Electro Magnetic Field

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Lecture - 41
Transmission Lines

In this lecture, I want to take a look back at what we have done. And then make the final step from what we have been doing as wave theory to something more practical which is the subjects of the transmission lines. Now, let us look at where we started we started with statics.

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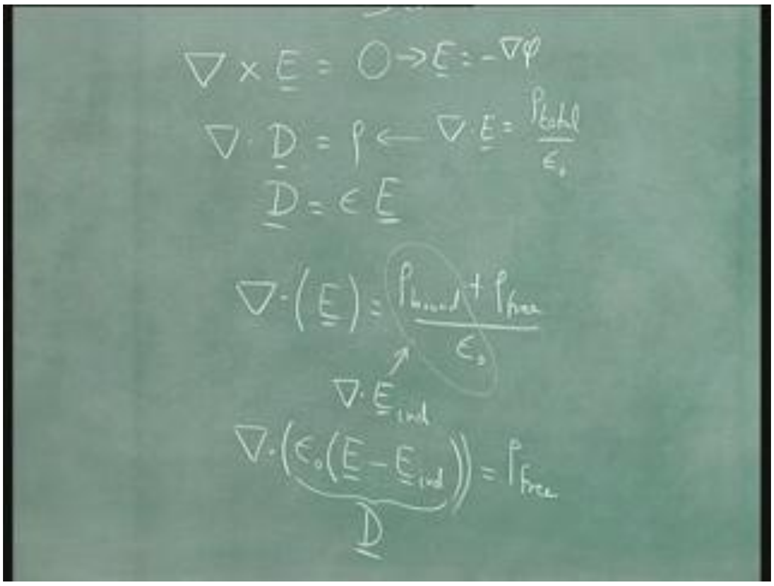

And we concluded that there where 2 equations which governed electric field 1 was curl of E was equal to 0. And the other was divergence of D was equal to row, because curl of E was equal to 0 this allowed as to say the electric field is derived from a potential. So, E was equal to minus grade pi. And we invented this concept of the displacement vector, because the any material has charges init which are bound to where they are to the atoms in which they belong. But in the presence of the electric field does not introduced polarization, and we work it out in this course that you can show that D is equal to epsilon E that is D is proportional to E.

So, the correct equation here is divergence of E is equal to row total over epsilon naught, but by introducing the concept of induced electric field. And saying that the induced electric field is proportional to the electric field itself we could write this as divergence of E is equal to row bound plus row free divided by epsilon naught. And we identified this row bound by epsilon naught this piece with the divergence of E induced. So, and you came up with the new equation which was divergence of epsilon naught times E minus E induced was equal to row free and that is this equation. So, this is nothing but D, so this was statics and the final achievement of statics was the poison equation which gave us that del squared pi was equal to minus row over epsilon naught.

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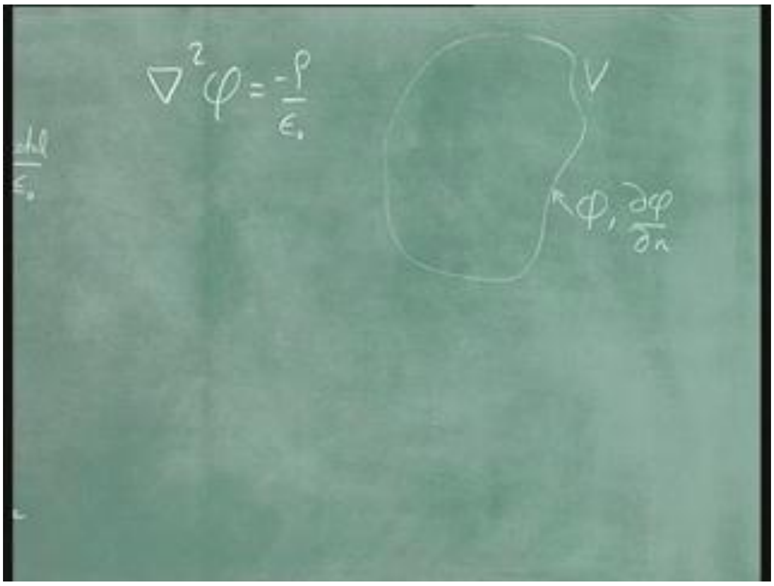

Now, that is something that I did not do here, because they were purely mathematical in nature. One of them was that you can prove that if you have any volume V and if on the surface of this volume on either any point specified either pi or del pi del n. Then the solution inside exist and is unique, so that means that this equation has is well defined and has unique solutions. So, that was statics that is electrostatics when you went to magneto statics and when you went to magneto statics the picture was exactly reversed.

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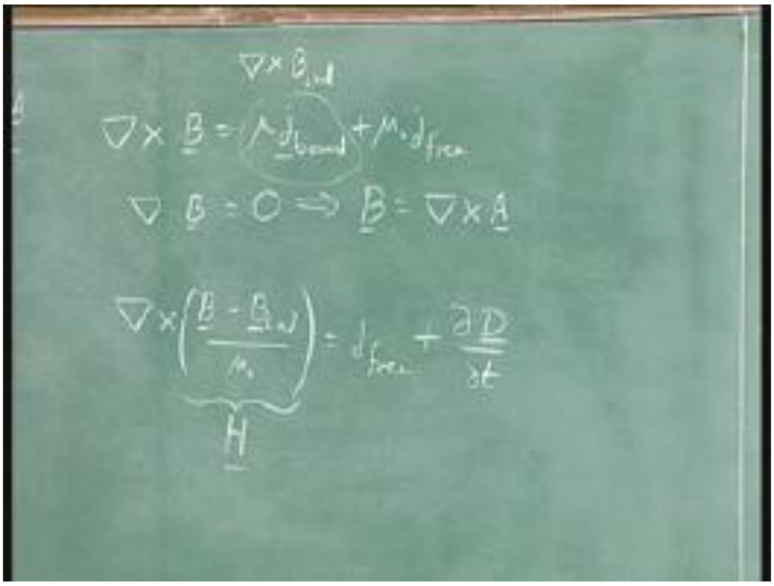

We found that we could write curl of B was equal to mu not j and we could write divergence of B was equal to 0 now, I discussed with you that the magnetic field is not a new field. In fact, if you go back and look through some of my older lectures the magnetic field is nothing more than the electric field when you apply the electivity to it. So, it is actually a side effect of the electric field. Now, you take this equation and once again apply it to materials this equations becomes mu not j bound plus mu not j free. And once again you can attach and implied field to this bound charge current. And you say that this is equal to curl of B induced which allowed as to say curl of B minus B induced divided by mu not was equal to j free. And we identified this as a new vector which we called H. So, we had 4 equations 1 was curl of E was 0 divergence of D was row curl of H is j divergence of D was 0. And these 4 equations took care of time independent phenomena.

