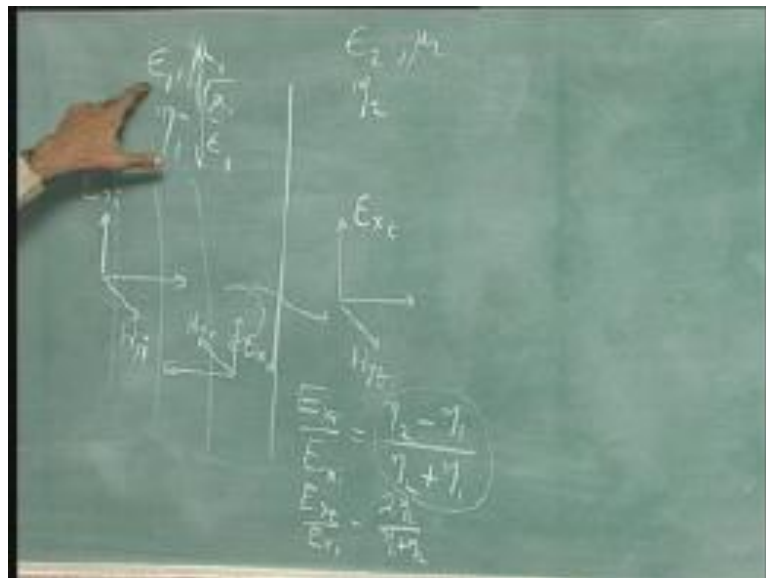


Electro Magnetic Field
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Lecture - 40
Reflection at Dielectric boundaries (Continued)

Last time we have been we were discussing the propagation of waves when they reach the dielectric boundary. So, I am going to continue that we had looked at a case whether they where 2 dielectrics.

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The first dielectric had dielectric constant epsilon and permeability or mu which meant that eta was equal to square root of mu over epsilon. And if this was region 1 region 2 had epsilon 2 mu 2 and eta 2. If there is conductivity it is observed into epsilon as we discuss the last lecture. Now, wave comes propagating in this direction it has an electric field and a magnetic field E_x H_y and it reaches this interface some energy reflects some energy transmits. Now, the question I looked at last time was how much energy reflects how much energy transmits? So, what we computed was that if the returning wave has an electric field E_{xr} and a magnetic field H_{yr} actually H_y would be negative where is this is the y direction.

So, if H_y pointing this way H_y is negative and it has a transmitted wave where E_{xt} and H_{yt} when we calculated that E_{xr} divided by E_x or E_{xi} . If you like incident was equal to

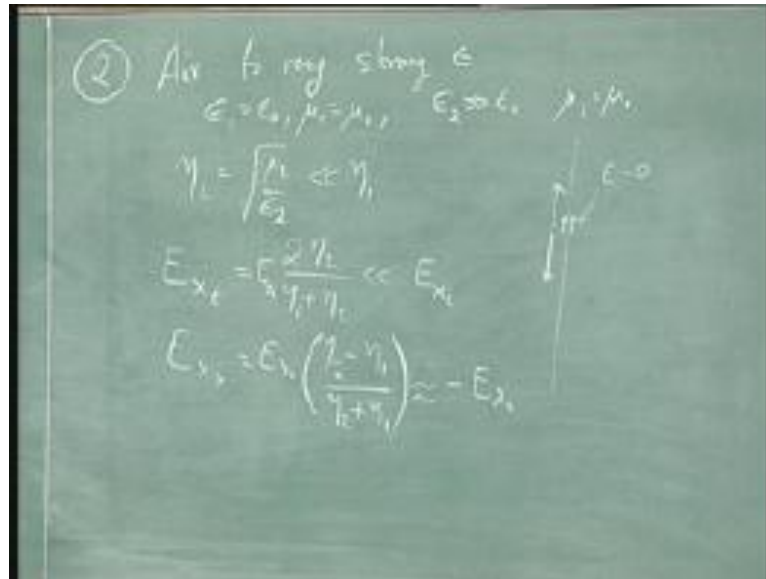
this eta of the medium 2 minus eta of medium 1 divided by eta of medium 2 plus eta medium 1. And similarly, E_{xt} divided by E_{xi} was equal to twice eta 2 divided by eta 1 plus eta 2. This quantity is called the reflection coefficient where is not quite the reflection coefficient, but it representing the amount of the wave that reflects given a wave coming in. If 1 volt per meter wave comes in how many meters wave will comes back? It is a complex number. Because it is entirely possible for epsilon to be complex and eta t represents how many volts per meter in the wave transmits.

