Electromagnetic Field
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Lecture - 3
Introduction to Vector

Good morning!

This is the third lecture on the electromagnetic field for EEE students. In the last two lectures we had covered vectors basically and this lecture I want to introduce the first application of vectors, namely Coulomb's law.

So we had already seen Coulomb's law before in many courses, so I don't have to go into it. If you put two charges Q1and Q 2 then the charge Q 2 feels a force from the presence of charge Q1and the force is on a line joining the two charges. And it is pointing away from Q1when it is applied to Q 2. The same two charges apply the force opposite to it, that is, Q1feels a force from charge Q 2. It is an equal and opposite force. How large is this force? Well if this distance, we call this r12 - when I label things like r12, I mean put the foot of the vector at 1, draw an arrow to 2 and that vector is called r12. Then the force is proportional to the strength Q 1. It is also proportional to the strength Q 2 and it is inversely proportional to the square of the distance, that is, it is inversely proportional to r12 square. This is the well known Coulomb's law and I am sure you met it many times before. This is the force and forces are vectors, which mean that this is the magnitude of the force. The force also has a direction and the direction of force is given by the direction of r12. When I put a cap on top of any symbol it means unit vector along the direction of that symbol.

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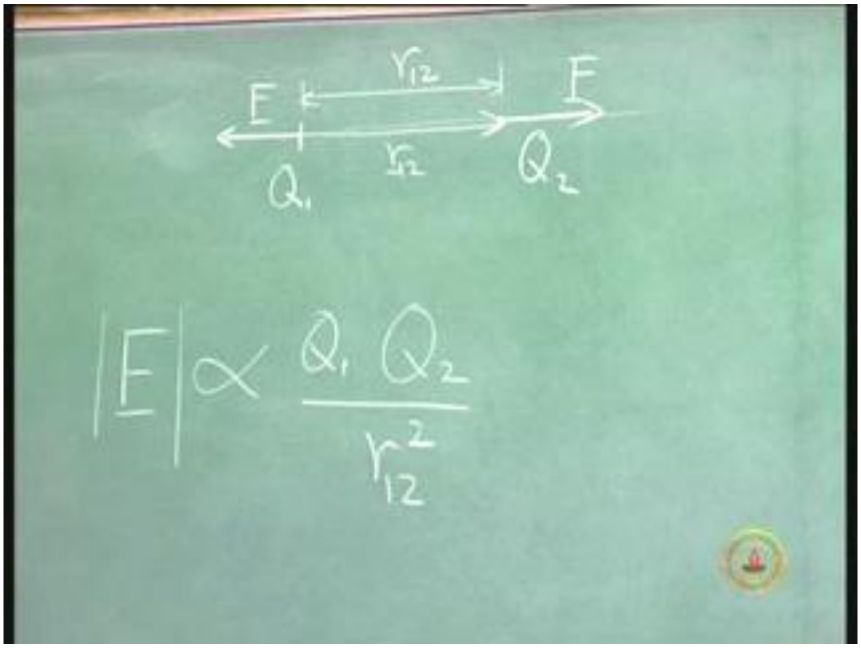

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So the force is along r12 and it has a magnitude that is proportional to Q1Q 2 over r12 square. Now the proportionality is completely arbitrary. There are coordinate systems, namely the CGS system, where we would actually write F is equal to Q1Q 2 over r12 square, but in this course we will be using the SI system, the Standard International system, and in the standard system this proportionality constant is not 1, it has got a value. So the full equation looks like F, which is a vector, is equal to this proportionality constant times Q1Q 2 over r12 square along the direction r12 and for SI units, this proportionality constant is called1over 4 pi epsilon 0.

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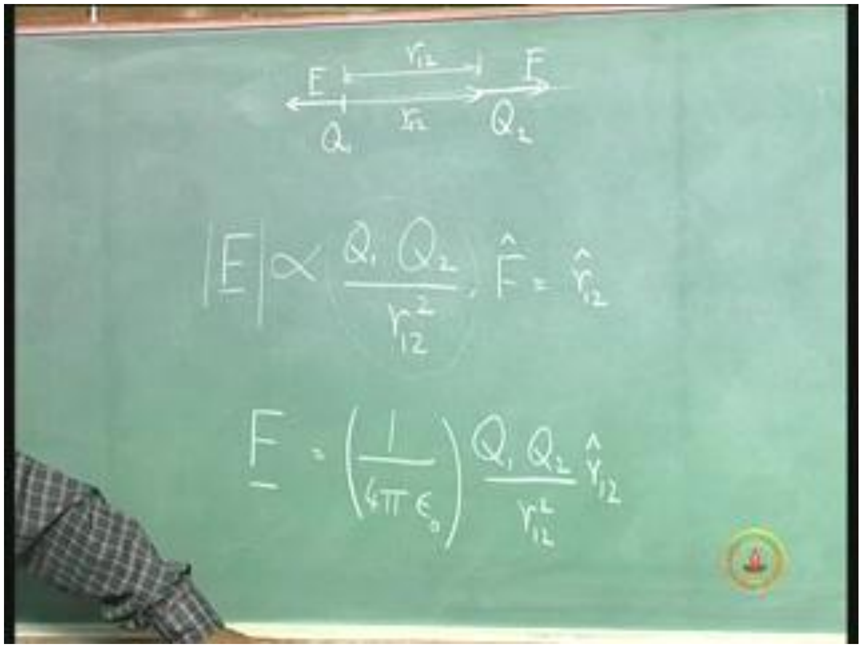

This, as I told you, is completely arbitrary. In another coordinate system then this proportionality constant would be different. For now what is all important to know is that epsilon, which is called the permittivity of free space, has a value which is “eight point eight” into ten to the minus twelve. And if you look at the units that it must have, you can take r square to this side, Q1Q 2 down, so what will you get? You will get the 1 over epsilon has the units of newtons meter square per Coulomb square. So epsilon naught must have the units of Coulomb s square per newton meter square.

