

Electro Magnetic Field
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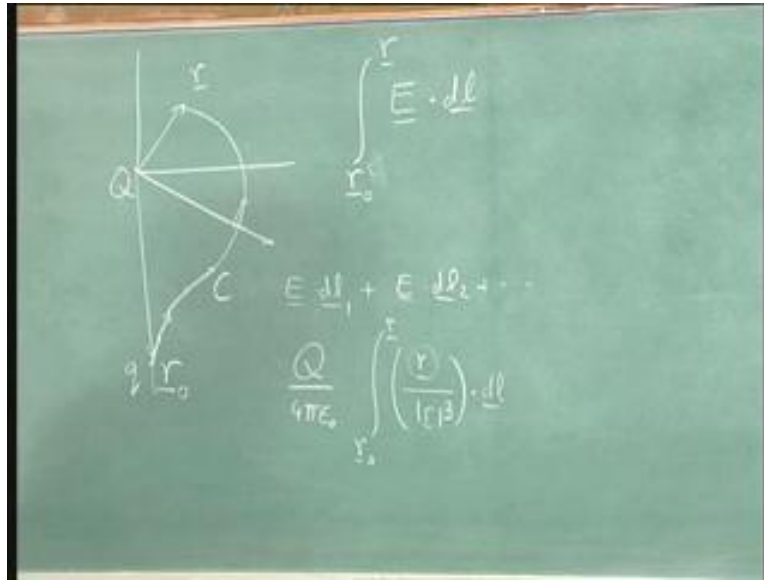
Lecture - 6
The Gradient

Good morning. Last time I had introduced a new quantity namely the electrostatic potential, by showing that a particular integral of the electric field depended only on end points. Let us review. If you have a charge and it is sitting at the origin charge Q and if I took another charge small q and took it on some path from a position r_{naught} ... so, r_{naught} is this vector, to another position r and you went however you liked, then the electric...then you can write an equation which says, go from r_{naught} to r along this path c .

Usually, you do not write the end points if you write c and you work out what this integral is. What this integral means is I choose a lot of points along this curve and I draw little arrows and on each arrow I find the local electric field and I compute $E \cdot dl$. Then I add $E \cdot dl_2$, etcetera and I take that whole sum and then I let dl_1 , dl_2 , dl_3 , etcetera, become very small. In other words, I put more and more points on this curve uniformly so that these arrows become very small. When I do that, I am essentially having an integral and that is the integral that is meant here; $\int_{r_{\text{naught}}}^r E \cdot dl$.

Now in general, this integral would depend on how I went from r_{naught} to r . However, we saw last time that if you work this integral out for a charge placed at the origin, you find that this integral becomes the charge Q over $4\pi\epsilon_0$ times $\int_{r_{\text{naught}}}^r \frac{1}{r^2} \cdot dl$.

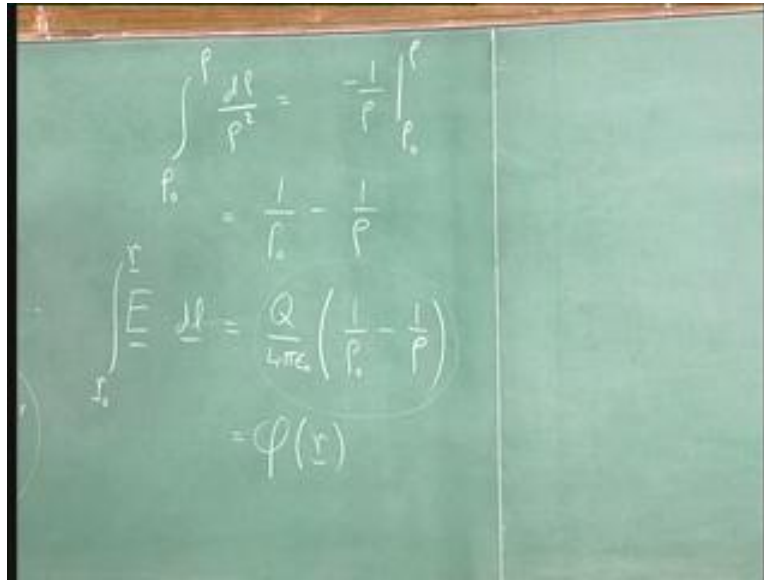
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But this direction r at any point is pointing outwards. This $d\underline{l}$ points in some general direction. But $d\underline{l} \cdot \underline{r}$ only cares about that part of $d\underline{l}$; that is, along the radial direction. So, you can take this entire integral and replace it with an integral that is only in the radial direction. So you find the radial part of \underline{r} naught. Call it ρ naught; radial part of r , call it ρ , and then you are really only now concerned with the radial direction $d\rho$ and r over r cubed $d\rho$.

So, this becomes a scalar integral and when you work it out, you find that this is nothing but integral ρ naught to ρ $d\rho$ over ρ square which is equal to 1 over ρ naught minus 1 over ρ . So here, ρ is nothing but the radial coordinate of r and what you are seeing is the electric field dotted with $d\underline{l}$ integrated from a point r naught to a point r along some path - it could be any path - is equal to Q divided by $4\pi\epsilon_0$ naught times 1 over ρ naught minus 1 over ρ , where ρ naught and ρ are the r coordinates of the beginning and the ending point. It does not depend on how you got there; it only depends on where you started from and where you ended up. I could have gone this way or I could have gone this way or I could have gone this way. It does not matter. All that matters is the beginning and the final value of the radial coordinate.

