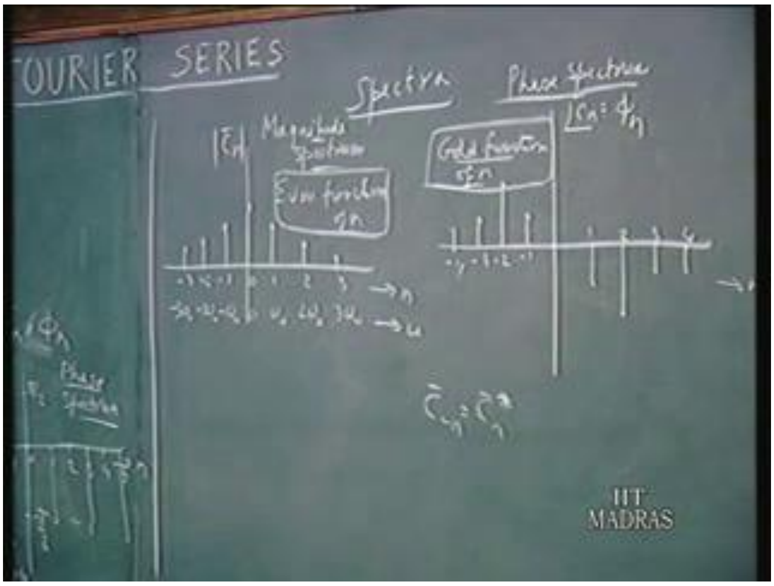
A periodic waveforms we have 2 kinds of spectra, the magnitude spectrum which is really a plot of the magnitude C n as a function of n or omega nought or omega and. So, in general, you have values for integral values of n or integral multiples of omega nought. You may be a little puzzled about this, meaning of minus omega nought what is a negative frequency. It is just the coefficient of the term t, in the exponential representation we write minus j omega nought t. It has the dimensions of frequency as I mentioned earlier. These 2 together constitute a sinusoid of frequency omega nought.

So, this is only a mathematical terminology that we use and you should leave it at that.



You should not try to find out a physical meaning for minus omega nought as a frequency. If you plot the phase spectrum.

(Refer Slide Time: 11:21)

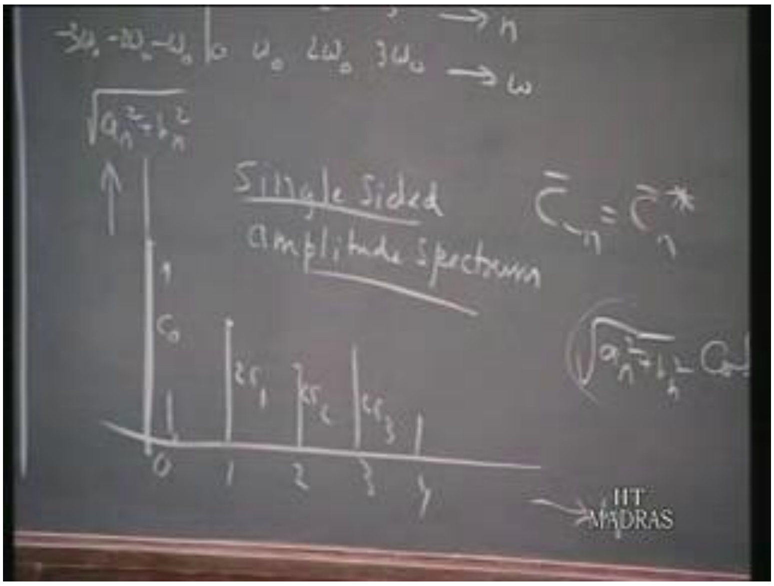


In general again you have some values. It is the angle of C n which is equal to phi n. The first spectrum turns out to be an odd function of n why because, we observed earlier that C n c minus n is C n conjugate which means: that the angle associated with C n and the angel associated with minus n are the negatives of each other. Therefore, for n equals 2 it has a phase angle, n equals minus 2 it is exactly the negative.

So, the phase spectrum is an odd function of n. On the other hand, the magnitude spectrum which is the magnitude of C n is a linear function, because the magnitude of c minus n is the same as the magnitude of C n. So, a magnitude spectrum is an even function of n. This is an odd function of n. So, these are 2 important properties of the spectra. The magnitude spectrum is an even function of n the phase spectrum is an odd function of n. And it is conventional to plot the spectra for both the positive and negative indices n, even though as you can see if, you give this information pertaining to positive n for C n and positive n C n you can reproduce, what is happening on the negative side utilizing the property of evenness of the magnitude and recognizing that phi n is an odd function of n.

It is also sometimes.

(Refer Slide Time: 13:46)



This is also done. Instead of plotting C n, 1 plots square root of a n square plus b n square, this being the amplitude of the n'th harmonic component. You recall, that in the trigonometric expansion of function of time, we can the amplitude of the n'th harmonic is square root of a n square plus b n square. And this is the coefficient of cos n omega nought t plus phi n function of time.

So, this can be plotted also 0 1 2 3 4 etcetera. This is a single sided, sometimes called amplitude spectrum. Because what we are really plotting is; the coefficient you recall that the n'th harmonic is represented by cos n omega nought t plus. So, we are plotting this as an alternative. So, instead of this magnitude spectrum, 1 can plot this, but in this case, we only have positive non negative values of n to deal with. Therefore, this turns out to be a single sided amplitude spectrum. This is an alternative way of plotting the magnitude spectrum, nothing more than this.

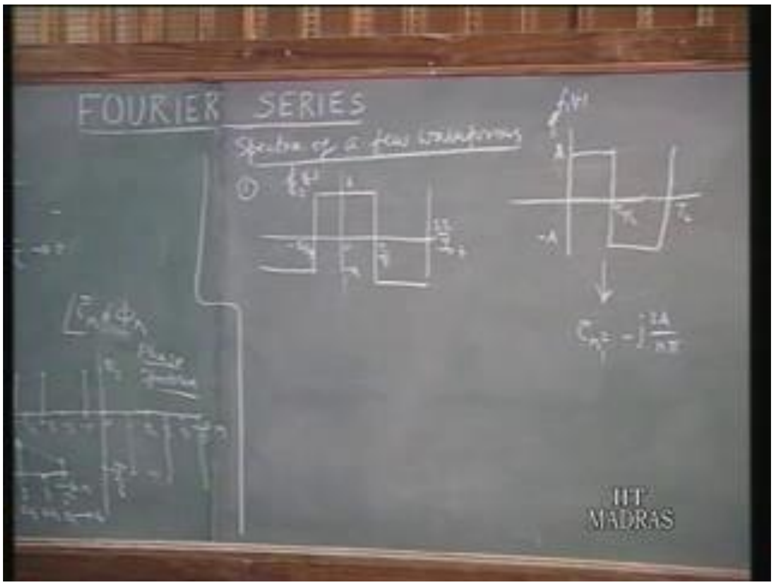
However, we normally confine our work to utilizing plotting the spectra in this manner rather than in this manner because, that will be convenient for us when, we go to the Fourier integral concept. You notice that if this is C nought, this is C 1 this is C 2. When you go to the amplitude spectrum, square root of a n square plus b n square as you know is 2 times c of n. Therefore, this will be 2 C 1 2 C 2 2 C 3 etcetera, but the d c term will be the same that is C nought.