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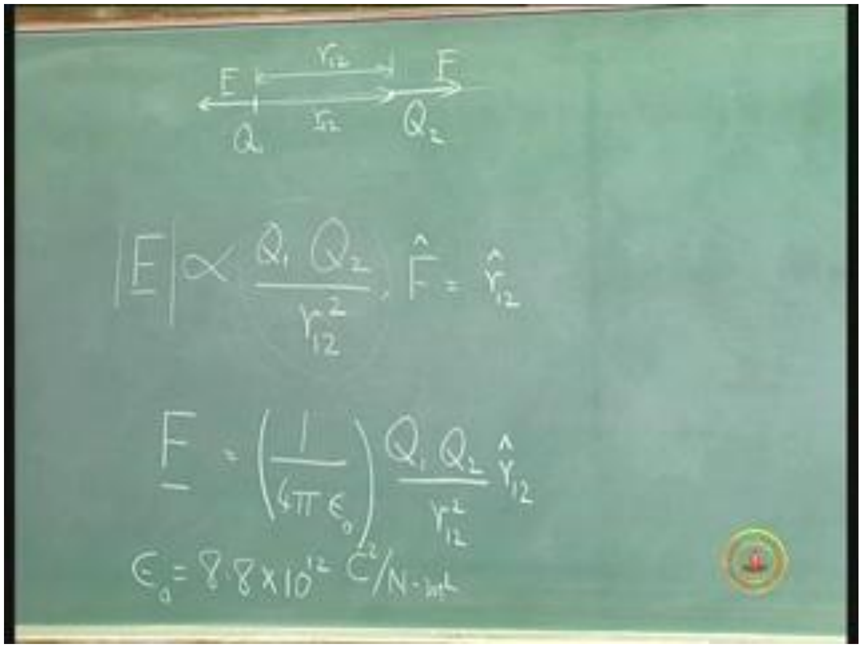

More conventionally it is defined in terms of capacitance but we will come to that later. So this is a vector equation. It defines a vector F in terms of another vector and some scalars. Now this F, which is the electrostatic force F, is a function of Q 1, the location of Q 1, Q 2 and the location of Q 2. How strong is this force? For this, let me go back to Feynman lectures; Robert Feynman, 1 of the most famous physicists in the twentieth century, he wrote three volumes; Lectures of Feynman, which is probably the finest introduction to physics that has ever been written, and in volume two he talks about electricity and magnetism. I strongly suggest you read chapter 1 of volume two and I am going to take an example from there.

You know that charges come in two types: there is positive charge and there is negative charge. Positive charges include protons, ions; negative charges include electrons and any human body contains enormous number of both.

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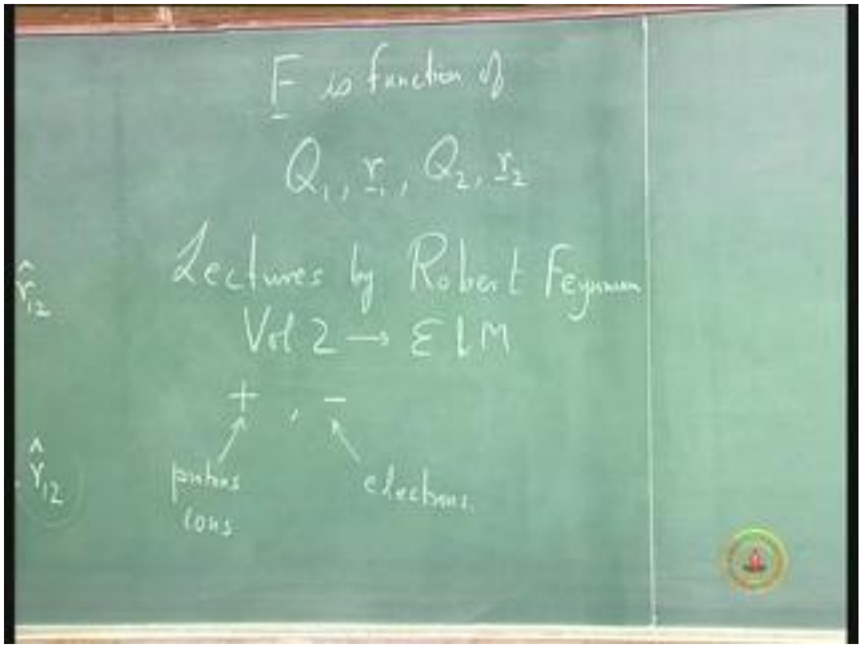

Now supposing humans were not did not have equal number of electrons and protons; supposing we had an excess of electrons: how much? Let us say we had a 1 percent excess of electrons? And to simplify all our mathematics, I will assume a human being weighs hundred kilograms. So roughly 1 kg worth of protons equivalent charge in electrons is excess. Human density is 1 gram per centimeter cubed, so 1 kg contains thousand centimeter cubed. Each centimeter cubed contains around ten to the twenty three protons or electrons. So we will say 1 percent corresponds to thousand times ten to the twenty three electrons. Now what does that give us in terms of a force? Supposing you had human being 1 percent excess electrons and this human being is standing below another human being who will also have 1 percent excess electrons. So each of them has ten to the twenty six electrons; that is ten to the twenty six more electrons than protons.

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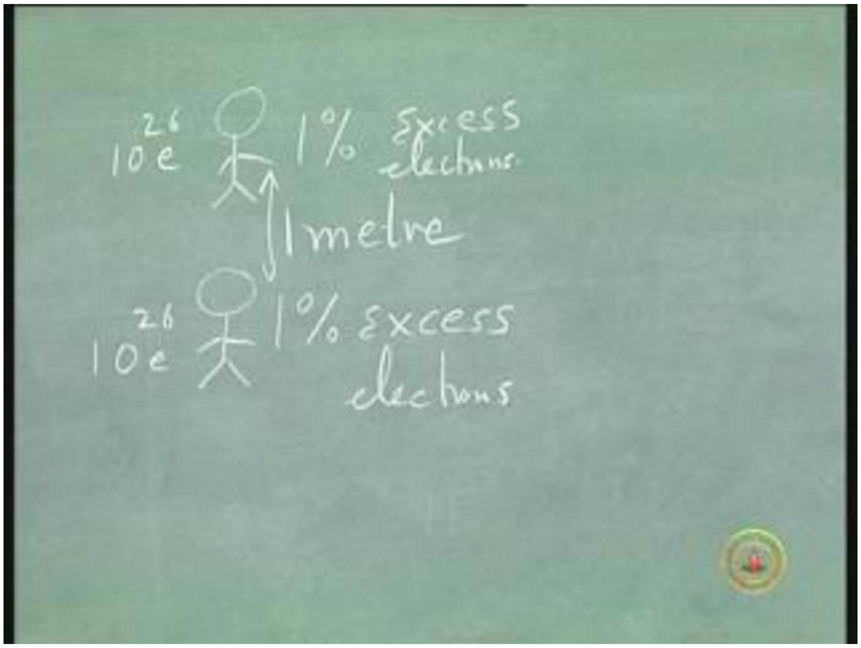

So we can apply Coulomb ’s Law. Let us say they are about1meter apart, so we will say that the force magnitude is equal to1over 4 pi epsilon 0 into 10 to the 26 times the charge of the electron whole square, because both of them have the same charge, divided by the distance which is1meter whole square, and this force should be newtons. We already know what epsilon is, epsilon is 8.8 into 10 to the minus 12. 4 pi is 12, so if you combine the two, this is around 10 to the minus 10. Charge of the electrons is 10 to the minus 19 Coulomb s. So if you put it all together, what do you get? You get 10 to the 26 minus 10 to the minus 19 that is 10 to the 7 whole squared, 10 to the 14 divided by 10 to the 10 minus 10 which is 10 to the 24 newtons. Having this, 4 times 10 to the 24. Ten newtons is the weight of1kg that is the force of gravitation is 10 newtons per kilogram.