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$$\int_{r_0}^r \frac{dl}{p^2} = -\frac{1}{p} \Big|_{r_0}^r$$
$$= \frac{1}{r_0} - \frac{1}{p}$$
$$\int_{r_0}^r \underline{E} \cdot \underline{dl} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_0} - \frac{1}{p} \right)$$
$$= \phi(r)$$

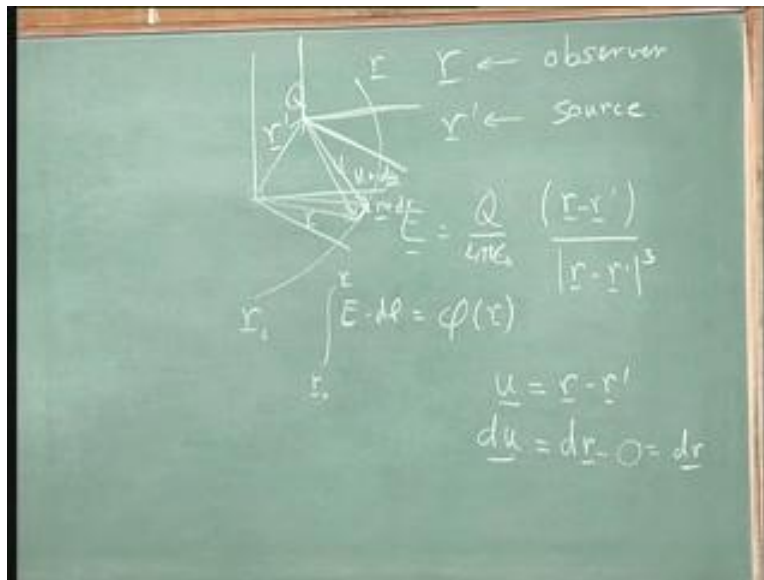
When this is the situation, then we can call this a function of the final point alone and is called the electrostatic potential, which is a function of position. In fact, in this case, it is a function of only the radial component of that position. So, this is what we call the electrostatic potential; but we have derived it only for a charge kept at the origin.

Supposing the charge was not at origin. Supposing it was somewhere else. Supposing my charge is sitting here. So the distance from the origin to the charge is a distance I will call r prime. It is very standard in electromagnetics that when I use a symbol r , I mean observer, and when I use the symbol r prime, I mean source. So, this Q is generating the electric field. It is the source of the electric field. So, I use r prime. Now I have got some curve going from r naught to r . This equation will not work because this equation assumes the charge was at the origin. However, what is this origin? The universe does not really have knowledge of an origin. Universe does not say this lab is centre of the universe. It does not say the centre of the Milky Way is the centre of the universe. If we put origin here, it is in our minds; universe does not define it. So, we might as well put an origin at the charge. Now, if you put the origin at the charge, we know the answer. We know that the electric field r naught to r of $\underline{E} \cdot \underline{dl}$ is equal to some ϕ which is a function of r and not of how we got from r naught to r .

So, how can we take this information and use it in general? Well, the answer is quite easy. What we do is we say, let us consider a new variable called u which is equal to r minus r prime. Now, as we move on this curve what is changing is r . So I will do a change of variables. du is dr minus zero because r prime remains constant. So, du equals dr .

So, what that...what does that say? That is saying that if I take a point on this curve... Earlier I had...this was r . Now I have, from the new origin, I have u and they are different vectors. But if I now go to the next point, that is r plus dr , that corresponds to u plus du and when we changed coordinates like this, r plus dr minus r , that is, the dr is nothing but u plus du minus u .

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So it says du and dr are the same thing even though I change coordinates. So, my coordinates are very different; but the change in u and the change in r are the same and it is only the change that I concern myself about when I do an integral. It is dl that comes; not l itself. So, let us see where this transformation of coordinates takes us. So we start with the integral r naught to r 1. I am changing this final point to r 1 because I used r for the intermediate point as well.

So, r naught to r 1 electric field dot $d l$; that is where we start. Now I will put...I will change my coordinates. I will go to this coordinate system, u equals r minus r naught. So, that is like working in these coordinates because this is u and this is u plus $d u$. So, when I do that, the integral becomes u naught to u 1 - the electric field as a function of u dot $d u$; and I already know that $d u$ and $d l$ are the same thing. That is why I am able to write this. u naught is nothing but r naught in the u coordinate system. That is, it is this vector starting from this origin to r naught and u 1 is this vector starting from the new origin to r 1. But in this new coordinate system, my charge Q is sitting at the origin which means I know how to write down the electric field. You can write it down. It is integral u naught to u 1 Q over $4 \pi \epsilon_0$ naught u over u^3 dot $d u$.

So, this is the electric field in the u coordinate system dot $d u$ along the curve; but this is the problem we have already done. We have already done the problem of integral of e dot $d l$ when the charge is at the origin and the answer we got for that was, it is equal to Q over $4 \pi \epsilon_0$ naught integral...I replace the vector by just the distance, the radial distance from the origin. So, without the line underneath, this is just a radial coordinate u naught to u 1 $d u$ over u^2 ; and the integral became as we saw last time, Q over $4 \pi \epsilon_0$ naught 1 over u naught minus 1 over u 1. But what is u naught? u naught is the magnitude of the vector u naught. It is the radial length; so that is the magnitude, but u naught itself is defined by this equation. So, it is equal to magnitude r minus r naught.

Similarly u 1 is equal to magnitude r minus r 1...sorry, I think I made a mistake there. It is equal to r naught minus this. This is r minus r prime, r naught minus r prime and this is r 1 minus r prime. So, I substitute these in here and what I get is Q over $4 \pi \epsilon_0$ naught 1 over magnitude r naught minus r prime minus 1 over magnitude r 1 minus r prime.

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$$\int_{r_2}^{r_1} \underline{E}(\underline{r}) \cdot d\underline{l} = \int_{r_2}^{r_1} \underline{E}(\underline{r}) \cdot \underline{e}_r dr = \int_{r_2}^{r_1} \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} \cdot \underline{e}_r \cdot \underline{e}_r dr$$

$$= \frac{Q}{4\pi\epsilon_0} \int_{r_2}^{r_1} \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_2} - \frac{1}{r_1} \right) = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{|r_2 - r'|} - \frac{1}{|r_1 - r'|} \right)$$

$$r_2 = |\underline{r}_2 - \underline{r}'| \quad r_1 = |\underline{r}_1 - \underline{r}'|$$

So once again, even though the charge is not at the origin, this integral turned out to be a function only of the initial position, the final position, and of course, where the charge is. It did not depend on how we went from r_2 to r_1 . That got taken care of by evaluation of this integral and the integral depended only on how, where we started and where we ended; it did not depend on the route.