So, the concept of spectra, is just a way of indicating the various harmonic coefficients in a pictorial way and since, each harmonic coefficient is associated with 2 quantities either a n or b n or magnitude of C n and phase of C n. For example, if you choose to plot in this fashion, the angle information is again once again this. So, as far as the phase spectrum is concerned you do not have any alternative. On the other hand, if you like, you can plot a n and b n separately, but that is an unconventional way of doing it.

So, since each Fourier coefficient is specified by 2 parameters, we need to have 2 spectra and usually we do this in the form of a magnitude spectrum and phase spectrum. Now, let us use this background, let us plot the spectra of a few representative waveforms. I would like to keep this waveform and this is the spectra.

So, let me see.

(Refer Slide Time: 17:37)

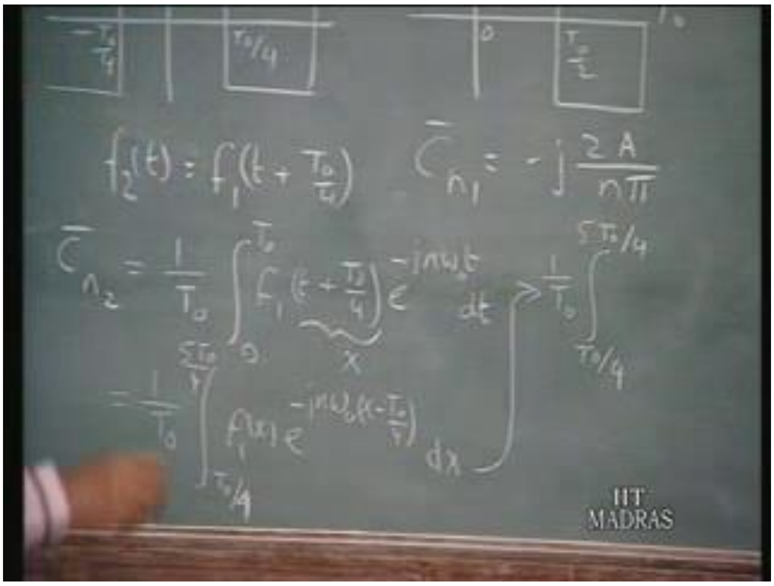


Suppose, I take a waveform, which is related to this, in the sense that it is a square wave all right, but shifted from the square that we have already dealt with. This will be 3 T nought upon 4. This is minus T nought upon 4. Let me call this f 2 of t. If the original square wave that we have is f 1 of t that means, for f 1 of t, I am considering this to be f 1 of t. We know that, this needs to a Fourier coefficient let us say, C n 1 because, I am calling this f 1 minus j 2 A by n pi, minus j 2 A by n pi.

Now, what would be the Fourier coefficient for this? You recall you can see from this, these 2 are essentially the same waveform except for a shift in the origin.

In fact, f of 2 of t is.

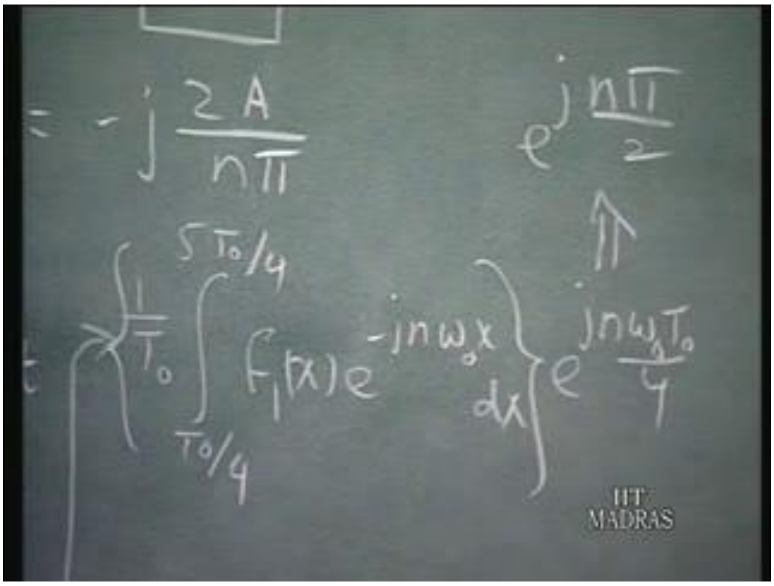
(Refer Slide Time: 19:32)



Obtained by advancing this by a quarter period. Suppose, similar events are made to occur t nought by 4 seconds earlier then, this waveform is obtained from that. So, a rise from a negative a to a plus a occurs at 0. Now, that occurs at t nought by 4 seconds earlier. Therefore, f 2 of t can be written as f 1 of t plus t nought upon 4. So, the whole thing is advanced by T nought by 4 seconds. C n 2 the fourier coefficient for f 2 of t will therefore, be, 1 over T nought 0 to T nought f 2 of t which is f 1 t plus T nought upon 4 e power minus j n omega nought t dt.

Now, in order to relate C n 2 with C n 1 let us, make the identification that t plus T nought upon 4 equals X in which case, we can express this as 1 over T nought. Now, substitute the variable t by the variable X. Dt will be dX and this will be f 1 x of course, and e power minus j n omega nought. T will be X minus T nought upon 4 and the limits of integration when t equals 0 X will be T nought upon 4 and when X is t nought this will be 5 T nought upon 4. X will be 5 T nought upon 4. And therefore, this will now we equal to 1 over T nought T

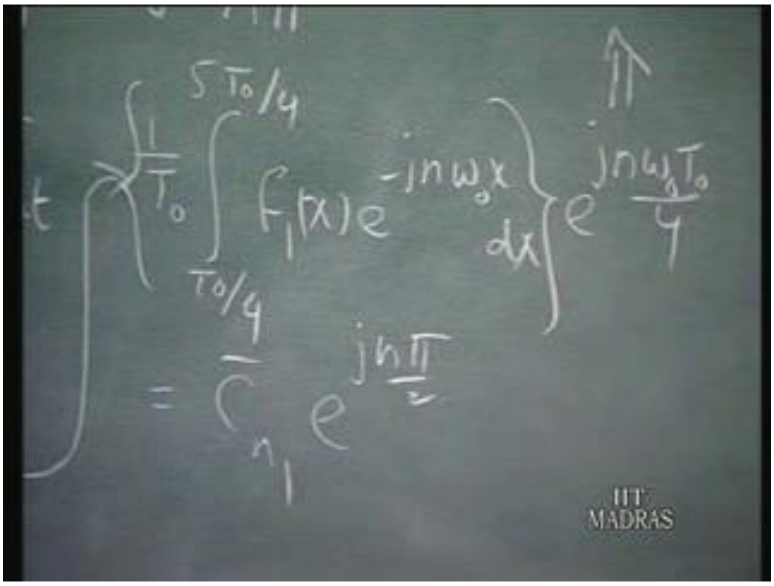
(Refer Slide Time: 21:54)



nought upon 4 to 5 T nought upon 4 f 1 of x. I can write this as minus j n omega nought X dX and the whole thing multiplied by a constant e j n omega nought T nought upon 4. E j n omega nought T nought upon 4 is of course, e power j n pi upon 2 because, omega nought T nought equals n pi upon 2.

