Electro Magnetic Field
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Lecture - 37
Radiation and Circuits

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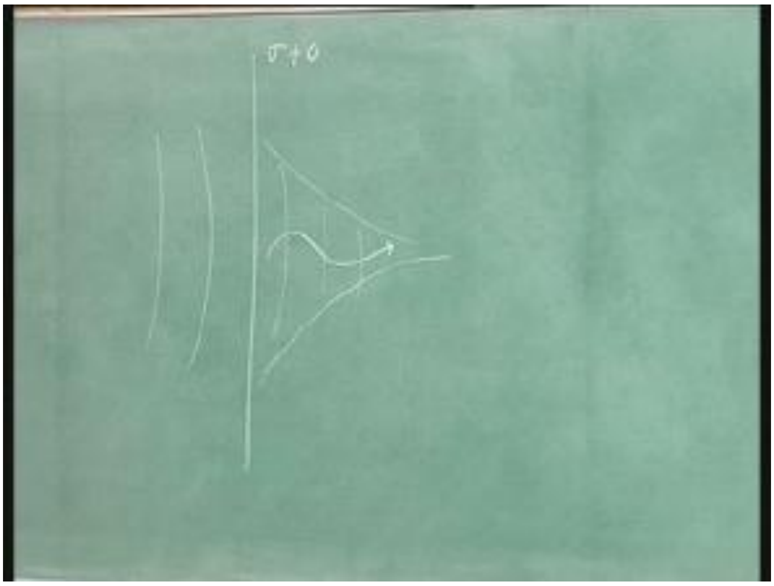

Good morning. Last time I hope I gave a fairly good field for how an electromagnetic wave enters the conductor and dissipates? And from the description I gave I hope you understood that an electromagnetic wave, when it meets up with a material let sigma not equal to 0 that it decades exponentially and also propagates. So, it has both the real components which give its a wave length and an imaginary component that gives it a decade. And my conclusion from all this was that materials body like circuits can observe these waves. And I just suggested that if they can absorb them they can also emit them. Now, what I am going to do is change tacts and go back to looking at the material body by itself. So, I am going to now look at the problem where I have for simplicity a bar.

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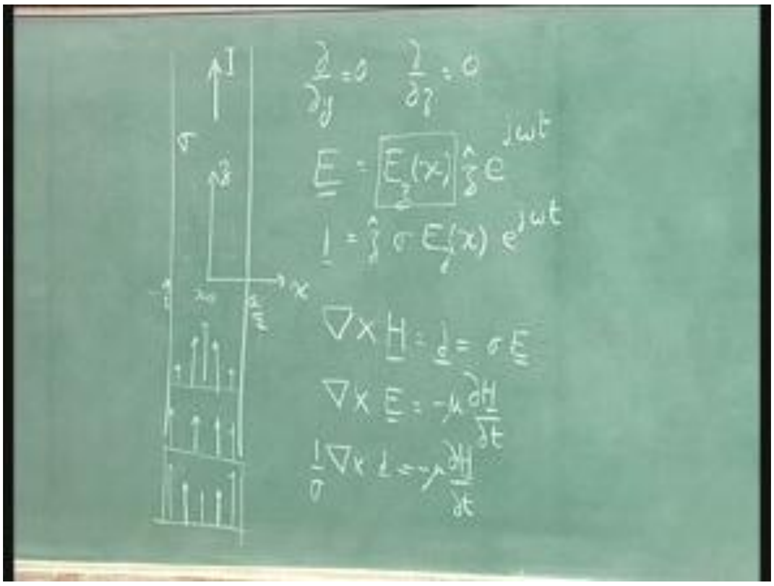

A plate the plate is or my coordinates x and z this is x is equal to 0. So, x is equals a over 2 and minus an over 2. And I have applied an electric field in the z direction and I have a current I. And I am assuming that this plate is long and wide that in the direction out of the board. So, that the only thing that present are variations along x that is variation along y is 0 variation along z is 0. Current is flowing in z, but it is not a different distribution here then here then here. It is a same distribution as you move in z what I do not know is how this current is moving? I assume that we applied electric field which must be uniform, because I was assume that nothing was varying in z takes the form the electric field E is some E z which is the function of x along z. I am allowing them to be functions of x, because this is the material and the material has the conductivity sigma. It means that the current density j is also in a z direction and is equal to sigma e which is sigma E z of x.

