Electro Magnetic Field

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Lecture - 42
Transmission Lines (continued) & Conclusion

This will be the last lecture in this course and I want to complete what I was saying about how electromagnetic leads us to transmission line theory. And then make the survey of what topics we have covered and what topic we have left out which you should cover on your own. So, the last lecture I had started with wave is plane waves and talked about what happens if you take a plane wave and put it into a pipe. And further I talked about a special kind of wave inside a pipe which still look like a plane wave namely I have some cable.

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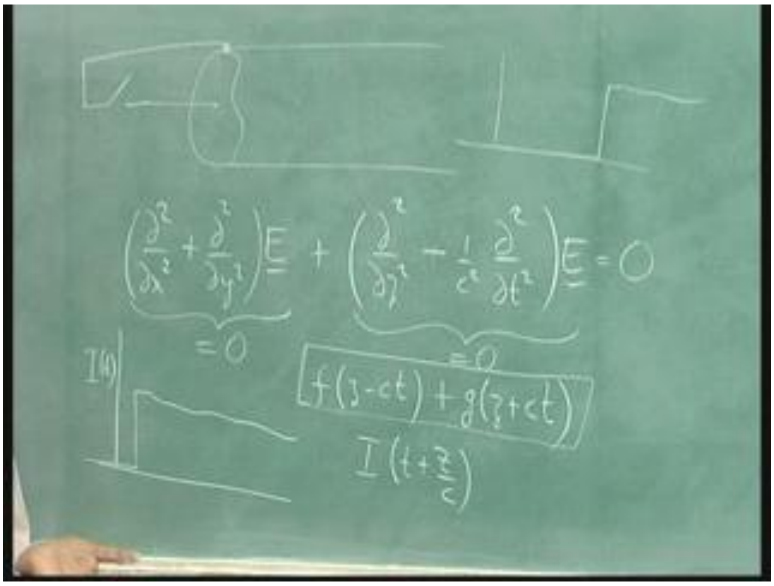

And I look for a wave such that the x and y variations and the z and t variations the couple that is to say this portion by itself is equal to 0. And this portion by itself is also equal to 0 if we could do this. Then there are several benefits one of the benefits is its back to your 1 d plane wave problem. And we have already seen that the 1 d plane wave problem has solution of the form f of z minus ct plus g of z plus ct. And this is tremendously important, because supposing I was using this in a practical circuit. And I have a switch whereas, just closed only goes to the middle the other leads goes to the outside conductor of my cable when this switch closed current went from 0 to some value and then continue to evolve.

So; there was a jump in current this jump of course, limited by the inductance of the system here and parasitic capacitance and things like that now what is receive at the other end? This expression tells us what is receive at the other end, because if I have a signal that looks like I of t. Then what is receive at the other end is I of t plus z over c, because the signal arrives later naturally it arrive later at the destination then it left. And it must be of this form which means that even at the destination the signal was 0 up to later time then went up and they whatever it they. So, it is it is the fact that we have solutions of this type that make for a very accurate transfer of signals through a cable. This is why when we buy expansive oscilloscopes you will notice that a lot of money has to be paid for those connectors.

The high quality connectors are not cheap that is, because they have to obey this kind of behavior. What is called distortion free carrying of a signal? They have to do that for very high frequencies even if the raise time is of the order of nano seconds this kind of behavior should happen. So, that is why you will see that an oscilloscope has a megahertz rating up to that frequency the oscilloscope can give you signal faithfully. And they will give you cables that you are supposed to use with the oscilloscope which in fact, also carry this information faithfully up to that frequency. So, it is very important that we be able to get plane waves in a pipe that is what it is this is the plane wave 1 dimensional wave. And I want to send this 1 dimensional wave through a pipe the only way I can do it is I have I can solve this problem. And thus we say last time the way to solve this problem is to look for the solution of this type.

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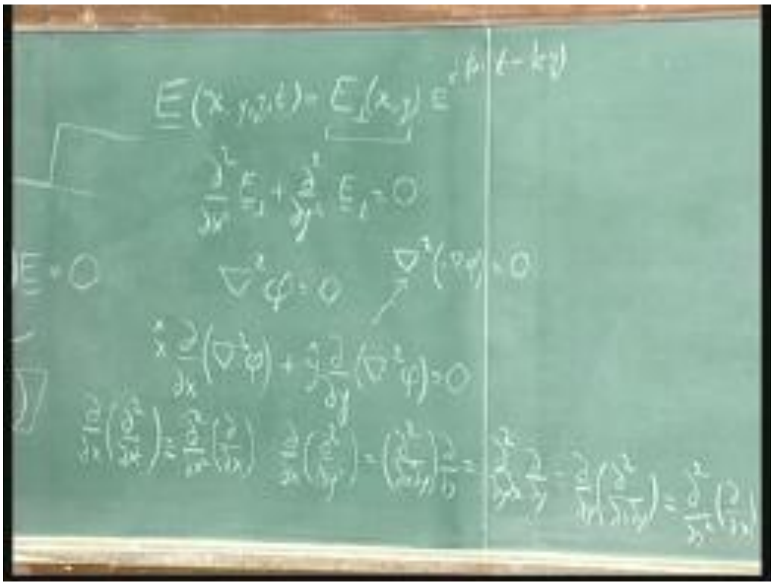

The electric field which is the function of x y z and t is some perpendicular function is a function of only x and y into e to the power of j omega t minus kz, of course you can also be j omega t plus kz. Now, what do we get from this? This you can do any way this is just courier transforming in omega and k you may not let have learned about networks and systems. But if you have an equation whose boundary conditions are nice. And which do not have coefficient that depends on z and t. You can always go to another way of representing quantities which is by representing them as complex exponentials, but in addition. We make one more requirement since, we looking for this term being 0. That amounts to saying del squared del x squared of E perp plus del squared del y squared of E perp equals 0 this is like a 2 d poison except it is a equation in E perp.