However, once you went to time dependent phenomena 2 of this, equations where found inadequate. So, we added extra terms first this curl equations turned out to have a correction to it which was minus del D del t. And the effect of minus del B del t could be seen, because if this equation was still correct which meant B was the curl of some other function A substituting if back in here you found that E was minus grade pi. But substituting this curl back into this you also got the minus del A del t. So, the electric field was not only due to a gradient of potential it was also due to a time dependence of the vector potential. Similarly, this equation was found to be unsatisfactory you found you needed a new term which was del D del t. So, that gave us the final system of equations and let me write them down once.

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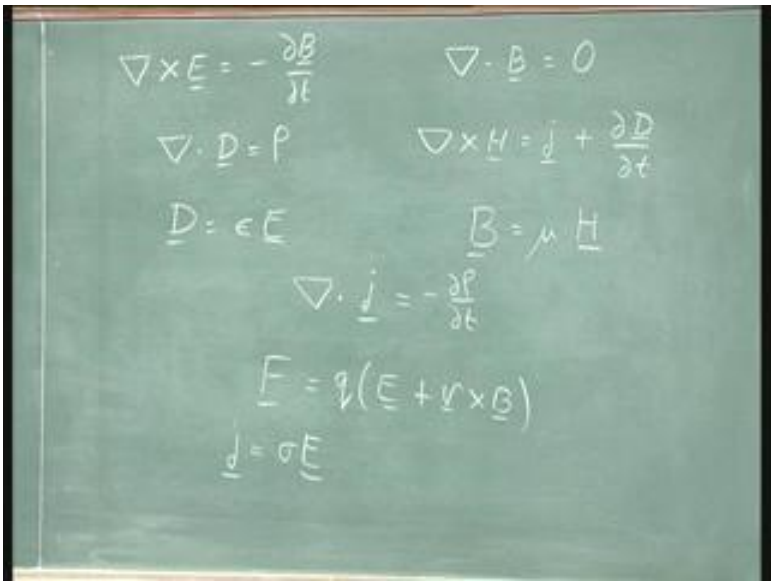

So, that we can see that curl of E is equal to minus del B del t faradays law divergence D is equal to the free charge gausses law divergence of B is 0. There is no such thing as magnetic charge curl of H is equal to the free charge plus del D del t. That is what we call Maxwell's equation D is equal to epsilon E where epsilon is the permittivity B is equal to mu H where mu is the permeability. And we have one more equations which connects row and j which is divergence of j is equal to minus del row del t. So, these are the equation, that define the field and there is one equation. That is coming outside the system which happens to be at least as important as these which is the force equation the force on any charged particle is q E plus V cross B.

So, these are the system of equations that we have been looking at through this whole course initially. We ignored the time derivatives and we did statics then we took that time derivatives into account. We got faradays law induced dmf motional emf. And then finally, we got waves we have spent quite some waves. Now, and what I want to do is to make those wave solutions interesting to someone was not going to be in communication theory, because really. If you are not going to do am and fm transmission why should you learn about the wave theory? Well, there is good reason for that that why you should learn. And it has to do with the fact that even if you do not do free space wave propagation every electrical engineer like it or not is going to have to deal with the transmission lines.

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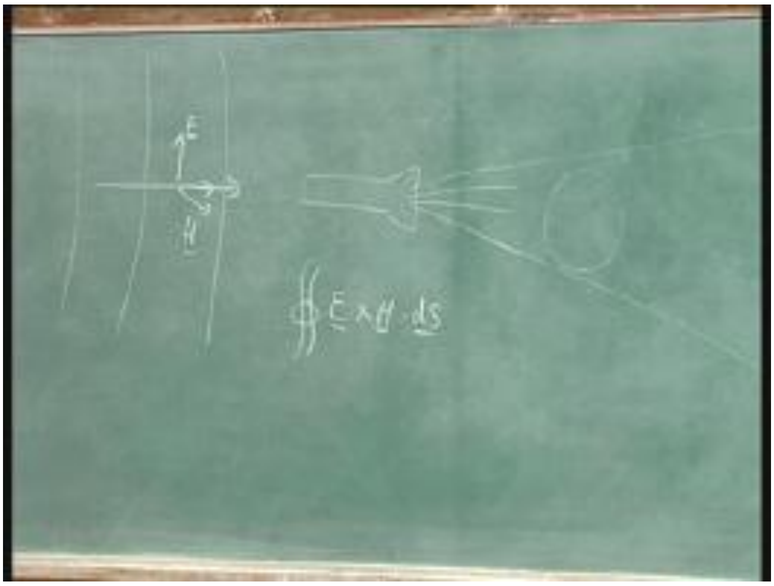

So, what is the transmission line? Up to now we have looked at waves which travel in 1 dimension. Basically we know that there are more complicated solutions, but we have looked only at 1 dimensional solutions you have the electric field in some direction magnetic field 90 degrees to it. And E cross H is the direction in which the wave is traveling. Now, the first things of interest versus that if you had a torch and you shown light from this torch you know that. The light spreads what; that means, is if you standing some where you shine a torch at the wall. The spot will appear on the wall if you shine the same torch at a far away building this part will be much bigger. And if you shine it at a mountain you won it see anything at all.

So, what it means is that this beam is spreading out and we work this out in terms of pointing theorem. We said that does not inverse square law for intensity namely volume integral of E cross H dot ds is constant assuming. That your light bulb is in 1 place you are asking how much how spread out is my electric. And magnetic field of my wave what you have is E cross H dot ds is independent of how far away I measure it if I measure using a surface like this. I will get 1 answer if I measure using a surface like this. I will get the same answer let, because the same wave is going through there are no sources here. So, I should get what ever came in must leave E cross H is power power density.

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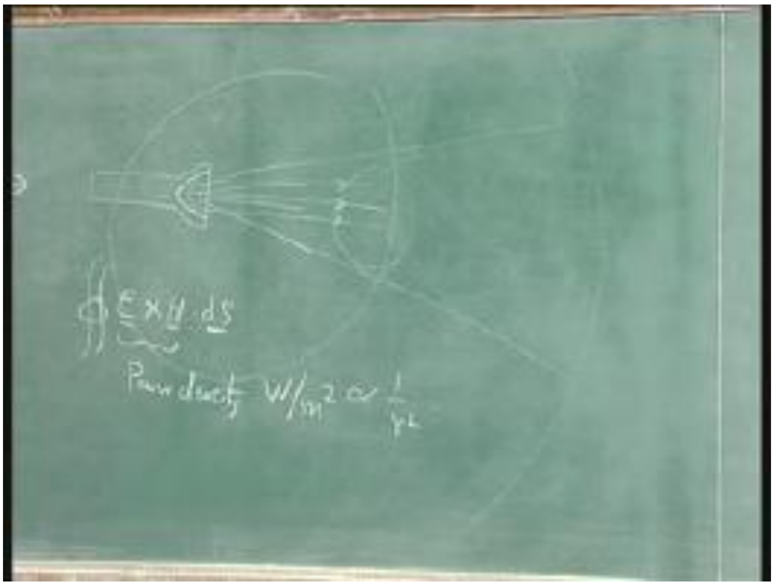

If you like we saw last time it has units of watts per meter square. So, this power density is inversely proportional to the area of the surface and the area of this sphere is proportional to the square of the radius. So, power density is proportional to 1 over r squared that explains a lot of thing. It explains the suns why the sun is not brighter, than it is how lights spreads out it explains the great number of things. But it is an inconvenience supposing I want information to travel from point a to point b without spreading out. Well of course, I can design very good optics and make sure that the light does not diverge. And we do that when you have point to point micro wave links are transmitter has 1 volt of power and links may be tones of kilometers apart. So, it is possible to have very low powered sources going through air without spreading much; however, that is not always the best approach. So, what we prefer to do is to have cables we will prefer to take this waves packaged it in to a cable and send it to from point a to point b like.