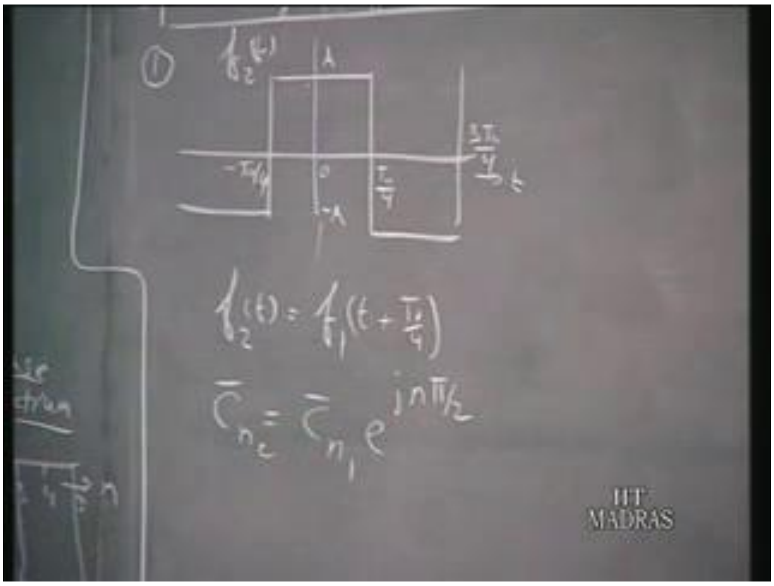
So, this will now become this multiplied by e power j n pi by 2. After all, X is a dummy variable here. You can go back to t. This f 1 of t e power minus j n omega nought t d t is exactly the formula for evaluating C n 1 because; we are integrating this for a complete period. Therefore this will become.

(Refer Slide Time: 22:53)



C n 1 multiplied by e power j n pi upon 2. So, e power j n pi by 2 has a magnitude 1 and its angle of course, depends upon the value of n. So, C n 2 happens to be equal to C n 1 e power j n pi by 2. The Fourier coefficient for this; C n 2 C n 1 e to the power j n pi by 2.

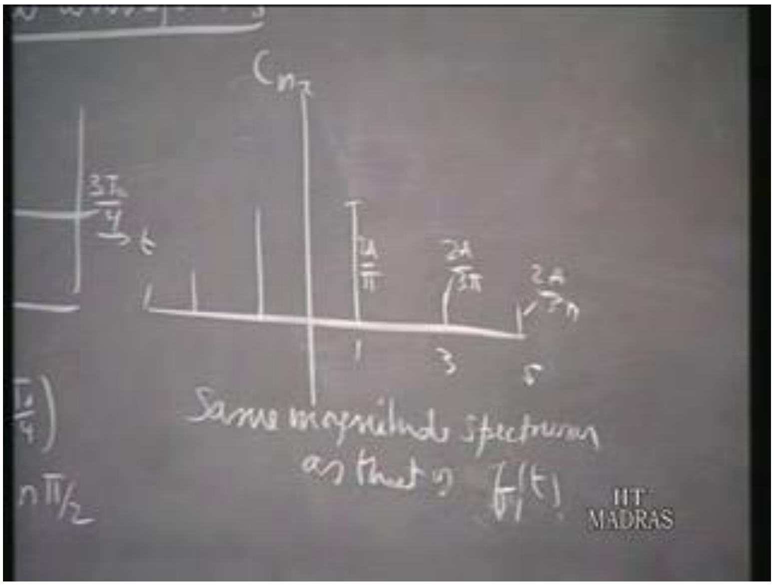
(Refer Slide Time: 23:16)



So, what is the spectrum of this? How does it look like? The magnitude spectrum will be the same as that of C n 1 why, because the magnitude of C n 2 is the same as the magnitude of C n 1 because the magnitude of this is 1.

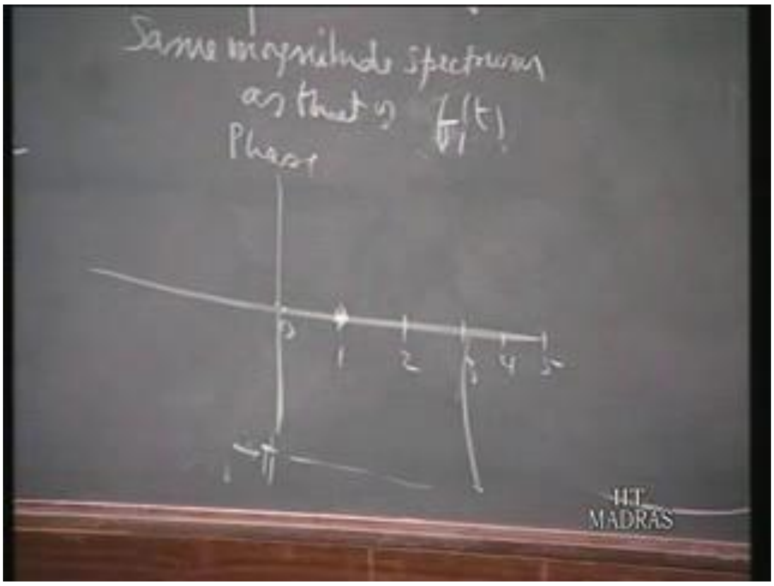
So, you have.

(Refer Slide Time: 23:44)



1 3 5 1 3 5. This will be 2 A by pi 2 A by 3 pi 2 A by 5 pi etcetera. So, the magnitude spectrum remains unchanged. Same magnitude spectrum as that of f 1 of t. So, this magnitude spectrum and that will be identical. Phase; however, get modified. So, instead of this phase you have at each component, you have an addition of n times pi upon 2.

(Refer Slide Time: 24:42)



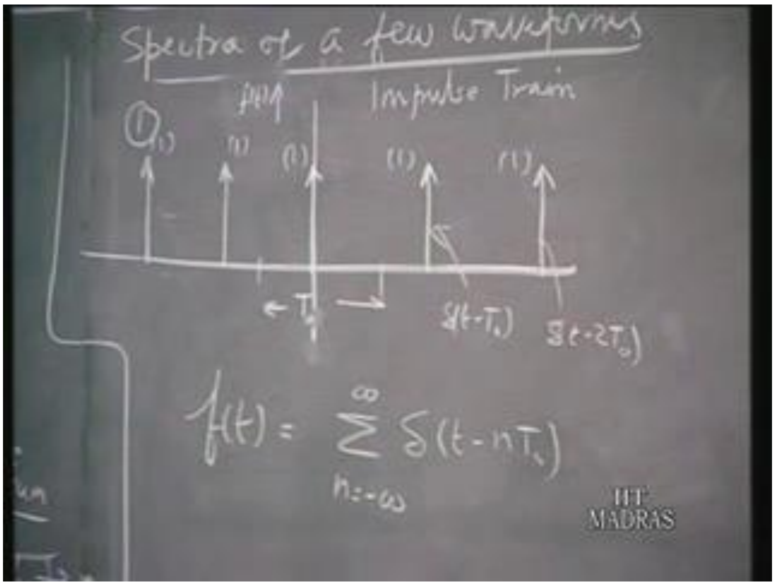
D c of course, is nothing. At 1 it is minus pi upon 2. So, add to that another pi upon 2. So, this becomes 0. At 2 there is no component to deal with. At 3 it is minus pi upon 2. To that we add 3 pi upon 2. So, that becomes 180 degrees. You can as well say it is minus pi. Plus pi or minus pi are 1 and the same and at 5 you add 5 pi upon 2. It is already pi upon pi upon 2 minus pi upon 2 you add 5 pi upon 2. It becomes 2 pi. Once again it is 0. So, you have a whole series of angles 0 or pi alternately and this is the phase spectrum.