And not in pi the poison equation you have seen is Del squared pi is equal to 0 that is lap losses equations, but this is lap losses equation for electric field and not for potential. The wave that we get round this is we say that if this is true then let us calculate what taking the gradient of this equation is. So, I will get Del del x along the x direction of del squared pi plus along. The y direction Del del y of del squared pi must also be 0, because del squared pi is 0 this must also be 0. But these del squared is nothing, but second derivative in x plus second derivative in y and this is derivative with respect to x. So, it is always true that del del x of del squared del x squared is the same thing as del squared del x squared of del del x. This derivative commute similarly del del x of del squared del y squared can be written as del squared del x del y acting on del del y.

So, second partial derivatives they commute, so I can write this as Del del del squared del y del x acting on del del y. But now I take it in the other way Del del y of Del squared del x del y do the same thing I did here. And I get Del squared Del y squared of Del del x, so I can push this Del del x right through to the inside which means that if I take this expression I can go from this expression to del squared of minus grade 5 is equal to 0 which is nothing but this expression. So, what all this means is that if I want to solve Del squared of E perp is equal to 0 all I have to do is solve lap losses equation for potential. And I can automatically get solution for electric field we did this last time. And obtain what is called tem mode. At the end of last lecture what I did also was to proceed to the circuit formulation of the transmission line. And I want to spend the little more time there because it is a fairly shuttle thing supposing.

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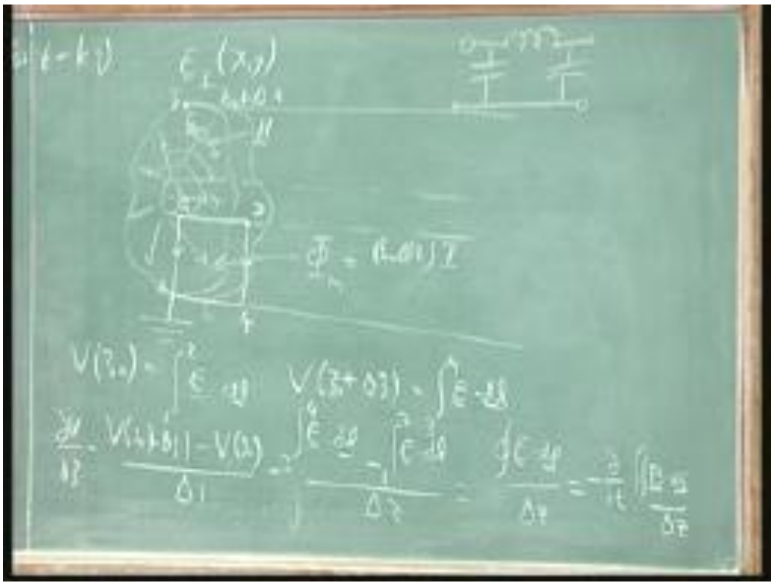

I have the cable and this cable is a bnc cable, so it is has an inner core and an outer shield and let us say that the outer shield is grounded now, I applied this approach to it. So, I believe that my electric field E E perp is a function of x and y only. I can solve for capacitance for this is a correct approximation my electric field will be mostly radial. And if I want to solve for the magnetic field well there is a current through this central wire. So, the magnetic field will be in the theta direction and therefore, E cross H which is radial cross theta is in that z directions. So, it is a consistence picture we have now, the question is can I go from this picture to my transmission line picture which is this picture. So, what I did last time was I said consider a 0.1 and a 0. 2 consider another 0.3

and a 0.4 1 and 2 are at the same z collect z naught 3 and 4 are at z 1 are z naught plus delta z.

So, I did an integral I want to find the voltage here V of z naught is integral 1 to 2 E dot dl that is, because electric field is derived from a scalar potential V of z naught plus delta z is equal to integral 3 to 4 E dot dl. So, there are 2 line integral I can do and these 2 line integrals give me the voltages, but these 2 voltages are not the same. And I like to know the derivative in voltage that is Del v Del z is equal to v of z naught z not plus delta z minus V of z naught divided by delta z as delta z goes to 0. That is the meaning of derivative, take the difference of voltage and divide by the difference in position and take the difference to 0. But that is equal to integral 3 to 4 E dot dl minus integral 1 to 2 E dot dl over delta z that is I have to go 3 4. And then I have to go 2 1 and I can add this 2 segments because electric field any way is radial, so E dot dl is 0 on those 2 legs.

So, I can write this as loop integral E dot dl over delta z now I did my strokes theorem applied faradays law. And what I got was this is equal to minus Del del t of surface integral B dot ds over delta z. We just faradays law now, the B dot a, is magnetic flex through this rectangle. And the magnetic flex through this rectangle is clearly proportional to delta z. Therefore, delta z is going to cancel out how much flex is there pi magnetic if equal to L which is the inductance per meter times dz times I magnetic flex is Li. But in this case the inductance is the distributed inductance, it is not a inductance that sitting in a 1 points. So, the amount of inductance between the z and z plus dz is L dz.

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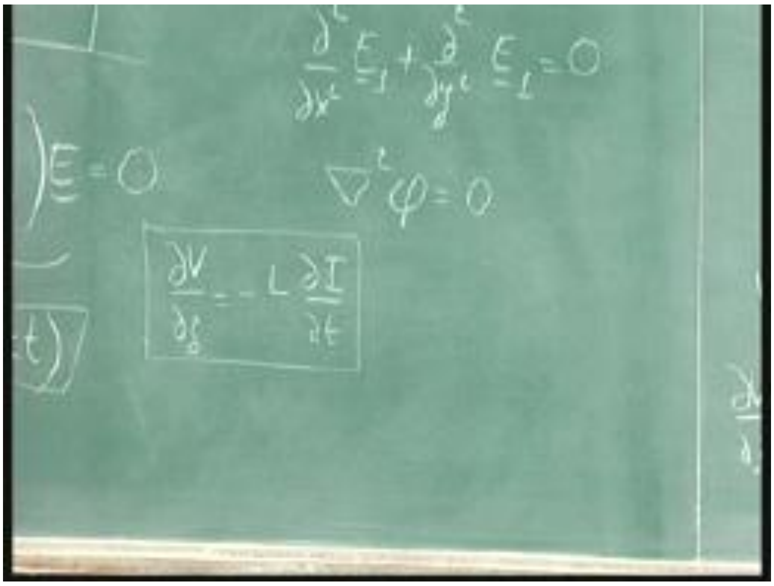

So, I should write this as delta z, so the delta z cancels out and what we have left with is the first equation, which is Del v Del z is equal to minus L del I del t, so this is the first transmission line equation. And it comes from looking at strokes theorem applied to this rectangle now, for the second equation we have to be little more careful.