Now, additionally I am assuming both of these are functions of time up to now when we did this problem we did it in static. Now, let us do it in time varying systems, so I am assuming E to the j omega t. So, it is a phaser. So, naturally I would take the real part or the imaginary part of this to get the actual electric field and the actual current. So, the statement I am taking here is assuming sigma is the real quantity the current and the electric field are in face. Now, what I want to do is to determine this function E z of x that is I want to know whether the current is flowing like this or is it flowing like this or is it flowing like this. And you know the answer of course, you know it is the last 1, but I like to get it out of the equations. Now, when I made this assumption that the electric field is only in the z direction.

I am assuming that there is no electric field that is in the x direction which means that there is no accelerating motion of charges which are breathing outwards and inwards. It is a good approximation and it is true for the systems we are looking at. So, I want to go further into it, but you can relax that the assumption and you will find that this is the interesting solution. So, now, you want to get the equation and the equation is the same equation, but let us get. It I have 2 equations 1 is amperes law I have dropped the del D del t terms. So, I have gone back to looking at ac low frequency ac systems and the other is faradays law. Now, since I am interested in the current I am going to get an equation for that current rather than for the magnetic field or the electric field. The way I do it is of course, to understand that j is equal to sigma E. So, I am going to say this equation is really curl of j with a 1 over sigma outside is equal to minus mu del H del t. So, I have substituted for E as j over sigma. I can see that there is an another equation I can used to eliminate this H, so I will take curl of both sides.

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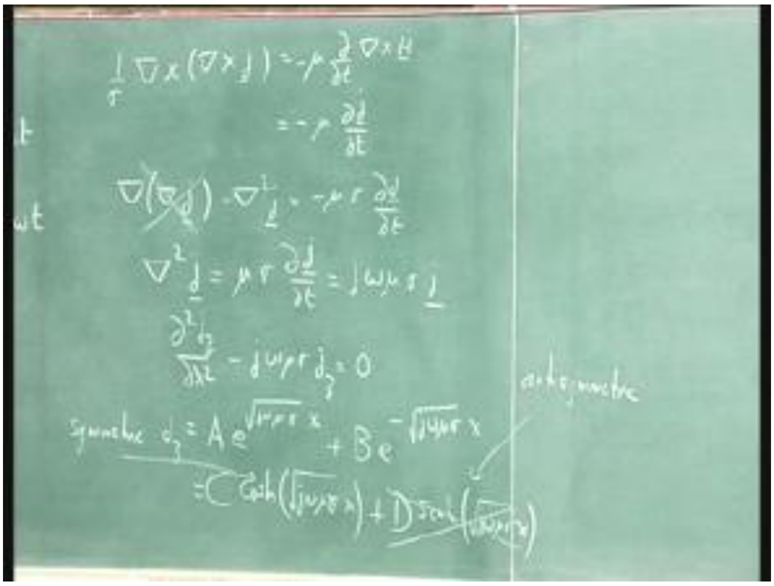

So, when I take curl of both sides. So, I get 1 over sigma curl of curl of j is equal to minus mu del del t of curl of H which is equal to minus mu del j del t. Now, we have used this curl curl operators we know what it is? This is equal to gradient of divergence of j minus del squared j is equal to minus mu sigma del j del t. But if you look at this j is in the z direction, but depends on x therefore, divergence of j must necessarily be 0. So, This comes to the fact that we assuming there is no Ex there are special cases where Ex is present, but we do not worry about it. So, this terms goes away and we are left with the same equation the same skin depth equation that you got last time namely del squared j is equal to mu sigma del j del t which is j omega mu sigma j, because j is go as e to the j omega t.

Now, do not get confuse between this current density vector and square root of minus 1. I have to draw the right this in a different way, but I am hoping that you want get confused. So, I want to solve this equation which is the same equation. So, we know that we already seen 1 solution of it, but now, I want to solve it in a different sense I wont to solve it for this geometry. Now, since I am driving current through this plate there is no reason why what is happening at this wall should be different from what happening at this wall? I expect a symmetric solution. So, I do not expect the solution that will look like that kind of solution is not going to happen. Because I have electric field pointing 1 way to the current cannot point 1 way to the left and 1 way to the right.