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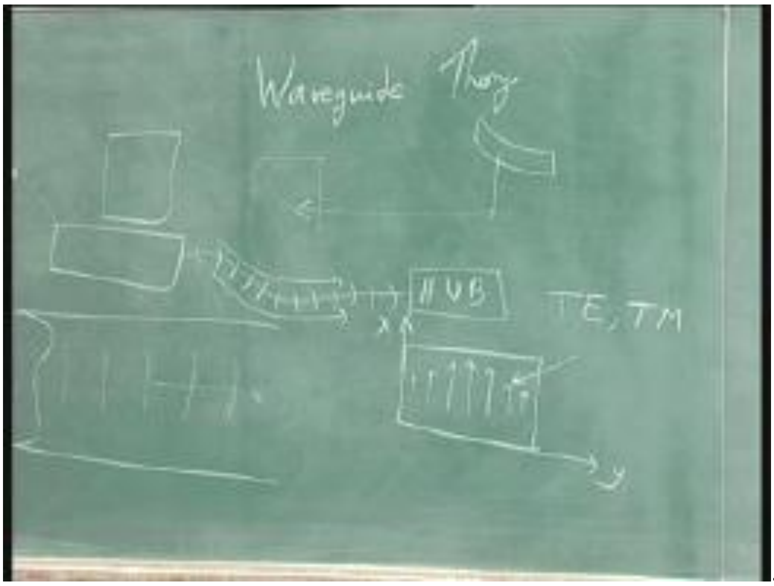

We have a computer and we have a cable an Ethernet cable and it connect to a hub. This cable is also taking time dependent information 100 millions bits per second from this point to this point. And the wave that is traveling through this cable is force to stay inside the cable and. So, does not obeys the inverse square law that is the strength of the wave at this point is not inversely proportional to the distance it attenuate. There are losses in this cable, but otherwise if it where loss less the amount of energy here would be equal to the amount of energy there left here. So, what we really want is that same wave that travels through the space except we want to bottle it into a pipe. We like to look at what solution at one can have if I take a wave and I make it travel inside a metallic pipe unfortunately. There is a bad news lets look at this along the axis, so its got some shape this is x this is y.

So, this pipe has some cross section now my electric field. And my magnetic field are what they are, so let say I have my electric field this way. And my magnetic field is this way now you can see that there are places where the electric field will be tangential to the wall. Now, you have already discuss that tangential electric field is always continues since is always continues. The electric field inside the metal is also what it at the wall which means you have a very strong electric field inside the conducting material. Similarly, if you look at this point your magnetic field has a normal components and just as we show as the tangential components of the electric field is always continues. The tangential the normal component of magnetic field is also always continues.

So, you find that what ever the magnetic field is here is also inside the material. But you know that amperes law tells us that if you have an electric field you have current if you have a magnetic field you have an induced electric field. And therefore, you have again lots of current. So, if your current conductivity your sigma is extremely high you cannot really afford to have a large electric field which means that this kind of solution is not possible, if you try to package what is called a plane wave. The kind of wave we will be looking at and put it into a pipe the plane wave will be destroyed in centimeters it will just not survive.

If a plane wave is to survive it must change it must be a different kind of a wave it must actually vary not only in z. But it must also vary in x and in y and how much did vary it must vary by becoming 0 wherever the electric field tries to be tangential. And the magnetic field must become 0 wherever it is trying to be normal. There are such solutions and the study of waves flowing through pipes. So, it is a very important subject its called wave guide theory for instance if you are doing radar. And you have building wave you have generated your strong radio frequency signal. And you have your rador antennae well there will be 100 meters are few 100 meters between the antennae. And the place where generating the signal, so this high frequency signal has to be taken there. And it may be 10 kilowatts of power that must be given to this, antennae it has to go through a pipe.

And such wave that go through a pipe and driven rador antennae these are waveguide. But I does the problem with the waveguide problems with waves flowing in pipes is that your electric field. And your magnetic field, are not the simple fields that we have looked at so far there are more complicated fields for instance if you had the cross section that was rectangular. So, you had x you had y then you could imagine that you would have an electric field that was 0 at the 2 ends. And slowly becomes bigger biggest and become smaller such a solution is quite possible. And these kinds of solutions exist and they are called transverse electric.

And transverse magnetic modes you will see the names TE and TM modes that is all, its means. It means that there is variation in x and y direction, but transverse electric mode is 1 which does not have any electric field in the z direction. The transverse magnetic mode is the mode that does not have any magnetic field in the z direction unlike all, this mode which have quite complicated. And not really of importance to low frequency application there is one special mode. And this special mode can travel at low frequencies it can travel through pipes. And it looks like a plane wave and it that wave I want to talk about today.

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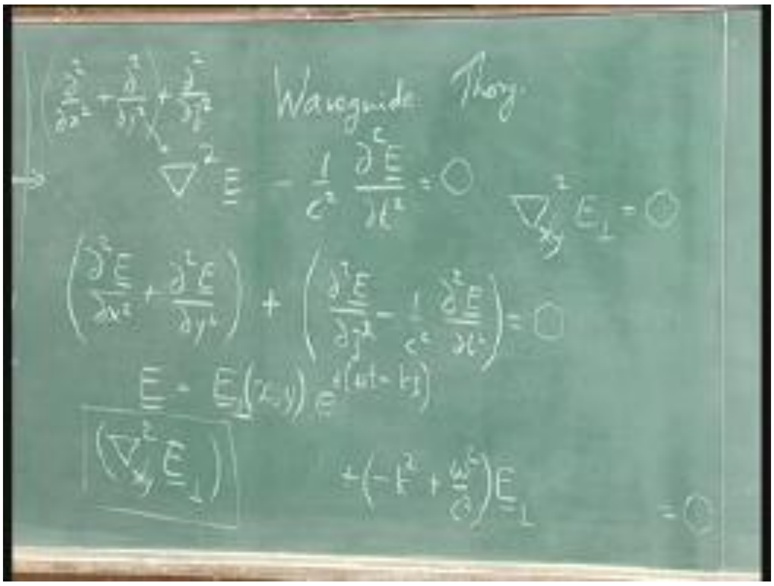

Let us go back to the wave equation, so we had Del squared E minus 1 over C squared del square E del t squared is equal to 0. Now, I am going to write this Del squared as 2 parts its actually equal to the second derivative with respect to x plus. The second derivative with respect to y plus the second derivative with respect to z that is what Del squared is, so I am going to separate out these 2 pieces. And keep this separate, so del squared E del x squared plus del squared E del y squared plus del squared E del z squared minus 1 over c squared del squared E del t squared is equal to 0. I just put brackets this 2 equation are identical, because I just written out the del squared collected 2 terms. And put the third term with the time derivative now I am going to make a guess I am saying I am having a long tube.

