Electro Magnetic Field

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Lecture - 35
Skin Effect

Good morning. Today, we are going to take final step where you are going to talk about how the pointing vector allows radiation on the one hand. But we are talking about as our 1 dimensional wave equation and circuits on the other hand how they talk to each other. I will only touch this topic, because it is not properly part of this course at all. This course is suppose to stop short of antennas; however you cannot understand the rest of electromagnetism till you know where antennas are suppose to fit in.

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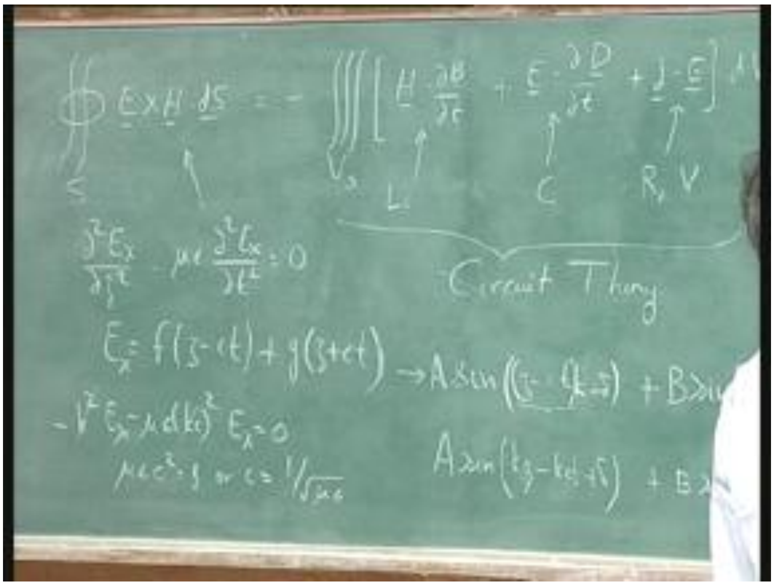

So, last time we have returned down the pointing theorem the pointing theorem said the surface integral of E cross H dot ds over a surface is equal to negative of volume integral over the surface of magnetic energy. The flex of magnetic energy the rate at which the magnetic energy is increasing with the minus sign. So, rate at which magnetic energy is decreasing similarly rate at which electric energy is decreasing. And the rate at which energy is getting dissipated, and as I told you if you look at this piece this is basically what we call are inductor L. And this is what we call are capacitor and this peace is both resistors and voltage sources. So, the entire right hand side of this equation is circuit theory. And what this equation is saying is when circuit theory fails the failure is represented by this surface integral of the quantity E cross H dot ds. So, as long as this conservation theorem holds you are basically satisfying kvl and kcl why do I say that we are satisfying a kvl and kcl? Because it is this term the E cross H that includes the terms that break down E equals minus grad 5 except in capacitors and V is equal L dI dt except inside inductors.

So, inside an inductor you have this and inside a capacitor you use stored energy, but outside this components. If you look at what is happening inside the circuit what we are saying is that we can do well addition of voltages. And this holds that is loop integral E dot dl is equal to 0 that is what this is saying and that is what this is saying as well the place where this fails is inside an inductant. But, with explicitly take inductors into account we do that because inductors are become low frequency elements. So, we can take care of them without invoking waves, so magnetic stored energy electric stored energy and dissipation and batteries is already part of theory of circuits. Now, you want to add 1 more piece now to prove that we need to add this piece I am going to talk about a particular problem. We already done the case we found one solutions to this wave equation.

Let me remind you the wave equation as solved was del squared E x del z squared minus mu epsilon del squared E x del t squared was equal to 0 we solve this equation. And we found that E x is equal to some function of z minus ct plus another function of z plus ct and other I discuss with you this f and g can be any functions. So, I use things like triangles and rectangles just to make that point, but in electrical engineering. We have some favorite functions the function that is easy to understand and manipulate. And, so we usually talk about sins and cosines. So, we say that E is some A sin z minus ct plus. In fact, I will give an angle also plus delta plus B sin z plus ct plus delta prime as my function for E x since. This can be arbitrary function they can also be this function there is nothing in there that they cannot be only thing that important is there should be functions of z minus ct which tells us they can actually be some z minus ct times.

Something I am going to call it k plus delta and z plus ct times k plus delta prime as long as the functions are z minus ct and z plus ct. They fit this form you write this out what you get A sin kz minus kct plus delta plus B sin kz plus k ct plus delta prime. So, it some function of z some function of t through this things now we go back to this equation.

And substitute that what we get the second derivative of sin of anything gives me minus sign of the same thing. So, I will get minus k squared times the entire function E x minus mu epsilon by take the time derivative I will get minus kc squared. And plus kc squared here which are both kc square times E x equal 0 which tells me, because k squared cancels out that mu epsilon c squared is equal to 1 or c is equals 1 over square root of mu epsilon as you worked out earlier. So, this satisfies this equation now, I am going to thing about particular problem the particular problem I am going to think of is that I have a slap of copper.