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① Air to Air
 $\epsilon_1 = \epsilon_2 = \epsilon_0, \mu_1 = \mu_2 = \mu_0$
 $\eta_1 = \eta_2 = \sqrt{\frac{\mu_0}{\epsilon_0}}$
 $E_{xr} = E_{xi} \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) = 0$
 $E_{xt} = E_{xi} \left(\frac{2\eta_1}{\eta_1 + \eta_1} \right) = E_{xi}$

Now, I stop last time when I was looking at some asymptotic cases that is some cases which were already known to us let states 1 case, which is air to air, so in that case epsilon 1 equals epsilon 2 equals epsilon naught mu 1 equals mu 2 equals mu naught. So, both eta 1 and eta 2 are equal to square root of mu naught over epsilon naught and, so they are equal, so what do we get for this 2 coefficients. We get E_x reflected is equal to E_x incident times eta 2 minus eta 1 that is eta minus eta over eta plus it is the same eta, because is the same medium. So, it is equal to 0 and E_x transmitted is equal to E_x incident times twice eta divided by eta plus eta which is equal to E_x incident. So, this case shows as that no energy is reflected all the energy is transmitted which is what you expect we can always draw a line. But unless properties changed when you went from one side of the line to the other side we do not expect anything special to happen there. So, the wave just kept going. And that is what we see here.

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Now, in other case would be air to very strong epsilon. So, epsilon 1 is equal to epsilon naught mu 1 equals mu naught epsilon 2 is much greater than epsilon naught say mu 2 equals mu naught. So, in that case eta 2 is equal to square root of epsilon 2 over mu 2 is much less than eta 1 sorry now, we have to find out what happens. And we have to look at this 2 expressions now, when we look at this 2 expressions what we get is that E_x in the transmitted region is equal to twice eta 2 over eta 1 plus eta 2 is much less than E_x incident. So, very little of the wave is getting transmitted whereas, E_x reflected is equal to E_x incident times eta 2 minus eta 1 over eta 2 plus eta 1. Since eta 2 is very small this is roughly equal to minus E_x .

So, most of the energy now, bounces and what is more that energy bouncing gives me a phase change. That is the reflected wave is negative in amplitude to the transmitted wave or the incident wave why would I have this? Basically what I need at the interface is that electric field be continuous on both sides now, the electric field on this side is nearly 0. So, the electric field on this side must also be 0, but the electric field on this side is due to the incident wave plus the reflected wave. So, if the sum of the incident and the reflected wave is going to be nearly 0 the reflected wave must be in the opposite direction. So, that is what we see. Now, I am going to take an example and o, an example taken from your own book. And the numbers will tell you something about what is actually happen.

But before that let me emphasize 1 thing each medium is characterized by 2 numbers epsilon and mu and an epsilon as observed within it sigma; however, when you talk about behavior at an interface epsilon and mu do not separately effect what happens? Actually the only thing that matters is the ratio of epsilon to mu. If the ratio of epsilon to mu is the same on both sides there is no as for as the wave is concern there is no change in a medium the waves keeps going. So, you can have a situation where epsilon goes to twice epsilon mu goes to twice mu So, therefore, eta is the same on both sides no reflection, because we saw that case if eta is the same there is no reflection, but what does changes the wave length.

Because speed of light in the 2 media in 1 case v_1 is equal to $1/\sqrt{\mu\epsilon}$ and v_2 is equal to $1/\sqrt{4\mu\epsilon}$ because 2μ times 2ϵ . So, the velocity here is 1 half the velocity here frequency being the same wavelength has dropped. So, λ_1 is equal to v_1 divided by frequency λ_2 is equal to v_2 divided by frequency the same frequency because after all. If the wave oscillate with the certain frequency cannot oscillate with the different frequency when it enters the new medium. We will come back to that point just in a little while, but what you can see is if v_1 is twice v_2 λ_1 is twice λ_2 see you have rather peculiar situation where the medium has changed.

But there is no reflection it is important to realize that it is only eta that determines reflection. It is not separately epsilon and separately mu, but separately they do effect the wave. One of them talks about a material property which we call impedance and the other talks about the velocity. So, when we talk about velocity we require the product of mu and epsilon and we talk about eta we require the ratio of mu and epsilon. So, clearly you can have materials which fix 1 and vary the other or fix the other and vary the first. And that gives you a fairly rich kind of set of behaviors that we see. So, let me go straight to an example as I told you the example is as follows.

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The chalkboard contains the following handwritten equations:

$$E_{xi} = (100 V/m) e^{j(\omega t - kz)}$$

$$H_{yi} = \frac{E_{xi}}{\eta_1} = (1 A/m) e^{j(\omega t - kz)}$$

$$E_{xr} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) E_{xi} = \frac{200}{400} E_{xi} = (50 V/m) e^{j(\omega t - kz)}$$

$$H_{yr} = \frac{E_{xr}}{-\eta_1} = (-0.5 A/m) e^{j(\omega t - kz)}$$

$$E_{xt} = \frac{2\eta_2}{\eta_1 + \eta_2} E_{xi} = \frac{600}{400} E_{xi} = (1.5 E_{xi}) = (150 V/m) e^{j(\omega t - kz)}$$

$$H_{yt} = \frac{E_{xt}}{\eta_2} = \frac{150}{300} = (0.5 A/m) e^{j(\omega t - kz)}$$

Parameters: $\eta_1 = 100 \Omega$, $\eta_2 = 300 \Omega$

Let suppose we had the incident wave is equal to 1 volt per meter into of course, e to the j. So, it is equal to 1 volt per meter into j omega t minus kz. And I am going to assume that in my first region eta 1 is equal to 100 ohms. Now, I will get back to the units of eta in a little while and in the other region eta 2 is equal to 300 ohms. I promise this last time and I still in got to it the dimension of eta is something to be discussed. Now, I want to calculate what actually are the various components of the system Hyi is equal to Exi well I think the example has 100 volts per meter, because we will cancel out nicely Exi divided by eta 1 let because E is equal to eta H and therefore, it is 100 by 100. So, it becomes 1 amp per meter into e to the j omega t minus kz. If you look at the reflection coefficient Exr it is equal to eta 2 minus eta 1 over eta 2 plus eta 1 times Exi eta 2 is 300 eta 1 is 100.

