Electromagnetic Fields
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Lecture – 27

Faraday's Law

Good morning. The last 25 or so lectures we have been talking fields that are not varying in time. First we introduced the electric field coulomb's law, the divergence theorem poisson's equation and then we introduced the static magnetic field talked about ampere's law the biot savart law. And we have established a large number of equations which seem to all describe a single consistent picture. So, for context I am going to write down those equations. First of all we have the electric field is one over four pi epsilon naught volume integral row. This is the charge density at another point r prime and this electric field is at a position r.

So, r minus r prime divided by r minus r prime cubed d v prime. This is our basic coulomb's law. If I write the same equation for magnetic field, we have magnetic field is equal to mu naught over four pi volume integral j of r prime. This b is b of r prime of r sorry, j of r prime cross r minus r prime divided by r minus r prime cubed. It is the same dependence, but in one case the electric field is due to a scalar multiplying this operator. The magnetic field is due to a vector j cross this same operator. Now we know that r minus r prime over mod r minus r prime cubed is equal to minus the gradient of one over and this quantity I tend to call minus gradient one over r 1 2. It is just notation.

In fact it is not even standard notation. The correct way of saying it is one over the magnitude of the vector r minus r prime. So, from this definition we went to potentials. We defined the scalar potential phi of r which was 1 over 4 pi epsilon naught volume integral row of r prime divided by r 1 2 d v prime. It is a scalar field because row is scalar and r 1 2 is again a scalar field for the magnetic field. We defined a vector potential a of r which was equal to mu naught over pi volume integral j of r prime divided by r 1 2 d v prime. Symmetry is there again the scalar source creates a scalar potential. The vector source creates the vector potential. Both of them are connected through the r 1 2 field. The way potential and electric field connect is electric field is equal to minus the gradient of potential. The way the vector potential in b connect the magnetic field is equal to curl of the vector potential. This is the difference. Magnetic field is intrinsically at right angles to everything whereas the electric field is intrinsically in the direction of everything. This is essentially saying electric field is a central force. That is the force is along the line joining the location of the source and the location of the observer along that line.

Whereas, the magnetic field is not a central force because you have the line joining r and r prime and you have the source current the magnetic field is created in a direction, that is 90 degrees to both. Because, the electric field is a gradient we could show that curl of electric field which is minus curl of gradient of phi was 0. Because, the magnetic is a curl divergence of b is divergence of curl of a, is 0. The symmetry continues since b is a curl it is the divergence that goes to 0. Since e is a gradient, it is the curl that goes to 0. Now, there is a very remarkable thing about the electric field which is that if you take the divergence of the electric field, the divergence of the electric field is dependent only on the charge at the point where you are measuring the electric field.

Even though the electric field may be due to charge in all sorts of places when you write the divergence of the electric field it is equal to charge at the same point divided by epsilon naught. This same remarkable feature is also present in the magnetic field. If you take the curl of the magnetic field the magnetic field may be created due to currents in many different places but the curl of the magnetic field only responds to the current at the place where you are measuring the magnetic field namely mu naught j of r. So, the divergence of the electric field is a function only of charge density at r itself not at r prime. Curl of b is dependent only on j at r not at r prime.

All of this is essentially a complete description. It is a complete description for vacuum and it is a complete description for known charges and known currents but we also have materials and when we have materials, we have that row is equal to row free plus row bound. Meaning, the charges we actually place and the charges that are induced in materials. Similarly, currents j is equal to j free plus j bound. The currents we actually place and the currents that are induced in materials. Once you have this the row bound and the j bound are not known. They are the materials' response to the applied electric field. They can even be non-linear. Similarly j bound is the materials' response to magnetic field. It is almost always non-linear. But they are material properties and supposing we knew that we could define we could take this row bound put it into the definition of epsilon and we could define a new field which we call d is equal to epsilon e and then we would find that our equation for d divergence d was equal to only row free.