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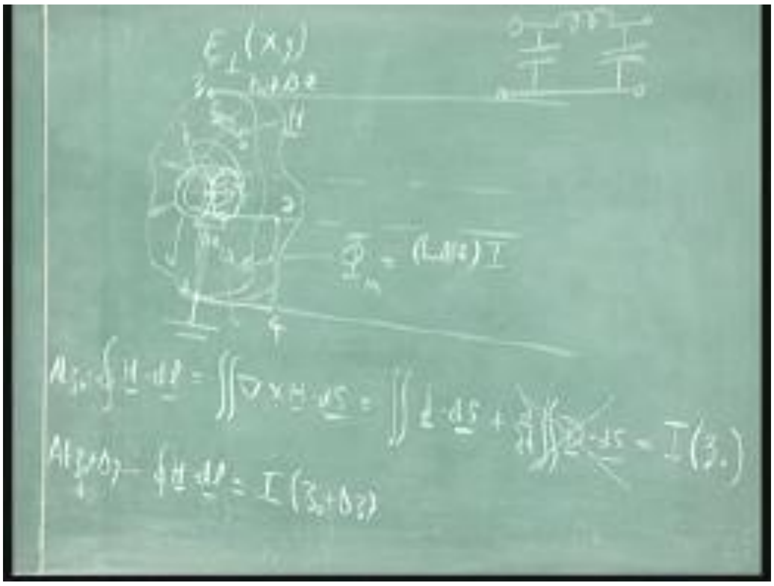

Now, what I want is to look at a loop going round the inner core looping it in the positive sense if I do H dot dl there integral H dot dl is therefore, equal to surface integral curl of H dot ds strokes theorem. So, this is curl of H on this surface this surface is in the x y plane, because it is all at z equal to z naught let me apply Maxwell equation here. It is equal to surface integral j dot ds plus Del del t of surface integral D dot ds. Now, whatever electric field I have is in the x and y direction there is no z electric field which means D dot ds is 0. And integral j dot ds is correct, so it is I if I do this same calculation at the at x plus delta z again in the same direction. So, this is at z naught at z naught plus delta z loop integral H dot dl is again equal to I now this current is current at z naught this current is current at z naught plus delta z let us work here itself. So, that we have this equation in place what I am going to do is I am going to make a surface out of this I have 2 line integrals. But, now I am going to take this surface the surface that connect the first line integral. And the second line integral the surface integral of D dot ds the displacement vector.

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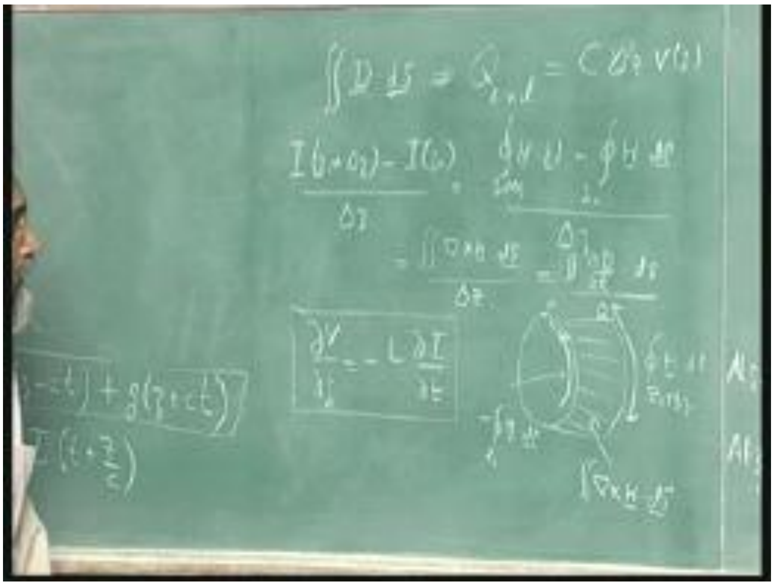

Now, this surface is just outside the conductors, so that is no j, so this D dot ds is directly equal to charge enclosed that is clear, because divergence D is equal to row. So, if I apply gausses law that gives me that this surface encloses a certain amount of charge how much charge does it enclose that charge is nothing but CV. But this capacitance and the charge enclose clearly depends on delta z. So, I can write this a C delta z times v of z. So, this is the displacement vector that I if I integrated on this surface this surface that I have showed note that there is no displacement vector d dot ds on this surface or on the other surface. Now, what I am going to do is I am going to take construct Del I del z. Because I already have this 2 results I of z naught plus delta z minus I of z naught divided by delta z is equal to loop integral at z naught plus delta z of H dot dl minus loop integral at z naught of H dot dl divided by delta z let just from this 2 derivations. So, I am saying that this loop integral of H dot dl contains the current enclosed. But if I look at what this loop is if I look at a ring like this. And I ask what is the boundary of the surface which I have mark with a horizontal lines? That surface has a boundary which corresponds to going on that on that edge and going on this edge in opposite senses.

Because I if you like the way to think about it is supposing I break this and make ring. That is not a loop when the edge of the surface is going to be going round this inner surface going radically out coming back down. And then coming back on the return path if I join this 2 these 2 parts of H dot dl will go away. Therefore, for a complete surface this surface integral of curl of H dot ds is nothing. But loop integral of H dot dl as z naught plus delta z minus loop integral H dot dl at z naught. So, in other words this piece can be written as surface integral curl of H dot ds divided by delta z. This surface is the surface I shaded as horizontal lines maxwells equations is there, now there is no conduction current all, the conduct ant current is inside the wire. So, this must be equal to surface integral of del D del t dot ds divided by delta z I can pull the del del t out. And you can see that I already have D dot ds is C delta z v of z. So, what do we get there is a sin problem that will be there let me draw the ring again.

