Electro Magnetic Field

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Lecture - 30
Magnetic Energy II

In the previous lecture, we had been discussing an easier way of calculating vector identities. And I am going to continue that, and immediately use it to derive, what we have been struggling to discuss the last 2 lectures namely, magnetic energy.

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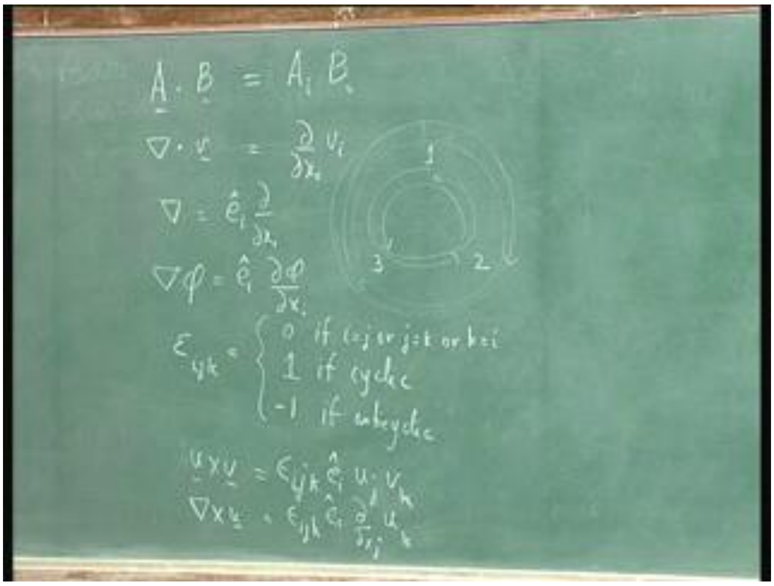

So, what I had done, that I introduced this notation, where if I have any dot product vector A dot B vector B i denote that as A sub I times B sub I meaning, I sum over I equals 1 2 3. So, A x B x plus a y by plus A z B z, and if I take a similar thing namely, divergence of A vector v i write that as del del x i of v i. So, the gradient vector is the vector unit vector along the i direction multiplied by del del x i. So, this dotted with the vector v gives me this dot product which is nothing but the divergence. Similarly, gradient of a scalar becomes, E i del del x i of the scalar which is the gradient. Now, when it comes to the curl, I added a new twist to this. I defined something I call the unit anti symmetric tensor epsilon i j k, which was equal to 0. If i equals j or j equals k or k equals i and is equal to 1, if cyclic and equals minus 1, if anti cyclic. And what I meant by cyclic was that if I draw a circle put 1, 2, 3 as if it were a clock.

Then if I went clockwise in my numbering say 1 2 3 or 2 3 1 or 3 1 2 that is called cyclic if I had to go 1 3 2, 3 2 1 or 2 1 3 that is called anti cyclic. So, epsilon i j k is equal to 1, if it is cyclic and minus 1. If it is anti cyclic and as I showed you last time by just working it out the determinant of a matrix is nothing but epsilon I j k of the various rows of the matrix namely rather than writing it out. Generally, which tends to be complicated, I will directly apply this to the curl, which is u cross v the cross product is equal to epsilon i j k E i u j v k. And curl of u is equal to epsilon i j k E i del del x j of u k. Both of these are determinants, they are the determinant of x hat y hat z hat u x u y u z v x v y v z. And when you take the determinant, this I showed last time is equivalent to writing it this way. Now, what is the advantage of all this? What I am going to do is I am going to use this notation and derive a general expression for magnetic energy.

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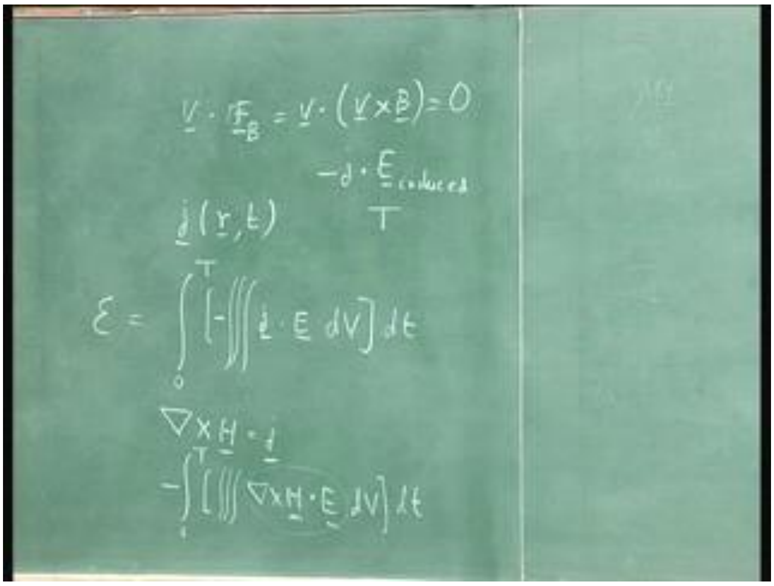

The way I will do it is to go right back and ask how does energy enter the magnetic field? And we know, that the magnetic field cannot do work by itself. Because v dot B is going to be v dot f b, which is equal to v dot v cross B is equal to 0. So, the magnetic field by itself cannot do any work. And therefore, where did the energy in the magnetic field come from. It had to come from the induced electric field. Now, if the electric field points in a certain direction. And if there is current in the same direction, then we know that. This current represents movement of charge in the direction of the force, which means, the charge gains energy kinetic energy and the field must, therefore lose field energy. Therefore, if energy is going to go into the field, I must be talking about minus j dot E induced. This is what must be giving me magnetic energy, J dot E induced would be the energy given up by the field in order to push particles.