It is not a function of row bound at all. Similarly if we take this bound current and put it into our definition of mu, well h is equal to b over mu then we find that curl of h is equal to j and only the free current corresponds to h. So, the symmetry is remarkable. Actually for every equation in magnetostatics there is a corresponding equation in electrostatics. For every equation in electrostatics there is a corresponding equation in magnetostatics. Finally we can also write down what are the boundary conditions.

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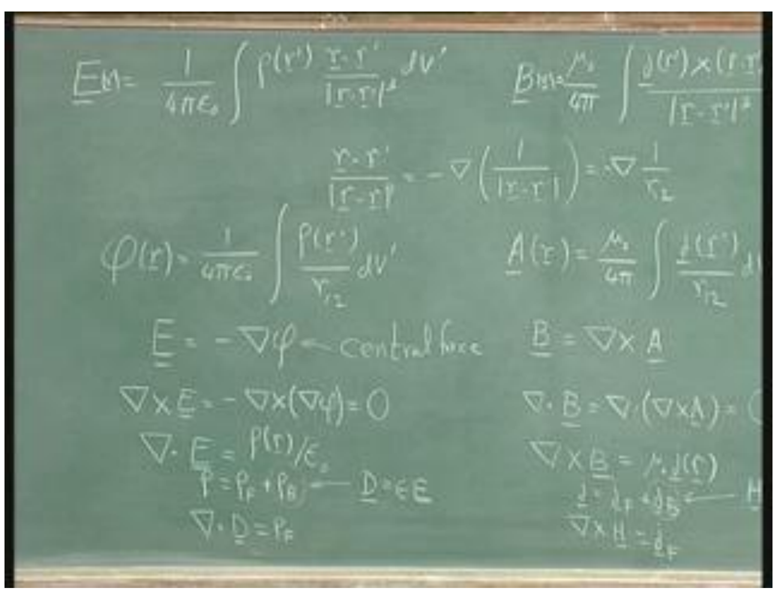

The boundary conditions we had were d normal is continuous, e tangential is continuous in magnetic field case b normal is continuous, h tangential is continuous. They are switched in the sense that e and b are the real fields. They are the fields that respond to the total charge of current free and bound, e tangential is continuous but only b normal is continuous. The reason is the curl, the e and b are 90 degrees in related by a 90 degree shift in direction.

So, if e tangential is continuous you should expect b normal to be continuous. That 90 degree effect comes in and it is normal and h tangential that are continuous. We have a force equation. For a charge it is q e plus q v cross b and you can see the force only cares about electric and magnetic fields. It does not care about d and h, d and h are conveniences. They are imagined because we need to get rid of dealing with bound charges and bound currents. But the real fields are e and b. So this, I think is a summary of everything we have done in the first half of the course. And you can see that really the equations are quite pretty and it looks very much as if electrostatics and magnetostatics are going hand in hand, they are really talking about very similar things. So, what is there to add?

Now let us take a thought experiment. I have a magnetic field. Let us say it is a uniform magnetic field b along the z direction and I have a charge q and I have another charge q prime. Now these charges are at rest. Because, they are at rest if you look at the force equation the force on charge q is zero because of the magnetic field. Because the charge q is not moving, but it is non-zero because of the electric field, there is an electric force e. Now I have not drawn this properly. Then we put the charge this way. So, the electric field is not along b.

Electric field is in that direction and because of that electric field there is a force. So, the force is equal to q, q prime over four pi epsilon naught and if this distance is d or r 1 2 it Is equal to r 1 two divided by r one two cubed. So, we know this is coulomb's law. Everything makes sense. Now, supposing I, as the scientist was doing this experiment, gets into a car and this car comes moving with a velocity v. The charges are not moving but the car is moving and when I come here out of the window I stick out my measuring instrument and measure force on the charge. Now common sense tells us that the fact that I have a car does not change the amount of force felt by the charge. I could be in a rocket. I could be in the next galaxy. Whatever force the charge is feeling is the same. But if you look at my point of view sitting in the car as far as I am concerned, I am stationary. I am inside the car. This charge is moving with a velocity v in the opposite direction.