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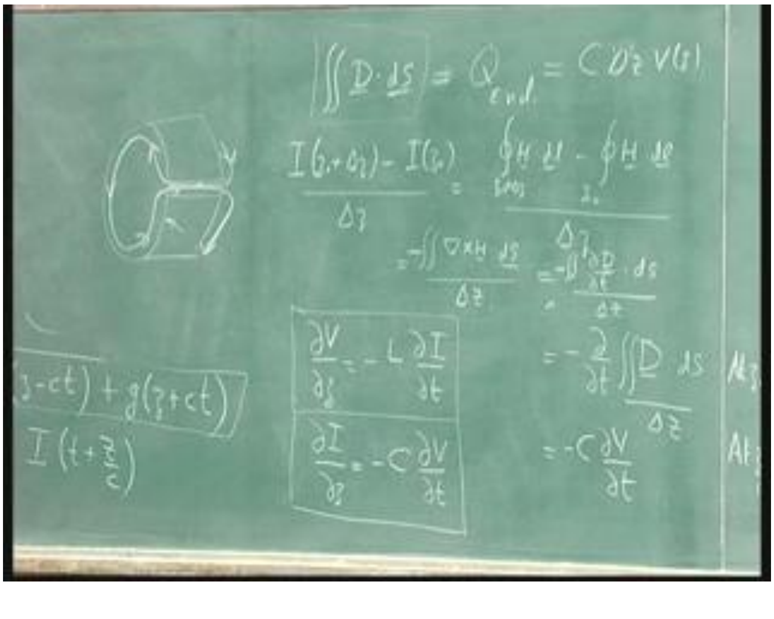

I am going to break the ring here. So, I have a loop that goes this way if I have such a loop the when I do the right hand rule I am talking about flex entering. This surface where as what I count by D dot ds is the flex leaving. The surface therefore, there is a minus sign when I connect up curl when I connect up the loop integral H dot dl and curl of H dot ds. So, what I have here H dot dl z plus delta z minus H dot dl at z is actually minus of curl of H which is minus of del D del t dot ds you can see this. Because if you go around you have use your right hand rule you find that you are talking about invert pointing flex of curl of H. But invert point what we need, because D dot ds is talking about outward pointing flex of D. So, what you get is therefore, minus Del del t of surface integral D dot ds divided by delta z I.

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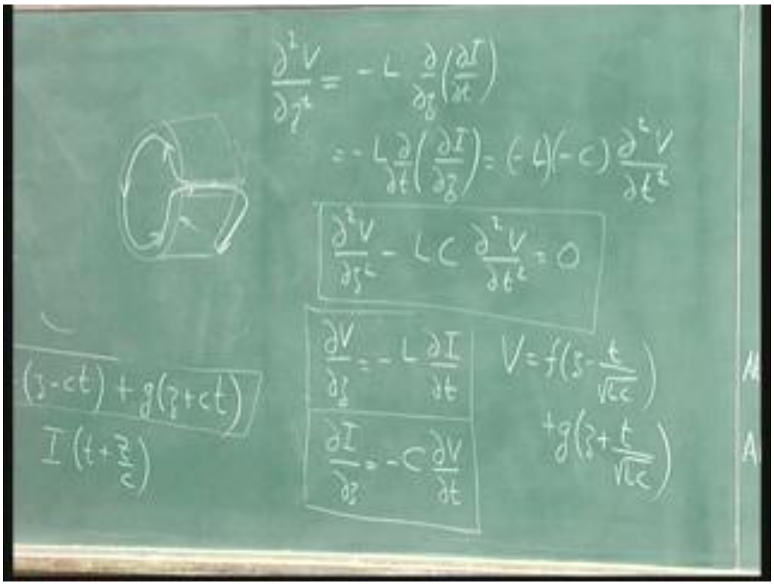

Now, apply this, the delta z cancels out, so that gives me minus C del v del t, so that gives me the second equations which is del I del z is equal to minus C del V del t. It is important to realize we have used a lot of the ideas in tem waves. But instance when I did this integral I said dz is 0, otherwise when I did the integral at 1 particular z not. And I got curl of H is equal to I, I could not have drop this D dot ds similarly when I do the E is equal to the minus grade pi I could not have done that unless dz is 0. So, all of this, idea has been used, but at the end of it we get 2 equation which are purely circuit equations.

And these circuit equations are what we use when you want to talk about transmission lines I now take the z derivative of this equation. So, what do I get del squared v plus del z squared minus is equal to minus L del del z of del I del t I can interchange z and t. So, I get minus L del del t of del I del z apply this equation I get minus L times minus C times del squared V del t squared. So, we get the final transmission equation line which is del squared V del z squared minus LC del squared v del t squared is equal to 0. And this is where the electro magnetic is leading up to, because you can see the amount of work that is actually hiding behind such a simple equation.

We are no longer talking about the electric fields we are no longer talking about the magnetic fields. In fact, we say magnetic field is nothing more than current electric field is nothing more than voltage yet. The final equation is deceptively simple this equation has solution which say V is again equal to f of z minus vt where v is 1 over square root LC plus g z plus t over square root LC. And, so once again any signal send on such a transmission line will go through without being distorted. Now, of course, any real transmission line is not ideal, so where are the non ideality is of a transmission line 1 non ideality you have is that this medium.

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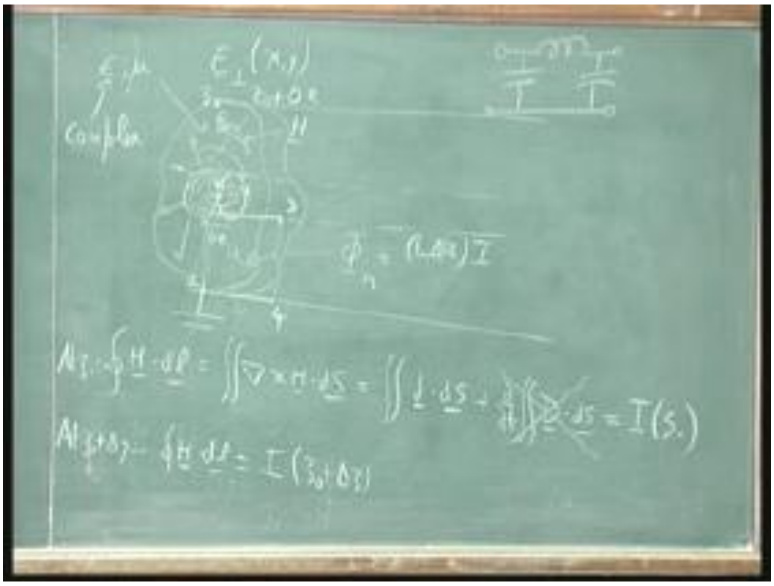

Which is characterized by epsilon and mu may be lossy, so epsilon may be complex the epsilon is complex we know that you will start having a leakage current. So, the equivalent circuit will not just be a capacitance they will also be a resistor in parallel to the capacitor you will start having a resistive current between the inner wire and the outer wire.