So, 300 minus 100 is 200; 300 plus 100 is 400 into Exi and Exi is 100 volts per meter. So, that is 50 volts per meter of course, again into e to the j omega t minus kz. What is the magnetic field corresponding to this well it is again divided by eta 1. So, Hyr is equal to Exr divided by minus eta 1 minus, because the direction of propagation is opposite, so they have to be a negative sign. So, it becomes 0.5 with a minus sign amp per meter into e to the j omega t minus kz. Now, what about the transmitter side? Well in the transmitter side we have Ext is equal to twice eta 2 divided by eta 1 plus eta 2. So, this is 600 times Exi is equal to 600 divided by 400 Exi which is Exi is 10 times 1 and half. So, that is 150 volts per meter e to the j omega t minus k prime z why k prime? Because when eta when

the material changes epsilon and mu have changed. So, the velocity of light is change. Now, we do not have enough information to know what k prime is because k prime is related to 1 over square root of epsilon mu.

And we only know the ratio of epsilon to mu not the product of epsilon and mu. So, the value of k prime is not known that will know if you have additional information what about Hyt it is equal to Ext divided by eta 2, because we are now in material 2 I have to divide by eta 2. So, its 150 divided by 300 which is in which is equal to 0.5 amp per meter times e to the j omega t minus k prime z. So, you can see that there are some interesting features here incoming value of the electric field is 100 reflected is 50. So, 100 is 50 is less than 100 that seems to be a sharing, but what is transmitted is 150 and 150 is greater than 100. So, the amplitude of the transmitter wave is actually greater than the amplitude of the incident wave thus something rather beside here. So, what is going on well we should not be talking about amplitude of the electric field. What matters is power from our pointing vector diagram of last time we know that total power must be concerned. So, what is the power in?

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The incident power is $E_{xi} \times \hat{x}$ cross $H_{yi} \hat{y}$ this is in the z direction and how much is it? It is 100 volts per meter times 1 amp per meter which is 100 volt amperes which is 100 volts per meter square as you know E cross H is a power per unit area, because I have to integrate E cross H over a surface area to get actual power. So, the input power is

hundred volts per meter square. Now, what is the reflected power? Reflected power is equal to its coming in the negative direction, but I am not taking the sign into the account, because we already know that it is $E_{\text{ref}} H_{\text{ref}}$ which is equal to 50 volts into 0.5 amperes, so 50 into 0.5 equal 25 watts per meter square. So, watts per meter square there is a minus sign in the minus sign means it is coming in the reverse direction, but I am not writing it here, because we already know it. What about the transmitted path that is the worrying one after all the amplitude of the electric field is actually greater than what was incident.

So, what is happening to the power? The transmitted power is $E_{\text{trans}} H_{\text{trans}}$ along the x direction H_{trans} transmitted along the y direction is 150 times 0.5 which is 75 watts per meter square. So, we see after all power is concerned 75 plus 25 watts is equal to 100 watts. So, the total amount of input power is equal to the total power reflected plus the total power transmitted which is what we need? Actually whether the electric field is larger or smaller is a detail is just got to do with the medium's way of connecting electric and magnetic fields. Some regions magnetic field is emphasized some regions electric field is emphasized, but power is what matters as far as the power if concerns input power equals to output power and this will always be true all right. So, there are basically 2 things to understand one is medium properties are characterized by η .

And of course, the velocity now I have been saying η is units of η I will discuss later. Let us go back and now discuss little bit about units. You already know that potential has the units of volts and since electric field is like $-\nabla \phi$ as the units volts per meter. So, the question is what are the units of H ? Now let us go the curl equation $\text{curl } H$ is equal to j plus other things curl is again a derivative. So, this is something like $\nabla \times H_y \hat{z}$ is equal to j_x j is a current amperes per meter squared. So, it is a current density therefore, this has the unit of H divided by a length is like amperes per meter square or per length square. So, H as the units of amperes per meter that is what I have given here ampere per meter and that is why $E \times H$ as the units of volt amperes per meter square which is watts per meter square.

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The chalkboard contains the following handwritten equations:

$$F \propto \frac{q^2}{r^2}$$
$$\epsilon = \frac{Q^2}{F r^2} = \frac{C^2}{N m^2}$$
$$C = \frac{EA}{d} = \frac{EL^2}{L} = \epsilon L$$
$$E = F/m$$
$$\eta = \frac{E}{H} = \frac{V/m}{A/m} = \frac{V}{A} = \Omega$$

Now, that is about electric field and magnetic field, but what about eta well when we look at what an epsilon is... Epsilon comes from the relation force is proportional to charge square divided by epsilon r squared. So, epsilon as the dimension of Newton's sorry coulombs Q squared over force r squared which is coulomb squared per Newton's meter square. Now, usually we do not use this unit for epsilon rather we go back to looking at capacitance. We know that capacitance sorry it is called coulombs squared capacitance is epsilon A over d and capacitance is farads the unit of capacitance is farad. And if you look at this is like epsilon times L squared over L which is like epsilon L.