Since the charge is moving with a velocity in the opposite direction, there is a force q v cross b. So, the force according to me sitting in the car will call this force car is well there is the electric field, q q prime over four pi epsilon naught r 1 2 over r 1 2 cubed right. But there is also this magnetic field force which is plus q v cross, b naught which is along z. So, I can put z hat. Now, this force is the same. It is a new force and that is very strange because what this is saying is if I measured the force while I was walking around if I measured the force sitting in a car, if I measured the force sitting down I am going to measure different forces. Now right at the beginning of the course I am made a very important point. All the theory of electricity and magnetism is a theory.

The only real thing is the force that we can measure because it is the force that deflects a needle. It is the force that actually causes your electronics to respond. So, it is the forces that are real. The electric field is not real. The magnetic field is not real, h is not real, d is not real, phi is not real, a is not real. The only thing that, real is f because this is the only one we can measure. And what I am saying is i am going to measure different things depending on what I am doing. Even though the charge is not in my car charge is staying where it is, but if I am moving relative to the charge the charge experiences different force, Now about 300, 400 years ago Galileo, the same Galileo who got into trouble with the pope and looked at the moons of the Jupiter that same Galileo proposed a fundamental principle of physics. He said whatever the scientist is doing cannot change, the laws of physics, if the charge is moving, yes. That makes a difference.

But if I am moving it, should not make any difference and he thought about that and he said that this amounts to saying that if I take any law of physics and when I say physics, I mean engineering also. Any law of physics is invariant under translation, what that meant was if I have, if I have measured some law of physics in x y coordinates and if I then say x is really equal to some w plus v t then my w coordinate y w coordinate is something else. This is w, this y w coordinate is going to move.

It is going to move with the velocity minus v. Now, whatever law of physics I propose for x y should also hold in w y because this coordinate system is my imagination. The electron does not know about coordinate systems. Electron does not know about what is the centre of the universe. It does not know about origin. It does not know about any of these things. It does not know about x y z also. It is just doing. What it is doing because it is experiencing a force and that force cannot depend on where we drew our axis and it cannot depend on whether the axis is moving with a steady velocity. So, it said any law of physics is invariant under translations, but this law of physics is not invariant on a translation. The force measured if I am standing still and the force measured, if I am sitting in a car are different, either Galileo is wrong or this equation is wrong. Well Galileo turned out to be wrong but he turned out to be wrong in a very sophisticated way.

Einstein improved on his invariance statement and made it even better. But the philosophy behind this invariance we believe to be completely correct. That is nothing in this universe is concerned about what the scientist is doing. It is only concerned about what the object we are studying are doing. So, somehow these two things must be saying the same thing. So, how do we do that? I mean how can we make the force be the same whether we have movement or we do not have movement.

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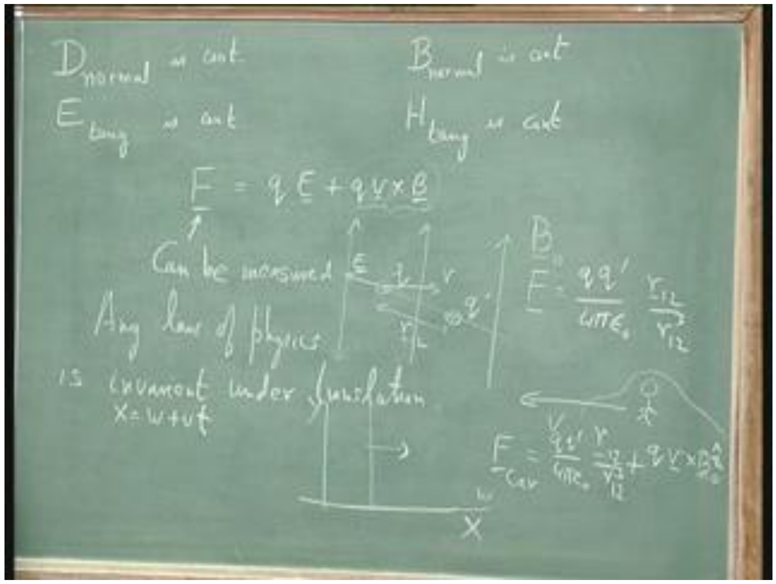