But minus j dot E induced is the energy that particles give currents give, when the build up the field. So, the total energy in the magnetic field, let us say j is a function of position and of time and it was raised from 0 to its full value in a time capital t. So, then I would say that the total energy in the field would be equal to integral 0 to capital t. I am integrating in time and integral over all space of minus j dot E d v. This is the total energy that I am putting into the field. So, I would like to get an expression for what this is equal to I mean, it is a complicated expression. It has 4 integrals in it, 3 space integrals and 1 time integral. But I luckily have ampere's law to help me; I know that, curl of h is equal to j, so I can substitute curl of h in here. So, this would be equal to integral 0 to capital t I will take the minus side outside 3 integrals over space curl of h dot E d v d t. Now, we immediately see that we have to do something with this. But this is one of those nasty vector identities how do we simplify this vector identity. So, what I am going to do is, I am going to take this curl of h dot E and apply my simple techniques to it.

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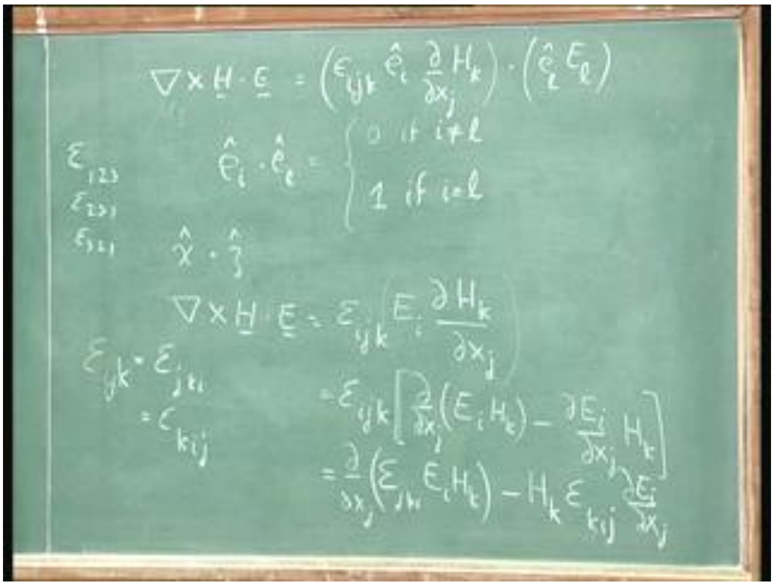

Well what is curl of h? It is equal to epsilon i j k E sub i del del x j of h k. That is curl of h dot E sub let us say, l electric field sub l, so this is the curl of h and it is dot producted with E. Now, E i dot E l let us, look at that E i dot E l, this is like saying x hat dot z hat. So, E i dot E l is 0 unless i and l are the same number. X hat dot x hat is 1, but x hat dot y hat is 0, x hat dot z hat is 0. Similarly, y hat dot x hat is 0. Y hat dot y hat is 1hat dot z hat is 0. So, I can write this, as it is equal to 0, if I is not equal to l, It is equal to 1 if I equals l. This is how we got dot product expressions, so this I mean; this must be very familiar to you. So, it means, that i can replace this l here by i itself. So, curl of h dot E is going to be equal to epsilon i j k, this E i dot E l is just 0 or 1, but this E l is now, E i capital E I, because if E l is not if l is not equal to I, it is 0 l is equal to I, it is 1. So, I just pull that E l and call it E sub I, del h k del x j.

Now, forget this epsilon i j k. If I only look at this piece i can apply my differentiation rules. I can say that this is equal to epsilon i j k times, del del x j of E i h k minus del E i del x j times h k. This is nothing but differentiation of a product is equal to the first term times. The derivative of the second term plus the derivative of the first term times the second term. So, it is you differentiation rule, but now, let us identifies each of these. If I look at this, expression I can write it the following way, del del x j. Now, this epsilon, i j k is also equal to epsilon j k I, because it is cyclic. It does not care whether i say 1 2 3 or I say 2 3 or I say 3 1 2. So, since all of them, are equal if I just cycle these numbers the value does not change. So, i j k is the same thing as j k I, so epsilon j k i times E i h k minus h k times again I would like to make the epsilon i j k have k as the first coefficient. So, it is equal to epsilon k i j, again cycling, so epsilon k i j del E i del x j.

So, I have done nothing here, all I have taken is, I have taken the curl of h dot E I have written out the curl. And I have taken that E dot del h k del x j and written it out as the difference of 2 terms, what I have done here is I have recognized that, epsilon i j k can also be written as epsilon j k i. It can also be written as epsilon k i j, what do I mean, by that. I mean to say epsilon 1 2 3 or epsilon 2 3 1 or epsilon 2 3 1 are the same thing. They are the same thing, because we drew the circle. I drew a circle and put 1 2 and 3, when I am taking determinants, it does not matter whether I start with the first. Then take the second and third or I start with the second and take the third. And the first or I start with the third column and take the first and the second. You know that you have go in a cyclic way, when you are doing determinants. That is all that epsilon is preserving, it is just storing the structure of determinants, so it does not care if I rotate it. So, I am rewriting epsilon, so that I can see a dot product coming out of it.

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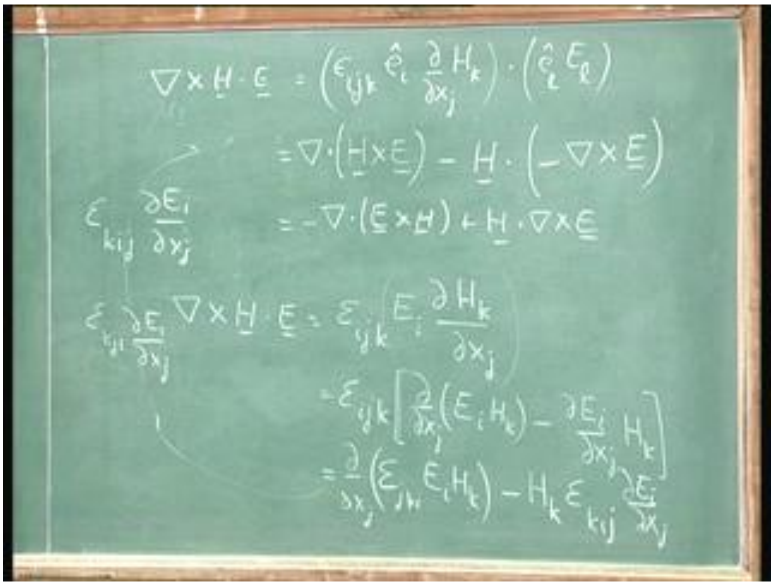