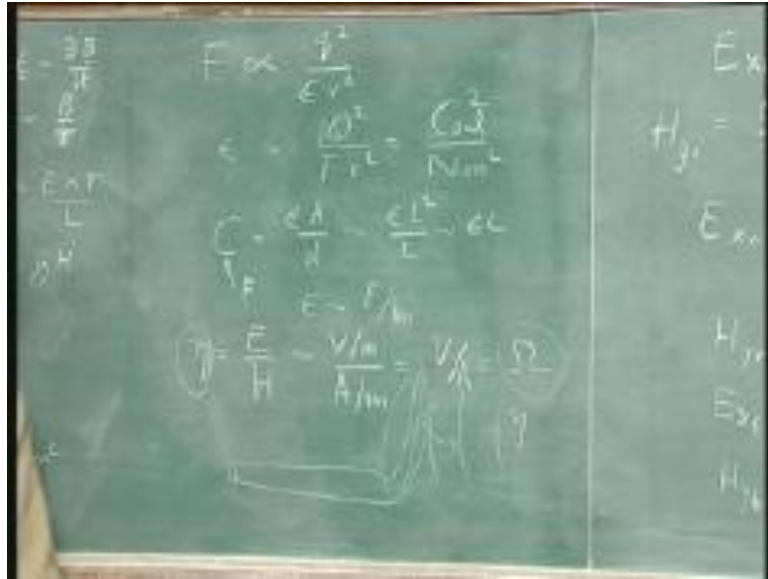
So, that is why epsilon is usually not given this units it is given the units of farads per meter, but does no nothing wrong with this unit this is the correct unit to apply. Now, as for as the unit of eta are concerned eta is nothing, but the electric field divided by the magnetic field E over H, but we already know the units of E and the units of H. So, we can say that this is like volts per meter divided by amps per meter which is volts per amp which is nothing, but the resistance ohms. And since we know the units of the expression for eta we can also get the units of mu. Units and the calculations of units is very trivial its. In fact, over emphasized I think the units just sort themselves out as long as you know 1 quantity.

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Ultimately all other quantity follows from it. So, Maxwell's equations are telling us all the units we need for example, if you want to know the units of magnetic field v . So, you just go back to faradays law curl of E is like $\text{del } B \text{ del } t$. What does that mean? Electric field divided by a length is like magnetic field divided by a time or magnetic field as the unit of electric field times time divided by length. And from that you can derive the units of magnetic fields, but magnetic field in turn is equal to μH which has already the unit of amperes per meters this has the unit of volts per meter that gives you the units of μ . So, all of the units are just interconnected and there is no nothing profound about saying that epsilon is this expression or that expression both expressions are completely correct.

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But practically speaking certain units are very important. And the unit of eta as ohms happens to be one of those very important units. Just as the units of potentials and the unit of electric field being volts per meter is very important. These are important, because they are practically come up again and again, now why is eta so important? It is because when you actually design circuits that couple to electric and magnetic fields you end up having to designs an antenna. So, you will have a wire and the wire will connect to an antenna and the antennae have to pick up waves. Now, the efficiency which with these antennae picks up waves is again going to be related to exactly the same equations that you have been writing all along. Namely there will be reflection of this wave and there will be a transmission in to the medium. And the reflection and the transmission coefficients are going to be not exactly the same as these because this is 1 dimensional treatment. So, 3 dimensional problem, but very similar.

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So, much, so there if you knew the eta of air and you know the eta of this antenna you can figure out whether they will be good matching or not. So, it is important to know this numbers and 1 of the numbers you need to know is eta of air. Eta of air is vacuum is nothing, but square root of mu naught over epsilon naught turns out to be 120 pi ohms. This is by construction you have to understand that the values of epsilon and mu are because of our mks system other systems would have other numbers. But this answer actually comes out of the symmetry of the Maxwell's equations. So, the impedance of air happens to have a number which is between 300 and 400 ohms. This number is what we need to match to when we have an antenna. And if you match close to this number it means that there will be lot of interaction between our antennae. And the wave in free space we have far apart, then what will happen is relatively little of the energy will get into the antennae or out to the antennae and most will just reflect.

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Handwritten mathematical derivations on a chalkboard:

$$E_{xi} = (100 \text{ V/m}) e^{i(\omega t - kz)}$$

$$H_{yi} = \frac{E_{xi}}{\eta_1} = (1 \text{ A/m}) e^{i(\omega t - kz)}$$

Boundary conditions at $z=0$:

$$E_{xt} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right) E_{xi} = \frac{200}{600} E_{xi} = (50 \text{ V/m}) e^{i(\omega t - kz)}$$

$$H_{yt} = \frac{E_{xt}}{\eta_2} = (-0.5 \text{ A/m}) e^{i(\omega t - kz)}$$

$$E_{xt} = \frac{2\eta_1}{\eta_2 + \eta_1} E_{xi} = \frac{500}{400} E_{xi} = (50 \text{ V/m}) e^{i(\omega t - kz)}$$

$$H_{yt} = \frac{E_{xt}}{\eta_2} = \frac{150}{400} (0.5 \text{ A/m}) e^{i(\omega t - kz)}$$

Material properties: $\eta_1 = 100 \Omega$, $\eta_2 = 300 \Omega$

No there is one more important topic to cover in this area.