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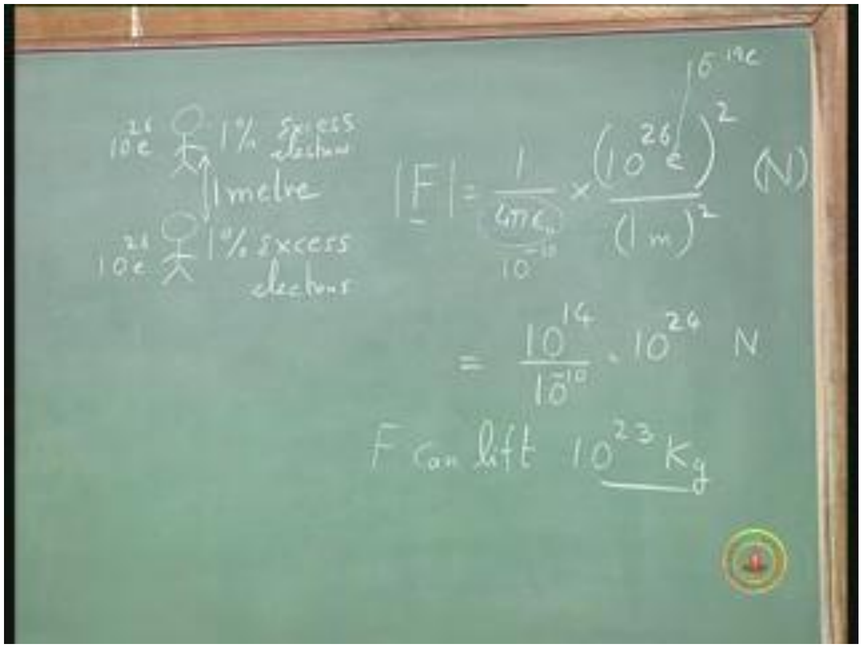

So this force can lift 10 to the 23 kilograms and you know what the weight of the earth is; the mass of the earth is 10 to the 24 kilograms. So if two of us happen to be having an excess of electrons of1percent, the force between us could lift the earth. That is how strong this electric force is. It is unimaginably strong. You can compare it with the gravitational force because if you have two people they attract each other gravitationally and they also interact through electrical forces. So supposing I take two electrons there is an electron and another electron, they are separated by a distance d. The force due to gravity is the gravitational constant times the mass of the electron square divided by d square. This is the force with which 1 electron attracts another electron, this gravitational attraction. The electrical attraction force due to the electric field is1over 4 pi epsilon 0, e square- the charge of the electron square, divided by the same d square.

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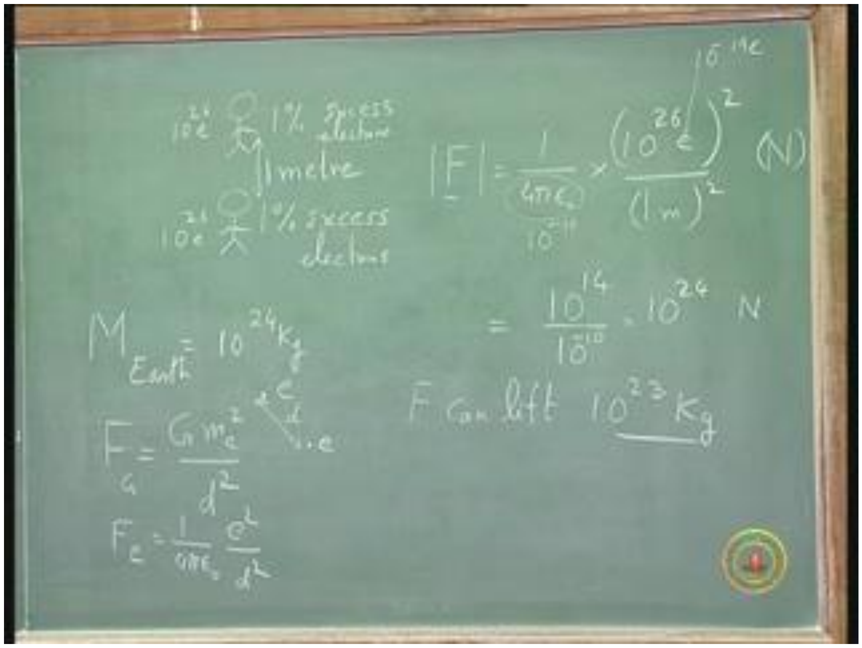

So we can work out how much stronger the electric field is compared to the gravitational field. How much is it? Well, the d square will cancel out, so what you get is F electric divided by F gravitation, is e square over four pi epsilon naught, divided by G times m e square. As before the electric charge is 1.6 into 10 to the minus 19 Coulomb s. The mass of the electron is 9.1 into 10 to the minus 31 kg. Now the gravitational field- the universal constant of gravitation, is 6.7 into 10 to the minus 11 and the units are again gotten from here which is newton meters per kg square.

Now you can see I am going to take 10 to the minus 30 square, 10 to the minus 60; and then I have a 10 to the minus 11. I am going to take it all to the numerator, I have here 10 to the minus 19 times 10 to the 10. So a huge number is going to come out. Well you can work it out and what you find is that this is of order 10 to the 43.That is, the electric force between two electrons is 43 orders of magnitude stronger then the gravitational force between those electrons. Now naturally the gravitational force is proportional to mass, whereas the electric force is proportional to the charge.

So if we took a heavier particle with the same charge, you will get a better answer. Well how much better? If you took the proton proton has a mass which is around 2000 times, well, 1800 and something times electrons. So in that case, this ratio gets divided by 2000 square, so then you will find that F E upon F G will be like 10 to the 43 divided by 10 to the 6, which is 10 to the 37. It doesn't get any better than this because the proton is basically a heavy particle, most of our body is built out of units which is the weight of the proton. So this is why Feynman says that the force due to electricity is a billion billion billion billion times strong.