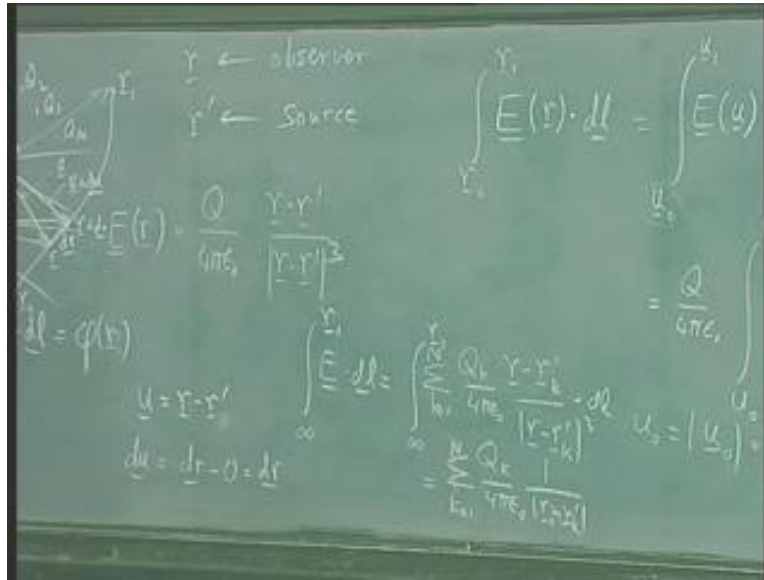
Now usually, we take this initial point such that it does not contribute at all. For instance, in 3 D we will choose this to be infinity, in which case, we get integral r_2 which is infinity to r_1 . Now I am going to just call it r , $\underline{E} \cdot d\underline{l}$ is equal to minus Q over $4\pi\epsilon_0$ 1 over r minus r' . What this means is that we can again define a function, a scalar function, which is derived from the electric field. So, we will define potential ϕ of r . This is equal to minus integral infinity to r of $\underline{E} \cdot d\underline{r}$. However we come from infinity to r , this answer will be unchanged. It only depends on the fact that we came to this point.

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So, this is the generalization. It means that regardless of where the charge is, whether it was at the origin or not at the origin, my potential function is the same. And now I have one more generalization. I can put many charges Q_1, Q_2, Q_3, Q_N and I can use this same technique and I can say integral from infinity...this is the symbol for infinity - an 8 that is turned horizontal to $\int \underline{E} \cdot d\underline{l}$. This electric field is due to all these charges.

So, I can write this as infinity to r summation over say k equals 1 to infinity, 1 to N $Q_{\text{sub } k}$ over $4\pi\epsilon_0$. Well, let me call this r_1 after all; $r - r_{\text{prime } k}$ divided by $|r - r_{\text{prime } k}|^3 \cdot d\underline{l}$. Now, each of these terms is nothing but this integral that we have already done. So, the answer must therefore be just the sum of all these answers. So it becomes equal to $\sum_{k=1}^N \frac{Q_{\text{sub } k}}{4\pi\epsilon_0} \frac{1}{|r - r_{\text{prime } k}|}$; and this is now the r_{prime} , the limit r_1 .

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So, I have now an expression for potential. It is actually the negative of this is what I want. So, minus, minus, minus; that became minus; so this is plus, phi of r. So, this electrostatic potential is the sum of electrostatic potentials due to each charge Q_1, Q_2, Q_3 , up to Q_N . Now, this potential is completely fictitious. At least as far as the electric field is concerned, we know there is a force. Therefore we can say, we believe there is a force per unit coulomb. But as far as the electric potential is concerned, there is no way of measuring the electric potential.

However, it is an extremely useful idea, because you see the electric field has 3 quantities in it. So, there is an electric field along x, an electric field along y and an electric field along z. So, it is a complicated field. It has three components in it, but the electric potential phi has only one component - phi itself. So, it is a much simpler concept than the electric field. So, I have gone down from having 3 pieces of information at every point to having one piece of information at every point, but there is more to it than that.

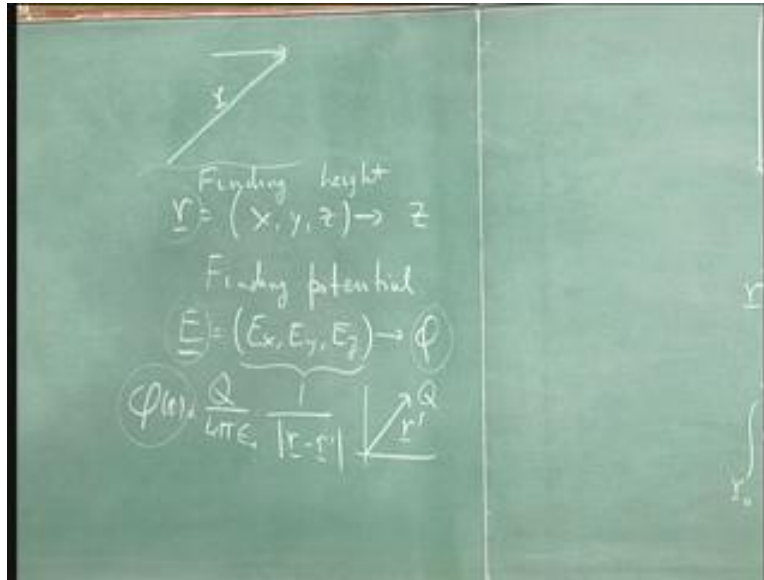
If this is all I was doing...for example, supposing a plane is flying over ground. So the plane has a location and this r would have x, y and z information. x and y talk about where horizontally it is; z is how much higher than ground. Now I could ask what is the

altitude of the plane that maps from this 3 D value to just z . So that also is like going to a very simple idea. So, you could think of finding altitude, finding height, which takes a 3 dimensional vector and gives me back an answer. It is one kilometre above the ground; and finding potential, I go from electric field which is E_x, E_y, E_z , to a single value ϕ .

So it looks like I am doing the same sort of thing. There is something that is very surprising. If I told you the height of the plane, you could not tell me where the plane is. You can only tell me the height of the plane. You cannot tell me that it is 100 kilometres east of Madras; but if you are trying to...if you are...if you give me the electric potential, it so turns out, I can tell you what all these three components are. It is the information that is inside ϕ is enough to reconstruct the entire electric vector. This is an extremely unusual thing. In fact, what it means is that the electric field is a very simple field. It is not really a vector field at all. That in some sense it is so simple, its entire information is captured in a scalar field.