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It is not what I am going to teach as part of the course, but more for your understanding. Supposing I have a wave that is coming at an angle I am supposing you have epsilon 1 mu 1 epsilon 2 mu 2 as the regions properties. The question is if this angle is theta what waves are excited when this wave hit the surface from optics? We know the answer we know that if you take a torch light and shine it on a glass window at an angle that the there will be some amount of light going back. This way and there will be some amount

of light going back going this way. And we know that this angle is θ whereas, this angle is not θ it is some θ' . I like to get this out of electromagnetic I do not want to I do not want to 2 separate subjects one called optics and one called electromagnetic.

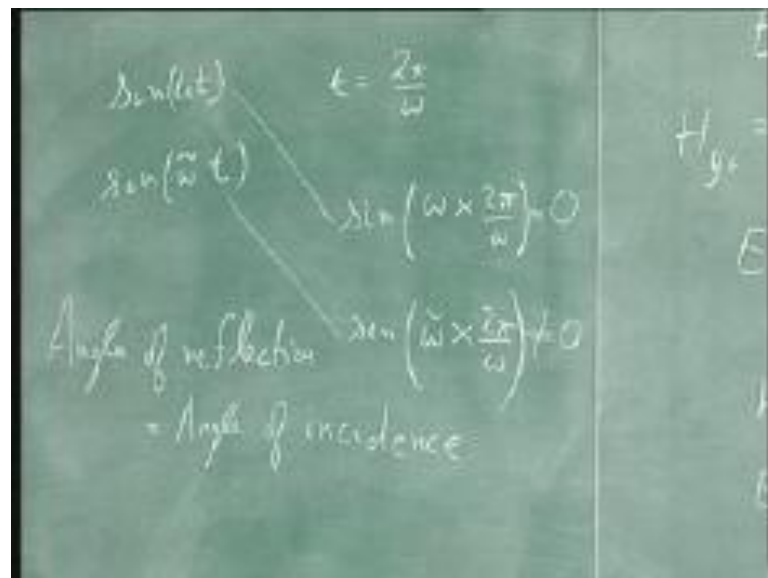
After all the whole point of the Maxwell's equations was that optics is in fact, just a part of electricity and magnetism. So, how is that we get such conclusions? To understand this we have to look at what is happening at this surface. We know that the electric and magnetic fields the tangential components of electric and magnetic field are continues on this surface. So, let us take a particular case whether electric field is out of the board magnetic field is the electric E cross H the magnetic field is this way electric field is out of the board. So, this is E and this is H , so that means, on this surface the electric field is tangential is coming out of the board and I can write this E tangential on this surface is a function of this direction.

Now, I have to give coordinates I am going to call this direction as x out of the board out of the board as x this direction as y and this direction as z . So, E tangential is a function of y as well as t clearly, because if you freeze time you will get for the electric field you will get that these are the point at which the face is the same. So, you will get a wave, but if you check at any other time the wave would have progress little further. So, this same wave would look like this. So, it is not just a function of y , so also function of t . This white curve is let say t equals 0 whereas, this round curve is e sum value greater than 0. So, if you sit at any point E tangential is a function of t whereas, you freeze time E tangential is also function of y .

Now, it is this E tangential that must be continues. So, I must construct the E tangential out of this wave and any other waves that are present due to it hitting the wall. So, the question I immediately I would like to ask is are there any wave in the system which have the same kind of E tangential. If you look at function it is clear that E tangential has the forms some $E \sin \omega t$ minus some $k y$. Now this is not same thing as k because k is in this direction, so if you look at this is λ . The distance between 2 crush is λ whereas, this distance is much larger than λ , because λ is this distance and. So, the diagonal is always greater than 1 of the sides.

So, the wave length along y is much larger than the wave length along the direction in which the wave is going. Therefore, this k_y is not same thing as k . k is 2π over λ . k_y is 2π over λ_y , and since λ_y is larger than λ , k_y is smaller than k . Now, the answer is fairly straight forward you know that if I took a wave which had the same angle, but going the other way. That wave would also have exactly the same k_y , because we drawing a triangle. The triangle could be drawn this way as well as then also it is the same triangle this would be λ and this would be λ_y . Since the angles are the same λ_y over λ would be the same for both of these cases the wave that went backwards from the interface with the same angle θ would have the same wave length. As this wave it would have to have the same frequency, because otherwise if you waited one time period and looked at the same wave. It should have not changed, but this λ_y would have changed by a different value that is to say.