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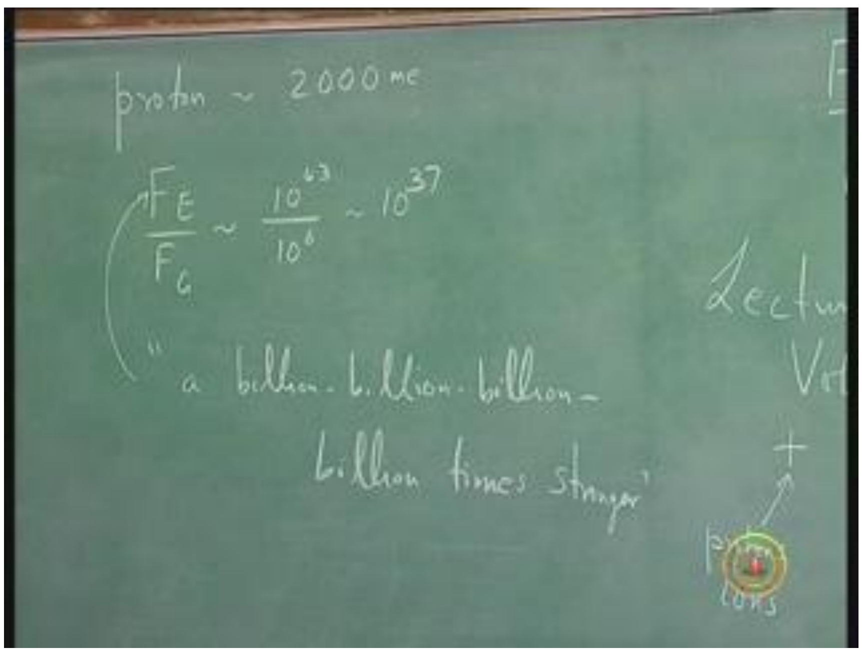

Four billions in a row, because a billion is ten to the nine. We put four of them next to each other, ten to the nine to the power of four is ten to the thirty six, this ratio is actually ten to the thirty seven. So the electric field is enormously more powerful than the magnetic field, than the sorry, the gravitational field. However, in our normal experience there is no electric field. In fact, if you look at the ancient records you will find they talk about gravitation but they don't talk about electricity. They knew lightening and the Greeks knew a little bit about amber- they knew amber could be rubbed, then it will it will attract things. They knew about some magnetic materials, that was it. This force that is a billion billion billion billion times stronger is missing. It is not there, so where does it go? How is it that when you get to larger and larger scales the electric field becomes less and less important, whereas when you get to larger and larger scales the gravitational field becomes extremely important?

Let us take a look at that. There are several answers that are given. Let us take them. Answer 1: equal number of electrons and protons, so the result is if you have an electron and you have a proton and they are both close to each other, then far away this electron is, let us say it is a positive charge, it is attracting the charge, the proton is repelling the charge, and so there is a very slight force only. And so all these huge forces that are being exerted by the body, they all cancel. There is a problem with that argument- if you see that problem. Let us work out what happens in such a case. Supposing I have a charge Q and I have a charge minus Q, that is I am thinking of the proton and electron and they are separated by a distance d, I want to know what is the field due to this proton and electron combination far away. Now if you look at this, you can construct what the forces will be. You will draw a straight line to this to this point and the force will be away from Q. We will draw another straight line connecting minus Q to this point and the force will be towards minus Q. Now the forces will mostly cancel.

However, if we write out what the forces are, you can see that this distance from the point P to minus Q is a larger distance than from the point P to plus Q. How much larger? Well, you can draw perpendiculars and see that this distance is equal to this distance plus a little extra, whereas this distance is equal to this distance plus again a little extra. So the distance from minus Q to P is equal to distance from plus Q to P plus this quantity and how much is that quantity? It is a standard calculation; you have d1 it in your optics- when you do interference if you take this angle as theta that angle is the same as this angle theta. So you can say that the difference in distance is two d sine theta. So d sine theta, the total distance is d so this distance is d sine theta.

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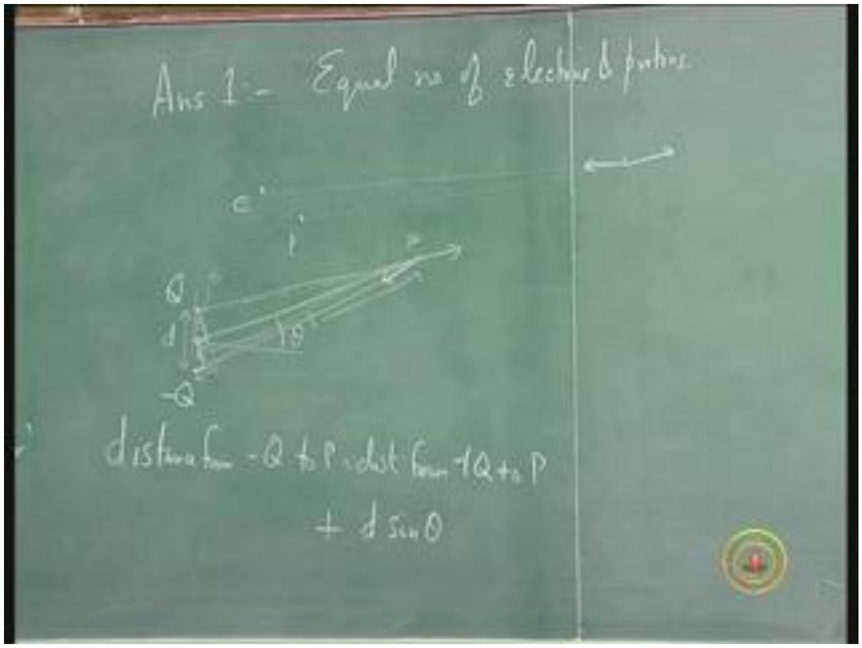

So the distance from minus Q to this point P is the distance from Q to this point d plus this extra distance and this extra distance is nothing but this d sine theta. Sine is nothing but the opposite side over the hypotenuse. So now if I want to find out what the force is, well the force- I am only calculating the force along the radius; there is actually a force in this direction- we will work that out later when we work what the electrostatic potential is. But the force along the radius, so F sub r is1over 4 pi epsilon 0 Q over- I will call this r 1- square minus Q over r 2 square so this is r 1, this is r 2.

In other words, the force along the radial direction is due to the fact that one charge is nearer, one charge is further away. So the one over r square is not the same. So I can write down what this will come out to. The radial component of the force is1over 4 pi epsilon 0 times Q over r1square minus Q over r 2 square times, of course the charge of the at the point P there is a small charge q it should also be multiplied.