Let me redraw the picture. I have my magnetic field b. I have z this is x this is y and let us say I have two charges q q prime. Now what I am going to do is I am going to say that supposing I allow electric field to be different because electric field is just something I have defined. Only thing that is real is force. Supposing I will have my electric field and magnetic field to be different to be defined differently depending on whether I am sitting still or I am moving. So, in that case I will define f is equal to q e at rest and I will define f is equal to q e plus q v cross b. But it is not the same e and the same b. So, I will give it a dash when moving.

Now, we already know that the magnetic field came out of relativity. If you go back and listen to my earlier lectures, you know that you can derive the magnetic field using coulomb's law and Einstein's theory of relativity. So, the magnetic field is basically okay, it is already got current built into it. So, when you move if currents happen to change magnetic field changes. It is the electric field. Therefore that has to change. These are saying the same force. Then, it must be that q e is equal to q e prime plus q v cross b. So, when I move there is a change in my electric field such that the changed electric force plus q v cross b is equal to the original electric force or you can look at this formula and say the modified electric field is equal to the original electric field minus q v cross b sorry minus v cross b. Now this is very non-intuitive. It does not agree with coulomb's law. As far as coulomb's law is concerned e is e, e has been defined and we defined it here. There is no scope for putting in a velocity in here. Yet somehow when you look at the required electric field the required electric field should change so that there is an adjustment for velocity. In the presence magnetic field the electric field somehow modifies itself. Now this is just coming from taking electrostatics magnetostatics and the most fundamental requirement we have about science. The most fundamental requirement is that the universe did not create itself for us. It does not change its laws depending on whether I am walking or I am standing. It does not know about me. If it created a law called coulomb's law, it created it without any regard for human beings. We as human beings are looking at these laws, but the laws do not change because we look at them and the laws do not change depending on how we look at them. If we just put in that requirement we are getting this change in the definition of electric field and it is a change that is not compatible with coulomb's law. Something new has appeared and it is very strange.

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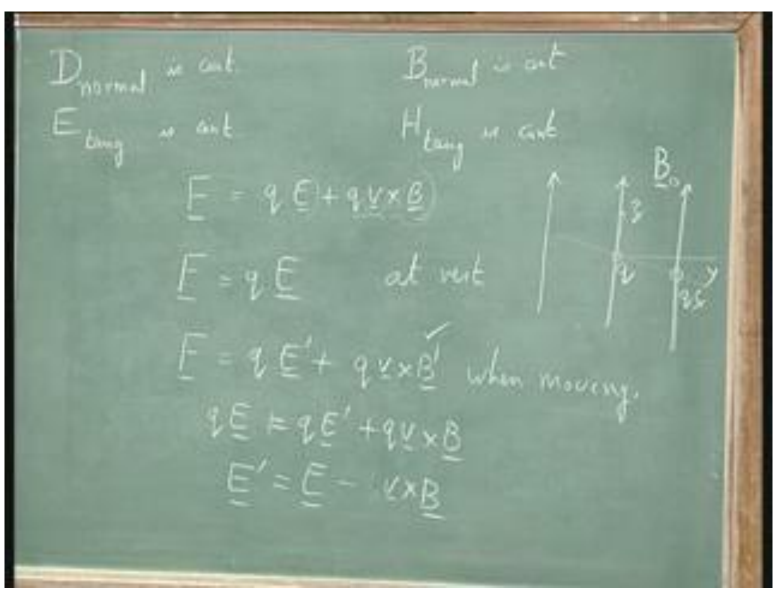