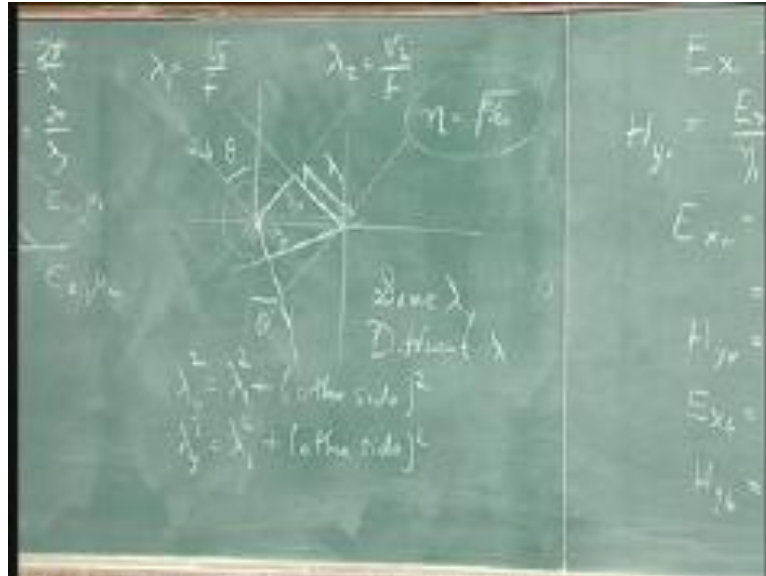
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Supposing the incoming wave was $\sin \omega t$ and the reflected wave was $\sin \omega t$ some other ω . There could be equal at $t = 0$, but now, if I go to $t = \frac{2\pi}{\omega}$ then this term becomes \sin of ω times $\frac{2\pi}{\omega}$ which is 0 again. But this term becomes $\sin \omega t$ times $\frac{2\pi}{\omega}$ which is not equal to 0 is not equal to 0 unless this also had the same frequency. So, it is necessary that the waves goes back has the same frequency and has the same wavelength along y. And having the same wavelength along y means it has the same angle with respect to the knob. So, that is why

angle of reflection is equal to angle of incidence. This is nothing but your reflection theorem of optics.

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Now what about refraction? Let seems little more difficult. Well, when we look at refraction we have to understand that lambda in a medium is the velocity in a medium divided by the frequency. Now if 2 media had different velocities this would be equal to lambda 1 is equal to v 1 or f 1; lambda 2 is equal to v 2 over same f f must be the same, because otherwise you would not be able to match across this boundary. I would just have to wait for 1 time period and I will have a mismatch. So, is the same f, but different velocities? So, my wavelengths are different. So, let say I have got 1 peak here and I have got 1 more peak here. So, this distance is lambda 1 therefore, this distance is lambda y. Now I can draw a parallel line here and I want some other wave on the other side which has the same lambda y. It does not have the same lambda y I cannot match across, because e tangential is continues. So, therefore, E tangential has the same properties on both sides.

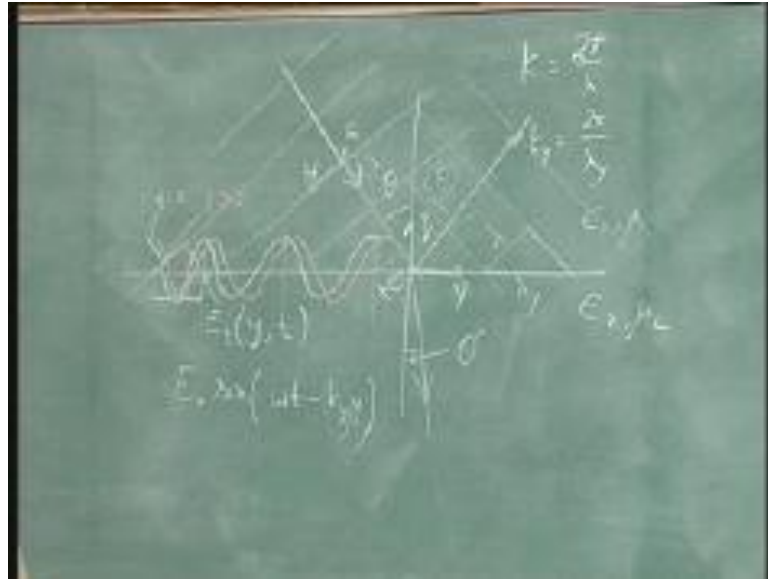
So, same lambda y different lambda different lambda, because different velocities. So, how do I do that? I can I have now, a campus earlier my campus said I have I can draw a circle of radius equal to lambda. Now unfortunately my wave length is changed I have to draw a smaller circle. This is my lambda in region 2; however, I want the still the lambda y. So, I want to draw a tangent to the circle that gives me the same lambda y and if I

now, draw the line through that that is the new direction in which the wave is going to go. It has the wave length that is the λ_2 , but its perpendicular wavelength is still λ_1 . So, you can see what is happened is the smaller the λ the closer to the normal. The wave is have to go, because otherwise the tangent will not give me the same λ .