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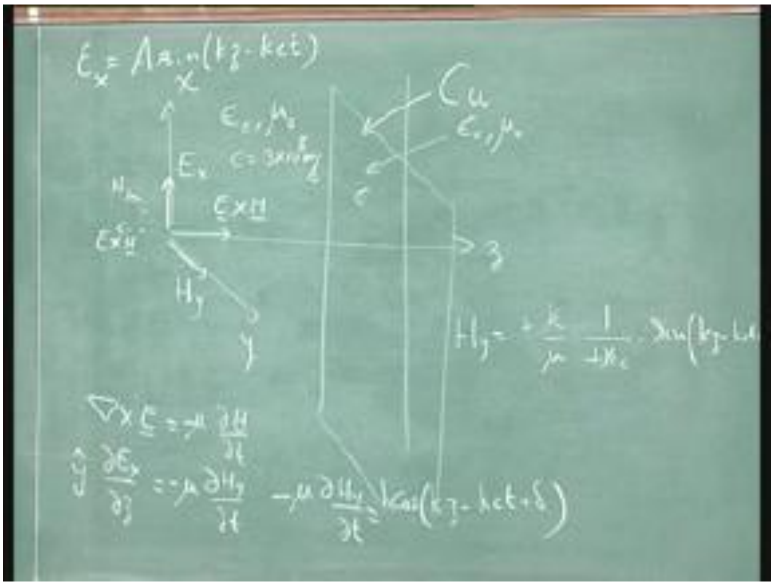

And this is at z direction; this is the x direction this is the y direction. So, I have this solution for the electric field arriving at the copper plate electric field is this way magnetic filed is this way. And E cross H is in the z direction now of course, you know that the magnetic field is along plus y for this problem, because when we do and you want to solve for the magnetic field we use paradise law. So, we will say curl of E is equal to mu the minus sign del H del t we are working in the y direction E is along x derivative along z. So, it is y hat del E x del z is equal to minus mu del H del t, so we take del E x del z that will take a sin and make it cosine.

So, you get that minus mu del hy del t is equal to cosine of under a derivative k times the cosine of kz minus kct plus delta, if I now take the time integral of both sides that should give me H y. So, that will tell me that H y is equal to minus k over mu times the integral of this with respect to t 1 over minus kc times sin of kz plus kct plus delta which is the same function of as for as E x. We know that H y and dx looks the same the k cancels out the minus sign cancels out. So, H points along the plus y direction, but you look at this case will be dividing by plus kc. And so there is a minus sign left over and H will point the other way in which case E cross H will point in a reverse x z direction.

So, this is the forward wave this is the reverse wave, so for now let us not worry about the reverse wave we only worry about the forward wave. So, I have the wave that is A sin kz minus kct I can always observe delta into t I shift my time and delta can me made 0. So, that is my wave E x is equal to A sin kz minus kct this is vacuum, so it is epsilon not mu not. So, c is equal to 3 into 10 to the 8 meters per seconds square per second let us assume here this copper is also epsilon not mu not, but copper is a conductor. So, it has a sigma now this wave is going to come and land on this copper plate. What happens well, we at the end of the last lecture I had worked out most of the boundary condition the matching condition across an interface I had worked out that just does not static.

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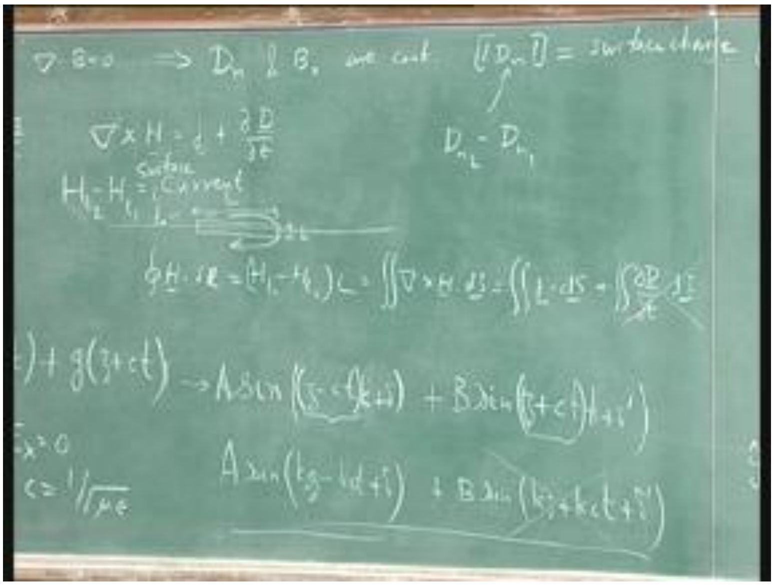

Because divergence D is equal to row and divergence B is equal to 0 D normal and B normal are continuous with only condition being D normal this is now we indicate a the jump in D which is Dn 2 minus Dn 1. So, you want to say Dn 2 is equal to Dn 1, but there is one condition which is the jump in D normal is equal to the amount of surface charge if you happen to have a system whether a surface charge is present. They knew when you do your gaussian surface you always include that surface charges value. So, the D normal will not be continuous it will jump according to the amount of surface charge you have that comes straight form this equation when we did the paradise law we immediately saw E tangential is continuous. And when you look at amperes law here a slight difficulty you have the material; you draw a strokes curve. You make sure that the height H is much smaller than the length L and we do a integration in some sense. So, I choose a direction in which I will do integral H dot dl integral H dot dl on this curve is hardly this 2 vertical line hardly contribute anything because H is much smaller than L.