So, whatever shape I have in x and that shape remains what it is as I go down in z. So, if I cut here I will still see the same shape and if i go further and i will see the same shape. So, I guess that the electric field has a dependence on x and y which is independent of how it depends on z and t. So, I write a electric field as some vector which depends on x and y alone times e to the j omega t minus kz. This is perfectly reasonable I do this because I can see that time and in z my structure is symmetric if I shift in z problem looks the same. So, I look for sins and cosines in z by shift in time the problem looks the same. So, I can looks for sins and cosines in t, but I cannot shift in x and y, because there where pipe.

So, that problem is more complicated if I substitute this in here and I will call this E perp I can write. This as Del squared xy acting on E perp multiply by the e to the j omega t minus kz plus the Del del z squared will act on k kz. So, I will get a minus k squared plus omega squared over c squared e perp e to the j omega t minus kz is equal to 0. So, all I have done is I substituted this form into this equation this part. I do not know about is going to do is just going to act on this unknown function. But this part I know what is going to do because i know the form of the dependence of time. And the dependence on z, so I pulled it out let me get rid of the exponential, because it is common, so I remove it.

Now, if I want to compare this problem with the plane wave problem I would need to get rid of this piece without this piece this is nothing. But my 1 dimensional wave equation this piece is what is going to complicate my life. So, I look for solutions where this piece goes away. So, I try to find solutions where the Del squared xy acting on this amplitude which depends on only xy is equal to 0 if I could find it. Then the rest of it is done this is 1 dimensional wave equation. So, it is my plane wave bottled into a pipe and packaged. And then send now the question is how do I solve this problem? I have gone from one problem to another. But the second problem does not look any easier then the original problem.

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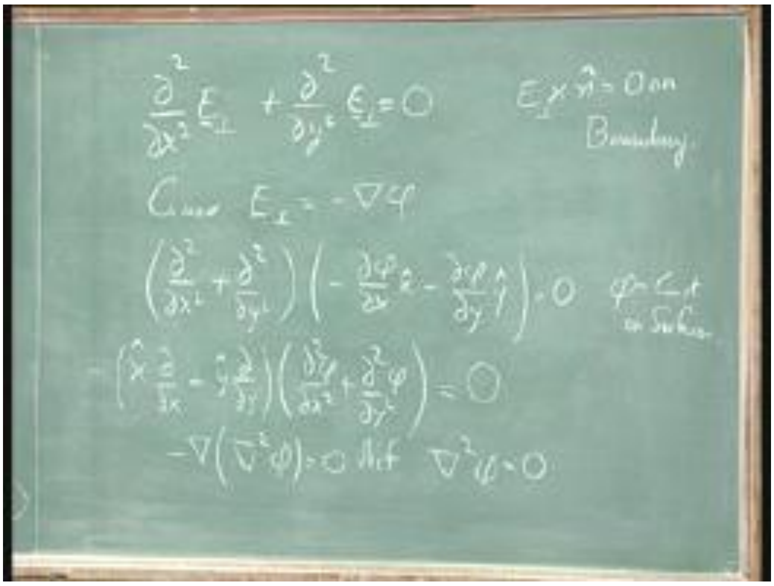

So, I want Del squared Del x squared of my E perp plus Del squared Del y squared of E perp is equal to 0. And E perp cross the normal to the surface is equal to 0 on boundary, because the tangential components must be 0. Now, what I am going to do is I am going to look at this and say this does not depends on time at all. So, I will guess E perp is a vector is equal to gradient of some scalar after all it looks like electro statics to me. So, why not try it even this looks like electrostatics its say there is no tangential electric field,, so the wall is a eqi potential. So, let me put back into this what do I get I get del squared del x squared plus del squared del y squared acting on minus del pi del x x hat minus del pi del y y hat is equal to 0. And in addition pi equals constant on surface, because I need tangential components to go to 0. So, pi is constant,, so tangential component is also 0. Now, because these are all derivatives without any coefficients I can always commute them I can pull this Del del x outside and I can put these inside.

So, I can write this as x hat del del x plus y hat del del y acting on del squared del x squared plus del squared del y squared of pi, because after all second partial derivative is commute. So, it does not matter in which order I do it del y del x del del del y del x or del del x del del y. So, the order does not matter I will do this operation. And I will do this operation after I do this operation. But, when you look at this equation what does it say it says minus gradient of del squared pi equals 0 which can be easily satisfied its if del squared pi is equal to 0. So, I am really looking for solving this problem I am looking for something very simple I am saying. That the guess that the electric field looks like this some function of x y times E to the j omega t minus kz. And this function of xy is derived from a potential and this potential satisfies lap losses equations let us look for an example. And see whether all of this mix in or not.

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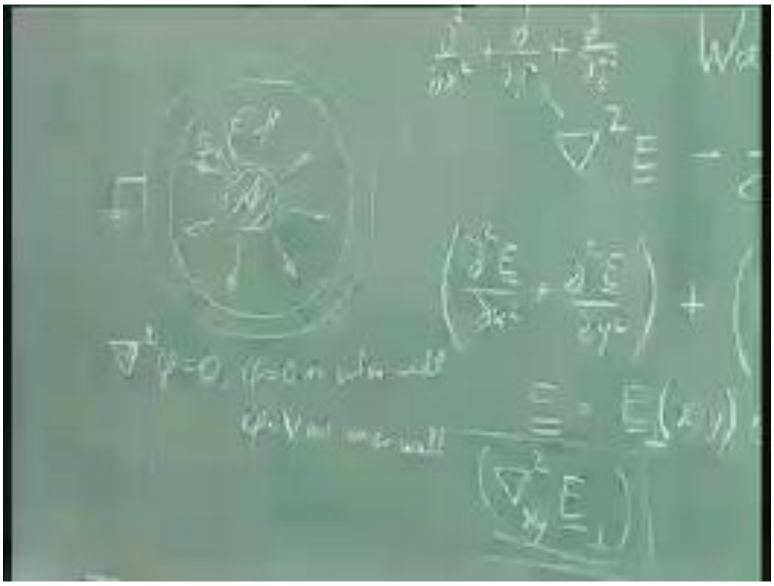

I will start with the central conductor and a metallic pipe the outside I am going to hold that ground inside such sum voltage v. And there is dielectric now really I should say I am holding it around voltage v its going to be carrier wave, which mean that the voltage is not constant the voltage is function of z. But when I look for the solution of this problem I am looking for del squared pi is equal to 0 pi equal to 0 on outer wall pi equals v on inner wall this is a well known problem. We have already done it this is nothing, but what we did for calculating capacitance of 2 cylinders. So, you can easily find the solution and you know that there is radically symmetric Er. This electric field Er will satisfy del squared pi is equal to 0. There is no tangential electric field on the outside surface there is no tangential field on the inside surface. So, its satisfies all the conditions that we want and therefore, del squared xy of e perp is 0. And the boundary conditions are satisfied what does that it means for that particular form we are solving only this equation.