Let just look at the triangle construction if I have a larger radius then the angle that I need to satisfies my pythogerous theorem is smaller. If I was smaller radius the angle I need is larger. Basically I have λ_2^2 is equal to λ_1^2 plus this third side. So, I have the same λ_2^2 , but in 1 cases λ_1^2 in the other cases λ_2^2 plus other side squared. So, if λ_2 smaller than λ_1 and this side is the same this side must be larger. Otherwise I cannot make it out this equation I need. I need it should be the same wavelength along the medium along the boundary wavelength in the region are different. How can I have that? I can only have that if this other side in medium 2 is longer than in medium 1.

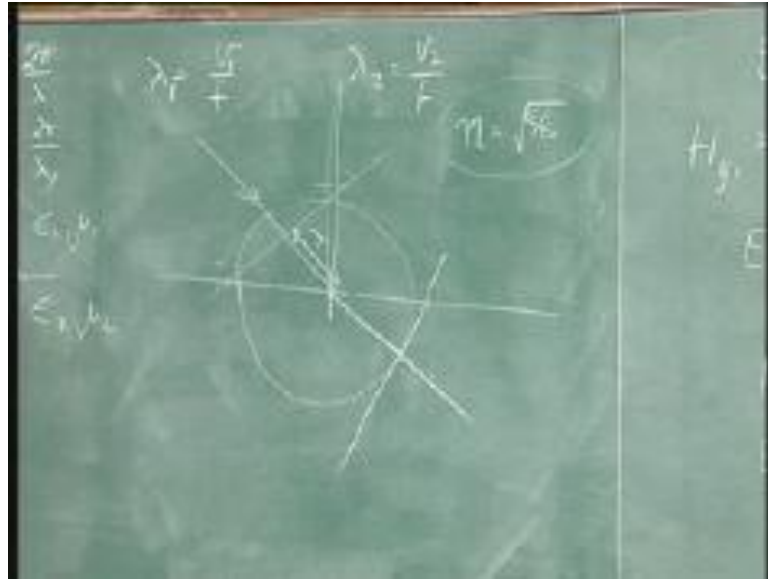
So, this is ultimately why you end up with refraction. That is the wave came with a particular angle θ , but that θ would not have been given me the same λ . So, the waves have to be bend and it gave me a θ' . It is very easy to prove that $\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_2}{v_1}$ has the ratio of refractive indices. You just have to know that the refractive index is actually the square root of ϵ over ϵ_0 . To use that definition it is very easy to prove the snells law. But snells law is not important what is important is to understand that? When a wave come the only thing that we have to adjust in the boundary is the boundary itself the parameter along the boundary. So, whatever wave comes on the other side must have the same property on the boundary as this wave.

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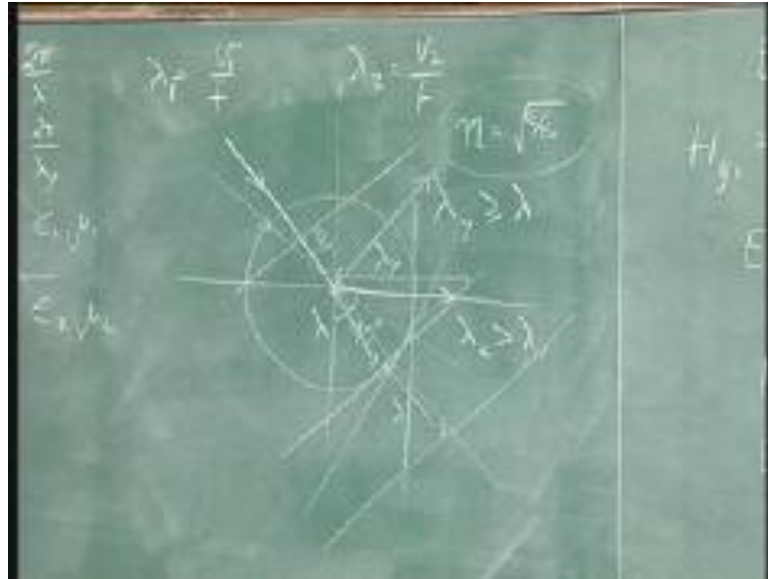
So, the wave on the other side must create the same E_y as the function of y and thus the function of t . The function of t means is the same frequency f and the function of y means is the same λ . And keeping λ constant while allowing λ to change, because λ is V over f that is what create snells law. The same argument can be used whether electric field is pointing out or magnetic field is pointing out. The equation gives you different and interesting results, but fundamentally refraction is simply the process by which the boundary condition is maintained even though the medium's property change. And it is clear that if the wave length in the second medium is smaller than the wavelength first medium then the wave will bend towards the normal. If on the other hand the wave length is larger than the wave will bend away from the normal sorry let me show you that case.

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You have a boundary your wave is coming and this is your wave length this is lambda. Let us suppose that you have the larger lambda. So, now, instead of having a circle of this radius I am having a circle of a bigger radius. Now, this bigger circle must well I have chosen 2 bigger circles and come back to that case later. This bigger circle must actually still give me the same wave length. So, this distance which is same figure as this distance? So, I must draw a tangent to a circle passing through this point which gives me a direction that points further away. Now you will have to pattern my drawing, because it does not look as if is very much further away that because of the way I draw my circle. My circle is not a very well drawn circle, but the basic point is that since I have the longer wave length I must tilde further towards the horizon in order to reach the same length lambda y.