Now, let us prove this. Supposing we have found potential. So we know that for a point source ϕ is equal to Q over $4\pi\epsilon_0$ 1 over magnitude r minus r' , this is what the potential is. Potential is itself a function of r and the point source Q is at the point r' , alright? Now, I want to go from knowing this potential to knowing this field and I am claiming that unlike in the case where I am given the height and asked to find the plane, and I cannot do it, in the case where I am given the electric potential, I can find the entire electric field. Let us see how we do it.

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First things first. If I have got an electric potential ϕ of r , this is a scalar field. It means there are numbers associated with every position. This electric potential came from an integral. Now I am sure you know that when we integrate things, we get smooth functions. In other words, integral of any function is continuous.

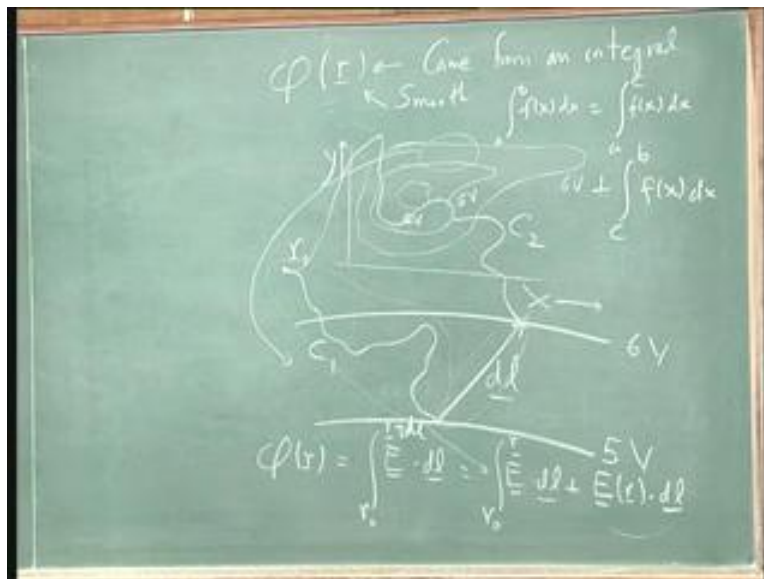
So, what it means is ϕ is smooth. Now what does that mean? It means that ϕ is like temperature, ϕ is like the height of a mountain. I can draw contours of ϕ . So I can...if I work only in x and y , I can draw contours. This is x , this is y and I say that ϕ is 4 volts, 5 volts, 6 volts and so on and so forth. So I have...because it is a smooth function, I have the ability to draw surfaces on which potential is constant. Now, what is the advantage of this? Supposing I look at this part alone. I am going to blow it up. The 5 volt surface looks like this. The 6 volt surface looks like this. They are both slightly curved. So I have drawn them slightly curved.

Now I am sitting here and I move in some direction; and after I move some distance, call it $d\mathbf{l}$, I go from ϕ of 5 volts to ϕ of 6 volts. However, we have a separate result. We know that ϕ of r , whether it is 5 volts or 6 volts, is equal to integral \mathbf{r} naught to r of electric field dot $d\mathbf{l}$ and it says it does not care how you got there. If you start at r naught

and reach r , however you reach it, its ϕ of r . So now let us see how that works. Let us say that my point r naught is here. To calculate ϕ at this point and find that is 5 volts, I have done an integral and come here. To find the ϕ at this point at 6 volts, I did another integral and came there and said the difference in potential is 1 volt; but this curve c_1 and this curve c_2 are completely arbitrary. I can choose any curve I like for c_1 and c_2 .

So, specifically when I choose c_2 , I do not choose this one. I am going to choose the curve that looks like this. I am going to retrace c_1 itself, go along c_1 , come to 5 volts and then I will move along d_1 itself and reach there. So, what does that mean? It means I did integral $\mathbf{E} \cdot d\mathbf{l}$. In order to do this integral, I am going to do it as integral from r naught to r to this point $\mathbf{E} \cdot d\mathbf{l}$ plus this piece $\mathbf{E} \cdot d\mathbf{l}$. What have I done? I can calculate this potential anywhere I like. I can go on any curve I like but I choose... chose to go to that curve which touches r before it goes to r plus $d\mathbf{r}$. This part of the curve is this and this additional part is this; and you know that if you have an integral a to b of some f of x dx , you can always write a to c f of x dx plus integral c to b f of x dx .

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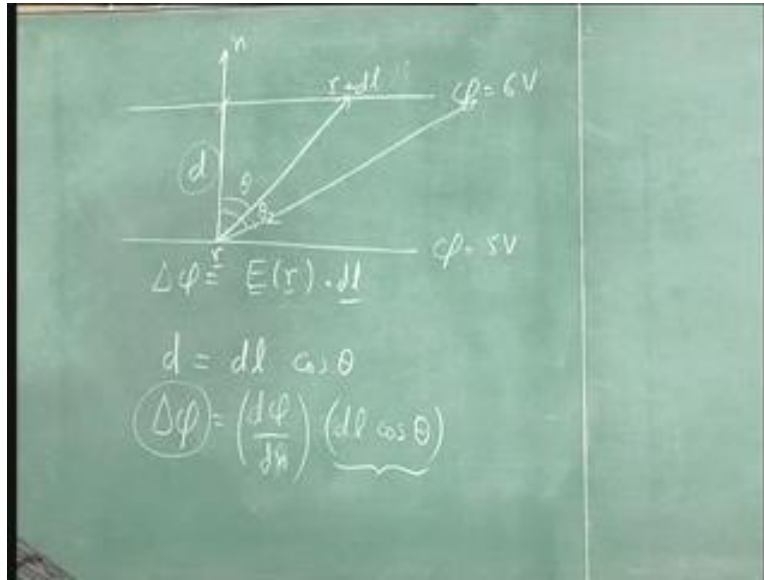
So, that is what I have done. I have broken it up into two pieces. The second piece is very simple which is $\mathbf{E} \cdot d\mathbf{l}$; but this first piece, I know the answer already. This first piece

is nothing but ϕ at r . I am sorry...here, this should be ϕ of r plus $d r$. So, I have an expression which says ϕ at this point r plus $d l$, is an integral r naught to r plus $d l$ $E \cdot d l$. That is, by definition, I have broken it into two parts: one part which goes to r and $E \cdot d l$ and this part is nothing but ϕ of r itself.