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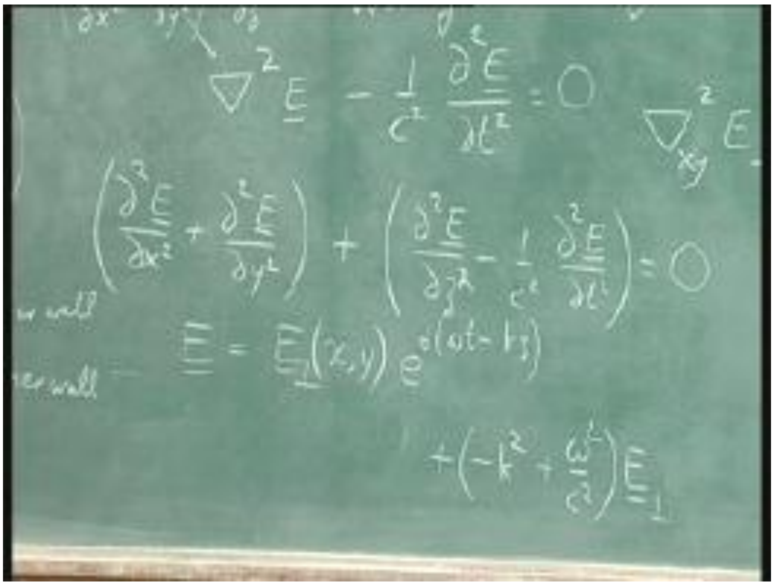

Now; that means, my solutions is going to be look like this the electric field which is the function of x y z.

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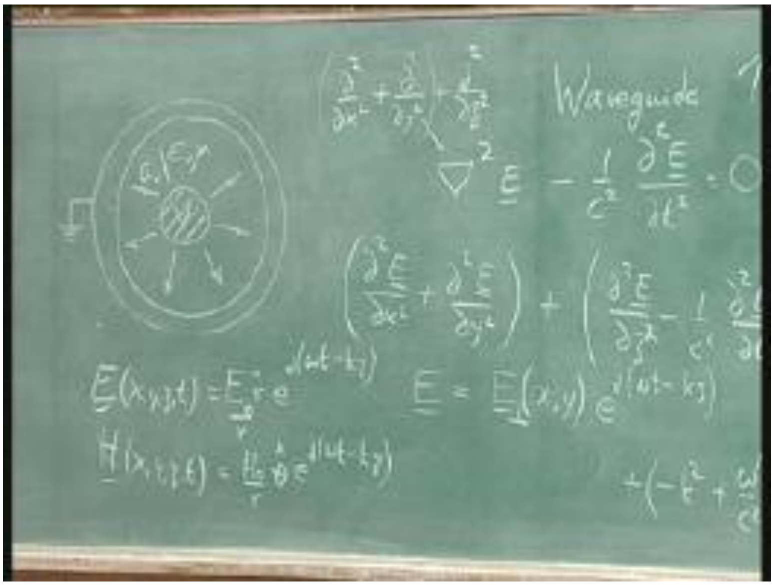

And t it really an Er along r direction that Er, we can work out is going to like 1 over r. So, you can say E not over r e to the j omega t minus kz what will the corresponding H look like? This will be the curl of E. So, when you look for the curl of E is going to be take the z derivative of Er. That will be in the theta direction H of x y z t will again look like some H not over r in the theta direction e to the j omega t minus kz.

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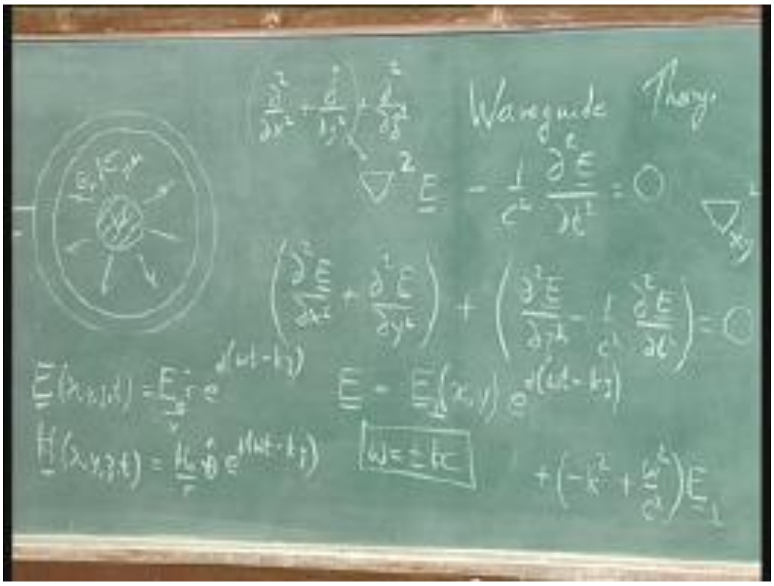

And when we put this into wave equation you get this solution which means omega is equal to plus or minus kc. Now, these are very special waves these are not the waves that are used for taking high energy radar signals. They have quite a few nice properties for one thing the line that connect omega and k. The curve that connects omega and k, if you plot it you find that it is a straight line.

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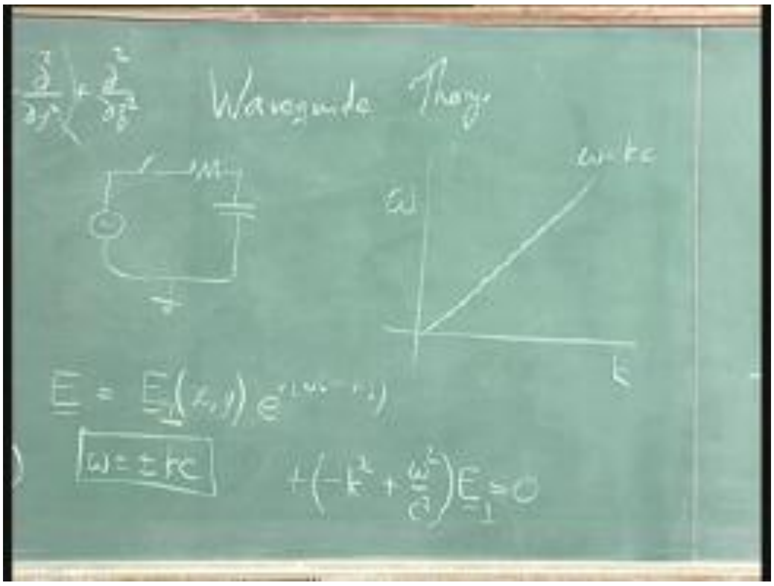

Now, this is very important, because if you look at what happens?