So, it is equal to H tangential 1 minus H tangential 2 times capital L because it is H dot dl over this section. And then minus H dot dl because the direction of the integration changed on this surface. So, you get H tangential 1 minus H tangential 2 times L, but it is equal to surface integral curl of H dot ds which is equal to surface integral j dot ds plus surface integral del D del t dot ds. The same argument that allowed as to through del B del t namely the area is L times H since H is very small area will be very small. So, this term goes away; however, if you have a surface current on the surface that surface current does not goes. Because ultimately what you saying is loop integral H dot dl is equal to current enclosed. And if the current is basically on the surface then you will have some amount of current capture. So, you end up with the condition that says H tangential 2 minus H tangential 1 is equal to current surface current at that point. Now, let go back here and look at what this is saying.

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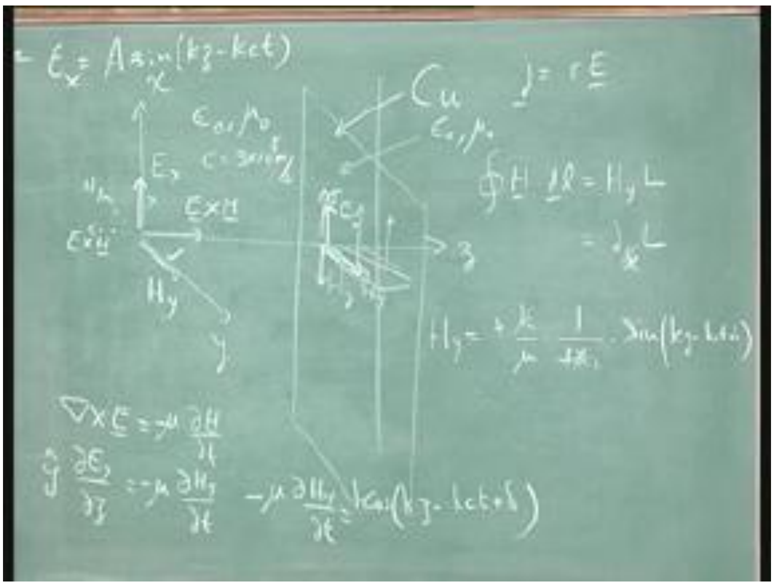

If we assume this conductor has a infinite conductivity then there is a surface current and this surface current will be connected to H y. So, in the wave reaches the valve there will be an electric field pointing up there will be a H y pointing this wave whatever current is on the surface must match H y, because H y on the other side is 0. So, if I draw strokes line strokes curve where is H this way there is no H this way, so my entire loop integral H dot dl is equal to H y times L. So, L is this length what is the enclosed current well the enclosed current let says in the vertical direction is equal to j surface in a vertical direction which will be j surface jx into the length L. So, H y is equal to jx now, this is the very good conductor which means that electric field must go to 0 if the electric field does not go to 0 electric field is continuous..

So, I will have a electric field inside the metal and already I know that electric field inside the metal means j is equal to sigma E sigma E is infinity and j I already know is some finite value given by H y. Therefore, E must be 0 you cannot have any electric field inside that matter, but I have electric field in the wave. So, how do I solve this problem the way electro magnetic solve this problem is simple it knows that H y is, but E x should go away. So, what it does is that add the wall it has the wave E x H y and it also adds a reflected wave. Now, remember that if we have plus sign here what that will do is here E x has a H y pointing the other way, but there is no real reason why I should take E x with H y pointing. The other way I can take E x pointing downwards and H y pointing the same direction both ways E cross H is pointing backwards let me repeat that.

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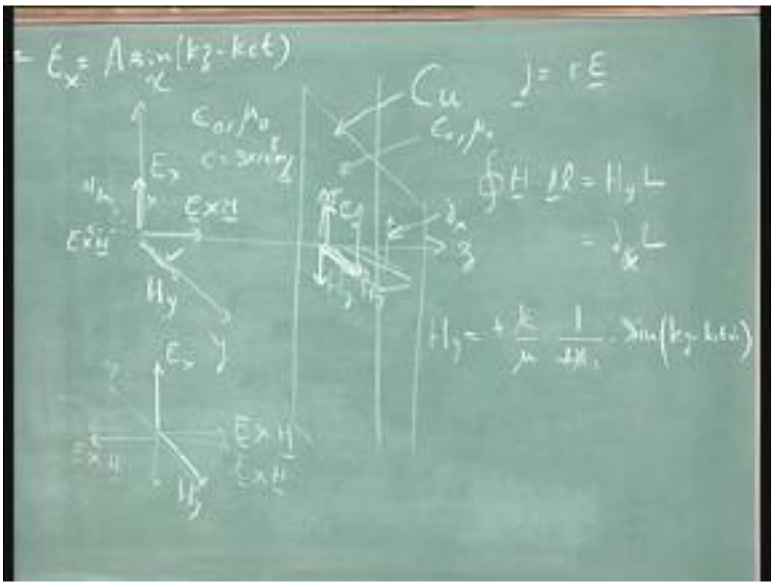

If I have the electric field pointing up E x magnetic field pointing side waves H y x cross y is z. So, I get E cross H in a positive z direction wave is moving forward if I have E x, but H y pointing the opposite direction that is minus H y. Then E cross H is in the negative direction if I have E pointing down wards and H pointing in the negative x y direction. Then in fact, I have E cross again pointing forward, but E is pointing downwards and H is pointing along plus y then H backwards. So, it depends on not just the direction of E, but the direction of E relative to the direction of H. This E cross H has to do with the cross product you have to take your right hand rule and apply it. And see in which direction you will tighten a right handed screw driver to make it in the direction of E cross H. So, the direction that I want is this direction now, what happens at the wall the plus E x and the minus E x cancel I will end up with E x is 0 which is continuous. And, because it is continuous inside the metal there is no electric field, but the H y is add up I get twice the H y which supports the current. So, inside the material I am going to have jx, but I am not going to have any electric field and my j dot E will be 0. Now, let us look at that where did the energy go I have my pointing theorem.