So if we write it out now, we have an equation that says ϕ of r plus $d l$ is equal to ϕ of r plus $E \cdot d l$. Of course it assumes $d l$ is very small so that second integral is $E \cdot d l$. I can take this ϕ of r to the other side and I can write it as ϕ of r plus $d l$ minus ϕ of r is equal to $E \cdot d l$. This is the $\Delta \phi$. Now if we divide it both sides by $d l$, then in a certain sense we get E . I mean, if you look at this equation, it is sort of saying...I am going to put this within quotes - $d \phi / d l$ is equal to minus E . Well, there is a minus sign coming somewhere. The whole thing has minus, alright? So, ϕ is minus of this. So, there is a minus sign. So, it is minus E , except we do not know how to define this quantity. We do not know how to say derivative of a function with respect to a vector. It is a very funny kind of derivative. We understand things like $d f / d x$, $d f / d y$, but what does it mean to say $d f / d l$.

Let us look at this diagram again. I had 6 volts, I have 5 volts. I started at a point r . I went to r plus $d l$ and I am saying that this $\Delta \phi$ is equal to the electric field at r dot $d l$. Now, there is another way of calculating $\Delta \phi$. Supposing these were straight lines. They are...for very small distances, they are straight lines. So if they are straight lines, you can draw this diagram as ϕ equals 5 volts, ϕ equals 6 volts. This is my $d l$. This is r . I can draw a normal between these two planes and I can take the angle between them. That is θ . If this distance is d , then it is clearly true that d is equal to $d l \cos \theta$ because this is the diagonal, this is the base. So, d over the diagonal is $\cos \theta$. So, d is equal to $d l \cos \theta$.

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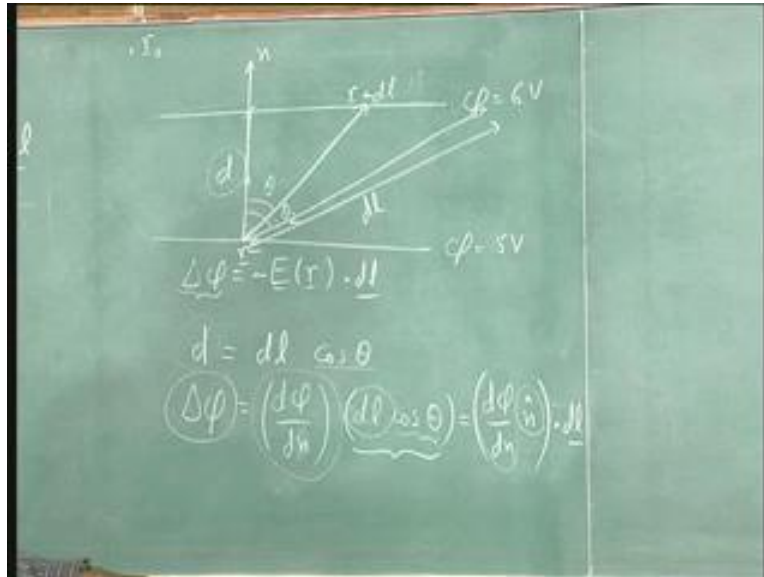


So in a certain sense, when I look at delta phi, I am saying, move along any direction you like till you go a distance along the base which is equal to d, because if I went at a different theta, I went along an angle theta 2, I would have to go to a different distance d l; but I would have to go that distance d l such that my base was equal to d. So, I can write the same relation as delta phi is equal to d phi d l times...sorry, d phi d n times d l cos theta, meaning, go in this direction. This is the n direction. Find...go in this direction. How far? You go in this direction, this length cos theta, you would have gone therefore a distance d. The answer is, you would have gone a distance. So you changed phi by delta phi. Now if you look at these two, this equation, you have one magnitude, you have a second magnitude and you have cos theta. And we know a vector operation that introduces cos theta. We can call this d phi d n along the normal direction dot d l, alright.

I want to repeat the steps I made so that you are with me. First of all, we have a reference somewhere, r naught. When I calculate potential at any point, I am doing integral r naught to r of E dot d l with a minus sign. I have...I have not kept the minus signs properly. So I integrate from r naught to r and I integrate from r naught to r plus d l. They give me different answers for phi and the difference in answers is what I call delta phi. Now I can calculate this delta phi in a clever way by saying the phi at this point is equal

to calculating this integral $\mathbf{E} \cdot d\mathbf{l}$ up to this r and then integrating along $d\mathbf{l}$. That gave me that the change in delta phi is only this piece - $\mathbf{E} \cdot d\mathbf{l}$ with a minus sign, minus sign.

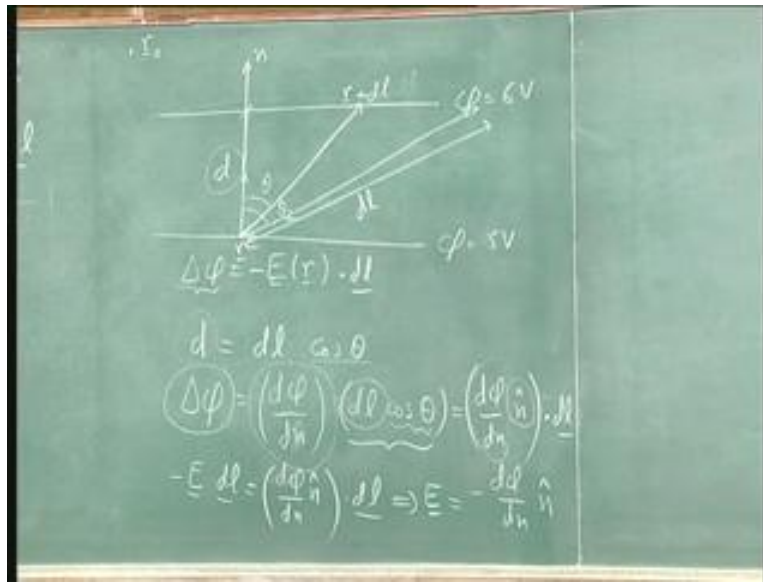
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So I have delta phi is equal to minus $\mathbf{E} \cdot d\mathbf{l}$ and I can calculate the same delta phi in a different way namely, I can say that if I move in any direction, I move a distance $d\mathbf{l}$. It is equivalent as far as the change in phi is concerned, with moving in a normal direction a distance $d\mathbf{l} \cos \theta$. That is, the normal distance I move is the sloping distance I move times cos theta. The change in potential is nothing but the change in the potential as I move in this shortest direction times this projected distance, which can be written as a vector operation. That is, I am constructing a special vector, a vector whose direction is the direction of the normal, whose magnitude is the derivative along this direction.