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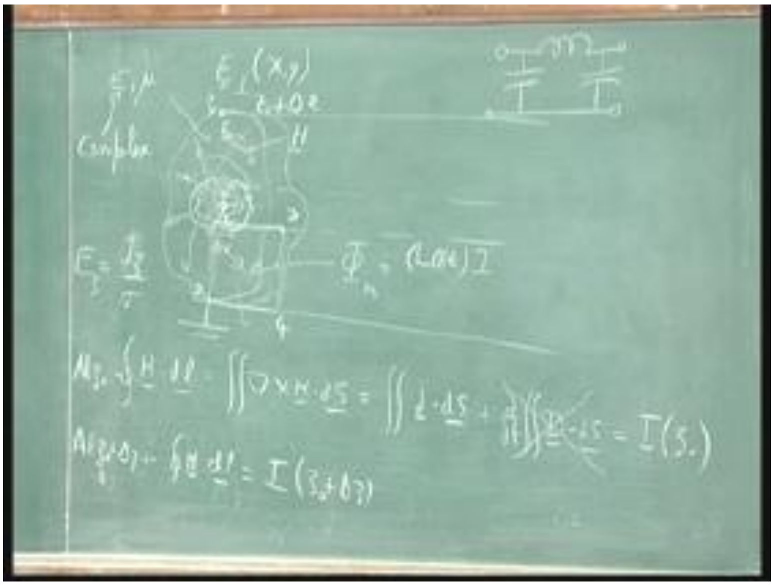

So, in place of just having capacitors each of which value is c over to Dz you will also have a sun resistor. The second kind of loss you have is in the wall itself, because you having currents this currents of course, traveling on the very edge of this wires. Because of skin effect and has the flow you; obviously, have an electric field you have an Ez which is equal to I or j if you like jz divided by sigma where jz is equal to iz divided by the volume over which the current is flowing. But this is violating whatever we said about these waves this waves have only x and y directions.

But we require a z direction electric field, so it means that in an truth tem wave is not possible. The moment you are wire has finite conductivity if it is a super conductor we can manage. But, if it is not a super conductor then we can not have such an electric field. We must have such an electric field we can not have it in this kind of formulation. So, what happen is that we must adjust for this formulation we must take a little bit of a electric field in the z direction most of the electric field in the x and y direction. And see how much that distort our picture of this plane wave in a pipe? It can be done it has been done of course.

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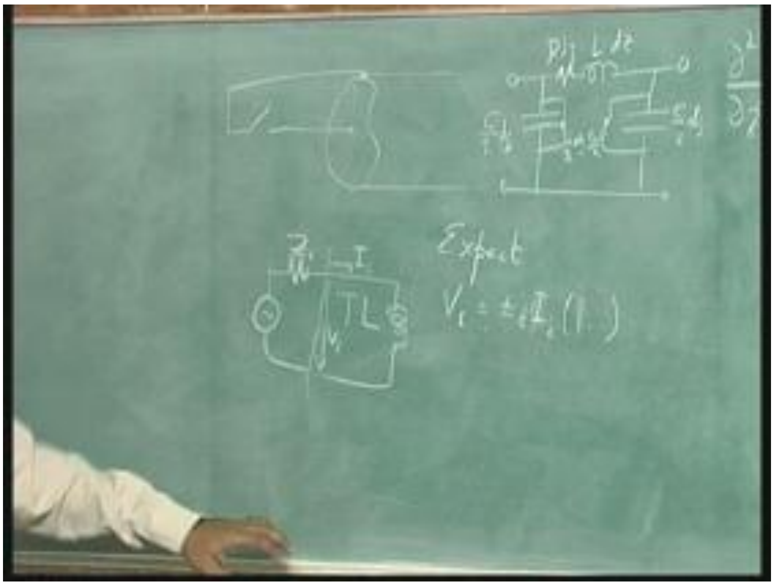

And what you get is that we add a very small series resistor to the inductance, so you have L dz and you have a little bit of resistance. So, these are called the sunk impedance is co admittance is called g series resistance is called r series impedance is called L series inductance service is called L. And the capacitance is called C these are standard symbols that are used. But this is the kind of working model equivalent circuit that describe what is actually happening inside such a cable once. We put these circuit elements in more or less you can through away electromagnetic this equivalent circuits. Now, contains all the things we need of it, but there are something you must bad in mind here 1 another thing you must remember is that if you look at a picture like this. And if you know that the equivalent circuit is dominated by inductors and capacitors.

Now, what I would expect is that I would have an ac voltage source forget the switch for the moment I have my internal zi. And I have this transmission line I would certainly expect that given that is mainly inductors and capacitors. But, this can best model as an effective induct ant or an effective capacitor depending on which ever 1 is more important at some frequencies. It may look like an inductor at some frequency it may look like a capacitor which would imply that the voltage across. The input leads would either lead or lag the current by 90 degrees, but you expect time some impedance you expect that the either. The voltage leads or either the voltage lags the current, but this is not the case in fact, we go back to this picture.