Now I can write these in terms of each other because I have already written the distance from minus P to P is the distance from plus P to P plus d sine theta. I will write that out, it is1over 4 pi epsilon naught Q over r 1 square minus Q over r 1 plus d sine theta whole square times small q.

Now what does that mean? I know that I am looking from a point far away, so this r1is much larger than this d. So I can say r1plus d sine theta whole square is approximately equal to r1square plus two d sine theta times r. That is, I am neglecting the d square sine square theta because it is very small. Then I substitute that here and I expand, so my F r becomes capital Q small q over 4 pi epsilon naught times 1over r 1 square times 1minus1over- I have pulled out an r 1square- so 1 plus 2 d sine theta over r 1.

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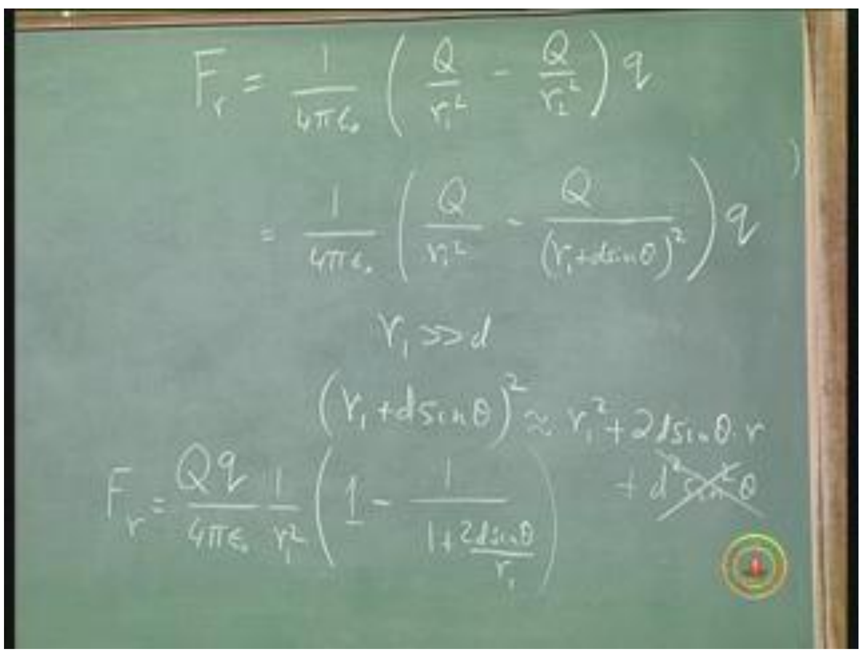

It’s a very small quantity because d is small compared to r 1 so I can expand this, and I will expand it in case when I will take it to the numerator this1plus two d sine theta over r 1 will become 1 minus two d sine theta over r 1. It becomes 1 minus 2 d sine theta over r 1. The 1 will cancel out and what you are left with is Q q over four pi epsilon naught. There is1 r 1 cube and a 2 d sine theta so ultimately what have we gained the Q q over 4 pi epsilon naught. r 1 square is Coulomb 's law. You have an extra term that goes like 2 d sine theta over r 1 times F Coulomb.

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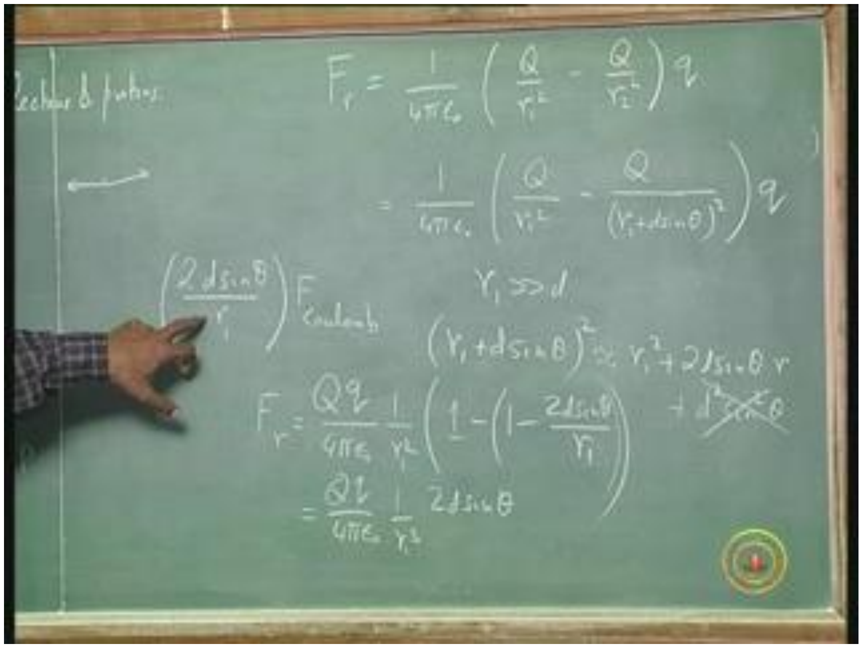

So the fact that you had electrons and protons close together buys you this factor. It means that if electrons and protons are on average d distance apart then at a distance r 1 the force is smaller by this factor. But does that explain why gravitation dominates? Let us see the numbers. We have the numbers here already. We have that gravitation dominates by a factor of ten to the thirty six, thirty seven. Now typically electrons and protons are 1 angstrom apart, so d is of the order of ten to minus ten meters. We are talking about a distance 1 meter away for this calculation so r is 1 meter. So what does that factor give you? It gives you a factor of two d sine theta over r, is roughly equal to d over r which is ten to the ten.

Now ten to the ten can reduce ten to the thirty seven to ten to the twenty seven ten to twenty seven is still a large number which means that this talk about equal number of electrons and protons does not explain why gravitational dominance. If it were really just that then electromagnetics would have dominated and you would have never noticed gravitation. We would all have been floating around like charge dust and you would never have noticed that there is a huge planet which is trying to pull us to itself. But that’s not what happens.