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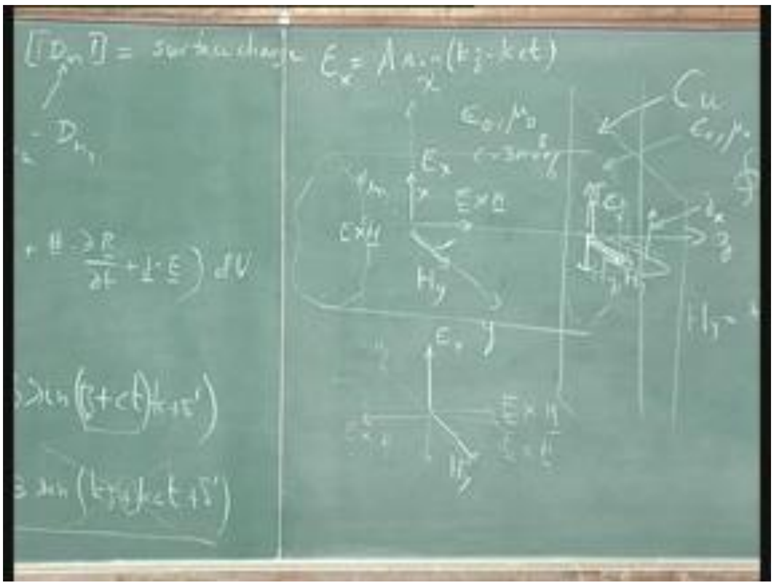

My pointing theorem said surface integral E cross H dot ds over a surface is equal to volume integral of basically E dot del D del t plus B dot sorry H dot del B del t plus j dot E dv this 2 terms are essentially study state. If I average over 1 time period nothing is happening j dot E is 0 perfect conductors. So, this right hand side is 0 which means the left hand side has to be 0 how can that be? Well, you look at it the surface we are talking about is a cylinder that is piercing this slap E cross H is in the z direction. So, E cross H dot ds does not contribute to the sloping sides electric field is 0 inside the wall. So, there is no E cross H. So, all the E cross H is here, so this is surface integral over the surface 1 surface 1 is just this piece of E cross H.

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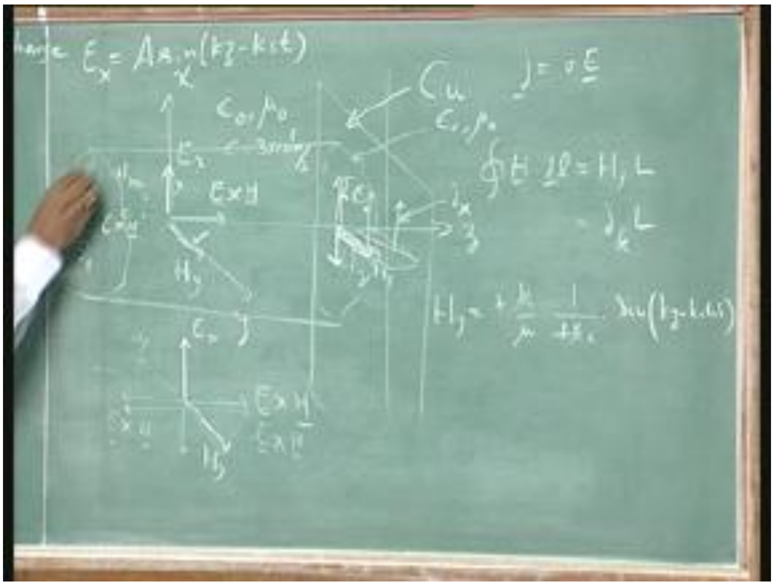

And I am saying that should be a 0.

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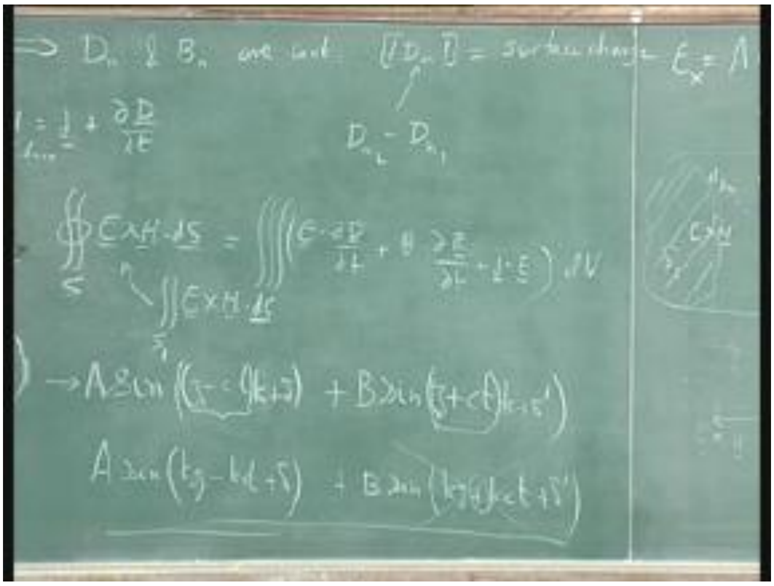

Well, it is 0, because they are 2 waves.