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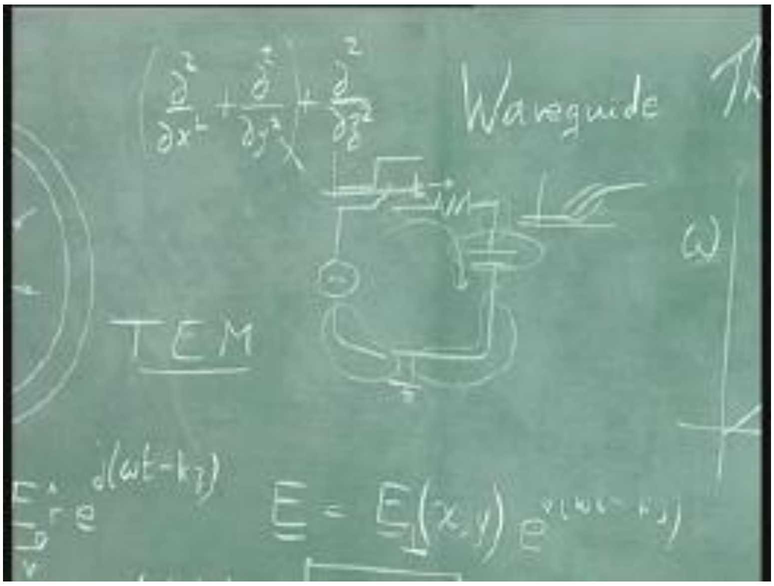

If I have a circuit and I have a switch and a closed switch, when I close this switch current will start to flow, now the speed with which the signal that the switch is closed goes through. These circuits depends on how well the signals propagate through this wires if the signal propagate at different velocities at different frequencies. Then what will happen is out here in time the signal is switched just closed and instantly the signal came on. And continue, but out here it may very well be that the signal will look like this that is to say the fact that. The switch closed was not very sharp start in the current starts gradually there is a raise time to this signal. And this raise time due to the signal comes, because the wire is not carrying all frequency with the same velocity. But this particular solution we have with a special solution it does. In fact, care carry all a frequency that the same velocity, because the velocity is nothing but omega over k which is equal to c, so all frequency travel together.

So, if I take the lap loss or courier transform of this signal its a spectrum every part of the spectrum travels together again take the inverse transform here. And I will get back the same shape it would not be 1 1 frequency is delayed with respect to the other which is the main cause of this kinds of effects. So, that is one important advantage of such waves and these waves are called TEM waves transverse electro magnetic waves. And they are the waves that live in what we call transmission lines. Now, in a certain sense transmission lines has come to rule electrical engineering no mater which branch of electrical engineering you are in that is. Because the state of the art has kept progressing till; however, passed to, however slowed your circuits are they are pushing the limit. And the limits are always the limits of signal propagation through circuits. So, you can see this happening through the different technologies that are happening in different branches of electrical engineering for instance naturally in communications.

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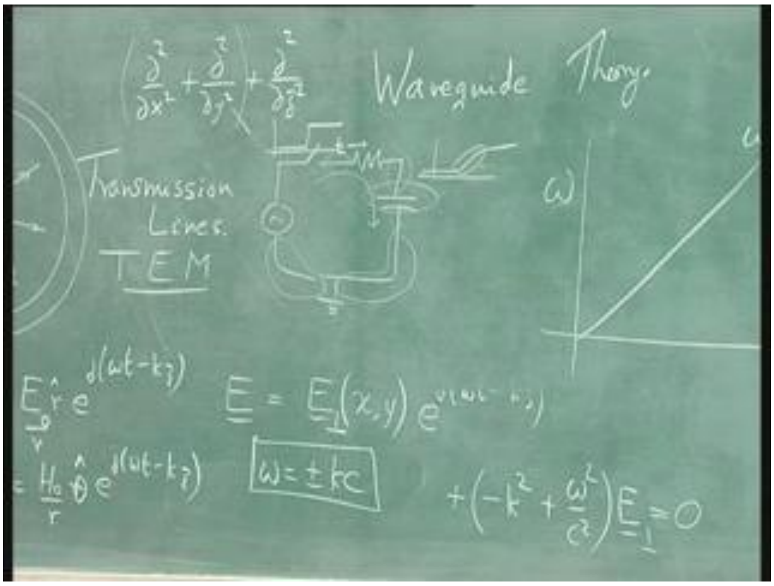

Transmission lines are required wave gyres and transmission lines are automatically required, but you look at for example.

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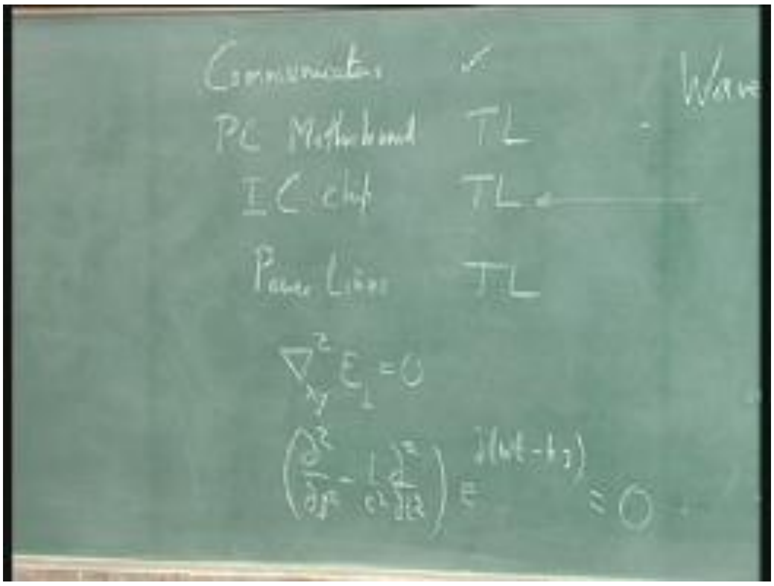

A PC main mother board now that does not look as not communications after all what about it well the size of lines on a PC mother board a few centimeters. And the speed of the clock on the mother board which are 100s of mega hertz combine together. So, that in fact, signal delay will becomes crucial. So, once again you find the transmission lines are crucial you do not take proper care of delays you do not properly understand. The transmission line, you can not design a good motherboard whatever in IC chip well these distances of lines are now in tens of microns to 100s of microns.