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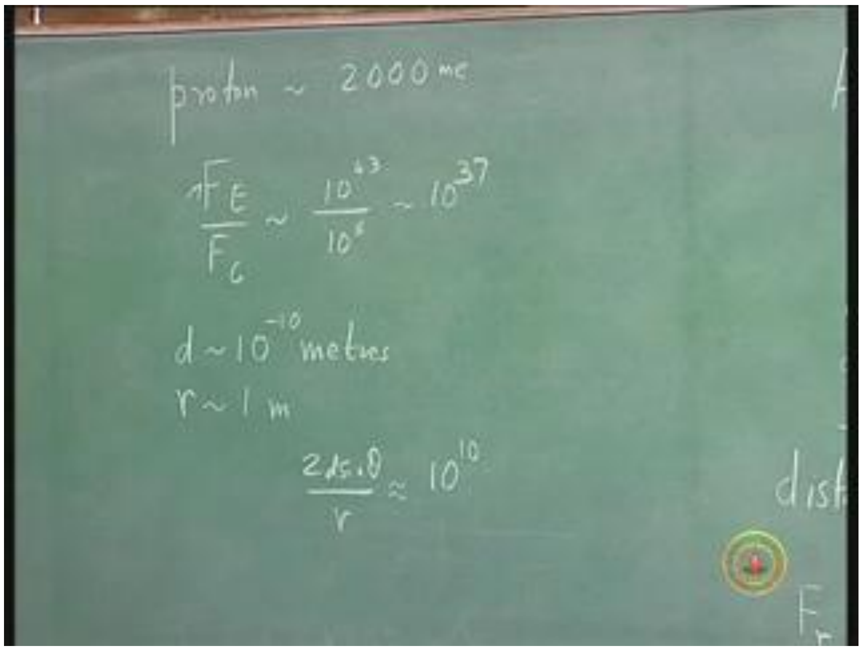

Now there is an answer to this. The answer has to do with a slightly different explanation in this and to explain this we have to really go back to all the forces. If you look at the universe there is gravitation that’s the weakest force, then you have magnetism that’s the next weakest that we know of, electricity and then you get to the nuclear forces and these are the strongest forces we know about. There may be stronger forces than these but we don’t know.

Now it turns out that when we talk about electricity and magnetism we don’t take into account nuclear forces. When we talk about gravitation we don’t consider electric fields. Sometimes we consider magnetic fields but not often. In fact gravitation is by itself. Electromagnetism is by itself. Nuclear forces are by themselves. That’s how you will find out in textbooks. The reason is there is such a large range of force here. Each is unimaginably stronger than the other that they cannot complete. If you see electric forces it means nuclear forces have cancelled completely, if you see magnetic forces in normal materials it means the electric forces have cancelled completely and if you see gravitation it means all these forces have been cancelled completely.

So how does this cancellation work? Supposing you take a gas containing charged particles, so there are positive particles and negative particles- electrons and protons. So they all are busy exercising Coulomb’s law and Coulomb ’s law is ten to the thirty seven or ten to the forty three times stronger than gravitation. Now such a gas for example is found in astronomical nebule- nebules that are heated by nearby stars, they are all charged. See you will have enormous amount of charged particles in these gases so you would imagine that electric forces are dominating, but that’s not the case.

Now what happens inside is these charged particles adjust themselves to other charged particles. By that I mean a charged particle has only as much energy as its temperature allows so its rattling around and if it has hundred degrees of temperature or thousand degrees of temperature that limits the amount of rattling it can do.

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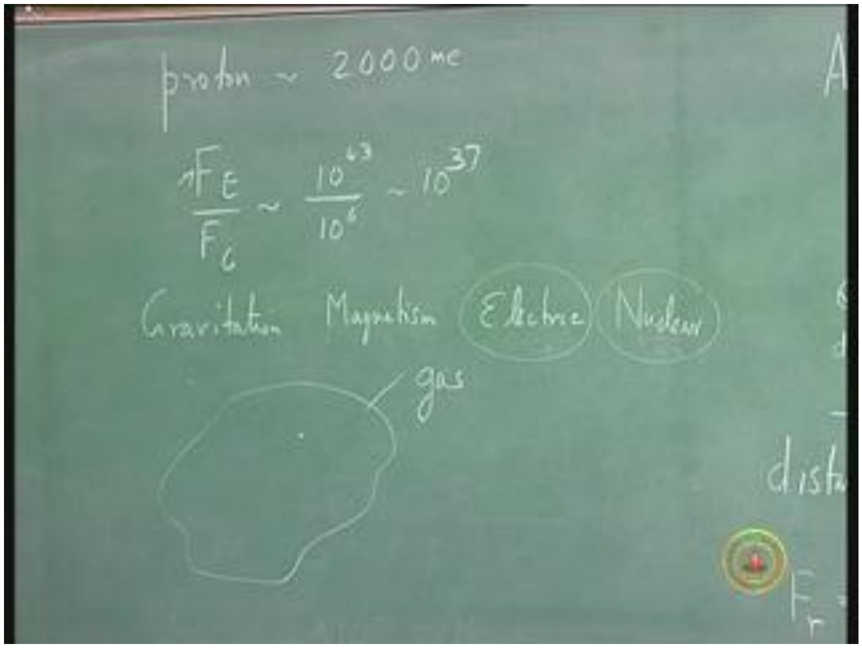

So if a very strong electric field develops, all other particles move and in moving they cancel that electric field and they do this at every point in this gas- everywhere till finally there is no electric field at all. Now we know one case of this which is a conductor. We know that inside a conductor you cannot have an electric field, you cannot have forces. The same idea happens here as well. In fact if you look inside a gas you can show, not going to show, but you can show that the force due to Coulomb actually decays like e to the minus r over lambda. It does not go as inverse square law at all; it goes exponentially small. This value of lambda depends on the material; it can be very large but usually it is very tiny.

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So now if you now take this model and you try and compare our body magnetic- I mean gravitational versus electric- the answer becomes very different because you are no longer comparing inverse square law to inverse square law; you are comparing exponential decay to inverse square law. When you do that you can ask when does e to the minus r over lambda divided by r over lambda square becomes approximately ten to the minus thirty seven. If that happens, then the ten to the thirty seven extra number will cancel out. Well you can plot this and what you find is the following. You can plot e to the minus r over lambda divided by r over lambda square versus r over lambda. This is a logarithmic plot; it's a semi log. You will find more or less a straight line that intersects 1. It starts out at ten to the thirty seven over table number, so this is ten to the thirty seven. It reaches 1 at around eighty to ninety.

So around ten to the two around a hundred r over lambda, the electric field has become as weak as the gravitational. So it depends really on the value of lambda. For most materials lambda is very small for conductors- it is angstroms. It is small for a weak conductor- it may be micrometers. So within a centimeter the gravitational field has taken over from the electric field. Now this is really the reason why we never see the electric field in large scale. Normally it is very rare to see that the moment you have electric fields from many many particles, the particles adjust to cancel out that electric field.