Now let us just assume that this is so and let us see what it would do and there is a very standard experiment that you would have read about in your electricity and magnetism course earlier which is you have a rail stationary rail and you have a moving rod. This rod moves with the velocity v and let us say there is a magnetic field b uniform and to make things interesting we can put a light bulb. So, normally if this rod is stationary we already know that nothing happens. There is no battery and therefore the light bulb is off. Now what happens if this rod is moved? Now according to what that equation tells us if this rod moves then inside this rod there is an electric field. How much is that electric field? It is equal to the original electric field was zero. The new electric field is this v cross this b and with a minus sign. So, the new electric field is in this direction e prime.

Now what will this e prime do? What it will do is, it will push electrons. The presence of this e prime means a lot of electrons will come there leaving behind a lot of positive charge here and this negative and positive charge build up. What it will do is, it will cause a current to flow and this light will glow. Needless to say, this experiment has been carried out. You probably have done it in your labs and the light does glow. If you take a circuit and you have a sliding bar conducting bar of course and if there is a uniform magnetic field and this bar moves, there is an induced electric field and it is completely outside whatever coulomb's law had talked about. There is no charge that was there to create this electric field. After this electric field was created, a charge build up happened which caused the current.

But the charge build up is notional because the moment the charge builds up it flows away as current. This electric field is only there because galileo demanded it and it is called motional electric field. Now this electric field was detected first by faraday and that is why in fact the law that comes out of it is called faraday's law and he did it in different ways. Basically what we were doing was something slightly different. He took a circuit and he applied a magnetic field and he found that if you changed the magnetic field a current would flow.

That is, again you put in a bulb. If you changed the magnetic field made it stronger, made it weaker a current would flow. Both of these could be captured by one statement and the statement was an induced e m f electromotive force is created when the magnetic flux through the circuit changes. That is to say, if you take this circuit or this circuit and you calculate surface integral b dot d s. This is what we define as magnetic flux phi m. If you take if you calculate the magnetic flux and if phi m is not a constant in time if it changes with time an induced e m f is created. Furthermore it is also observed that this e m f has a direction that opposes the change in phi.

What do I mean by opposes? Well a current is induced the moment does an induced e m f. For example you got an electric field, so a current flows, now imagine in your mind, this current is flowing. As a result of this current flowing there is going to be a magnetic field. What is the direction of that magnetic field? Well the direction of that magnetic field you apply your right hand rule. The direction of that magnetic field is like this it is downwards. Now if you look at the flux, the flux is increasing because the velocity is in this direction b dot d s integrated is increasing but if you take into account this magnetic field as well b is reducing, areas is increasing, b is reducing. So the direction of the current is that, direction which is tending to reduce the change in magnetic flux had the electric field been the other way. Then you would have created a magnetic field upwards which would mean not only does the area increase, but so does the field strength which means phi magnetic would increase even more.

But what faraday’s law is saying is the change tries to oppose the induced e m f tries to oppose the change so much. So, it is trying to keep magnetic flux constant. Now this is an observation. Faraday actually did experiments and observed. This on the other hand is just a theory based on how we believe the universe works and it is surprising, but true that both of these are say exactly the same thing. In fact if you take just this statement and you ask, what is that statement saying? Well you can look at this. In one second this rod would have moved to a new position. It would have gone a distance of v metres all right.

Now let us also assume that this magnetic field is not straight upwards. Let us say it stay up like this. Now, the velocity has moved v metres. Therefore the area has increased by this length l v metres square. Now the area you know has a direction. It is a normal. So, the area direction is l hat cross v hat. It is either this or minus sign of this depending on whether you make it point upwards or downwards. Now, the amount of magnetic flux has to do with b dot the area. So, what will be the amount of the magnetic flux that has increased? It is a constant magnetic field. So, the amount of flux that came through this part is constant. It is only this part that has given me new magnetic flux. So, how much is that? Well, it will be l cross v. That is the area with if this is l and this is v it is downwards dot b. That is the amount of flux that is piercing this new area.