Now, let us see what that gets me? I am going to stay on this board. So, let me erase a few things, so this is equal to this is del del x j of something. So, it is a divergence of now, what is this epsilon? j k i h k E i. So, it is h cross e, it is just a cross product. Cross product of h and E and this term says minus h dot well, this is epsilon k i j del E i del x j, but really, what we want epsilon? k i j del E i del x j. But what I really, would like is epsilon k j i del E i del x j, because I want the second index to correspond to the derivative third index to correspond to the vector. But again, I know that if this is 1 2 3 this is 1 3 2, which means, it flips the cyclic anti cyclic. So, this is minus of this, so minus curl of E. If you write it out this becomes, minus divergence of E cross h plus h dot curl of E. Now, this is a vector identity, we got this vector identity, just by looking at how this product rule works. There is nothing else that we did. Everything else we did was simply, manipulating this epsilon i j k and that is all that is there in all vector identities that you use calculus.

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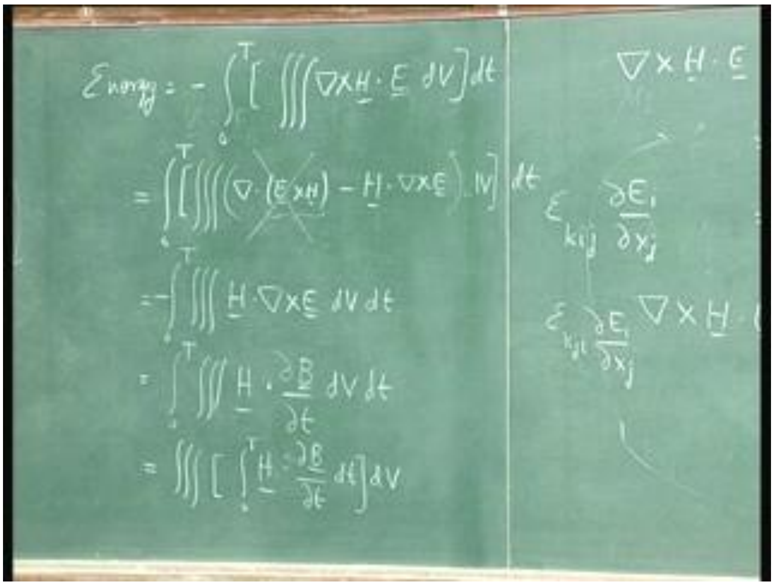

So, we now need to calculate the total energy, which is equal to minus of integral 0 to t volume integral curl of h dot E d v d t, but we already, have an expression for curl of h dot E. That is exactly, what we have done here. So, let me put it in, I have a minus sign outside. So, it is integral 0 to t volume integral this, minus curl of h dot E becomes, divergence of E cross h minus h dot curl of E d v d t. Now, this piece is a very important piece. It is, in fact, the poynting vector and we are going to learn more about it in the later part of the course. But for now, this poynting vector can be thrown out, because we know that the electric and magnetic fields are going to go to 0, if you far away enough, since this is over all space this is a divergence. So, this whole integral becomes. a surface integral at infinity of E cross h dot d s.

So, if whatever, we are doing is caused by localized charges and localized know that localized charges can create an electric field. That goes as one over r square and localized currents cause a magnetic field that goes like 1 over r square. So, this goes like 1 over r to the fourth, so this is 1 over r to the fourth and the surface integral is going like r square. So, the whole integral goes like 1 over capital r square. So, until we get to radiation, we do not need to worry about this term. We can throw it out, but it will come back and disturb us once we go to the next part of the course. For now we can throw it out, so we are left only with this term. So, it is equal to integral 0 to t volume integral minus sign outside h dot curl of E d v d t. But what is curl of e? That is where faraday's law comes to. Faraday's law tells us curl of E is minus del B del t.

So, it is equal to integral 0 to capital t volume integral over all space h dot del B del t d v d t. We have not made any approximations here, I have not assumed, that B is equal to mu h. I have not assumed that B is equal to epsilon e; none of those assumptions have been up to this point. All I have assumed is that I have localized currents and localized charges. So, that I can throw this out, now I will take this time derivative time integration inside this space integration, because it does not matter which way I do it. So, I will get integral over all space of integral in time h dot del B del t d t d v. Now here is where we will make an approximation, supposing it is true, that we are working with a linear system. Supposing B is equal to mu h if B is equal to mu h, then we can simplify that integral. If B is not equal to mu h this is still true, and In fact for magnetic materials. This is exactly, what we will have to do, because what you will find is that when you, if you cycle in time the magnetic field make it rise up and make it come back down. That this does not give you 0, because there is some laws in magnetic within the magnetic material itself.