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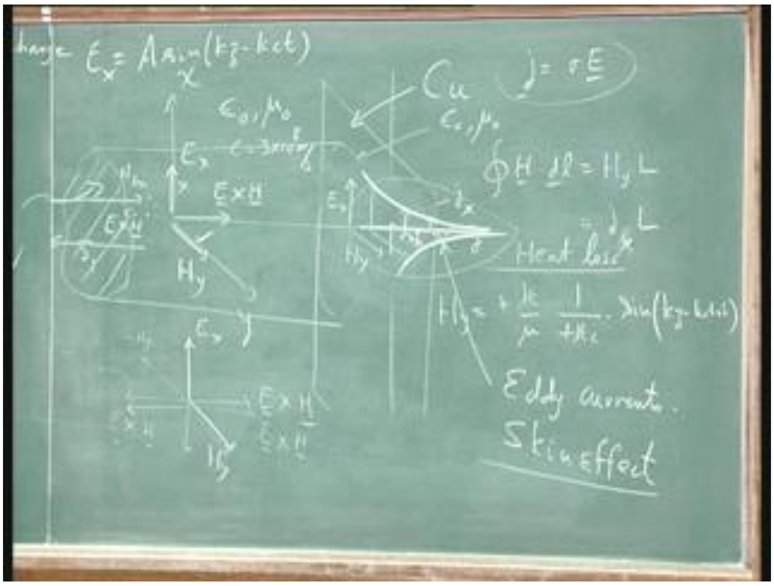

There is a wave which I will call E cross H forward plus an E cross H backward dot ds that is this piece and this piece I know that one of this is pointed in plus z direction. And the other is pointed in the minus Z direction, therefore when I dot product this 2 with ds. And ds points in z direction I am going to get the amount of power injected by the forward way minus the amount of power taken away by the backward wave.

But my in order to make electric field 0 inside the wall I have reflected wave and the incidental wave this 2 waves equal in power otherwise. This E x would not have cancel this E x and at I had some electric field inside the copper. So, it is means these 2 E cross Hx must be equal and they have opposite sign, so that is why this also becomes 0. So, this is the case where I have an electric field entering and have an electric wave entering and a wave leaving. And if this wave brought in 1 volt of power this wave took 1 volt of power and, so there is no net transfer of energy form the wave to my material. Now, let me take the case the sigma is not infinity supposing sigma is some reasonable number not very large not very small. What happens this is still true pointing theorem is a general theorem that we derive without making any approximations.

So, the entire theory must still holds what happens is that at this interface the incident electric field E x H y. They go in to the material and, because j is now not that is sigma is not infinity and j can be it requires an electric field to exist. Now, in fact, when H y is present inside the materials you need an E x to be present as well what happens is the electric field goes into the material magnetic field go in to the material. And at that point the presence of magnetic field implies the current jx and this jx implies an E x, because j is equal to sigma E. But jx stands sigma E x is the dissipation it is heat this is ohms law and therefore, the energy in the wave starts reducing the j reduces. And therefore, the amplitude of the electric field and the magnetic field reduce first. This is the smaller j as the result the amplitude of the electric field and the magnetic field further reduce.

And you have an exponential dk of the electric field and the magnetic field this effect whereby the wave enters the materials anticipate. It is energy comes under different names 1 name in which you see it which is eddy currents another name in which is see it is skin effect. This is the common name we usually to give skin effect and from our point today the important thing to understand is this represents heat loss the wave induced on this plate. And heated it up because j dot e means energy is gone into the material and the plate is getting hot which means that there is power being transferred by this wave to the material. So, it means that this, the solution that we have not only exists, but it is capable of carrying an energy with it. So, now, we have several characteristic to this thing let me enumerate them.

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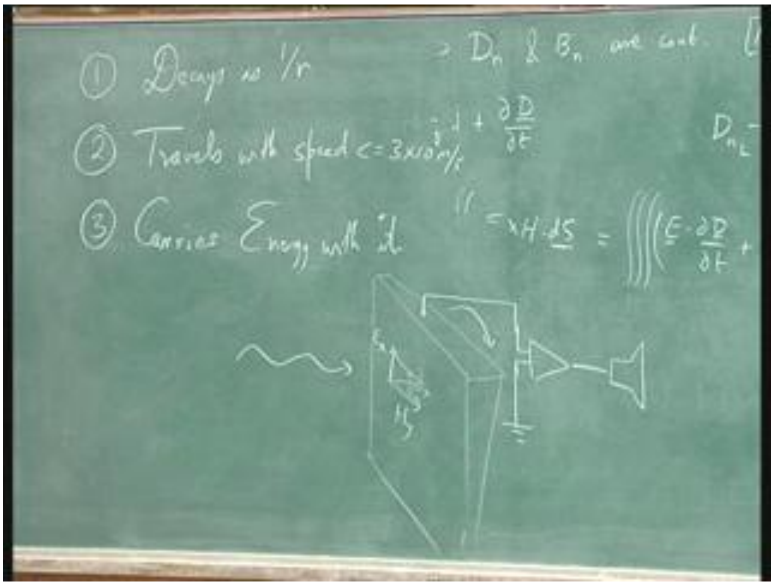

So, that you can see where we are going decays as 1 over r travel with speed C equals 3 into 10 to the 8 meter per seconds carries energy with it. Now, if you look at this table of observation you can see that these are all the thing that light satisfies, if you look at the sun. And you look at light going out from the sun we know that the total intensity of the light which would corresponding E cross H this proportional to 1 over r squared. And our case also E cross H is proportional to 1 over r squared light of course, travels with a speed of light. And light carries energy with it otherwise the suns light could not be giving energy to earth could not be heating it up could not be giving energy to plants and giving us a photosynthesis. So, we have almost conformed that this kind of solution corresponds to what we see in optics.