Now, look at these two equations. Both are equal to the same thing. So I can write minus $\mathbf{E} \cdot d\mathbf{l}$ is equal to $d\phi \hat{n} \cdot d\mathbf{l}$, which means I can write that the electric field is equal to minus $d\phi \hat{n}$ along \hat{n} .

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Now, the textbook gives the algebra. So you do not really have to understand the algebra that much, but what is important here is that from the electric field I did an integral, I got a scalar quantity. I can do that for many fields. As I told you, if you tell me the location of a plane, I can tell you its height. That is like extracting a scalar field out of a vector field; but given the height, I cannot locate the plane; given the height, I have only the height. I do not have the x and y coordinates. But this is a case where given the scalar field phi, I have now gone back and found the vector field E.

So it says, the electric field is particularly simple. It is so simple that given the electric potential, I can calculate back the electric field. This is one of the reasons why the electric potential is so useful because by working with the single scalar quantity, I have found out a vector quantity. By studying a function of...with one component, I have actually solved for three components and therefore there is a great saving in labour.

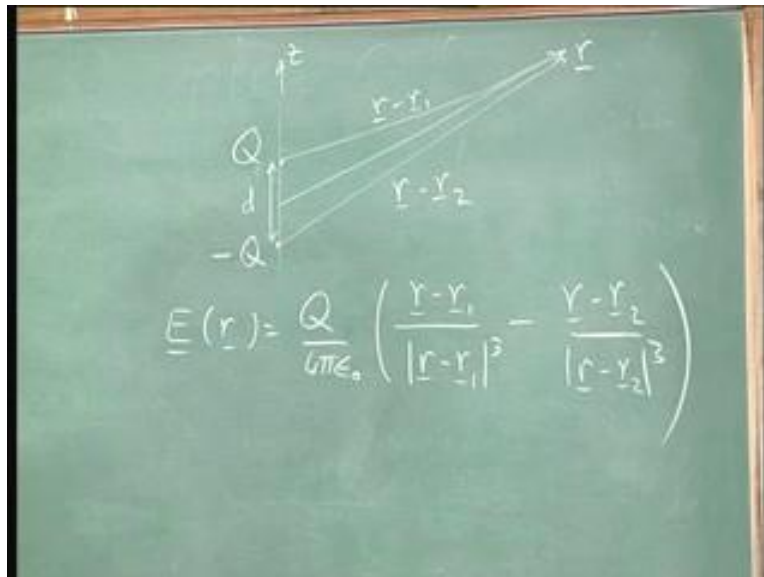
The best way of showing that it is a useful idea is to work out a problem. Couple of lectures ago, I derived what the dipole field...the field due to a dipole is. I did that because I wanted to show that the coulomb field does not fall fast enough. If you

remember, I was looking at the problem that the Coulomb field was 10 to 36 times, 36 orders of magnitude times stronger than the gravitational field.

So, one of the explanations that is given is, there are equal numbers of positive and negative charges. So I say, let us assume that these charges are separated by a distance of one Angstrom and let us work out what the field would be, and at that time I used Coulomb's law and calculated the field. Let us do the same thing with the electrostatic potential. So, I have a charge, capital Q, I have another charge minus Q. This is the z direction and some distance far away is an observe.

So I am putting my origin here and this is the position r. So, this distance between the charges is the distance d and I have...this is r minus r 1; this is r minus r 2. As we worked out, the electric field at r is Q over 4 pi epsilon naught times r minus r 1 divided by magnitude r minus r 1 cubed minus r minus r 2 divided by magnitude r minus r 2 cubed. It is a vector quantity. It depends on r, depends on r 1 and it depends on r 2 and at that time I did not want to do all the algebra. So, I only worked out roughly what the r dependence of this vector field is going to be.

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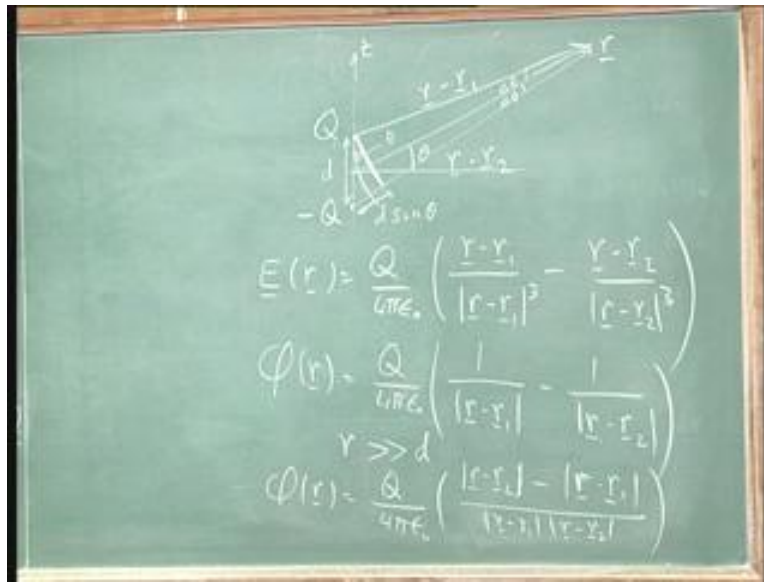


Now I can work out what the electric potential of this is. We have already worked it out. ϕ of r is equal to Q over $4\pi\epsilon_0$ naught 1 over magnitude r minus r_1 minus 1 over... That is, the electric field depended on the direction and the magnitude of each of these lines. The electric potential depends only on the distance. It does not care about the direction. It is an enormous simplification.