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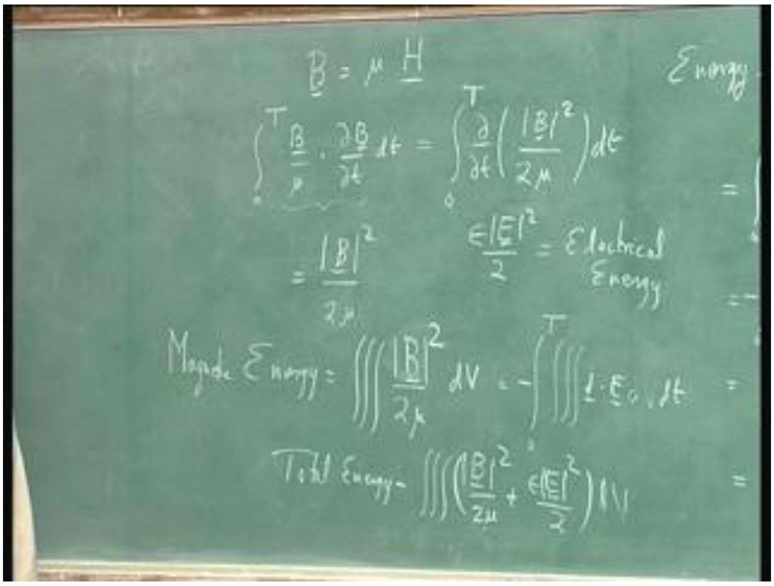

But for now let us assume that B is equal to mu h. If B is equal to mu h, then I can write that more simply, let me look only at the inner time integral. Integral 0 to capital t, B is equal to mu h. So, h is equal to one over mu b, B over mu dot del B del t d t. This can always be simplified. This can be again written in the simple notation, we have B sub I over mu del B sub i del t which is nothing, but del del t of B sub i times B sub i over 2 mu. that is B sub i square, I am going to put a sum on I, because there’s no longer 2 different things over, which I am summing. But this is well known to us, what is this? This is just B dot B or mod B square, so I can write this down. This becomes, integral 0 to capital t del del t of mod B whole square divided by 2 mu. An integral in time of something that is a total time derivative is a trivial integral. It is just this quantity at capital time t minus this quantity at time t 0.

So, it is equal to the final value minus the initial value. But I will assume that the initial value is 0, because the current started from 0. So, this is the value of this time integral, which means, energy is equal to volume integral B square over 2 mu d v. So, this is in fact, the expression I have been writing down again and again without proving. Now, I am sure that this is actually, more satisfying than the previous proof, which said that as you increase the currents, there is a changing magnetic field. Because of the changing magnetic field there is an induced E m f, and therefore the currents have to work against the induced E m f which introduces stored energy. That is exactly, what we did. We started with minus j dot e, then we converted j to curl of h, so minus curl of h dot E. Then we did this, vector identity and converted minus curl of h dot E to a poynting vector term and minus h dot curl of E.

Then we used faraday's law that is the induced E m f. The electric field is actually, due to a changing magnetic field which finally, gives us this term. So, I think I have done this enough times, so I hope you believe that, the magnetic field does. In fact, store energy, as I said last time this energy, that we stored in the inductor came from somewhere. It came from the outside supply, of electrical energy, but this energy rises to a peak, when the current is maximum. But when the current comes back to 0 all that energy returns, which means this device is not a resistor. This device is a storage device; it stores energy in something. Now, that energy cannot be the electric field, because if the magnetic field reaches a constant value there is no induced E m f.

So, there is no stored electrical energy that can account for this energy. It must be the stored energy in the magnetic field. So, that is where this final result comes, that the starting value which is integral 0 to capital volume integral minus j dot E d v d t. This is the total amount of work that we have done on the system shows up as B square over 2 mu. So, just as we said electrical energy is epsilon E square over 2 this is electrical energy. This is magnetic energy and it is when you have both of these, and of course your total energy is equal to volume integral B square over 2 mu plus epsilon E square over 2. So, you can see that the total energy is a sum of the energy in the magnetic field and the energy in the electric field. We have one more trick to learn about in the vector calculus techniques, but I will postpone that for now.

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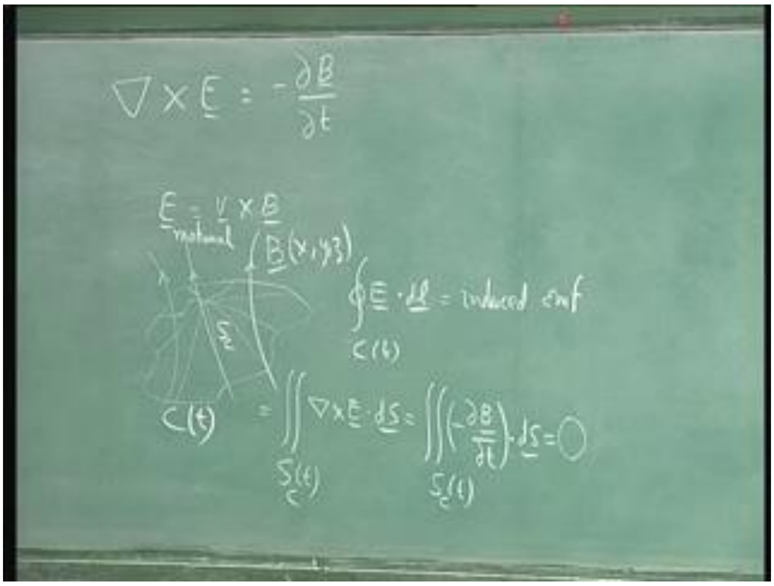

Let us first look at another important idea, in the last few lectures and the first part of this lecture, what we have been doing? Is we started with coulomb's law and magnetostatics put time variation in it and we came to Faraday's law. In the next part of the lecture what I would like to do is to reverse that process. I would like to start with faraday's law and go back and recover motional E m f. So, faraday's law said curl of E is equal to minus del B del t and where I would like to reach is E is equal to v cross B. That's motional. . So, what I will do is I will start with the loop. I call it c and I will apply faraday's law to it. Now, this loop is moving in time. So, the c of t, so loop integral E dot d l on this loop, which is function of time is, what I would call E m f and I want to find out what that is when the loop is moved. Well, I apply this equation, so this loop integral is equal to surface integral of the surface that connects this loop.