So, this kind of solutions is not possible all solutions that I get must be symmetric. Now, if I look at the equation and substitute that the fact that the only variation is in x i get del squared j z del x squared minus j omega mu sigma j z is equal 0. If you look at this equation it is fairly clear that the solutions are exp1ntials. So, j z is equal to A e to the power of square root of j omega mu sigma x plus B e to the power of minus square root of j omega mu sigma x. Now, this is perfectly correct, but I want to take a advantage of the fact that I know I have symmetric solutions. So, I can write any combination of exp1ntials in terms of hyperbolic equations hyperbolic functions. So, I write this in straight as some C cosine hyperbolic square root of j omega mu sigma time checks plus some D sin hyperbolic j omega mu sigma times x we did this earlier when we were talking about bounded system.

And solving leplosses equations cos h is nothing but E to the j omega mu sigma x plus E to the minus j omega mu sigma x over 2, sin h is the same thing with the minus sign. So, using these you can always construct this and using this you can always construct this. So, is the same whether I write it with general variables A and B or write it over general variable C and D. The advantage of doing at this way is that sin hyperbolic is anti symmetric and cos h hyperbolic is symmetric. Since I know the solution I am looking for must be symmetric I can through away the solution directly. I do not have to worry about it. The sin hyperbolic will look like this. It will be 1 sign for positive x 1 sign for negative x.

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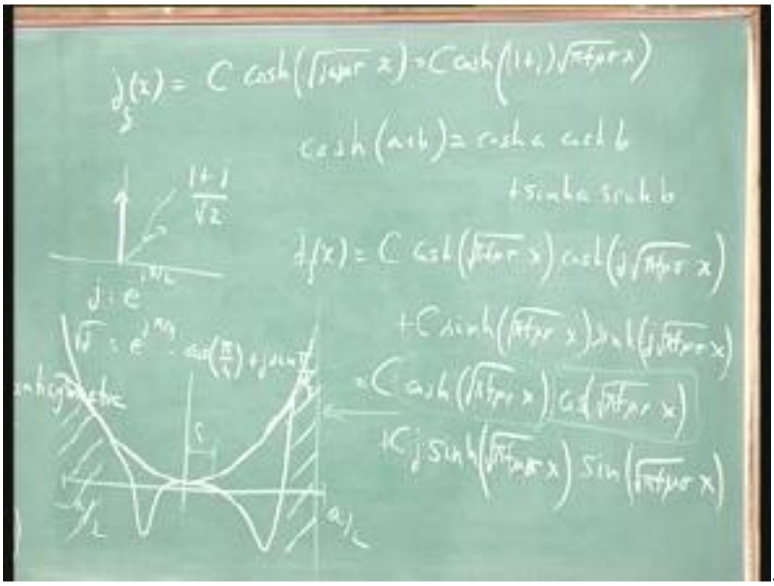

So, my solution in j, j z of x is some constant times cos h hyperbolic of the square root of j omega mu sigma times x. Now, you write this out square root of j if you done this to 3 times now, square root of j as unit amplitude, so it will be 1 plus j over root 2. Another way of seeing this is j is equal to e to the j pi over 2. So, square root of j is equal to e to the j pi over 4 and e to the j pi over 4 is nothing but 1 plus j over square root of 2. Because that is equal to cos h pi over 4 plus j sin pi over 4 and each of this is 1 over root 2, so that where this come from. So, I can put that in there is equal to cos h the 1 over root 2 I can take inside. So, that is gives as of similar skin depth 1 plus j times root pi f mu sigma times x. Now, cos h a cos h b; sorry cos h of a plus b is equal to cos h; just the addition rule for hyperbolic functions, so I will substitute that in here.

So, I will get that j z of x is equal to some unknown constants C times cos h of square root of pi f mu sigma times x cos h of j root pi f mu sigma times x plus C times sin h of pi f mu sigma x. I hope I am not making a mistake sin h of j root pi f mu sigma x I am just applying this formula here. Now, cos h of j anything is equal to j cos h of the something sin h of j anything is equal to j sin of that we can confirm that cos h of j x is e to the j x plus e to the minus j x, so there is no j sorry divided by 2. So, that is nothing but cos h sin h of j x is e to the j x minus e to the minus j x over 2, but to make it sin I need a 1 over 2 j. So, I multiplying and divided by j, so cos h of j x is cos h x sin h of j x is j sin x. So, substituting all that what do I get? I get that this is equal to C cos h of root pi f mu sigma of x cos h of root pi f mu sigma x times j sin h of root pi f mu sigma x times sin of root pi f mu sigma x where I we reached what we are seeing here is that j is a complex phaser.