So, the conclusion we draw from this is: if you have a square wave, whether you put the origin here or origin somewhere here, the magnitude spectrum will remain the same. So, if you have a periodic function, the amplitudes are of different harmonics or the ratio of the amplitudes of different harmonics will, remain the same irrespective of where you pitch the origin, because that is the fundamental property of the waveform; how different harmonics can go together to compose the given wave.

So, the magnitude spectrum remains invariant under translation in respect to time, but the phase spectrum undergoes a change depending upon, where you put the origin. Usually, we are interested only in the magnitude spectrum because, we would like to know the relative proportions of the various harmonic components and therefore, we can say that the magnitude spectrum is invariant under the operation of shift in the time axis.

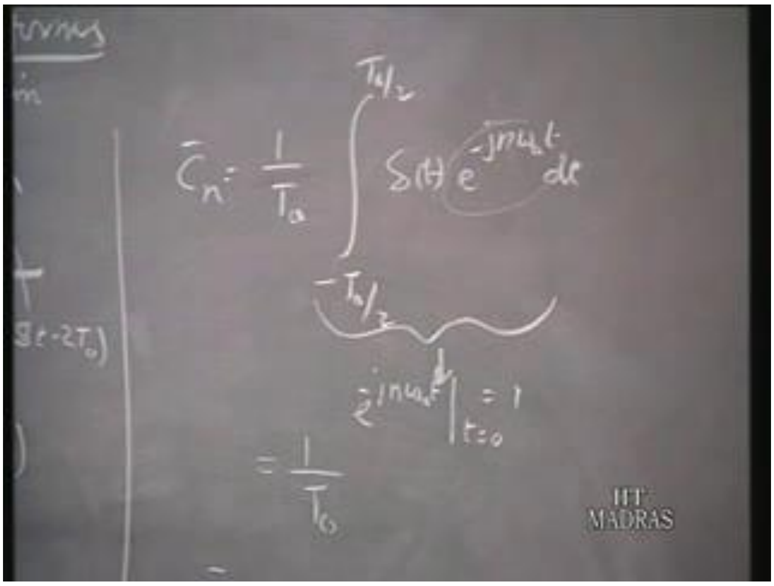
Let us take a second example.

(Refer Slide Time: 27:03)



Suppose, the periodic function that we are talking about, is 1 which is a periodic impulse train. So, each is of impulse delta function of unit magnitude, unit strength. So, f of t is a periodic waveform with 1 impulse sitting in each period at the centre. So, we can write formally that, f of t consists of a number of impulses delta t minus n T nought n going from minus infinity to plus infinity. So, for each value of n there is an impulse; at 0 this is the impulse. At n equals 1 this is after all delta t minus T nought and this is delta t minus 2T nought and. So, on. So, all these impulses go together to constitute this f of t. Now, what is the Fourier Spectrum for this? So, you have to find out the Fourier coefficient to start with, and this can be done with surprising ease because, whenever we have impulses inside the integration sign then, the integration becomes quite simple as we had already seen. So, C n here will be 1 over T nought say minus T nought upon 2 to plus T nought upon 2

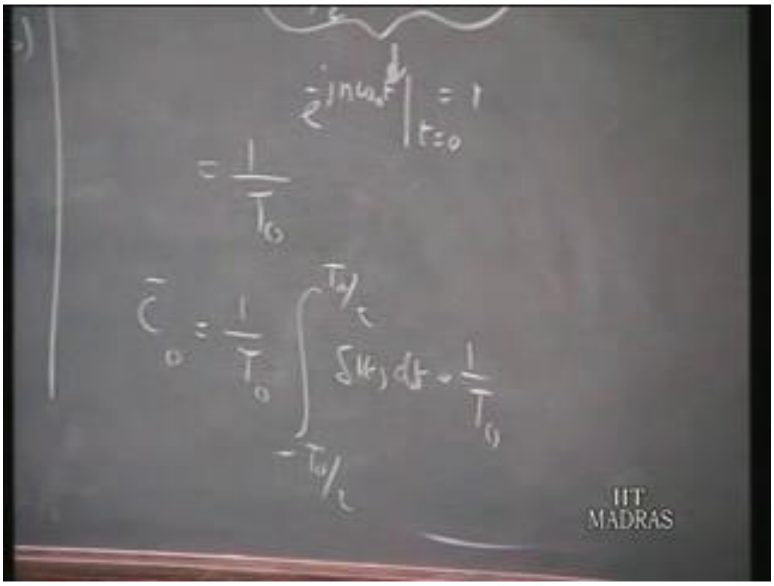
(Refer Slide Time: 28:42)



So, inside this range, function that we have got is only delta t.E to the power of minus j n omega nought t dt. Now, what is the result of this integration? Whenever a delta t multiplies a certain function of time and you integrate over an interval which contain that impulse the value is simply, the value of this function at t equals 0 because, the impulse it sitting at t equals 0. Therefore, the value of this function is 1. Value of this is simply this quantity evaluated at t equals 0 and that of course, is 1.

Therefore C n nought is 1 upon T nought. Normally, when we evaluate the Fourier coefficients it would be advisable for us, to verify if the general expression for C n is also valid for n equals 0. So, if you want to do it independently this is 1 over t T nought.

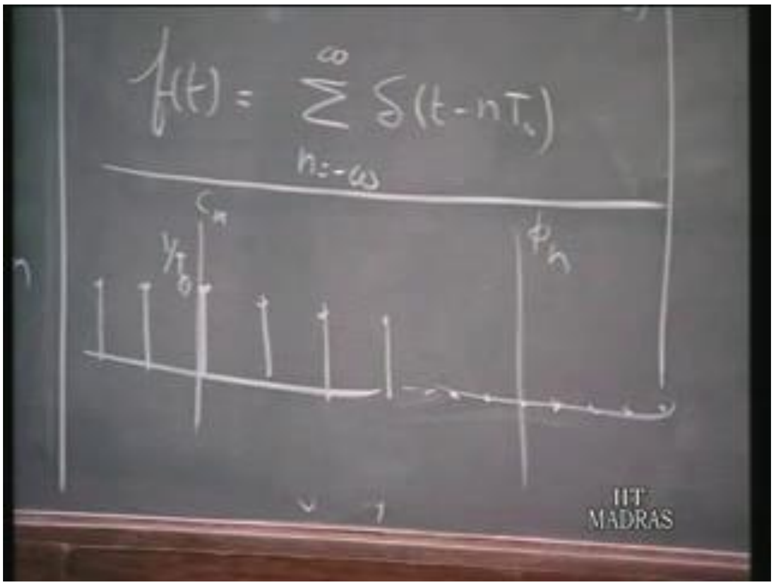
(Refer Slide Time: 30:03)



Say minus T nought upon 2 to T nought upon 2 delta t dt. Certainly this is equal to 1. Therefore, this is 1 over T nought.

So, what do we have for the spectrum?.