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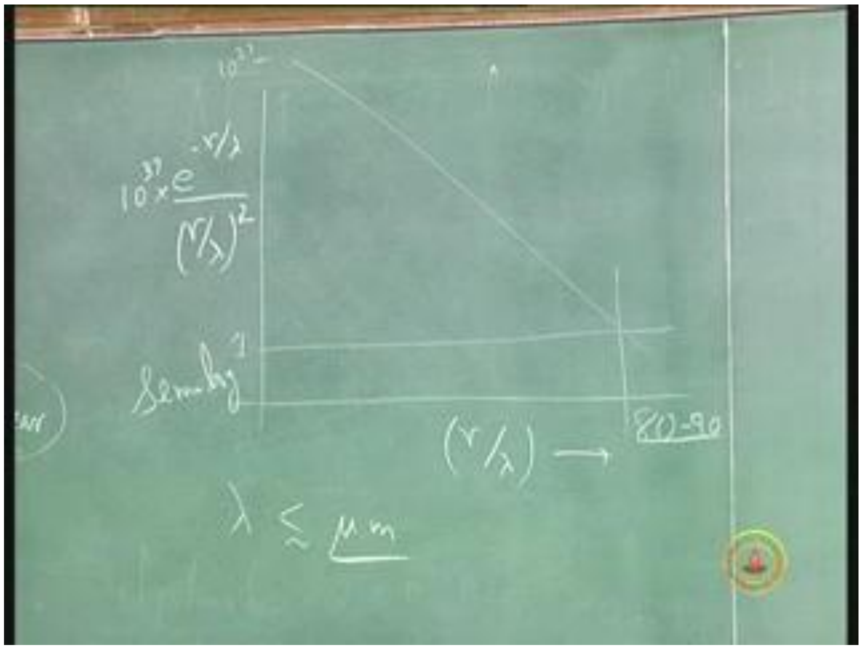

But particles cannot adjust to cancel out the gravitational field. The reason is there is no positive gravitational field and negative gravitational field- all masses attract each other. You don't have a negative kind of marks which repels. So gravitational field is phenomenally weak but it always adds. The electric field is phenomenally strong but it cancels as you go to larger and larger scales. The electric field becomes unimportant, so we can draw a diagram now explaining the various forces. We had nuclear forces, they form the nucleus. The kind of scale length on which they work is ten to the minus fifteen meters. Once you get away from ten to the minus fifteen meters the nuclear forces become attenuated, that is, they tend to adjust and cancel themselves out the electric forces. They work at the scale lengths of chemistry that is at angstrom distances, so due to nuclear forces we get elements. Due to electric forces, we get chemistry. We get molecules. We also get metals and other kinds of properties. The magnetic forces they have much longer scale lengths, in fact they can have galactic scale lengths.

So they are very large scale lengths but they are very weak. Much, much weaker than the electric forces. For example the earth's ionosphere is dominated by the magnetic field. The entire solar system is dominated by the solar magnetic field. Entire galaxy is dominated by galactic magnetic field and finally we have gravitation. Ultimately the weakest force gravitation becomes the strongest force and it actually forms the universe. So there is a very peculiar kind of ordering.

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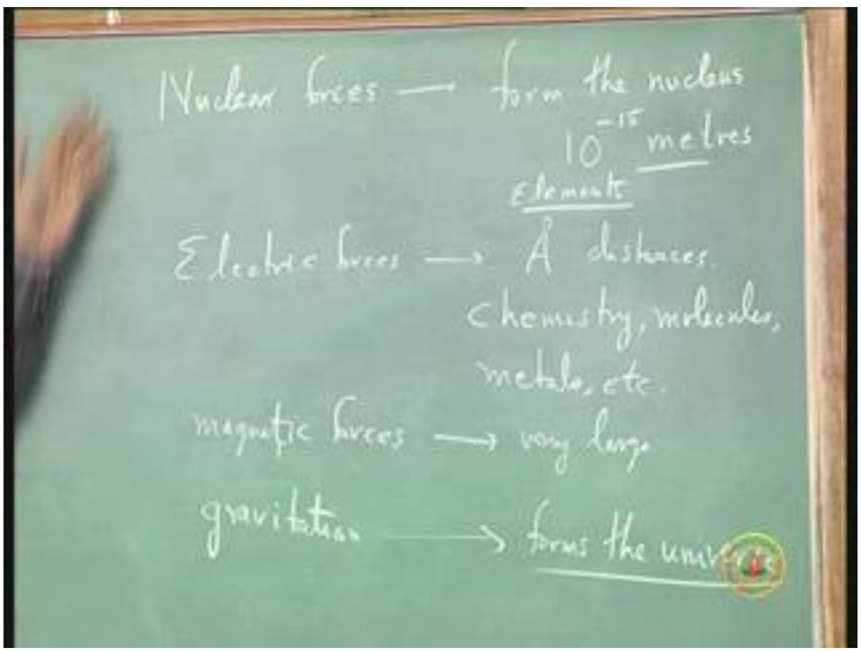

The strongest force cancels itself out earliest. The next strongest cancels it itself out next. the next strongest works up to a galactic scale length and gravitation finally conquers everything, and in this picture we are looking here we want to understand electric forces and we want to understand magnetic forces.

Now anticipated myself several times but still I will repeat just to make sure I have got my point across. We have the Coulomb force, it's a vector, goes like 1 over four pi epsilon naught charge 1 times charge 2 divided by r 1 2 square along the direction of r 1 2- this is the definition. This is the definition if you have two charges, but supposing I had a collection of charges and I want to know the force on a charge here. So I have Q 1 Q 2 Q 3 Q 4 and I have a charge q, what is the force on charge q? Well Coulomb not only found this law he also found out how this law works when multiple charges are present. He found out that you ignore all but one. Find out what Coulomb's Law says about that one.

Now you ignore all but another. Find out what Coulomb's Law says about the next and then vectorially you add them up. The result is the total force. If you want to write this mathematically, what it says is the force is equal to 1 over 4 pi epsilon 0 this charge q times Q 1 divided by r 1 square along r 1. So this distance is called r 1, this is r 2 r 3 and r 4 plus 1 over 4 pi epsilon z0ero q Q 2 over r 2 square r 2 plus 1 over 4 pi epsilon zero q Q 3 over r 3 square plus 1 over 4 pi epsilon 0 q Q 4 r 4 square r 4.

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Now you can see that q is common, so I can pull it out and when I do that, I can write the formula as follows: F is equal to small q over 4 pi epsilon 0. Summation I is equal to 1 to 4 capital Q sub i r sub i unit vector divided by r sub i square. This summation is simply saying substitute i equals i substitute i equals 2 i equals 3 and i equals 4 and add up all the results.