Why is it a simplification? Supposing I wanted to find out what this field is for r much much greater than d . So, I am talking about a point which is far away. Then, what is going to happen? These two lines... Let me redraw this because it is not drawn very well. These two lines are nearly parallel. By that I mean, this angle, $\delta\theta$ is nearly zero. So these lines are nearly parallel; which means that if I drew an arc with this line as my, as my radius, actually it will look like this. And this...it is essentially a straight line. How much is this distance? Well, this position r has an angle with respect to the horizontal θ . That same angle shows up here and so this distance becomes $d \sin \theta$. If you chose your angle the other way, which is what the book does actually, you will get $d \cos \theta$.

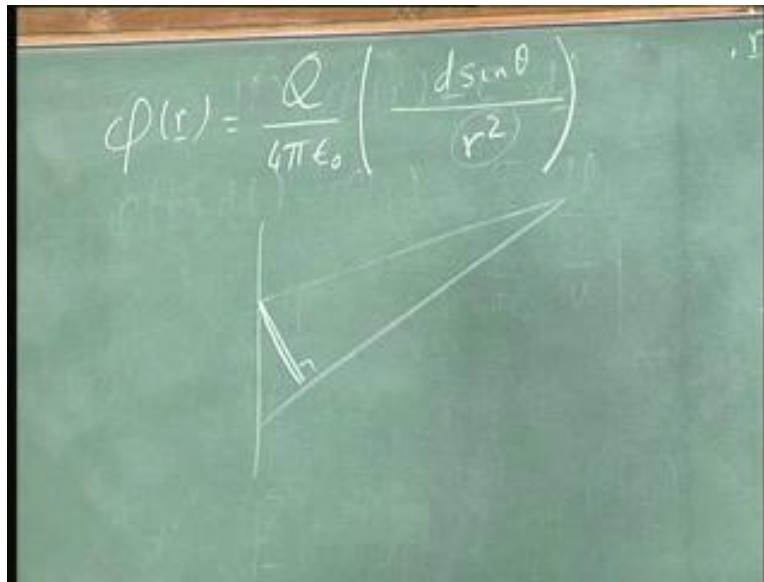
So now, let us see if we can work out what ϕ should be. I am going to take common denominator. So, ϕ of r equals Q over $4\pi\epsilon_0$ naught times $\frac{1}{r} - \frac{1}{r_2} - \frac{1}{r_1}$ divided by the product of the two - r minus r_1 r minus r_2 . So, it is the difference in distances between r and these two charges divided by the product of the distances.

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Now, each of these distances is more or less equal to net distance r . So the denominator can be simplified. So, what you get is ϕ of r is equal to Q over $4\pi\epsilon_0$ times...the difference in distances, we have worked out. It is $d \sin \theta$ divided by...the denominator is nothing but r^2 . It is an approximation. Two approximations are involved. One is that if I put right angle line here, it is equivalent to saying, it is an arc. This is a very good approximation. The second approximation is that the product of these two distances is approximately r^2 . So both of these are very good approximations for charges that are far away.

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Now, why do I call this an advantage? I mean, I have calculated phi but how does it help me? Well now, all I have to do is to go from phi to E. So I need to do...electric field is equal to minus d phi d n n hat. I can plot out this phi. I can find the, what are called equipotentials of phi and if I navigate along the normals to those potentials, I get the electric field out of it. Now, there is a standard mathematical machinery to carry out this action. Let us try and see how we can do it. If you, if you look at what we did earlier, we said delta phi is equal to d phi d n n hat dot d l. So in a certain sense, what this is saying can be written out. This piece, I will keep d phi d n n hat dot d x along x plus d y along y plus d z along z.

So this quantity has an x component, it has a y component and a z component. So it is equal to d phi d n along x d x plus d phi d n along y d y plus d phi d n along z d z; the component along x, the component along y, the component along z. But we also know that this same quantity is equal to minus E dot d l. So the same equation can also be written as minus E x d x plus minus E y d y plus minus E z d z, which means that E x is nothing but the x component of this normal derivative, E y is nothing but the y component of this normal derivative and E z is nothing but the z component of this normal derivative.

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$$\phi(r) = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r^2} \right)$$

$$E = -\frac{d\phi}{dn}$$

$$\Delta\phi = \left(\frac{d\phi}{dn} \right) \cdot dn = E \cdot dn$$

$$= \left(\frac{d\phi}{dx} \right) dx + \left(\frac{d\phi}{dy} \right) dy + \left(\frac{d\phi}{dz} \right) dz$$

$$= (-E_x) dx + (-E_y) dy + (-E_z) dz$$

Now, if you know your partial derivatives, you know that if we have any function f of x , y , z , then you know that df is equal to $\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$. This is the standard result in mathematics and here we have $d\phi$ is equal to $\frac{d\phi}{dn} dn$ along x dx plus $\frac{d\phi}{dn}$ along y dy plus $\frac{d\phi}{dn}$ along z dz . So, if you just look at these two, it becomes obvious how to calculate this $\frac{d\phi}{dn}$.

So, the...so I have three equations. Let me write them all here. This $d\phi$ is equal to this. It is also equal to $\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz$. It is also equal to $-E_x dx + -E_y dy + -E_z dz$; and they are all equal to each other, for any dx any dy any dz ; which means these coefficients are saying the same thing, these coefficients are saying the same thing and these coefficients are saying the same thing. So I can write down E_x is equal to $-\frac{\partial \phi}{\partial x}$, E_y equals $-\frac{\partial \phi}{\partial y}$ and E_z equals $-\frac{\partial \phi}{\partial z}$.