Now you can rewrite this. It is triple product. So, any triple product can always be written as you can rotate. So, it is l dot v cross b. So, you can see that same v cross b has come. Now I would really like to take l as one metre because when I want to talk about electric field, it is a local quantity. It does not know about distances. So, I would say l is one. So, this v cross b represents the amount of electric field picked up if the rails are one metre apart. So, the same v cross b is also saying that this v cross be is also saying that, this is rate of change of magnetic flux phi m if rails are one metre apart. So, you can see that the same statement is coming back at us.

We got here not from faraday. We got here simply by looking at coulomb's law and saying coulomb's law looks funny because if we sat in a car and moved we got different answers. So, we said it must be that the electric field is changing. It cannot be the magnetic field that is changing because the magnetic field was derived based on relativity itself. So, magnetic field is safe. Our definitions took into account moving things but the electric field did not. Electric field had coulomb's law. So, the electric field must change and now we find that if you take a picture like this, this v cross b is nothing but the rate of change of magnetic flux if the rails are one metre apart.

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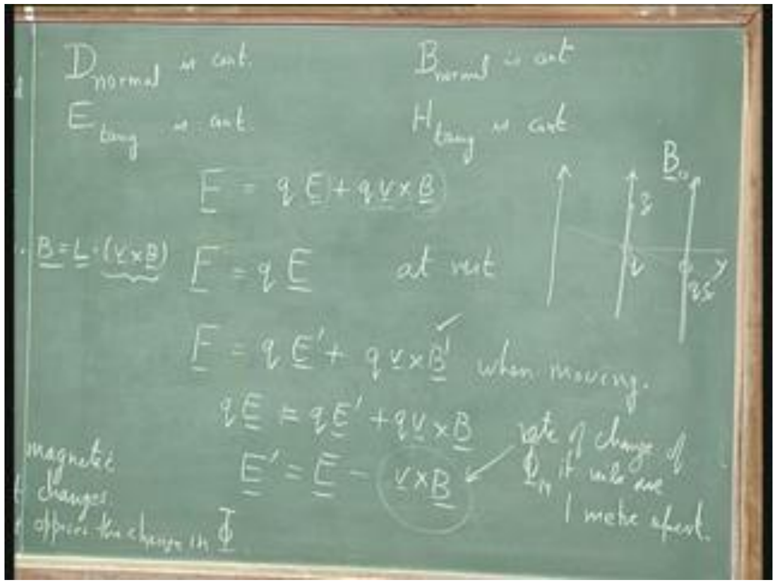

We will come back to that one metre apart business later. So, in a sense now both of these statements are saying the same thing and what are they saying? They are saying that an induced e m f is present if the magnetic flux changes.

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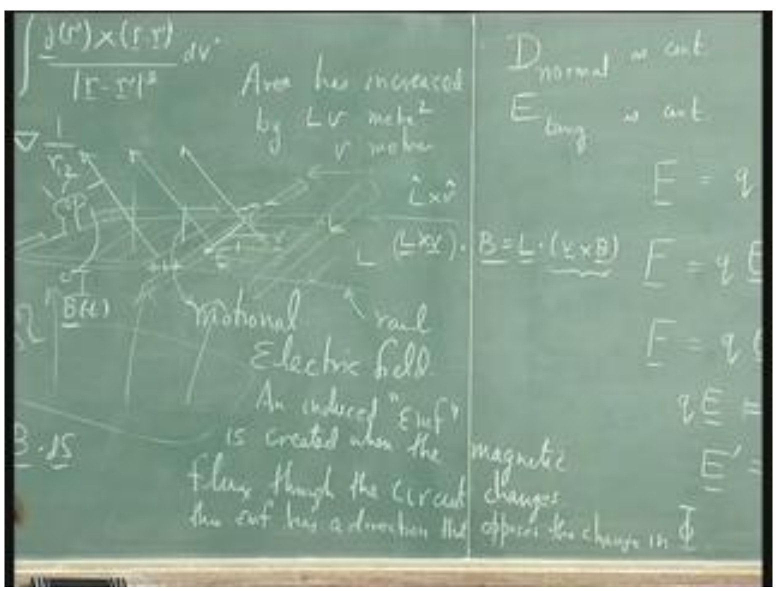