Now, you want to take the imaginary part of this spacer, because you started with the phaser representation that either you have to take the real. Or the imaginary part that, you decided at the beginning whether you are representing in cosines whether you representing a sins. So, take one or other of this when you finally, open a real quantity, but look at this expression there is a piece that represents of growing or decaying amplitude and there is a piece that represents in oscillation. So, let us assume we are taking real part of j as our real function. If you plot this quantity this is an over 2 this is minus a over 2 this cos h remember this is 1 over delta is really a function that looks like this. It grows very rapidly and the scale length on which it grows is delta cos h on the other hand is an oscillatory function. And it results in this same function actually doing something like that.

In other words, the amplitude the envelope is given by cos h and the actual face is given by cos h. This is very similar to what we got last time, because last time we had a wave this is coming in from the wall into the wall. So, we started with an electric field along z direction magnetic field along the y direction and there was penetration in to the x direction and the wave decade exp1ntially. Now, we have got a different view point of this problem there is no wave any more instead the driving current through a plate. And we drive current through a plate we get exactly the same picture. Once again we have a wave that is both oscillatory and decaying except it has to be symmetric. Because there are 2 walls this no reason to prefer 1 wall over the other and what you find these are the current is mostly at the edges.

So, we have found out what j is and since E and j are proportional to each other wave also find out what electric field is. Now, the question that should occur to you is how can such a electric field exist how can it be to the electric field instead of being uniform. Because after all I am applying a uniform z direction electric field, how can the electric field be concentrated out here and not be strong elsewhere? The answer is that is faradays law in action your electric field does not just come from the applied voltage the electric field also comes from del f del t, so del h del t. So, what can happen is if you look in a middle the electric field is uniform which means if I take curl of E it is going to be 0. Because whatever electric field I. get here is the same electric field on this side therefore, my curl is going to be 0. Since the curl is going to be 0 I must have del H del t 0, but I have an e to the j omega t. So, del del t is not 0.

So, H itself must go to 0, so the magnetic field is 0 on axis. Since the currents are creating this magnetic field is quite reasonable to the magnetic field is 0 at the center. As you go further away the magnetic field grows, because it grows you start having this effect. The curl H means that if I take any loop like this magnetic field is 0 on axis magnetic field is not 0 at other values of z. And therefore, there is the j that j in fact, is the E where we supported. The result is you have a electric field the electric field creates a current this current creates the magnetic field which in turn modifies the electric field what does this what is the effect of this modification? The magnetic field is into the board or out of the board therefore, what is happening here is the magnetic field being out being in the y direction causes a force a j cross B force on all the electrons in the material.

Therefore, the electrons the moment they bare current or forced due to the j cross b force to move outwards. The result being that the current preferred to go along the side of the wall leaving the middle empty that is what we are seeing here. Because we are self consistently solved this problem we took curl of H in to a account when we derive this equation and when you are solve this we have automatically solved also the j cross b effect. And therefore, this results shows that the electric field created a magnetic field that magnetic field put the current out and the current choose to stay on the outside of the conductor. Let me remind you last what we learned last time we learn that if you take copper there is a constant 0.066 meters divided by square root of f. This is the skin depth in copper, so if I am talking about 50 Hertz's square root of 50 is around 7.

So, it is around 0.9 centimeters. So; however, pick a bar I have take a bar I have only the last centimeter on either side is going to carry a lot of current the rest of it is basically going to be current free. This is the picture that I have got is exactly the same as what we did last time there is no difference. Last time, we had a wave coming in and the wave decade as the penetrate the material. This time you are talking about a wave we just look at the material itself. And what we find is that the current prefers to set near the edge and tends to have oscillations as well. In fact, it exactly the same solution whether you had the wave or you did not have the wave now, you can do the same equation.