But they still 1s basic question left the basic question is under what conditions can I say I have circuit theory and an under what condition I have to add this extra effect of radiation. We know now that if our wave hits the plate the plate will heat up, so there is now, conformation that this kind of solution can actually interact the material. And can deliver power to it, but that is a different statement from how to generate it. And how to make use of it for example, how does it short wave ratio work how does the broad cast station send out radiation. And what makes the circuit describable by standard kvl and kcl and when does the circuit requires a better treatment? Then that, so it is a very important question, because it tells us the limits of circuit theory tells us when we have to be careful. And when we can just forget about all this stuff and go back to voltages currents inductors capacitors and resistors.

Now, to answer that question you have to go back to this problem. The problem of the wave that strikes the plate and you ask something slightly different supposing I have a wave and it is strikes the plate. So, it is a thin plate and at the top of this plate I connected a resistor to ground. So, this plate if you like is just hanging now what I want to know is is there a some combination of circumstances by which I will get a current for example, if this where actually a light bulb can I make this light bulb blow. So, that would be a situation where I have electromagnetic radiation come hit this plate the plates gives power to this bulb which again created electromagnetic radiations. But we are not thinking in terms of that I like to know if I can obtain this radiation and make use of it instead of putting a light bulb I could more usefully put an amplifier and connected to my speakers. And then I get my broadcast and listen to it and my home. So, the question is what kind of phenomenon will allow a current to flow in an outside circuit?

The first thing to understand is that phenomenon is not what I described earlier, because if you take the electric field E x and you take this magnetic field H y and this H y causes a current jx and this jx dot E x dissipates this waves. The wave decades naturally the problem is whatever energy was there in the wave got dissipated in the plate. The plate got hot, but ever plate gets hot it does not give me a current does not give me I can amplify and listen in a speaker rather I want to get this wave energy. And instead of dissipating it inside this plate I wanted to come into the outside circuit. So, the question is under what conditions is that possible? Well, to answer that we want to go a little deeper into how I wrote down those equations that did not write down any equations I just do pictures. So, I am going to look into this phenomenon we are calling absorption of radiation by material or skin effects.

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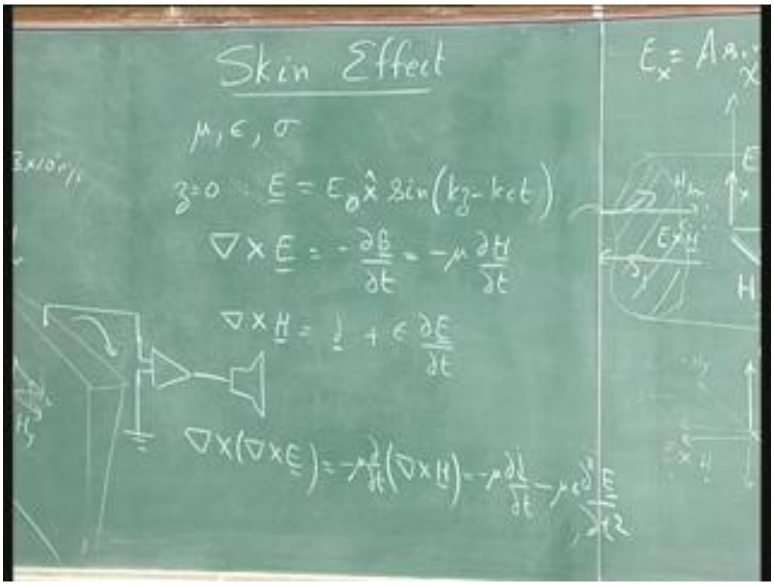

So, I have this wave this wave has somehow entered the material region .So, it is inside a region where permeability mu permeability epsilon and conductivity sigma. And at z is equal to 0 the electric field is E x or E naught in the x direction say sin kz minus kct plus delta. And I am removing the delta, because I can always adjust time and make delta 0. Now, to analyze this problem I have write down wave equation problem inside the material. So, let us look at that I start with Faradays law and del B del t is minus mu del H del t. Then I go to amperes law curl of H is equal to the current plus epsilon del E del t that is del E del t. And again I combine this 2 equations I can see that in order to eliminate H I must always take the curl. So, I get curl of curl of E is equal to minus mu del del t of curl of H, but curl of H is this. So, it is equal to minus mu del j del t minus mu epsilon del square E del t square is the same derivation I have done earlier except I have 1 additional term. Because the earlier wave equation I derive had no currents and no charges. Now, when you do this problem exactly large number of complication we have to make certain assumption.