That is s sub c and it is also a function of time and when I go from loop to surface, I put a curl the curl of E dot d s. That is stoke's law, but I know an equation for curl of E. So, it is equal to surface integral of that same surface, but minus del B del t dot d s. Now, if magnetic field is changing inside this surface then, this describes induced E m f and that is exactly, what happens in a transformer? In a transformer you have fixed windings. So, the coil is not moving, but magnetic field is changing and when magnetic field changes there is an induced E m f. So, this is our standard transformer relationship. But now, supposing I have a magnetic field b, but this magnetic field is not a function of time. So, B of x y z, but not t, that means, del B del t is 0 everywhere, because B is not a function of t. So, it is equal to 0, so we have a very peculiar situation, I started with Faraday's law.

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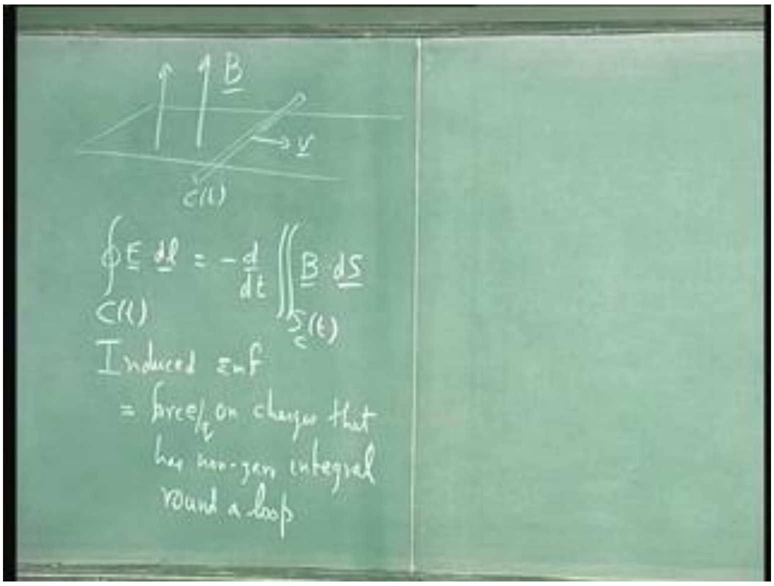

I took a moving coil, I tried to work out, what E dot d l is and I am getting E dot d l is 0. But I already, know from our previous taught experiment, which you can do in your lab that, if I have a rail. And I have uniform magnetic field B and I put a moving rod and give that rod a velocity v. This is therefore, a loop that is my c of t and as c of t changes, because of this velocity v we know there is an induced E m f. That is we also know that, this is not equal to 0, so there is something wrong. I mean, I started with Faraday's law I put in loop integral E dot d l. Faraday's law tells me 0, but I already know it is not 0.

So, there is something wrong with how I have done this and I already, know the answer, because I know that the current answer comes. If i assume E dot d l is equal to minus d d t of surface integral B dot d s. If I do this, I get the correct answer, because this is how I started? If I do this, I get the wrong answer. So, somehow I need to go from here to here there does not seem to be anyway for me to do it. The answer is a little tricky, what we call induced E m f is really, force on charges that has non zero integral round the loop. It is obvious; I am talking about the induced E m f. It is force charge unit charge on charges that have non zero integral when I go round the loop.

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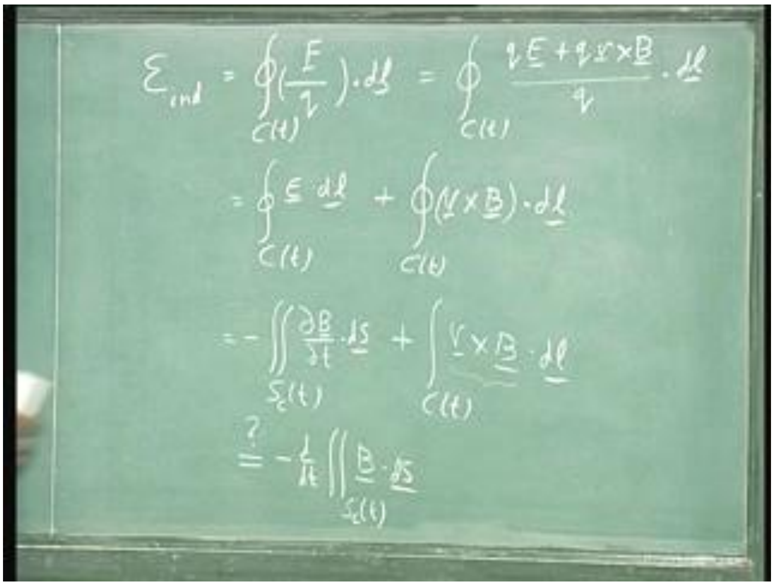

So, it really, when I say induced E m f I mean, loop integral around my c of t force on the charge divided by charge dot d l, that is what I mean, but what is this force. This force is loop integral on this moving loop q E plus q v cross B. That is the force and the charge divided by the charge itself dot d l all right. So, when I have velocity present and when I have magnetic field, present the force on a charge is not q E. The force on a charge is actually, q E plus q v cross B, magnetic field also exerts a force. So, I get 2 integrals not 1. There is an integral that is E dot d l on that loop and there's an integral on that loop that is v cross B dot d l. This is just coming out of a redefinition of induced E m f. Now, why do I say? Induced E m f is force by unit charge, just think of a circuit, I have got something. I do not know what it is, I am denoting it as a battery and I have a circuit. Let us say a capacitor and a resistor, what this battery does is it pushes charges in this direction. So, that they can fall down the potential and come back here as far as the rest of the circuit is concerned the effect of this battery is simply a force.

Because there is a negative force when charges go through these elements, because we are, so used to electrostatics we consider this force as being equal to q E. But this force can be any force, it could be gravitational force, it could be any force. So in fact, I should include a plus m g. It just turns out that m g will not give me a loop integral. It is also derivable from a potential, so when I go round the loop I would not get anything. So, I have 2 terms left and 1 of these terms is exactly, what I have used Faraday’s law for and that term gives me, the transformer part minus del B del t, so I can write that out. So, this term is equal to minus surface integral which is changing in time del B del t dot d s from faraday’s law plus a term c of t v cross B dot d l all right. And what I want to know? Put a question mark over that and then this is the same thing as minus d d t of surface, which is changing in time B dot d s. Now, this should be obvious; because we went the other way, but I will just remind you how we got there. So, that you can see, that this piece is the missing piece for motional E m f. we know it is, because motional E m f looks like v cross b.