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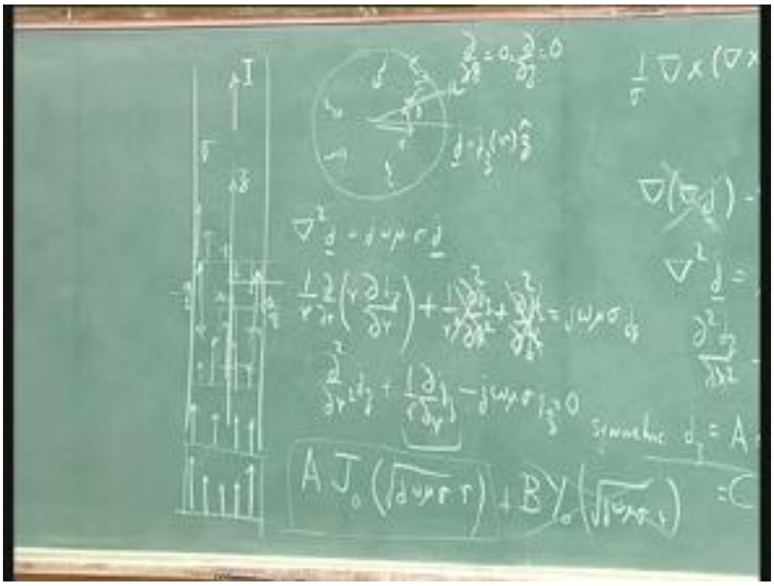

If you did with the round wire if you had a round wire for the radius a. You have the same equations this equation is still true. So, del squared j is equal to j omega mu sigma j where j is equal to j z which is the function of r, but not a theta and not of z and is along the z direction, but we have to write out del squared now. For a cylindrical system del squared we can look up your books is 1 over r del del r of r. So, this is the operator in the r direction there is no operator we need to keep the remaining operator 1 over r squared del squared del theta squared plus del squared del of z squared of j z. These 2 go away, because we are assuming just as we assuming for this plate we are assuming that del del theta is 0 del del z is 0 this is theta this is r and out of the board is z.

So, this is only part that remains of del squared z and it is equal to j omega mu sigma j z. It is almost the same equation let me write it out 1 over r del squared del r squared of j z sorry there is 1 over r there plus 1 over r del del z of j z minus j omega mu sigma j z is equal to 0. So, if you forget about this term is the same equation essentially it is the same equation. So, this is what cylindrical geometry has done for you. It has added 1 new term, because when you take this del del r and act it on r and when you get a 1 over r del j z del r. If you acted on del j z del r and you get this 1 which is the original term. The solution of this equation is actually in term of what I call Bessel function answers are J 0 of square root of j omega mu sigma r some A times this plus B times Y 0 of j omega mu sigma r. The J is like cosines the Y is like sins and more importantly the y actually blows up at r equal to 0.

So, similar to those arguments we threw away this one and we keep only this. And in detail you have to work out what this function looks like. But if your wire is large compare to the skin depth delta it turns out J 0 of j omega mu sigma r does not look any different from this solution it looks in fact, the same as this solution. So, there is some difference here, but this portion is identical whether it is the plane Cartesian or it is cylindrical. However, when delta becomes large compare to the radius of the wire then you have to take out the full expression and you have to work it out.

I introduced this only to mention that the cylindrical problem is no more difficult than the Cartesian product is the same problem. And in both cases what we are seeing is the wave entering in the from the sides to the middle. Now, if we try to do E cross H for this problem what do we find? There is an oscillation this oscillation does not dissipate any energy. It corresponds to E cross H going in and out part of the energy is getting dissipated due to j dot E. This corresponds to E cross H going in verse you will see both this terms present one part of the term will correspond to dissipation. And one part of the term will correspond to energy going in and out. This is generally true when you do pointing theorem.