But the speeds are much higher, because the speeds are much higher once again signal propagation delays are important. And you have transmission lines once again, so you comes lower in frequency and. So, you come to power lines. Now, you are talking about 50 60 hertz, but your power lines are very long your power line can be crossing a country. And once its starts getting approaching a large fraction of a 1000 kilometers once again you are a transmission line. So, the theory of transmission lines and more generally the theory of wave guide have come to dominate electrical engineering no matter which branch you are in.

And it is the same theory, the theory cannot change, because we are talking about power lines as suppose to communications in all these cases what is important is that you are able to separate the del squared of Ex and Ey. And the solution along the z which is Del squared Del z squared minus 1 over c squared del square del t squared of the behavior in t and z. So, I gave 1 example of this which was cylindrical 2 wired guide that is what you call a bmc cable it is also found in the tv coaxial cable. And all of those are machine to great precision, so that you can carry the signals up to very high frequencies. But since we are talking about low frequency effects we can also talk about power lines.

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Because you can consider 2 wires one of which is at 1 voltage and the other is at the, another. So, you have current flowing into this and coming out of this wire these are the written path of this current. Then we are solving for xy electric fields by solving this capacitance problem is its solvable problem. In fact, there are exact solutions available and if you solve this problem. And substituted back into this equation you got the transmission line that is used by power lines. And this is nothing, but the electro magnetic wave that travels through the 100s. of kilometers in which a power is transmitted. So, the power is actually going into the board. But, electro static and magneto static fields are set up due to the charges and the currents that get established on this, wires.

So, it is a important thing to realize that transmission line are a very unifying idea. And they are in fact, like you might say the final step in a basic electromagnetic course. You have learned about static field you have learned about magneto static fields. We learned about emotionally mf we learned about waves. And when we went to waves it was like when we went to a new subjects. But actually now, we come all the way back we have talking about using those waves inside circuits. We are talking about power lines carrying power we are talking about Ethernet cable carrying signals. And those circuit elements are in fact, waves again and perhaps most important part of it is to solve for this waves. You need electrostatics you are not using wave theory wave theory is the simple part you have to do electro statics.

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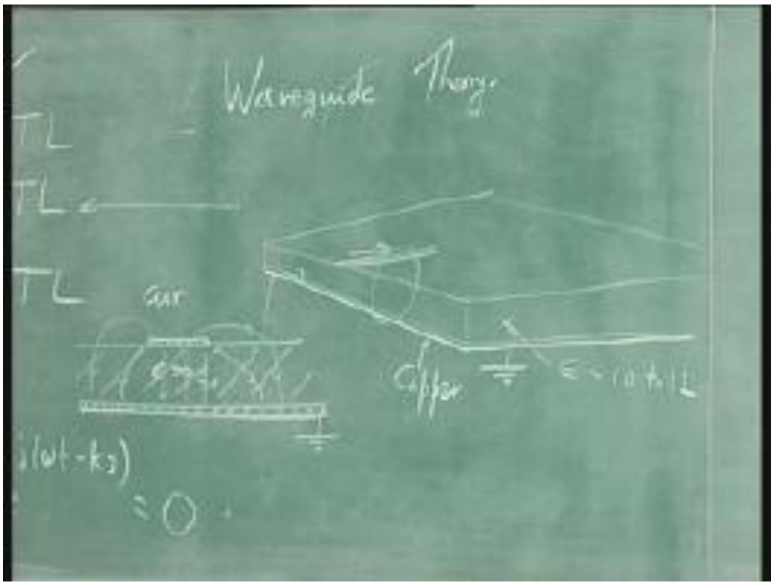

Now, you are doing PC mother board; you would have a high permittivity epsilon of may be 10 12 high permittivity material dielectric on which you have thin copper strips. The bottom part of this plate is actually fully copper and it is connected to the ground. This is what calls a 2 layer boat and of course, a standard PC mother board will not be 2 layers. It will 2 layer or 6 layer or higher, but this is the simplest way of looking at it. So, what happens is if you look at this structure? And look at it from side waves you have your dielectric at the bottom you have copper. And this copper is grounded at the top you have more copper current signal flow in into this copper strips. That is they are flowing this way that is in to the board in this picture when this when charges get built up on this strip you get capacitance. You have to solve this problem this problem would be the problem of having an air dielectric.

And epsilon greater than much epsilon naught with copper strip in the middle this problem corresponds to solving. This having solved it and actually there are some problems I would not go into them. So, it is only a imperfect solution you can make having solves them we have then left with the transmission line the transmission carries signals along. This, wire at essentially the speed of light actually half the speed of light approximately similarly, in an IC chip I mean. The same ideas go whether it is in a PCB and an IC chip are even in a micros strip antennae. Now, if you are done your transmission line course are even, if you have in when you get to it the language will be very different. You will not see the wave theory at all in fact, you will see a distributed rlc network.

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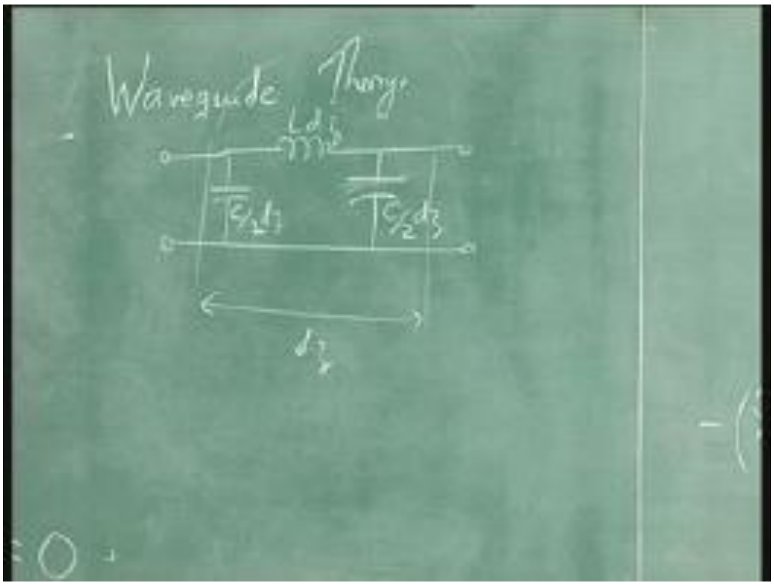

You will told that the transmission line can be viewed as a 2 port network and you can put can put an equivalent circuit. This distance would be dz this would be c over 2 c over 2 dz. And this is Ldz and you will be told that this represents the transmission lines an idea transmission for a short distance dz. And then you combine many many such section and you get the behavior of a transmission line which does not look like very much like. This equation at all this equation came from studying maxwells equations where as this is more of a equivalent circuit. So, where does this picture fit in? With this picture well the way to look at it is as follows imagine that.