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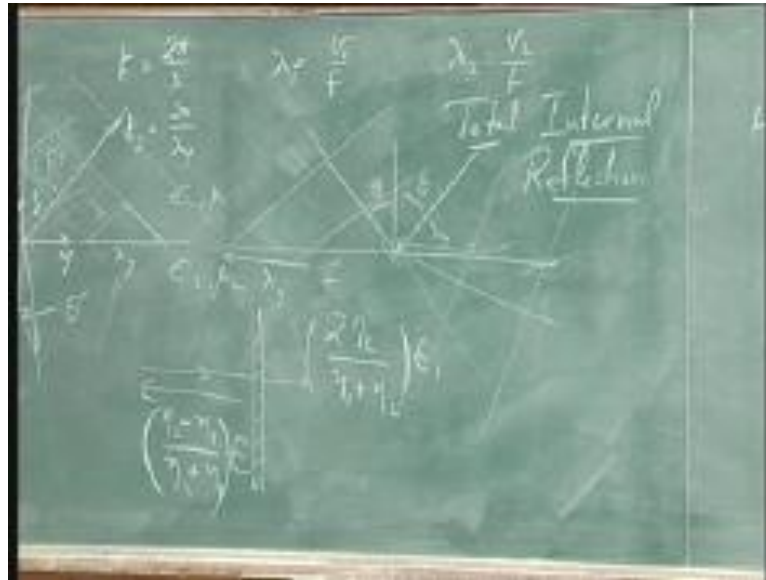


Now, this can fail and I will show you a case where it does fail I start with a wave and I choose a particular wave length such that λ_y is exactly equal to the radius of my circle. Namely λ_y is equal to λ in the second region what will happen? The only way I can have the same λ_y in both region is at the wave traveled exactly along the horizon. Supposing it travels this way the wave front will look like this which means I would had 2 larger λ_y . So, to have the same λ_y , so my wave front would have to travel like this. So, my incoming wave would have a θ my outgoing wave would have θ is equal to 90 degrees. This is the case; obviously, where λ_2 is greater than λ_1 for example, glass coming to air. Now, supposing I choose an angle which is greater than this clearly in that case my λ_y would be even smaller. And when my λ_y is smaller its within the circle that is there is no way I can create a λ along the boundary which is equal to what it is on the glass side, because the smallest λ_y i can make is λ itself.

You can see that from the construction if I have any direction then this is the direction in which wave fronts are λ apart this is λ this is λ . Then this would be λ_y λ_y is the diagonal. So, diagonal is always greater than this side. So, λ_y is always greater than or equal to λ . So, if I got a material where λ is, so large that it is greater than the λ_y that first side requires. Then I can never have a propagating wave on this side. Just any wave I create is going to not fit this requirement. So, it would not be able to consider this e of y comma t and since I cannot

create e of y comma t what will happen is, all the energy will reflect, so all of optics all of the optics we learned in school is nothing but a straight forward application of plane wave electromagnetic.

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When you have a wave coming in at a surface you must have that the wave length along the surface which is what I call λ_y and the frequency of the wave are matched on this surface. They are matched on this surface for a reflecting wave and they are matched on the surface for the transmitting wave. This matching requires that the reflected wave goes back with same angle with respect to the normal θ as the incident wave. For the transmitted wave what this means is that if the λ in the second material is smaller than the wave front have to tilde towards the normal. So, you will instead get a wave front like this and the tilt plus the smaller wave length will still give you the same λ_y . If on the other hand the second material had a larger wave length then the tilde would be away from the normal.

And now, the wave lengths are longer, but you still end up with your same λ_y . But if the wave length in the second material is, so large that is larger than this λ_y to begin with then there is no amount of tilde that will help you in which case there is no energy that goes into second region. All the energy reflects and that is call total internal reflection. Now, one thing that you get from this kind of analysis which you do not kept from optics. Is the amount of power that gets reflected versus the amount of power that

gets transmitted? The oblique case I am just introducing as a conceptual idea, but if you looked at the normal case we were able to obtain amount of power that got reflected and the amount of power that got transmitted.

Now if you did optics on this problem you would say that the ray of light. Ray of light hits the glass plane and it goes through snells law is satisfied law of reflection is satisfied, but you do not know what do with it. Ray optics does not tell you how much energy gets into the glass and how much energy reflects? However, if you do this kind of analysis we know that amount of electric field in the second region is given by twice ϵ_2 over $\epsilon_1 + \epsilon_2$ into E_i . And the amount of electric field reflected is $\epsilon_2 - \epsilon_1$ over $\epsilon_2 + \epsilon_1$ into incident electric field, because we know these 2 numbers we can actually. Now, design a system we can ask what can we do to make glass non reflective.

And in fact, it is possible to do that what you do is? You add a very thin layer of another kind of material and if you do the analysis with that kind of material you find that there is no reflection and there is only transmission. These kinds of analysis are possible only, because we are able to talk about the amount of feel that gets transmitted and the amount of feel that gets reflected. I will not be going any further into reflection and the refraction of waves in at a boundary. But it is a very interesting topic and you can read up on it there are number of very good books including your own book and learn about more interesting features in them.