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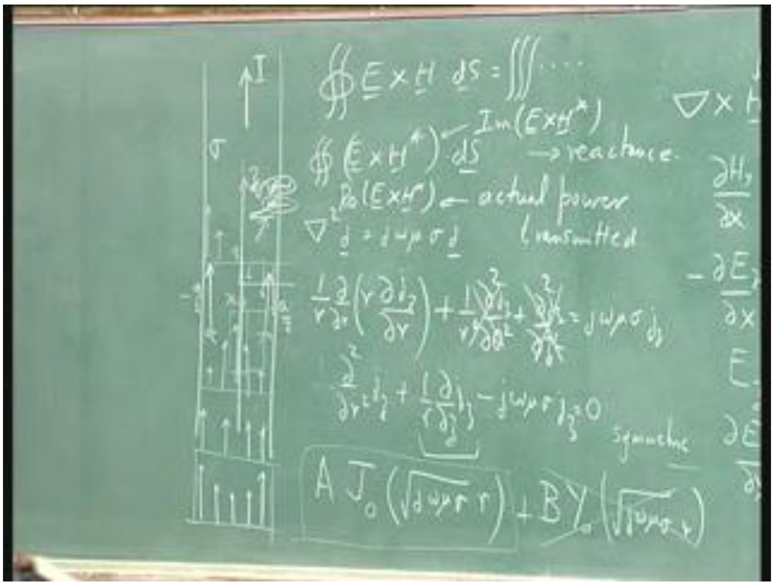

So, if you look at the pointing theorem expression we had an expression that look like E cross H dot d s is equal to volume integral various things. If you put phasers in here and we rewrite this equation I will do it next time, but I am just anticipating the answer what we will have is the real part and a complex and an imaginary part. So, we end up having to use E cross H star dot d s the real part of the E cross H star will give as the j dot E part. So, j dot E part is real part of E cross H star which is actual power transmitted. The imaginary part on the other hand represents oscillations.

It represents reactance we will come back to this ideas little later, but it is quite important and when I derive the phaser version of the pointing theorem. It will become quite clear, but you have to understand here itself there was both d k due to the amplitude and an oscillation due to the waves. So, when you take E cross H and average it what we find is that the piece that is matched in case, so the average does not go to 0 and that is the piece that 90 degree is out of face which does go to 0. And when you do the averaging they the piece goes to 0 represents reactance the piece that does not go to 0 represent power transmission. So, we have come up with 2 concepts so far.

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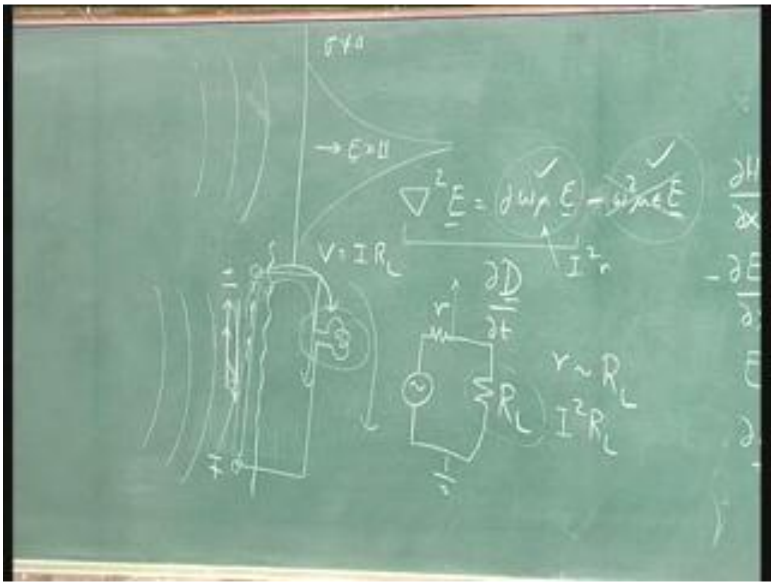

One is if you have plane waves and they reach the material rows conductivity is not 0 the wave get dissipated the decay as they entered the material. Second is when we look at such a system we find this is the direction of E cross H it is in to the material. Now, I actually want something useful out of this structure I want to make a bulb blow by having radiation come on this plate. Now, a bulb blowing is only a example if I can do this then I can put him electronic circuit here and make use of the radiation. For example, on the other side I could have a transmitter sending information and the electronic circuit here can interpreted that information. But what I really need is the way of getting this wave energy not into the plate, but into the circuit why is that a big difference?