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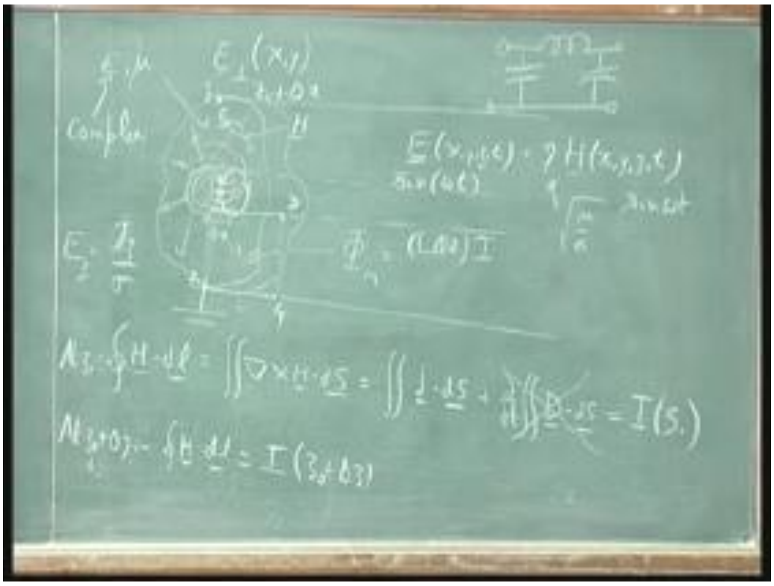

We know that the electric field which is the function of x y z and t is equal to eta times the magnetic fields eta is nothing. But square root of mu over epsilon what; that means, is that wherever the electric field is if this electric field is says sin omega t in dependence in time at a particular z. This magnetic field is also sin omega t it is not cos omega t, so the signals are in face inside a wave the electric field. And the magnetic field, are in face in an inductor or a capacitor circuit the electric field. And the voltage and the current ere out of face and this is a big difference which means that if you look if you use a transmission line you will not get this.

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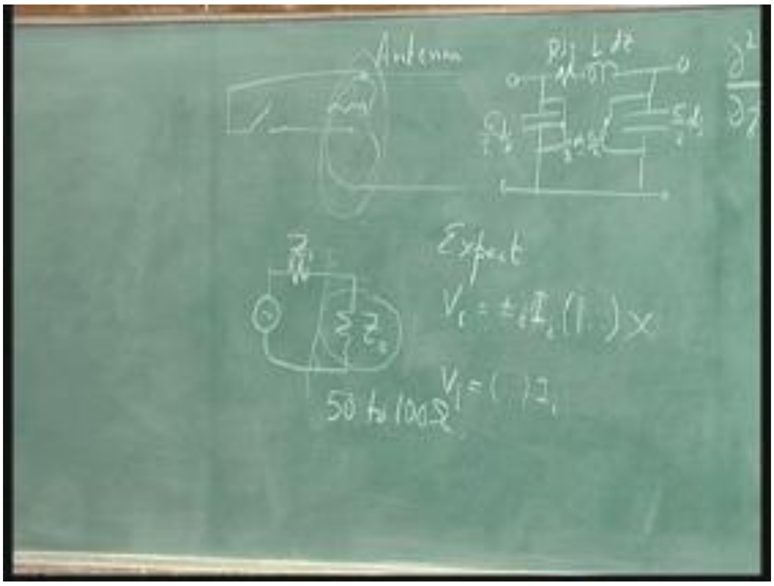

You will instead get VI is equal to sum impedance time Ii that is the voltage and the current will be in face. And this transmission line will look like a resistor and the theory of transmission lines starts with this namely that there is characteristic impedance to the transmission lines. It is the real number which means that it looks like a resistance and your source must drive this characteristic impedance of a transmission line. This is a very fundamental difference from circuit theory. In circuit theory when we see inductors and capacitors we expect immediately that voltage leads or voltage lags. The current that is not what you should expect our transmission line why is this difference there.

The best way of looking at a transmission line are any wave going through a pipe a wave guide is that you should thing of this lead connecting to the leads of the transmission line consider. This part as antennae you are delivering radiation into this pipe and you know that when we discussed what happens in an antennae. There is energy going away whatever energy coming is going through this transmission line. And eventually landing up in a load for away and therefore, if energy is leaving us thus there is one way. The energy can leave if there is a voltage and current in face if voltage. And current are exactly out of face what it means is that they wont be any real power delivered at all it will all be reactive power.

But I am delivering power through a transmission line if I send a spike that spike goes to the, from my computer to the next computer and vanish it which means energy went from where I send it to where it is needed. The energy must be send I cannot have reactive network I must have a resistive network. So, this z naught must be thought of as a kind of radiation resistance of my transmission line typical numbers of z naught are 50 to 100 ohms a blc cable has 50 ohms a twisted pair telephone cable has 100 ohms. So, this is the range of numbers that you will see the rest of this theory properly belong to a transmission line scopes. I do not wont to go further into it what I want to establish is this picture of being able to model a transmission line by inductors.

And capacitors actually have behind it the entire theory that we have study this course during this course. It contains electrostatics in order to establish the electric field it contains gausses law in order to calculate the connection between the electric flex, and the charge enclosed. It contains amperes law in it original static form when you want to link up loop integral H dot dl to current enclosed. It involves the Faradays law when you wanted to calculate the first of the 2 transmission line equations. And it involves the generalized amperes law when you wanted to calculate the second equation which got as here.

So, all of electro magnetic theory got used in this simple derivation let got as from fields inside a pipe to circuit equations it s all there. And in a certain sense you can understand how you went from a cable to these equations you have more or less understood. All that I wanted to teach you in this course that more or less bring to an end of this course I want to discuss for a few minutes what are the topics that I should have covered, which I have not, which you would benefit from reading further on your own. So, let us put down what we have covered and what wholes are left.