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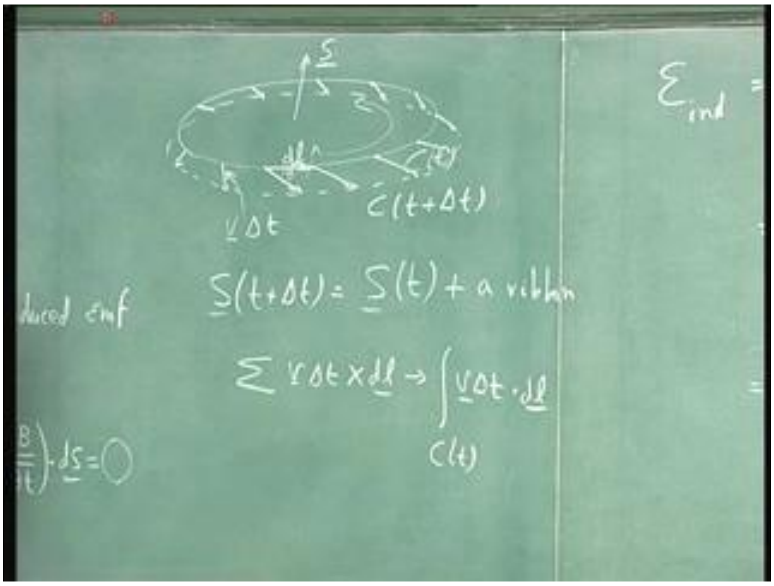

So, let us see, how we got it? What we do is we take a loop which is our c of d and we allow that loop to move. If the loop moves it came to a new position, this is c of t plus delta t. Now, in order to move every point on this original loop move with a velocity v for a time delta t and reached a corresponding point in a new loop. So, there is every point on the loop, there is a velocity vector, these vectors are v delta t. That is it moves a distance v in 1 second then at time delta t it moves v delta t. This distance is d l, now, if we take this c in this sense, then using your right hand rule you know, that surface normal points upwards, that is your right hand rule. Now, how can we construct the new surface using the old surface and this extra information? Well, we know that the s nu of t plus delta t is a surface at time t. I have put a vector here, but it is not really a vector.

It is a surface plus a ribbon, the ribbon is this extra piece that is showing the way s changed to become, s plus s of t plus delta t. Now, how can I describe this ribbon, this ribbon is nothing but summation on a lot of pieces all of whose normal must point upwards, again using my right hand rule, I know that it must be v delta t cross d l. Area is a cross product, because its magnitude of v delta t times magnitude of d l times sine theta, that theta is this angle. So, v delta t cross d l is an area element, I sum up over all the d l's that will give me the, are of the ribbon which has to be added to s of t to give me s of t plus delta t. A summation is really, an integral, so this becomes integral on this c of t of v delta t dot d l. That is the summation was over d l and if I make the d l smaller and smaller and more and more in number that becomes an integral. So, this is what I really have. So, now let me look at what this v dot v cross B dot d, l can be made to look like.

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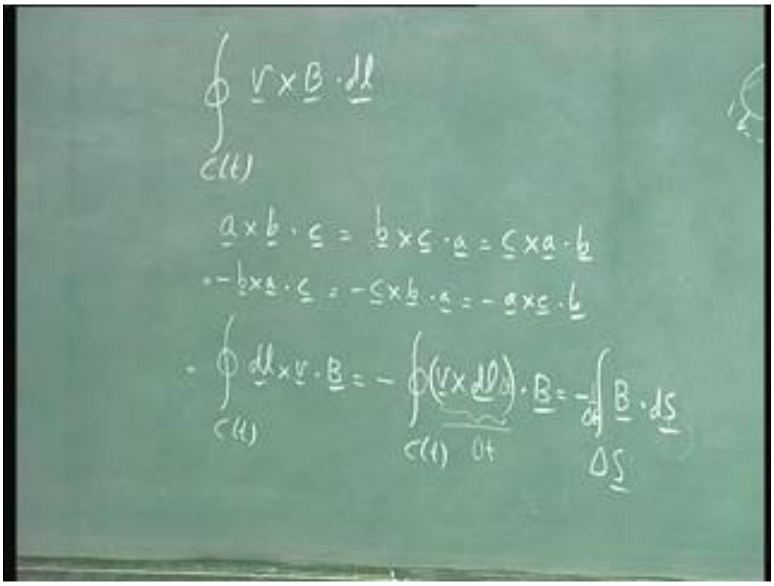

So, I have this integral c of t v cross B dot d l, sorry here it should be v delta t cross d l, it is a cross product not a dot product. So, I want v cross del d l, but what I have is v cross B dot d l, but I also know, that if I have a cross B dot c this is the same thing as B cross c dot a. And the same thing is c cross a dot B and it is also minus B cross a dot c minus c cross B dot a minus a cross c dot B. This is nothing but the same epsilon i j k tensor, that I talked about. So, if it is cyclic it is plus if it is anti cyclic it is minus. So, I want to change this into something that looks like v cross d l. So, let me push the B there and bring the d l here, so instead of v cross B dot d l I am going to do d l cross v dot B. This is equal to loop integral c of t d l cross v dot b, but I do not quite want that, I want v cross d l, because that is what I have here my change in surface area is v cross d l.