Well, if you look at this the energy of the wave goes into the plate and becomes heat. There was no information transfer all that the energy did was it made the plate hotter. It did not cause a of current to flow in the outside circuit. For a current to flow in the outside circle we need something more and what I want to discuss for the rest of the lecture is just what it is that you need. And it is going to give you a sampling a taste for what goes into the theory of antennas we have. So, for restricted ourselves to del squared E is equal to j omega mu E plus omega squared mu epsilon then a minus I guess E you can check the sin of this term. Now, we have ignored this term and you kept only this portion and we have done skin effect and analyze what happen inside the plate.

Now, the problem is if I want a current to flow in the rest of the circuit what do I need? I need that current flow up I need that voltage difference is developed between the top and the bottom which then appears as current in the rest of the circuit. Now, current flowing up seems reasonable all I need to do is to have my electric field in that direction and then the current will flow. So, if my electric field where upwards my magnetic wave where side waves that should happen. Well, there is only one problem if the current flows up and a potential develops what what does the rest of the circuits see as the cost of this potential? Rest of the circuit must see coulomb law is providing this potential. If coulombs law is going to provide this potential then there was a charge built up, so there must be positive charge here and negative charge here which is then driving a current through the bulb.

Or the electronic circuit that I have got there for the moment I have the positive and the negative charge here I have an electric field connecting this positive charges and this negative charges. And since this is an oscillating electric field this positive and negative charge is going to oscillate and therefore, this electric field is also going to oscillate which means I necessarily have the del D del t. I cannot escape it if I want to communicate with the rest of the circuit it is the del d del t that is going to allow me to communicate with it what this did was it converted things to heat? It did not give me useful output for useful output I really have to go back to this term and it is only this term I can make this plate look like a battery to me. Now, in a real system you have both of these you have conduction current and you have displacement current. So, how do we know what it takes to make an affective antennae?

Well, the way to look at it is the following. If you ask how much of my energy is going into heating and how much of energy is going into making a battery. The answer is somewhat like this I want to deliver energy here. I have a voltage source here and the voltage source has an internal resistance, which is represented by this j dot E. And we know very well that if you have a source and in internal resistance and in external RL that the maximum energy is transferred to RL when the internal impedance is equal to external impedance. Either R is larger or R is smaller you do not end up delivering power to RL. So, we require that r should be equal to RL of the same order if I am going to effectively transfer energy from my voltage source to my electronic circuits. So, I need this j dot E to be essentially representing the same amount of energy loss as this RL. This is one kind of I squared this is I squared r this is the other kind of value this is I squared RL. So, this term and this term should be equal .

Now, the problem with this is that RL is given to me it is something it is my electronic circuit. So, when I take this system that current that is going to flow here has this IV characteristic V is equal to I RL which means the amount of charge separation. I require is determine by the external circuit. Internally the amount of charge separation I have depends on ratio these 2 terms. So, how do I how do I adjust this? I mean I have 2 physical phenomena in 1 case I have a wave which is coming with its own characteristics. In other case, I have a circuit which has its own characteristics and yet i want the wave to talk to the circuit. So, I want to have 1 more adjustable knob. So, that I can adjust the ratio of the amount of j dot E produced to the amount of del E del t produced. Now, this is where antenna theory comes in the picture I am going to draw is not quite reasonable.

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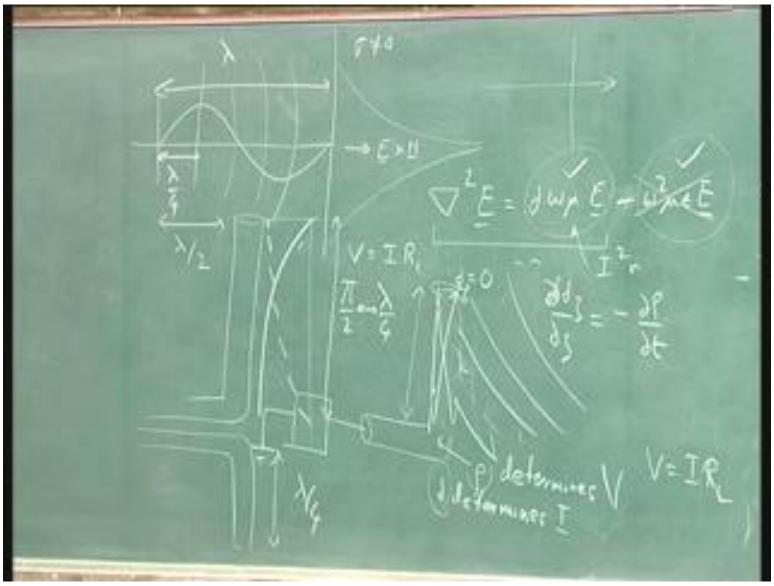