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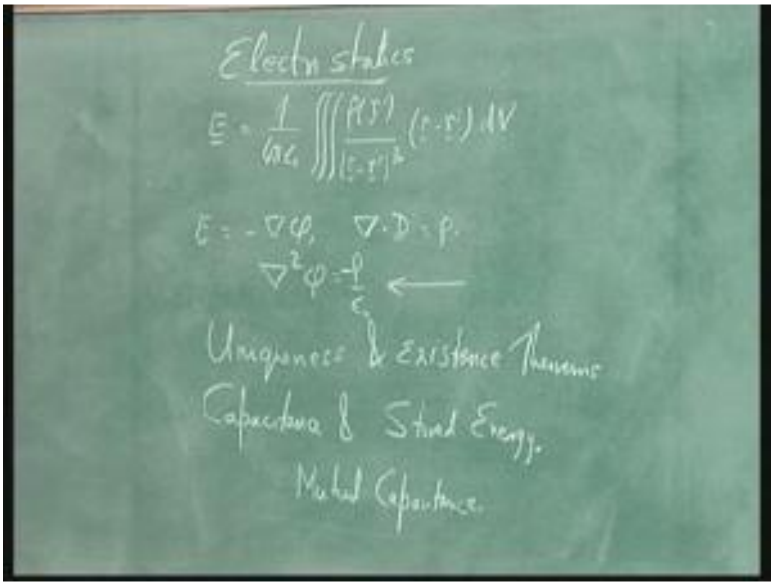

As far as electrostatics is concerned we adequately cover the first part which is to established that the electric field is coming out of charges. So, 1 over 4 pi epsilon not volume integral row of r prime minus r prime cube into r minus r prime. And we got from it that the electric field is equal to minus grade pi. And that divergence of D is equal to row at the end of this section we derived poisons equations. Now, we did not spend nearly enough time on this equation I just gave 1 or 2 examples where I solve this problem this equation. And worked out cases of capacitance with multiple dielectrics and 1 case where I had a cylindrical cavity.

And used this equation to solve for potential this is one area where I strongly urge you to go back to the text book. And look more carefully at the solution that you can get some more points inside this area are the uniqueness theorem this is the formal theorem. So, I have not really covered them because I do not think that I would do anything better than what the text book does already. But they are the foundations for doing the rest of electromagnetic theory. So, you should certainly read up on how it is that you can show that if I get a solution for potential in a region that solution is the only solution. That can be got that is it is a very important statement and its something that you should read up on your own.

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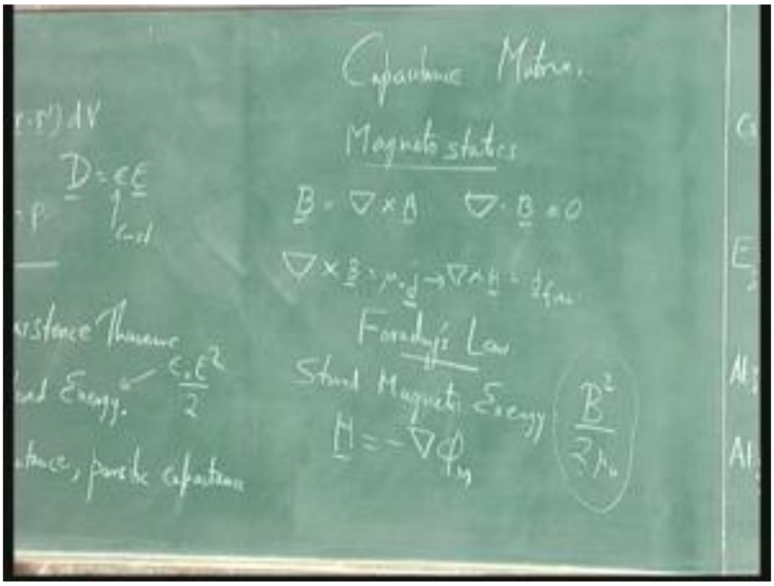

When we came to magneto static or thus one more point in the electrostatic which I did not do sufficiently I talked about capacitance and stored energy. However, I did not talk about mutual capacitance this is perhaps not very important for a power electronics. And triple e in general, because it really comes up more when you are doing micro electronics vlsi rf boards. Things like that where what happens is that you have a capacitor you have a device.

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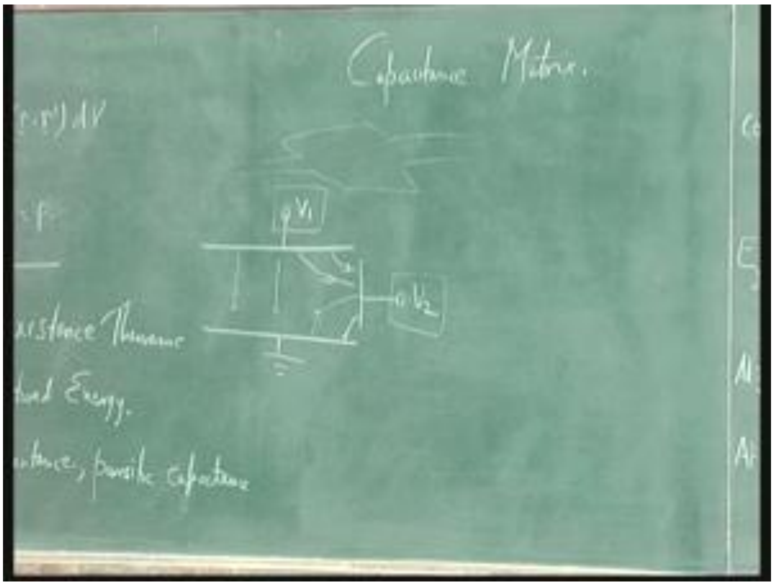

And the device has a capacitor somewhere and the capacitance occurs not only between say the junctions of your transistor, but also with the sub strict. So, there is something called parasitic capacitance and it has the strong effect on the frequency response of many devices. So, parasitic capacitance are mutual capacitances come from the possibility that electric flex lines do not always go from 1 plate to another it is possible. That you may have multiple plates may be I have a plate held at voltage V 1 another plate held at voltage V 2 in the third plate is grounded in that case instead all the flex lines going between V 1. And ground I will have some flex line going from V 2 and ground. And some flex line perhaps going from V 1 and V 2 in such a system I cannot define a single capacitance for one thing it is a 3 port device or 3 node device it is not a 2 node device. So, I cannot define a capacitance between 2 nodes, but what I can say is if I said V 2 to 0 what is capacitance between V 1. And ground if I said V 1 to 0 what is the capacitance between V 2 and ground. And if you built up such pair wise capacitances you can talk about the capacitance matrix.