Now this interesting formula is known as superposition. What it really is saying is the following; it is saying that whatever interaction there is between this charge small q and this charge capital Q, one is not affected by the presence of the other charges. Let me give you an example where this would not be true. Supposing I had my charge Q 1 and my q and supposing the medium in between were affecting Coulomb 's law and supposing the medium, its properties, change because of Q 2, for example, the medium has an epsilon, we have been writing epsilon naught but the medium may have a different epsilon and this epsilon may change due to Q 2. If epsilon changes, it means the force between Q 1 and small q is the function of whether Q 2 is present or Q 2 is absent. In that case, this would not work. This would be wrong, but luckily Coulomb 's Law has been verified and it is known that it is correct that if you do this experiment in vacuum that this superposition rule is accurate to some twenty digits.

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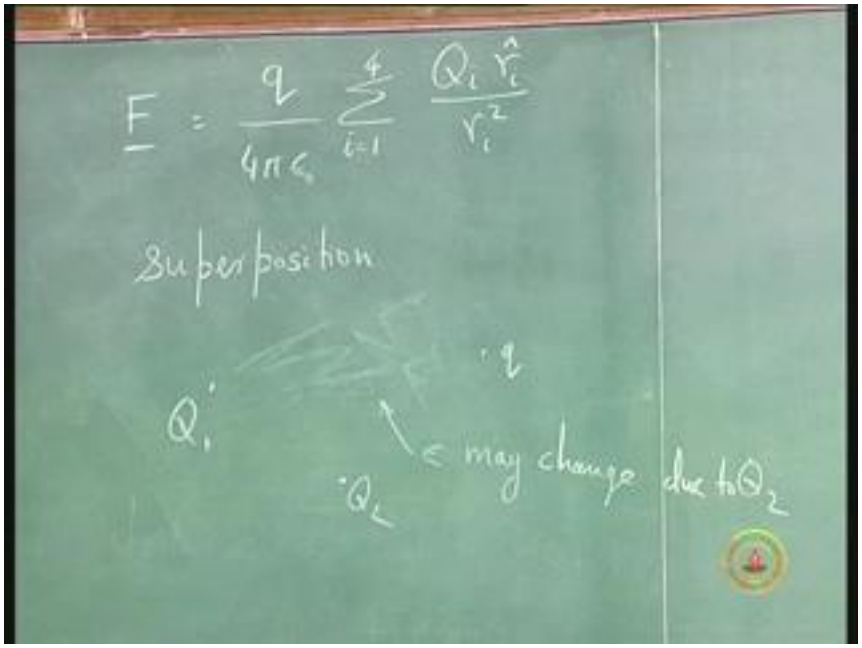

I will explain later with just how they come up with such statements, but in a certain sense Coulomb's Law is the most accurately known law in physics. There is no law that they know better. For instance we know the speed of light to about ten digits, we know the gravitational constant, it is about the similar number of digits. We are not at all really sure whether we understand the law of gravitation, but we know the power of two here correct to twenty digits.

So this is the most accurately known most precisely tested theory that we have in physics, so we have come up with superposition, and we can put point charges, and I already used this idea without telling you when I calculated the force due to a dipole because I had a charge Q 1 had a charge minus Q and I had a charge small q and I choose to calculate this force first and then this force next. That is only correct if superposition is true. However that's a very simple example.

Supposing I had a cloud of charge. Lot of charge and it is distributed over a region, and I want to know what the force on a charge q somewhere else is. Well this equation is still true. The force is equal to 1 over 4 pi epsilon naught. Summation 1 equals 1 to capital N Q sub i r i divided by r 1 square, but this number N may be very large. May be it is 10 to the 23. If you are talking about all the charges in a small amount of a metal then this will be a very large number and we cannot carry out practically such a sum. Luckily just when we struggle to carry out such sums we have a new way of looking at things which make things simple.

If there is a very large N then we can make a little box inside the volume. This box would have sides which are very small and we would count the number of charges amount of charge inside the box. So we could take box of size delta and we could plot. We could calculate the amount of charge in it so you could make a table and we could say, take delta F 1 centimeter 1 millimeter 1 micrometer and so on and work out what the charge is. Now obviously the charge will become less and less, however, if you plot charge per unit volume, that is, you calculate the charge and divide by the volume of this cube, divide by delta cube, well what do you expect? What we expect is like this- this is delta, this is Q over delta cube.

For very large delta, the fact that the charge is not uniform- there may be more charge out here, less charge out here, we are not- our cube is spanning all these different regions-
so you will expect some variation. Then as the delta becomes small, the distribution of charge inside that small region is more or less uniform. So we expect that the charge will stabilize.

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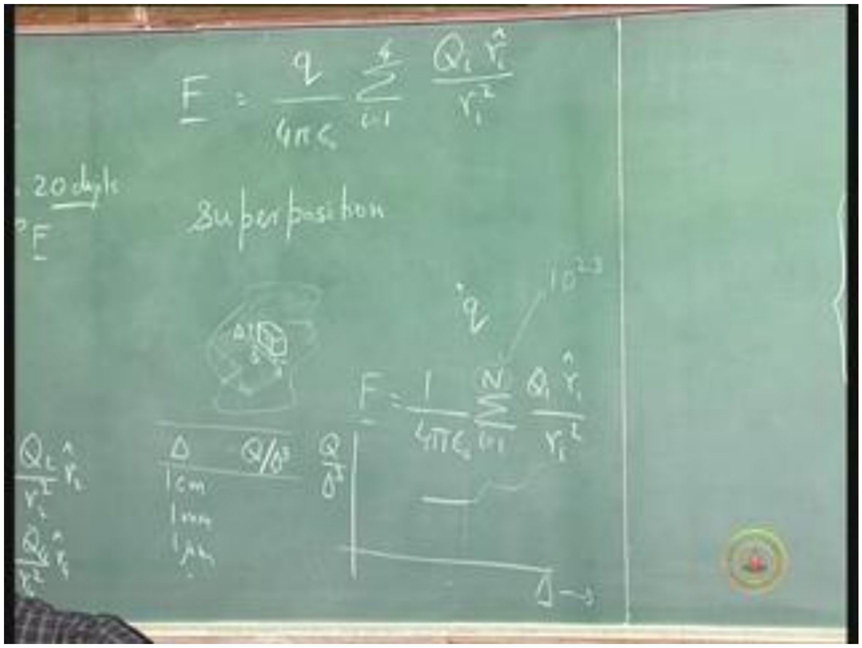

That is charge per unit volume will stabilize. Then when this box becomes very, very small so small that the number of charges inside this box becomes only a few, then you will start having noise. This is the kind of picture any experimentalist will tell you is present- we want a box around there, we want a box where we are not inside the noise but we are corrupted by the fact that our box is too big. If we take a box like that, we can replace a sum on individual charges via **some1 boxes,** and we all know what a **some1 boxes** is. This is nothing but 1 over 4 pi epsilon 0 volume integral charge density over the volume. I will continue this next time but the idea is basically here and we will continue next time.