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$$\begin{aligned} f(x, y, z) \\ df &= \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz \\ d\phi &= \left(\frac{d\phi}{dx}\right) dx + \left(\frac{d\phi}{dy}\right) dy + \left(\frac{d\phi}{dz}\right) dz \\ &= \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz \\ &= (-E_x) dx + (-E_y) dy + (-E_z) dz \\ E_x &= -\frac{\partial \phi}{\partial x}, E_y = -\frac{\partial \phi}{\partial y}, E_z = -\frac{\partial \phi}{\partial z} \end{aligned}$$

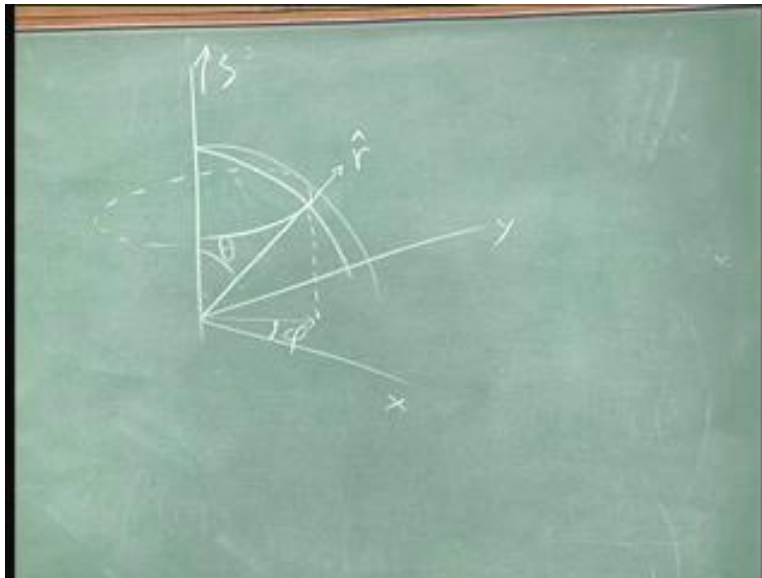
This procedure of extracting three values out of one function is given a name. It is called the gradient and it is also given a symbol. So we write, the electric field is equal to minus inverted triangle phi and when we write this, we mean this. We mean that the inverted triangle phi is a vector whose x component is del phi del x, whose y component is del phi del y, whose z component is del phi del z. So minus of that puts in a minus sign for each of these.

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$$\begin{aligned} f(x, y, z) \\ df &= \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz \\ d\phi &= \left(\frac{d\phi}{dx}\right) dx + \left(\frac{d\phi}{dy}\right) dy + \left(\frac{d\phi}{dz}\right) dz \\ &= \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz \\ &= (-E_x) dx + (-E_y) dy + (-E_z) dz \\ E_x &= -\frac{\partial \phi}{\partial x}, E_y = -\frac{\partial \phi}{\partial y}, E_z = -\frac{\partial \phi}{\partial z} \\ \text{Gradient} \\ \underline{E} &= -\nabla \phi \end{aligned}$$

Now, this is in Cartesian coordinates. What happens if we do not stay in Cartesian coordinates? Up to now, we have worked only with x , y , z , but it turns out that there are many many interesting uses of cylindrical and spherical coordinates. So, let us look at a case where you want to go away. Let us say we have spherical polar coordinates. Spherical polar coordinates that I remember talking about a little while back, you have the radial direction, you have a theta direction - that is the angle with the z axis, and you have a cone that is traced out by this vector if you rotate it around the z axis. That point...if you take this point and project it on to the x y plane, you get a vector in the x y plane. This is x , this is y . There is an angle here. This angle is called phi.

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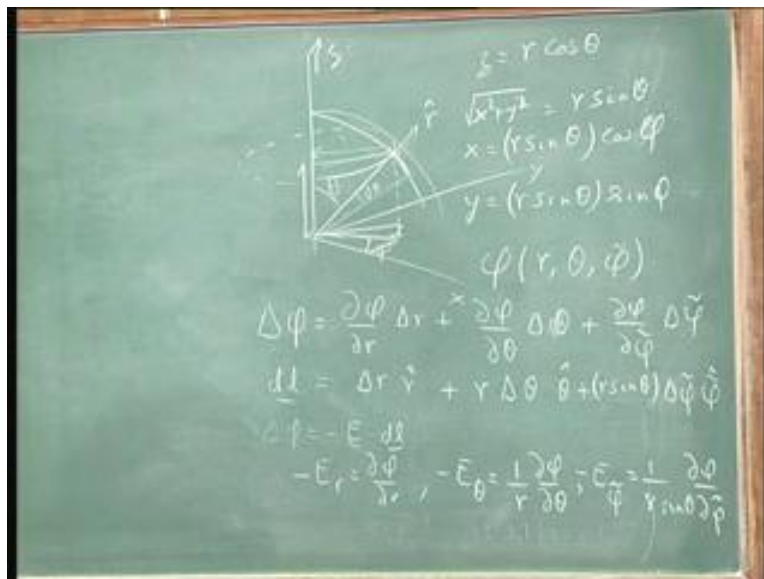


So you have z is equal to $r \cos \theta$, sorry. That is, it is the...this height; square root of $x^2 + y^2$ is equal to $r \sin \theta$. That is this length. The x coordinate itself would become $r \sin \theta \cos \phi$, because this length is $r \sin \theta$ and you have to multiply that $r \sin \theta$ to get... $\cos \phi$ to get the distance along x . Similarly, y is equal to $r \sin \theta \sin \phi$. This is an exceptionally useful coordinate system. You should know all about it; but in this particular case, what I am interested in finding out is if I wrote the potential ϕ as a function of r , θ and ϕ , what can I get if I want to get the gradient. Well, the answer is that I want to know what

delta phi is. Delta phi is del phi del r delta r plus del phi del theta delta theta plus del phi del... Well, I should use a different symbol, delta phi.

So this is nothing but...if I have a function of three variables, the change in the function is given by this expression. But I also want to know what is d l. d l is delta r along the r direction plus r delta theta along the theta direction. So along the r direction I just go; but along the theta direction if I increase in theta by an amount delta theta, this distance is r delta theta and then it is equal to r sine theta delta phi along the phi direction. I have put a twiddle over the phis to separate it out from the electric potential. Where did this r sine theta come from? Well, the projection on to the x y plane r sine theta and if I change phi, this distance is given by r sine theta delta phi. Now, if you look, I want to say delta phi is equal to minus E dot d l. So, I have to read off...this is the d l part, this is the delta phi. So, what is...what are the components of E? So, minus E r is del phi del r; minus E theta is equal to 1 over r del phi del theta and minus E phi is equal to 1 over r sine theta del phi del third angle.

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I will come back to this the next class but this will allow us to calculate the electric field out of a dipole. Thank you.