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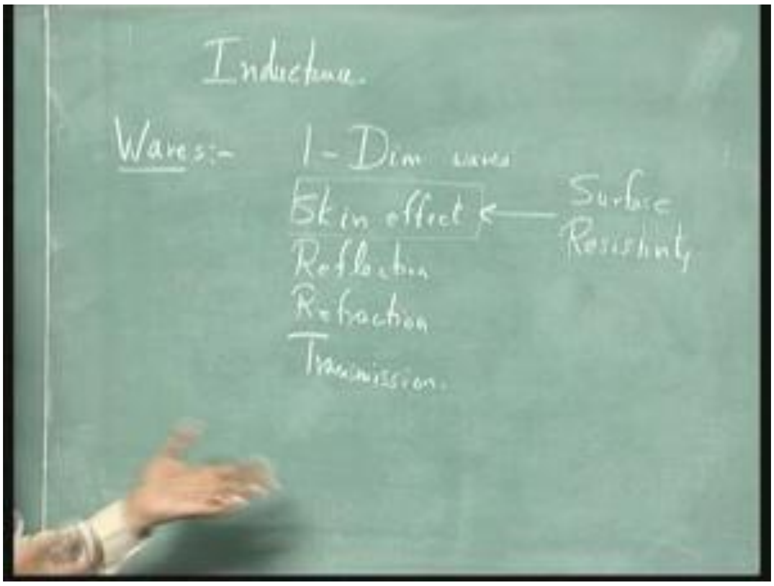

This is very similar to what we call the inductance matrix which we did do in a when we talked about magnetic fields when we came to magneto statics a lot of the materials was the same. So, we define magnetic field we defined the vector potential we said B is equal to curl of A, because divergence of B was 0. And divergence of B was 0 because essential B is driven by currents electric currents there is no such thing as magnetic charge. And then we came up with the amperes law and again discussion about materials. We generalize this to curl of H is equal to j free now really there was an enough discussion about the relationship between H and D whereas. The relationship between D and E is rather simple D is just epsilon E where epsilon is a constant. The relationship between B and H is quite complex, so B is a non-linear function of H not only that it depends on the history of how H changed.

So, it is it is you have historices, you have non-linearity you have everything inside these 2 equations. And most of the complication of magnetic systems comes here and I skipped all of that. So, I am hopping that that complication will be covered better in a, courses which you will do in connection with machines and transforms. We were not able to talk about magnetic stored energy till we introduced time dependence. So, we introduced faradays law and when we introduced faradays law we where able to talk about stored energy in the magnetic energy stored energy in the electric field. We showed was epsilon not E squared over 2 and if it is a material this epsilon not is replaced by epsilon. So, we got stored magnetic energy which was B squared over 2 mu not as usually if there is material it is B squared over 2 mu. But, this expression is suspect whereas, this expression is a good expression, this one should not be trusted, because it assume that B is equal to mu H and mu is a constant.

So, it will work only for very simple system all interesting system will fail this expression there was one topic in here which I did not cover properly. And that had to do with current free regions where you can derive H from a magnetic scalar potential while this is while I mention this I did not solve any problem with that. And this again is something that you will use a lot in your machines courses in yours transformers courses. So, it does not require that I talk much about it, but you should be aware that this comes from the fact that if curl of H is j free. And if j free is 0 then again you derive a H from a scalar potential, once we had faradays law we where we will be able to talk about inductance. And I introduced inductance and mutual inductance, and so we talked about the inductance matrix.

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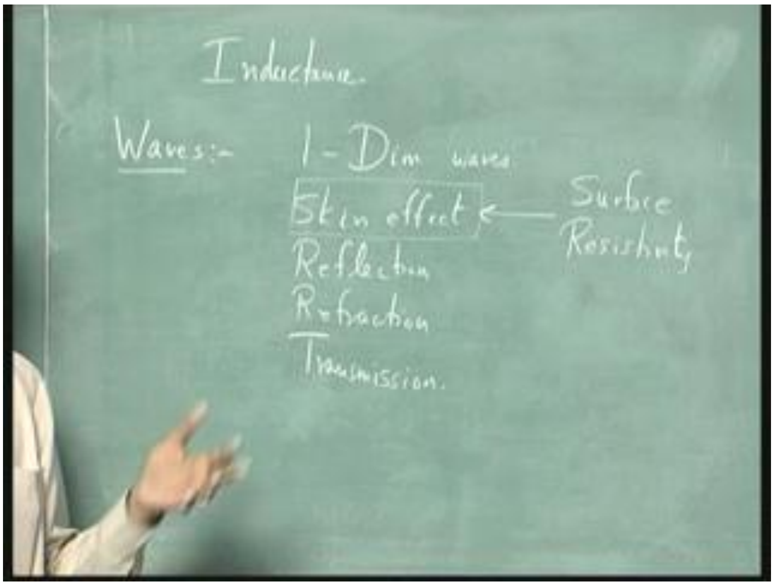

And I think that that coverage was fairly complete there are areas where you can do better. But again this is the core area of triple e I am sure it is get done in much better detail in other courses once we had completed that we came to waves now waves is a very large subject. And we basically touched the minimal part of waves we looked at 1 D waves and showed that they consisted of forward moving object and backward moving objects. And we looked at the properties of the 1 dimensional waves we look we got things like skin effect we got reflection refraction transmission etcetera. One topic I did not covered here was polarization it is a important complication of waves, but we will not cover it in this course. In the in skin effect there is an important concept by which I did not cover which needs to be covered.

And you should read out whatever your text books, says and something called surface resistivity. This is not something new it just saying if I have skin effect how can I take this skin effect and make talk about how resistors change the resistivity? And finally, using waves we went to transmission lines which is where we have been in the last few lectures. And transmission line is something you will spend a great deal of time doing because that is going to be your bread and butter. So, I must admit that there are, is only the sampling of the various topic that are there in electromagnetic theory. And each of the topics could have been done in the, for graded details. And what this course is suppose to taught you is not how to become an expert in electro magnetics, but to know what topics are there. So, that you can yourself study them in more detail if you go into the industry and you needed it. So, I hope you enjoyed the course and I will meet you in future courses sometime in later.

Thank you.