But it is the best way of looking at it. Supposing I had my wire and instead of connecting both end of my plate I connected only one end to my electronic circuit my wave comes and when my wave come of course, it causes current. Now, this current better be 0 here, because it cannot leap into vacuum. It is struck within an material it can be anything it is here, but it should be 0 there. At the same time my charge can be anything it likes here, but the charge whatever it is here is what is going to determine the voltage that I deliver.

So, row determines V and j determines I. Now, supposing instead of hitting this antenna flat supposing the wave hit it at an angle supposing it hit it like this. Now, you can see what would happen is that this j due to this part of the field has 1 sign, but due to this part of the field has the opposite sign due to this part of the field does the same sign.

So, within this wire the j would start having a shape. And similarly, row would have the same shape, because divergence of j del j z del z was minus del row del t. So, j and row would not be constant they would be oscillation functions of z provided this wave had not just propagation into the wire propagation along the wire also. Now, supposing that is possible what is the shape that we would want ideally? Well, let us easy to answer because we know that row determines V j determines I. So, and we know that V over I is RL. So, what we would want is the current that started at 0 groves to something. At the same time, because the current is 0 your charge density is maximum. The charge density is falling and you want sufficient length. So, that the ratio of row to j is whatever is required by ohms law. In practice this RL is very small.

That means, is that row should essentially be 0 and j should essentially be maximum what is that require? I have my wire j starts at 0 I want j to become a maximum row starts at maximum I want row to become 0. So, this distance if you look at this this distance should corresponds to pi over 2 in face pi over 2 in face means lambda by 4, because if you look at any wave this length is lambda corresponding to 360 degrees in face. This is the length we are talking about j goes from z to maximum. So, this is lambda by 4. So, if you want to effectively coupled from your wire are your plate which is receiving this electromagnetic radiation into an another electronic circuit. Then this wire has to have a length which is sufficient to change j from 0 to something close to maximum, the row from something close to maximum to something close to 0 which means that this wire should be one fourth of lambda. Now, actually there are 2 wires that is the wire and the wire to the ground.

So, when you look at an practical antenna you will see 2 wires each of which is lambda by 4 which means that the total antenna size is lambda by 2. And this is what is called a dipole antenna. The details do not matter what is the important to understand is if I have a plate and the wave goes straight into the plates exactly straight into the plate with no variation along the wire and just depositing heat. I am not giving any information to the circuit for me to give information to the circuit I must permit variation along this wire why? Because the ratio of j and row intern sick to the skin effect is not the ratio of j and row that external circuits wants. The skin effect gives me the 1 aspect of j and row and the external circuit requires another 1.

So, I need to have the matching and the best way of matching. In fact, the only way of matching I am I said is take advantage of little bit of variation in the wave along the direction of the antenna. So, the wave in fact, varies along the antenna as well as in to the antenna how much variation is required that is a practical matter it turns out that external circuits cannot match very well out here they need to be more like this. Current is at maximum the charge is at a minimum. So, the length of the antenna that matches well is lambda by 4. And you need 2 wires each lambda by 4. So, the length of any good antenna is lambda by 2. Now, thing about this you have your cell phone and your cell phone is 2 3 centimeter long. The frequency at which the cell phone works it about Gigahertz and Gigahertz in vacuum is a length of wave length of 30 centimeter which means lambda by 2 is 15 centimeters.

But your entire cell phone is so small kindly I am not sure how people talk it barley reaches out of ear how does the 15 centimeter lambda by 2 antennas fit into this. It will device what they do it should rap this antenna back and forth back and forth back and forth inside this cell phone details of those time belong move along to an antenna course to a design of an antennas course, so you want cover it here. But what is important to understand is that this fundamental requirement. That you must have variations along the wire in order to usefully extract power are usefully radiate power cannot be avoided. It is the fundamental requirement of all antenna design and no matter how advance of course, you go to the picture is not going to change. You still require that your current goes to maximum and your charge go to 0. And therefore, you require something like a lambda by 2 antennas.