So, I put a minus sign, because if I want a cross c instead of c cross a there's a minus sign required. It is equal to minus loop integral c of t v cross d l dot b, but I now, know that this is the change in surface area. And if I want to look at, what this is saying? This is saying nothing more than minus c of t. I should not say c of t, delta s of B dot d s, because each of these pieces with a delta t added and a delta t divided each of these pieces is at d s. So, 1 over delta t integral over the ribbon B dot d s is what this is? Let me just remind you how we got that. I started with v cross B dot d l. This v cross B dot d l could be written as d l cross v dot B. That is just, because of the property of the triple product which can be written as minus v cross d l dot B and multiply and divide by delta t. This piece is nothing but B dot d s, that's what we came to here? This is d s. So, I have 1 over delta t integral over delta s B dot d s. We are almost there, now let us finally get the answer.

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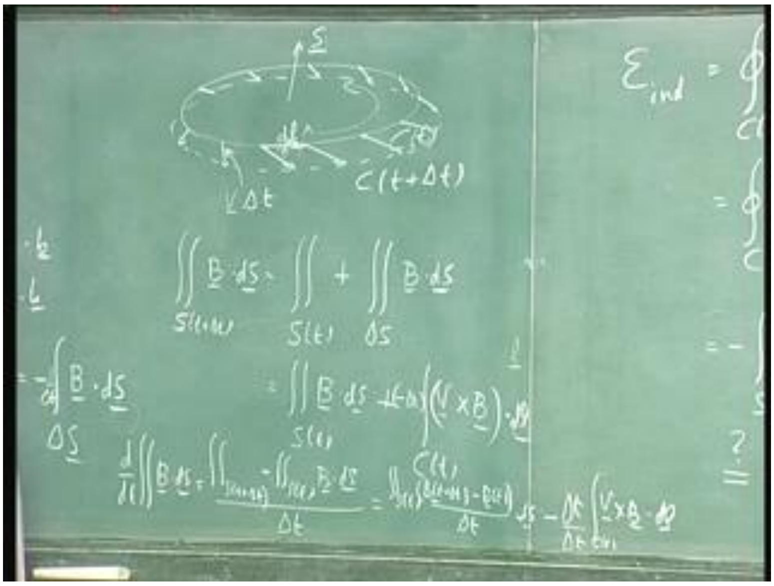

So, my integral over s of t plus delta t B dot d s, can be written as integral over the old surface s of t plus integral over the change in surface B dot d s. That is to get the integral over the new surface, I integrated over the old surface and then I integrated over the ribbon. This integral over the old surface B dot d s plus integral over the ribbon, that is nothing but integral over delta s B dot d s came from this piece. So, this piece can be written as integral over c of t there is a minus sign, because of this the delta t comes to the numerator B dot d s I mean, v cross B dot d s. Now, let us look at what the original starting point was that was here D d t of surface integral of B dot d s can be written as surface integral at t plus delta t minus surface integral at t of B dot d s divided by delta t. That is what derivative means; take this argument at t plus delta t.

Take this argument at t, subtract the 2 divide by delta t. But I already, know what this is, so this becomes, surface integral at time t of B of t plus delta t minus B of t divided by delta t dot d s. That is the first term, combined with this term. So, this term and this term together give me this minus delta t over delta t. That cancels out loop integral c of t v cross B dot d l, this piece is del B del t and this piece is the piece that came out of q v cross B over q. So, what it is saying finally is that indeed this is correct. I can start with just the force equation on a charge, Force equation day's q E plus q v cross B. The q E portion gives me transformer E m f, The del B del t over a fixed surface.

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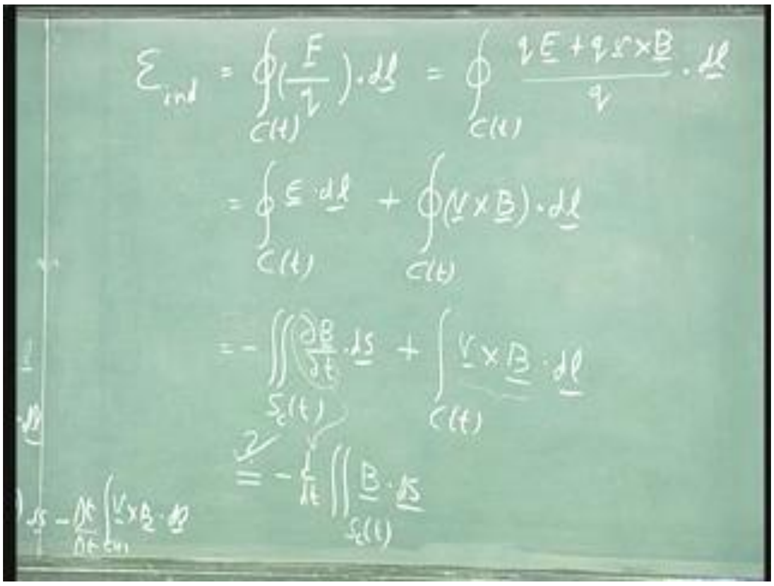

The q v cross B gave me a v cross B dot d l and that piece is precisely, the piece that will take this del del t and move it outside into a d d t. So. In fact, starting with Faraday's law and starting with the force equation. We can go back and infer motional E m f which is where we started. It is a very important thing to understand that all of it is one story, We started all of this saying that we have Coulomb's law, but if you move in time physics law should not change depending on whether I am in a car or I am standing. So, that made me believe, that there is a term that look like, this and then when I made the surface stationary I got del B del t and that gave me Faraday's law.

But when I take Faraday's law on a fixed curve and I have B is constant, I do not get back motional E m f. So, to get back motional E m f I must go back to the force equation at q v cross B and that in fact gives me back my starting point. So, Faraday's law has 2 phases. 1 phase is the transformer, that is del B del t and 1 phase is motional E m f which is what you see in motors, which is what you see in generators between these 2 they cover all of triple E. So, this is probably, the most important equation as far as you are concerned. You need to understand it thoroughly, I promise that I will continue and bother you with some more vector identities. So, here is the final one that you need to become completely independent of all hand books.