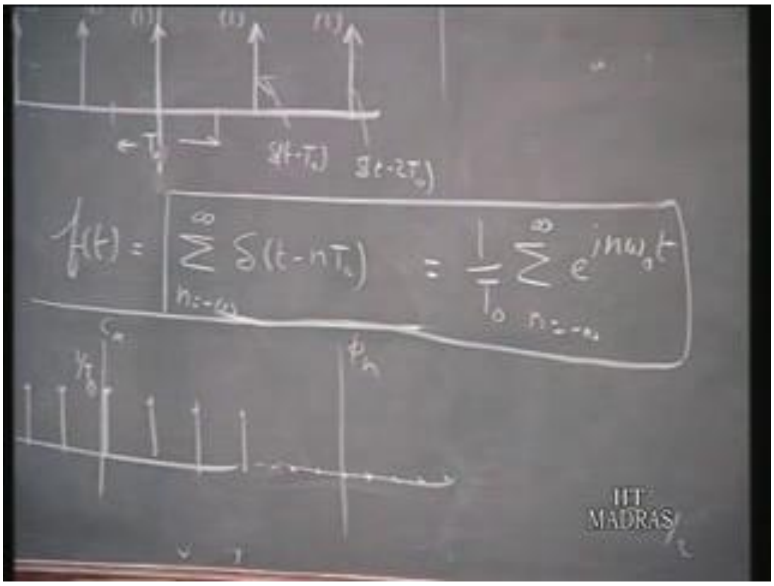
(Refer Slide Time: 30:18)



We have C n magnitude; spectrum flat all components of the same value 1 over T nought. Phase is 0 because C nought is 0 that means, phase spectrum. Very interesting result that we got now. You have a train of impulses of unit strength, occurring regularly at intervals of T nought. Then, the spectrum for that is flat here in the sense that, all harmonic components have the same magnitude. They do not do down as n goes up. They retain; that means, all frequencies; that means, we can say a train of impulses is. So, to say, impartial to the order of the harmonic all harmonics of equal strength, go together to compose this impulse time. This is quite an important fact which, will occur this particular property occurs, again and again in our future discussions.

So, we will observe that if f of t equals this train of impulses, the Fourier Series expansion for this is 1 over T nought e to the power of j n omega nought t n ranging from minus infinity to plus infinity.

(Refer Slide Time: 31:42)



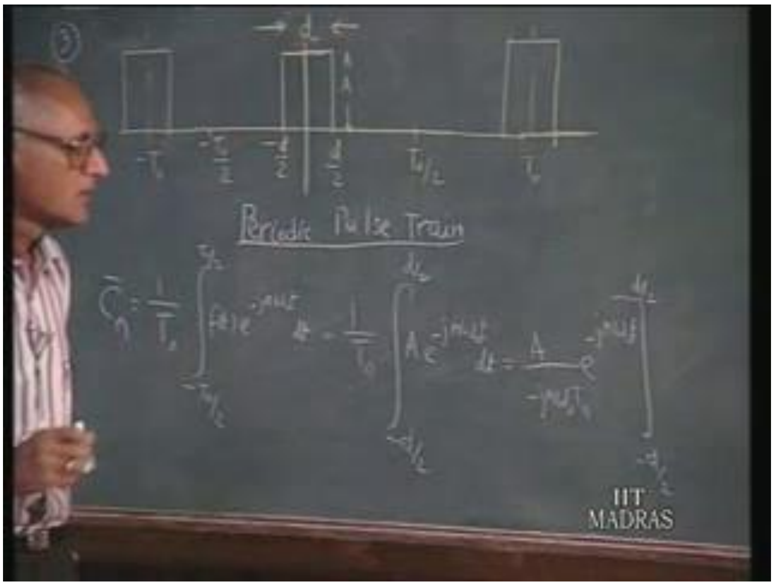
So, this is the result which is very compact and has a very nice look about it. You have summation from minus infinity to plus infinity of the impulses. On the other side, you have summation from minus infinity to plus infinity of exponential terms.

So, this spectra gives us an indication, how the various coefficients go remain the same. You have taken 1 extreme example of a situation where, the spectra magnitude remains constant right through. So, with this back let us go further and construct another spectrum. As a third example of finding out the spectra let us, consider a periodic pulsed train with a waveform depicted as here, where a rectangular pulse repeats itself for every T nought seconds. The pulse has a width d and an amplitude a and it is centered symmetrically with respect to the origin.

So, this is a periodic pulse train and the duty cycle is d upon t. That if for T nought it will last only for d seconds. So, C n for this will be 1 over T nought minus T nought upon 2 to plus T nought upon 2 f of t e to the power of minus j n omega nought t dt. This is the standard formula. But in our particular case, this f of t lasts only from minus d upon 2 to plus d upon 2 and the rest of the period it is 0.

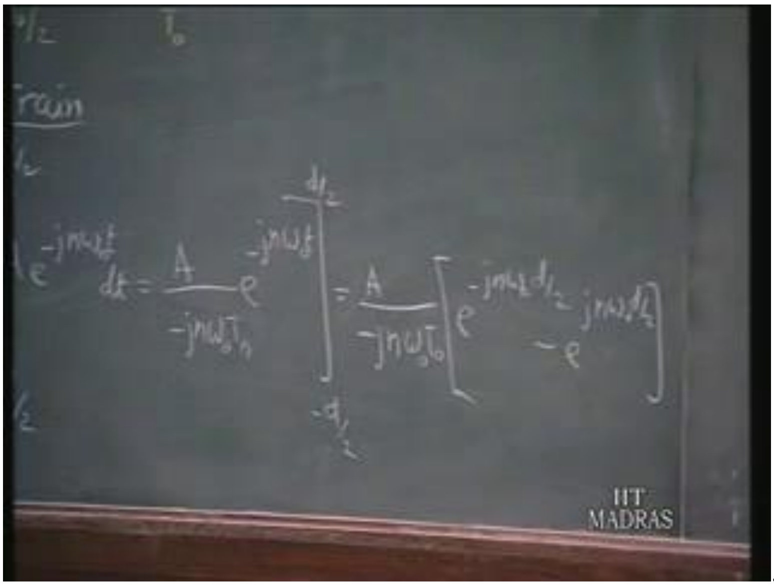
Therefore, we can write this as minus d upon 2 to plus d upon 2 and in this interval its value is A. A times e to the power of minus j n omega nought t d t and this can be written as A minus j n omega nought T nought e to the power of minus j n omega nought t. That is evaluated between the limits minus d upon 2 to plus d upon 2.

(Refer Slide Time: 34:34)



A minus j n omega nought T nought e to the power of minus j n omega nought T nought.

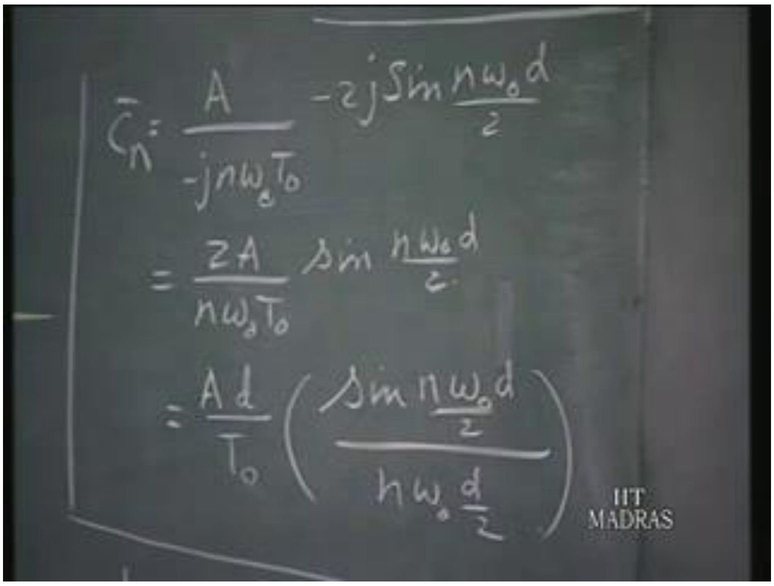
(Refer Slide Time: 34:40)



e to the power of minus j n omega nought d upon 2 minus e to the power of j n omega nought d upon 2, substituting the lower limit minus d upon 2.