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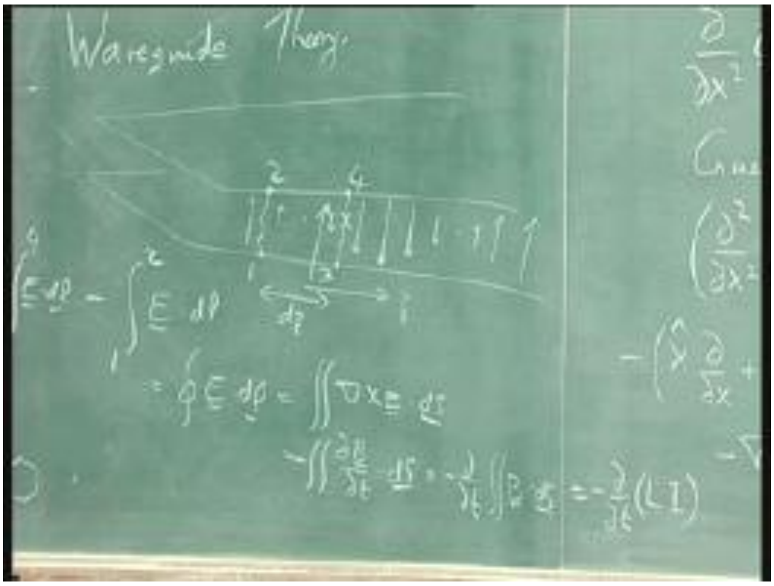

We have a parallel plate capacitor which is very long this direction is z and this direction the direction between the plates is x. And we do not care too much what is happening in y. So, we have again the possibility of tem wave, because you can solve for capacitance of the arrangement. So, we will have electric fields and then the electric fields will change direction and that is, what is our wave. The wave is going in this z direction now; I want to convert from electric and magnetic field to voltages and current, because that is what circuits know and that is what is the equation? The equation has do with the inductors and capacitors. Well, the idea is we have solved for a potential we looked for E equals minus grade pi. So, therefore, we can talk about the voltage across these plates.

We can calculate integral E dot dl from 0.1 to 0.2 we can also talk about E dot dl from 0.3 to 0.4 and let say that this distance is dz. So, in that case I can talk about E dot dl from 3 2 4 and E dot dl from 1 to 2. So, I can do integral 3 2 4 E dot dl minus integral 1 to 2 E dot dl which is nothing but loop integral E dot dl, because electric field is up. So, this 2 components sections do not do anything electric field is orthogonal to dl, what is that? That is nothing but surface integral curl of E dot ds which is apply Faradays law minus del B del t dot ds. I will pull the del del t out. It is minus del del t of surface integral B dot ds. But we know that surface integral B dot ds is nothing but plex and plex is li. We can write this as minus del del t of LI let see what we have written this L or this plex is the plex between of region a of dzy.

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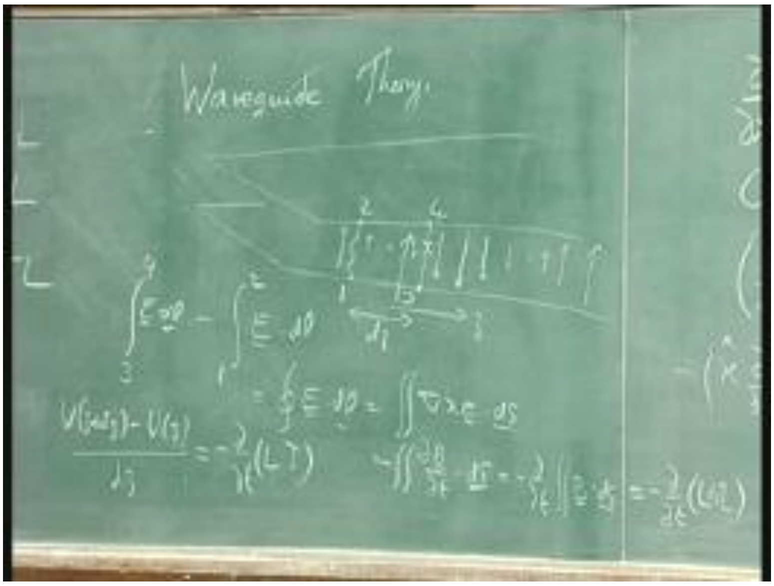

So, this inductance is Ldz and this difference is voltage at z plus dz minus voltage at z and I am going to take that dz out. And bring it this side divided by dz is equal to del del t of LI this is nothing but the z derivative of voltage which gives as a circuit equation. The circuit equation is del v del z is equal to minus L time del I del t. Similarly, you get another equation, the other equation will use Amperes law.

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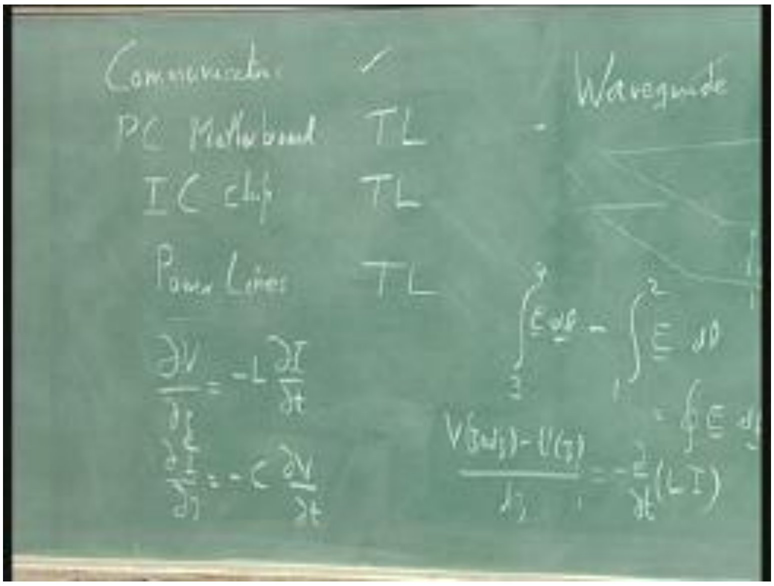

And it will give you del I del z is equal to minus C del v del t and these 2 equation together will give you the transmission line equations. The lesson to be learned here is all the work that you have done in electromagnetic theory statics motional emf waves as finally, reached here. It allows as to define a wave that is bottled into a pipe. And that wave that is bottled into a pipe a useful variant of that wave is what we call a tem wave. And that tem wave can be modeled by circuit concept you can talk about inductance per unit length. And correspondingly we can talk about capacitance per unit length. And you can instead of talking about electric and magnetic fields talk about voltages. And currents when you do that, this is nothing but the theory of transmission lines. And the theory of transmission lines is the practical aspects of electromagnetic theory.