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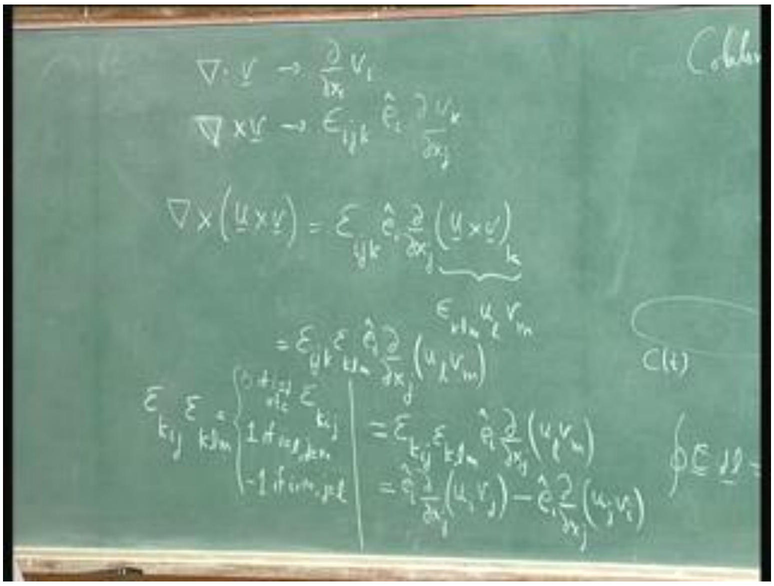

So, we already have that divergence of any vector becomes, del del x i v i. Curl of a vector becomes, epsilon i j k E i del v k del x j. Now, what does something like, curl of say u cross v become, we can see 2 curls are there I mean. 2 cross products are there, so this would be epsilon i j k E i del del x j of u cross v k. That is what it means, the curl of something is epsilon i j k E i del del x j of that thing k. But this, itself is epsilon k l m u l v m. The k th component of any vector cross product is epsilon i j k and you surprises the E I, because you want the i th component. So, here you want the k th component, so epsilon k l m u l v m. So, you get a complicated rather complicated looking mess. Let me write it out, epsilon i j k epsilon k l m E i del del x j of u l v m. Looks, in fact, downright terrible, is there something we can do.

Well, one thing we can do immediately, is we can rewrite this as epsilon k i j just by rotating it. Now, if you look at these this product let us look at it. It is epsilon k i j epsilon k l m, I know that i and j must be different from each other and different from k. Similarly, l and m must be different from each other and different from k. So, if k is 1 i and j can be 2 and 3 or 3 and 2, L and m cane be 2 and 3 or 2 and 2. So, really there are not many choices either i j is the same as l m or i j is the same as m l, so it means, that it is equal to 0 if i equals j etcetera. So, those if these indices are not totally, different it is 0, it is equal to 1 if i equals l j equals m and it is equal to minus 1, if i equals m j equals l why, because supposing this was 1 2 3. Then this would be 1 3 2, one of them is 1, other is minus 1. So, the product is minus 1, supposing this is 1 2 3, so epsilon of 1 3 2 is minus 1. If this is also 1 3 2, then it is minus 1 square it is 1.

So, it is either the same number square in which case it is 1 or it is 1 times minus 1 or minus 1 times 1 in which case it is minus 1. So, this rule simplifies any product of 2 cross products. So, let me apply it, what do I get? I get either j equals l k equals m with a plus sign or j equals m k equals l sorry, i equals l with a minus sign. So, let me first write it out in the proper form, epsilon k i j epsilon k l m E i del del x j of u l v m. So, if they are in the same order, it is plus sign, so I will say it is equal to E i del del x j times l is equal to i. So, u I, M is equal to j v j. And if they are in opposite sign i have to minus sign E i del del x j of now, l is equal to j and m is equal to i. So, u j v I, now, of course, I can now apply, my differentiation rule of a product and attack each of these in turn.

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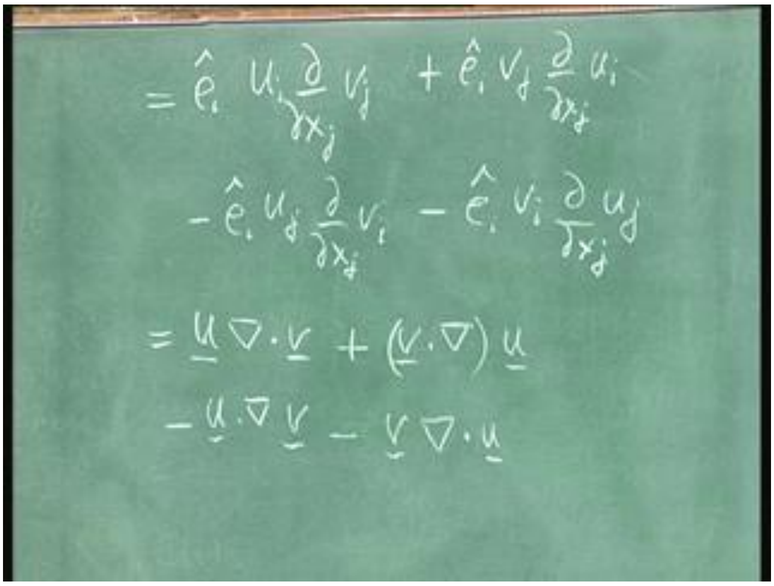

When I do that I get E i u i del del x j of v j plus E i v j del del x j of u i minus E i u j del del x j of v i minus E i v i del del x j of u j. All I have done is I have taken the derivative of a product and acted on individual terms. But each of these is now, completely recognizable, this is u divergence of v. This is plus you can see that this is v dot grad acting on u minus; this is u dot grad acting on v minus v divergence dot u. As usual you can check with your hand book and find out if it is correct, but this is an extremely powerful easy way of solving all interesting vector calculus problems. We will use this in the rest of the course, which is why I am introducing this, becomes very important when you are doing things like wave equation. Next time on I will introduce Ampere's law the generalized ampere law and we will go into Maxwell's equations.