So, if you pick up from here and go here

(Refer Slide Time: 35:21)



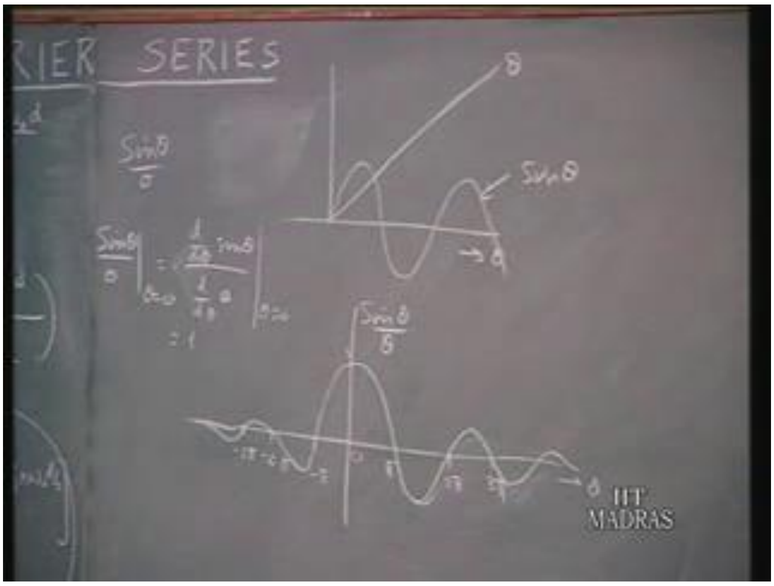
C n will now be A divided by minus j n omega nought T nought. This we notice, if we have reversed the side e to the power of j n omega nought d upon 2 minus e to the power of minus j n omega nought d upon 2 would be 2 j sine n omega nought d upon 2. The negative sign you reverse this thing. I can write this as minus 2 j sine n omega nought d upon 2. So, canceling this minus j on both sides you have 2 A n omega nought T nought sine n omega nought d upon 2. I will write this in a slightly different fashion.

Suppose, I write this sine n omega nought d upon 2 I establish a denominator n omega nought d upon 2. So, this 2 is taken over here. So, A remains and since, I introduce a d in the denominator i put a d in the numerator. N omega nought are already carried over here. So, all that remains is this. I put this in this particular form because, this function we have is of the form sine theta by theta which is, a function which is well known in mathematical literature and which comes up quite frequently in waveforms of this sort.

So, to plot this spectrum corresponding to this, let us fill ourselves upon some background information of the variation of sine theta by theta. So, if you are having a function of sine theta by theta, take theta. What we are interested in is to, find out the variation of sine theta by theta. So, let us plot this.

This is sine theta and theta will be like this

(Refer Slide Time: 37:24)

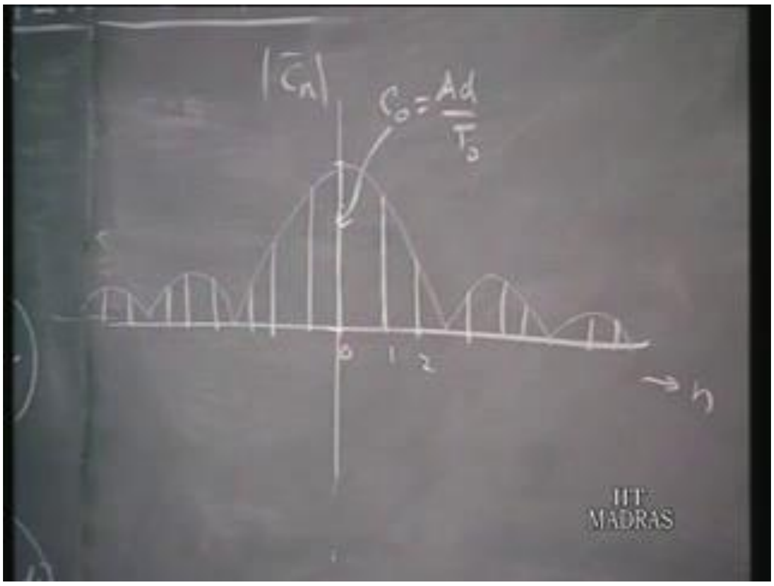


So, we are dividing this waveform by this. So, we can see that as the theta increases the amplitude of the oscillations decreases. On the other hand, at theta equals 0 this is 0 by 0. So, if you take sine theta by theta, evaluating at theta equals 0 we have to take the limit at theta goes to 0. That means; you have to take the derivative d by d theta sine theta in the numerator, d by d theta of theta in the denominator and evaluate at theta equals 0 using the l'hospital's rule and that turns out to be 1.

So, when you divide this quantity by this you get, sine theta by theta curve. It starts with 1 and we will now exhibit oscillations with decreasing amplitudes and since this is an even function of theta, both numerator and denominator are odd functions. Therefore the ratio is even function. So, you get something like this. This is the variation of sine theta by theta. This function occurs again and again in Fourier theory. So, we would like to see the properties of this.

So, at theta equals 0 it is 1 and it becomes 0 again at pi 2 pi 3 pi and. So, on and. So, forth. Again on the negative side minus pi minus 2 pi minus 3 pi and. So, on. So, this is the type of function that we have got. However, we do not have continuous variation of theta. We are interested in finding out the values of this, at specific values of theta which are given by integral values of n.

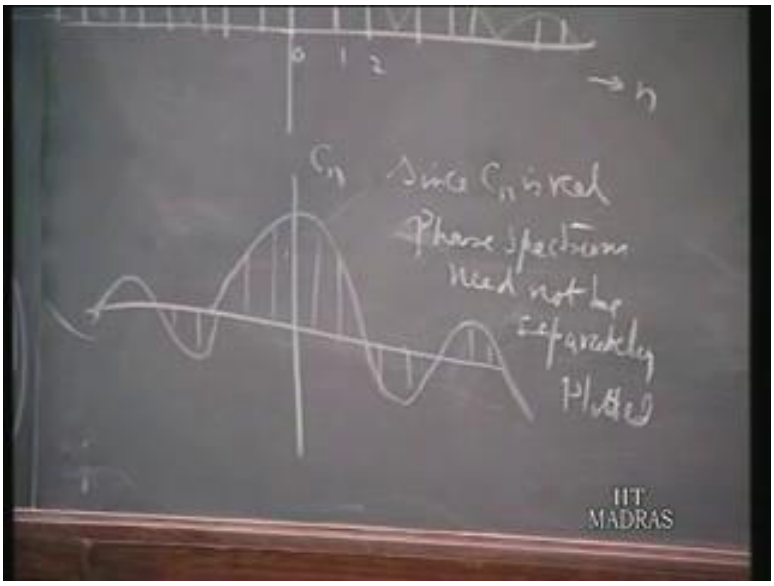
(Refer Slide Time: 39:54)



So, consequently the spectrum of this would be magnitude. We have an envelope like this, because we are considering only the magnitude. So, the negative loops are flipped over. This will be the envelope of your C n magnitude, but the lines, you will have only components like this and specific values of integral values of n 0 1 2 etcetera. In particular, C 0 will be A d upon T nought. As you can see, the average over a complete period is A d upon T nought. So, that is the C nought A d upon T nought and like this it will go.