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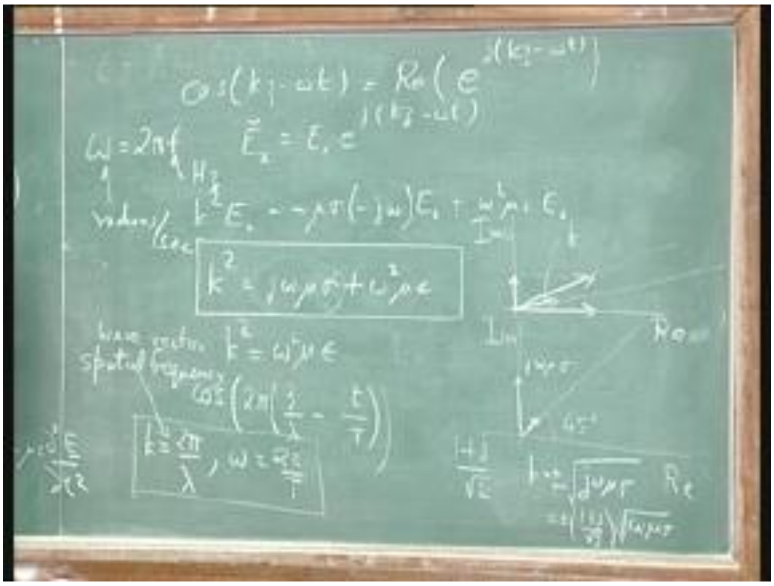

The assumption we are going to make which is most important perhaps is that the divergence of j which is minus del row del t is small. The idea is that in that case I can say divergence of E which is 1 over divergence of D which is therefore, row over epsilon is 0. How does that help us? Well, we can look at the left hand side you can go back in notes and how you derived it? You would find curl of curl of E is actually equal to gradient of divergence of E minus del squared E. This term would be present if you have charge density, but I am going to choose to drop it if there are situation where you have to keep it. But for understanding skin effect you do not need to keep it in fact, for most effect we can safely drop this term. So, then that gives you my new wave equation it does not have the charge density of it, but it does have current, but I am saying the current has the very small divergence. So, I get minus del squared E is equal to minus mu del j del t minus mu epsilon del squared E del t squared.

So, one new term I am going to specialized to 1 dimension and I am going to use the fact that my electric field has the very specific form. And incidentally I am going to call this case kc as omega just notation will understand what k and omega are in a little while. So, what do I get if I substitute 1 dimension? Then I will get minus del squared E x del E squared is equal to minus mu del jx del t minus mu epsilon del squared E x del t squared. Substituting from E x equals E naught sin kz minus omega t you can substitute in what do you get? You get k squared E x, because if I take the derivative of sin kx kz will become k cons kz minus omega t I take the derivative again I get k squared with a minus sign sin kz minus omega t. So, I get minus k squared this already a minus sign, so this plus k squared is equal to minus mu del jx del t plus mu epsilon omega squared E x, but we have a new term here jx.

And here I am going to use in the presence of conductivity if conductivity is present then I know the simplized jx is equal to sigma E x. So, I substitute that in here I can replace jx by sigma E x, so mu sigma del E x del t now here you will see a problem. Because when I substitute del E x del t I get a cosine my sin will become a cosine. So, I will actually end up with k squared sin kz minus omega t is equal to minus mu sigma omega cos kz minus omega t plus mu epsilon omega squared sin kz minus omega t. This is very difficult really to make sense of, so this is why when we do this, problems instead of using sines and cosines. We tend to use complex exponentials you have already been introduced I am sure to face us. And the purpose of using the faces is simply that the 90 degree shift is very easy to handle when we take a derivative of a cosine you get a sin. And then you have struck with lot of algebra.

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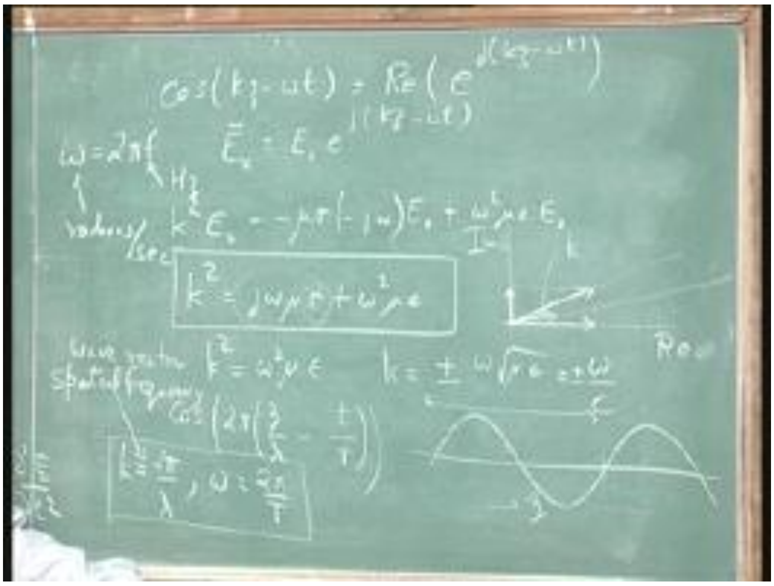

But in state we will use cos of kz minus omega t is equal to real part of e to the j kz minus omega t and then we will work with these cases. So, in terms of faces E x is equal to some E naught e to the j kz minus omega t. In that case the wave equation becomes k squared E naught is equal to minus mu sigma times minus j omega E naught minus omega squared mu epsilon E naught. So, all I have done is I have taken the second derivatives with respect to z, so I got a minus k squared and minus sign got observed. And I got a this also a plus sorry, I got a minus omega squared from the time derivative again the minus sign got observed here I get a jk I mean j omega, because I was single derivative of time in this equation. So, this is now gives us our final equation I can cancel the E naught out I get k squared is equal to j omega mu sigma plus omega squared mu epsilon.