So, the spectrum of a periodic pulse time will have this type of character and since, this is a real quantity, as far as the phase is concerned we can it is either 0 or pi and sometimes, when the C n quantity

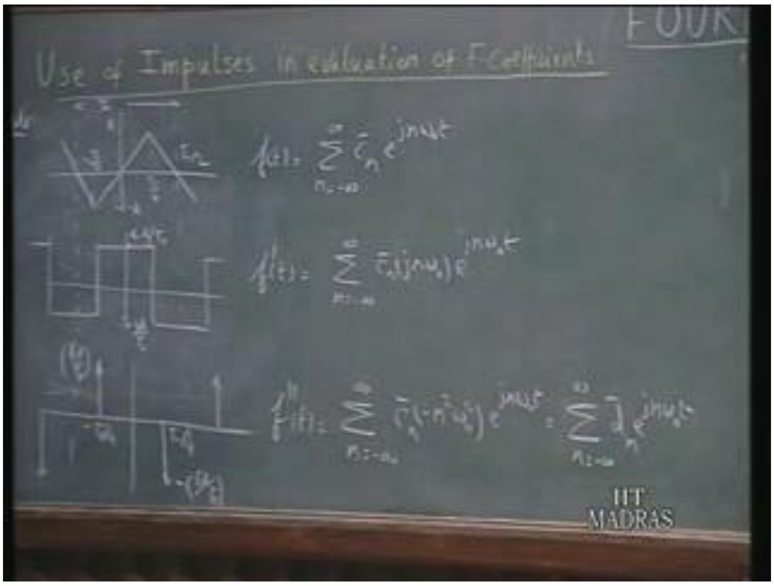
(Refer Slide Time: 41:04)



Y Turns out to be real, we can plot C n simply as like this. So, we do not have to plot the magnitude and phase spectra separately. So, C n is either real. It is either positive or negative and. So, that information can be given. Since C n is real, phase spectrum is not necessary. Phase spectrum need not be separately plotted. So, the amplitudes go down in this fashion.

We have seen in 1 of the earlier examples that, when impulse functions the evaluation of Fourier coefficient becomes quite simple, because integration with impulse is easy. So, I would like to take an example, where we make use of the impulses in the evaluation of Fourier coefficients. Let us take a triangular wave like this and suppose; I call this f of t and let its Fourier expansion be j n omega nought t n from minus infinity to plus infinity.

(Refer Slide Time: 42:20)



To evaluate this C n in this for this waveform requires, good amount of computational effort. But, let us see, what happens if you take the derivative of this, which is this. If you take the derivative of this, the slope here is constant. The slope here is constant, but negative. Therefore it is like this. So, if you take the derivative of f of t, it results in a square wave like this f prime t, but we would not stop that. We will take the second derivative of this. The derivative of this now as you can see, it is 0 everywhere except where there is discontinuity.

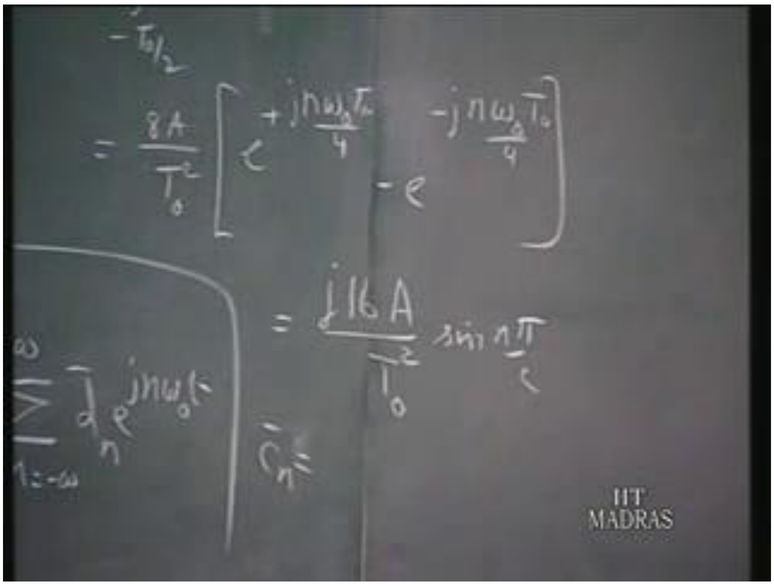
Therefore, there is an impulse here and an impulse here and what are the magnitudes of these impulses? The slope here is, it raises by an amount 2 A in T nought by 2. Therefore, it is 4 A by T nought. That is the magnitude here and this minus 4 A by t nought. Therefore, the jump that is involved in going from minus 4 A upon T nought to plus 4 A upon T nought is 8 A upon T nought.

Therefore, there are 2 impulses here. 1 positive and the other negative which tends equals to 8 A upon T nought and suppose we find the fourier coefficients for this periodic waveform, then can we related those 4 coefficients to the fourier coefficients of the parent waveform. That is what we want to do. So, if f t has this fourier series f prime t assuming that, we can differentiate under the summation sign. This will be C n j n omega nought e to the power of j n omega nought t. And you differentiate again, it will be C n again derivative of this. So, again you multiply with another j n omega nought.

So, it will become n square omega nought square e to the power of j n omega nought t. Let us call this d n. Let us call the Fourier coefficient for this to be d n, n from minus infinity to plus infinity of d n e to the power of j n omega nought t. So, if you calculate d n for this periodic function having impulses then, we know d n and C n are related by this C n square C n times minus n square omega nought square is d n. So, that is the plan of action.

We would find out the Fourier coefficient for d n.

(Refer Slide Time: 45:13)

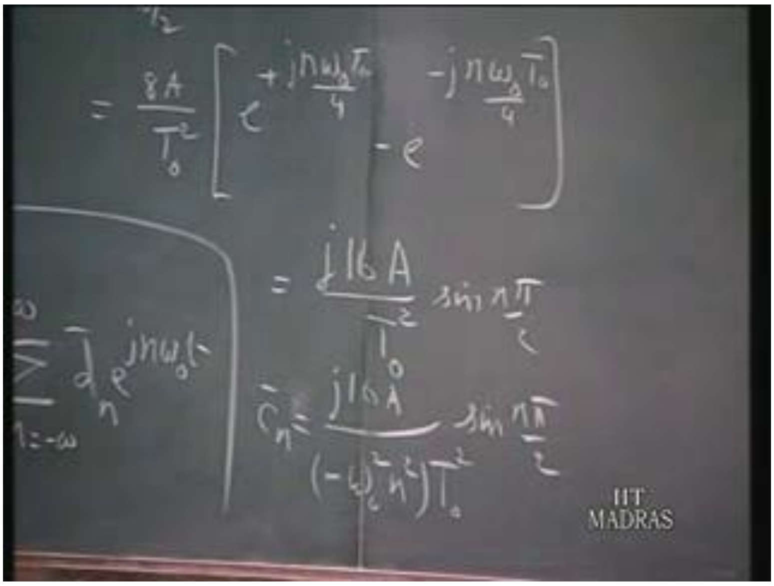


And then find out form that C n. So, d n is 1 upon T nought from minus T nought upon 2 to plus T nought upon 2. In this range of integration, we have 2 delta functions. So, 1 delta function at both magnitudes: 8 A upon T nought delta t plus T nought upon 4. That is this delta here at negative values of time and another delta function at t nought upon 4 with a negative sign.

Therefore, minus delta t minus T nought upon 4 e to the power of minus j n omega nought t dt. So, you have got 8 A upon T nought square. Now, this delta multiplied by this is being integrated. Therefore, this will be e to the power of minus j n omega nought minus t nought upon 4 evaluated at t equals minus t nought upon 4. That means, plus t nought upon 4 minus e to the power of minus j n omega nought t nought upon 4 and you can carry this out. Finally, you can show that is j 16 A upon T nought square sine n pi by 2. This is your d n.

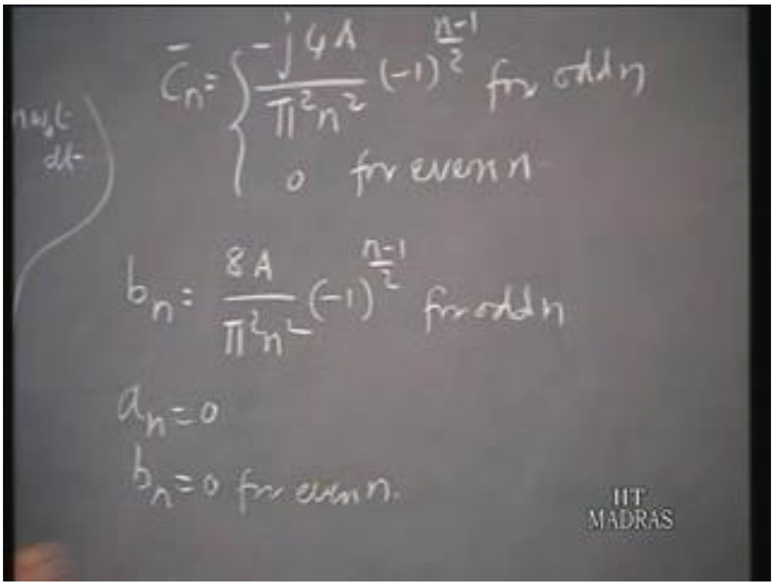
Therefore C n will be obtained by dividing d n by minus n square omega nought square.

(Refer Slide Time: 46:05)

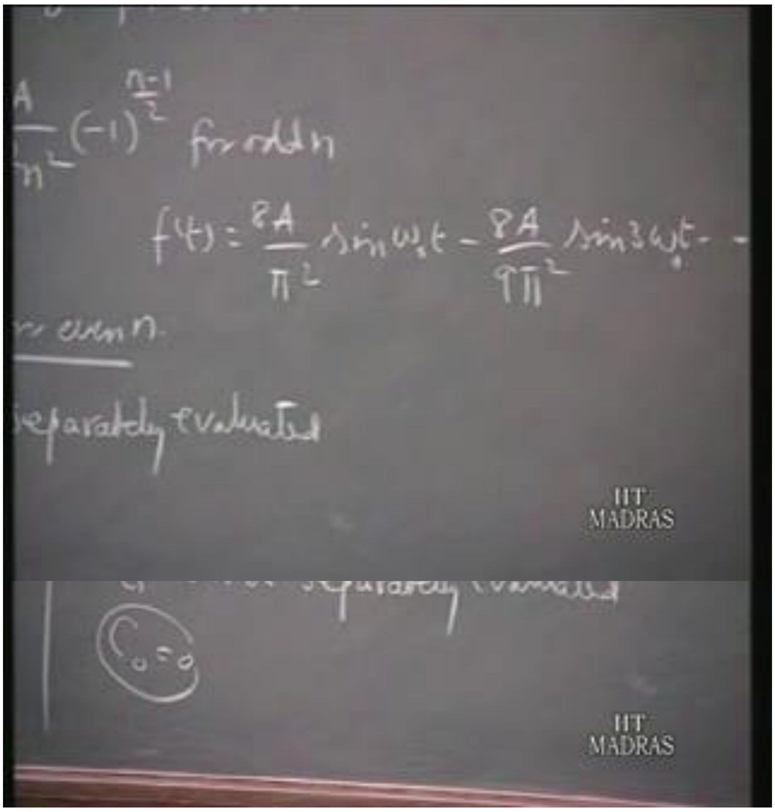


So, j 16 A minus omega nought square n square T nought square sine n pi by 2. And you can complete the work and you can show that this, will turn out to be minus j 4 A by pi square n square minus 1 raised to the power of n minus 1 by 2 for odd n and 0 for even n.

(Refer Slide Time: 47:27)



So, that is the Fourier coefficient for the original triangular waveform. Recognizing that C n equals a n minus j b n by 2 you can write, this contains b n is 8 A by pi square n



square minus 1 n upon 2 for odd n and we know a n is 0 b n is 0 for even n and what about C nought?.

(Refer Slide Time: 47:24)

C nought has to be separately evaluated. I will explain in a moment. C nought for this waveform; because it is 0 average the original waveform C nought is 0. Other C n had already been obtained. So, finally, f of t will be 8 A by pi square sine omega nought t minus 8 A by 9 pi square sine 3 omega nought t etcetera.

(Refer Slide Time: 49:18)

Now, let us see what happens. If you have a constant term here, when you take the derivative, the constant term disappears. Therefore, a d c term gets lost here and therefore, you cannot expect to that term to be present here. So, the dc term in the original function has to be separately evaluated because, that information is lost when you go to this. So, C nought in this case has to be separately evaluated, you cannot obtain from this information.

Now, what other terms can possibly get lost? If there is a constant term here, that can get lost when you take the derivative, but a constant term here means; that you have a term like k t here. Unless there is a term like k t you, cannot get a constant term in the derivative And if you have a term like k t in the periodic function that, completely spoils the periodicity of the function. Therefore, if you have periodic function you cannot have a k t term here. So, you cannot have a constant term here, as a derivative of this.

Therefore, there is no trouble on that score. The only trouble which can come because this, C nought that is here can get lost and that has to be separately evaluated and in this case the constant term is 0. So, this example illustrates; how the Fourier coefficients can sometimes be evaluated by taking the first or second derivative as the case may be and making use of a function which has only impulses as illustrated here.

So, in this lecture, what we have done is we have introduced ourselves to the concept of frequency spectrum, in particular the magnitude and phase spectra associated with this C n coefficients. We observed that these both are line spectra. The magnitude spectrum is an even function of n and the phase spectrum is an odd function of n and we have a series of examples. We saw how this spectra can be constructed. In particular we have constructed this spectra of a train of impulses, the square wave and a train of pulses. And also we saw, how the facility provided by impulses in the integration procedures can be availed of, in situations where either the first derivative or the second derivative as this case contains entirely impulses. So, in that case, it appears to take the derivative and find out the Fourier Series for that and use that information to find out, the Fourier Series or the original waveform as done in this case.