Let us go back to the case where thus we are in vacuum and sigma is 0. Then I get k squared is equal to omega squared mu epsilon or k is equal to plus or minus omega over square root of mu epsilon the plus and the minus referred to the forward wave and the backward wave. Because if the sines are the same the wave is travel in a minus z direction this, sines are different the wave is traveling in the positive z direction. So, that is what plus and minus mean, so this is nothing, but sorry plus minus omega over c. Now, what do this case and omega is represents if I look at this form or this form I am really looking at a wave which goes like cos of I would like to say 2 pi times z over lambda minus ct as it say t over capital T. Because I have a wave that looks like a sine or a cosine I know that if I move in z by a distance of 1 wave length that is of 2 pi phase I should get back this same point again, because cos is the periodic function.

So, if I go this distance, which are usually which is usually called a wave length you have seen wave and sea water waves and the sound. So, the same wave length concept if you go a distance lambda you should go a distance 2 pi in face. So, this is what I expect from z and if I go a time equal to 2 pi equal to time period t and again I should go a face of 2 pi. So, this is the form I expect this is the form I have, so I can immediately identify what k and omega r k is equal to 2 pi over lambda. And omega is equal to 2 pi over T k is called the wave vector or the special frequency omega is the normal frequency. It is the angular frequency of the wave, so k is to is like omega in the sense it has 2 pi in it you know that omega is 2 pi f f is in Hertz’s omega is in radiance per second, because it was 2 pi radiant is 1 full cycle which is 1 Hertz.

So, this this is the relationship between omega and f and k is related to 1 over lambda in the same way omega is related to 1 over T 1 over T is the very important quantity. So, we give it a name frequency 1 over lambda is usually is not given a name. In fact, the only name that is given is 2 k itself 2 pi over lambda. So, we have this expression relating the spatial frequency k in other way 1 over lambda square to the temporal frequency namely omega squared. And when sigma is 0 we get what we expect we get a wave, but when sigma is not 0. We get a something very different this j is actually going to give me a complex answer for k for given real frequency. We can see that if I look at this expression and I write it terms of real omega and imaginary omega or real and imaginary. I want to say real omega and imaginary omega omega squared mu epsilon is along the real axis j omega mu sigma is in the imaginary axis. So, when I add them up I am going to get a complex quantity that is not pointing along a real or imaginary axis, but in some general direction. And I want to take the square root of that, so when I take square root of that this, angle get bisected.

And the magnitude I take the square root that is my k, so in general k is quite complicated. However there is a very simple special case which is also the most important simple special case. And that case is the case where I can ignore omega square mu epsilon in favor of j omega mu sigma copper is an excellent conductor. So, when you go inside copper almost all the time this term dominates. So, this term dominate I have a situation that look like this this is the real axis this is the imaginary axis. And I have j omega mu sigma the square root must bisect this angle and it has the value that is square root of this value this angle is nothing. But 45 degrees if I had to write what that unit vector was it could be 1 plus j over square root of 2. That is a unit complex number with the same face 45 degree face as this vector. So, that is what I can write down. Now,, so my k which is plus minus the square root of j omega mu sigma is actually equal to plus or minus 1 plus j over square root of 2 times square root of omega mu sigma. Given that let says write down what the form of the wave is I will complete the rest of it later this is the wave.

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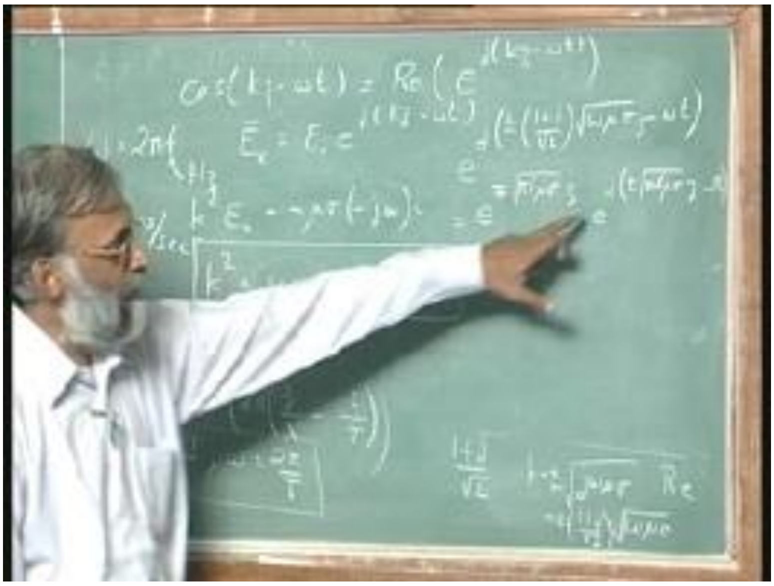

So, I got e to the j times this k plus or minus 1 plus j over root 2 times square root of omega mu sigma minus omega t if I see the j here and thus a j here and j squared is minus 1. So, this has the following dependence e to the minus plus square root of pi f mu sigma times z times e to the j times plus or minus pi f mu sigma z minus sigma t. So, there is a piece that has the special frequency and temporal frequency. And this of course, that the decade with z I will complete this derivation next time. And then talk about what it takes, what it is saying about making an antenna making an receiver? That can actually make use of the waves that arrives at the circuit in a certain sense that. What I will does you have light electro magnetic radiation coming on plants and plant cleverly take that radiation and make use of it.