And if you take that length l into account, the induced e m f, induced e m f remember is a voltage, okay? The induced e m f must be equal to integral electric field dot d l and e m f is a voltage. Electric field is the force per unit charge. If you want to get to voltage you have to relate it by e is equal to minus grad phi. Now, this is a suspect equation but that it where it comes from. So, I am taking volts. I am taking a derivative in space. So, volts divided by length is field. So, if I have an e m f then the e m f is related to electric field by integrating in distance. So, I have my final answer now. This is the magnetic flux. This is the e m f and the induced e m f is trying to resist the change in magnetic flux. So, it is saying e dot d l is equal to minus, it is trying to resist d d t of b dot d s.

There is only one thing that is important to note. This integral is not really from here to here, e m f is not so much a point to point thing. It has to do with a circuit because after all you are talking about a surface and the surface connects to a complete closed circuit. So, you cannot talk about an e dot d l till you define the full circuit. Therefore, this is a closed integral e dot d l and this is nothing but faraday's law. I want to emphasize faraday's law is really derivable from coulomb's law. It is that simple.

It is a measured phenomenon but we started from coulomb's law just required that what galileo said namely what charges do in the presence of electric and magnetic fields in the presence of other charges and other currents cannot depend on how fast we are moving. The moment you require that you find that the electric field has a correction term to it which is v cross b, you take that v cross b and you find that v cross b is nothing but change in magnetic flux and you can write down this equation.

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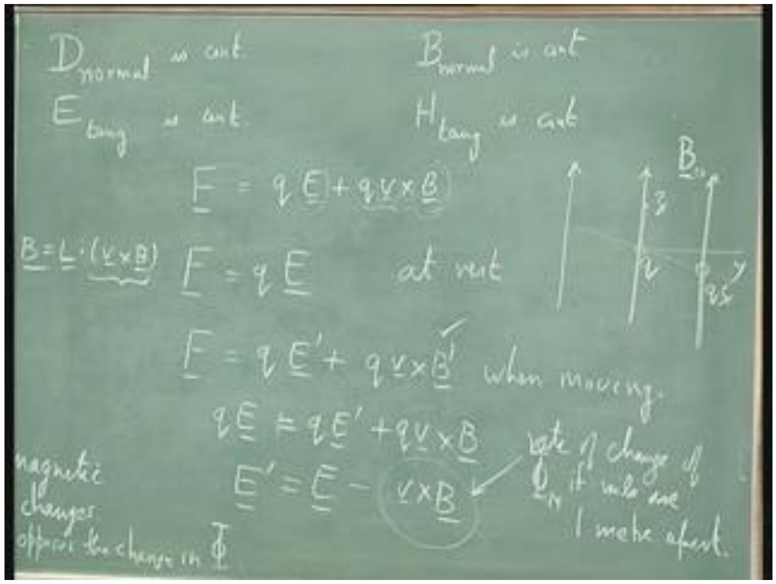

And that rate of change of magnetic flux if rails are one metre apart is basically saying that if you took one metre then this is nothing but e because, it is integrated over unit distance. So, the per meter amount of electric field, that is induced is equal to minus v cross b but if we integrate over the whole circuit then that v cross b is generalized to minus d d t of b dot d s. This is a completely general equation. It is relativistically correct. It is remarkably we have not been able to find any case where this equation is not true and yet we got to it just starting from coulomb's law. Now little bit of mathematics and we are done.

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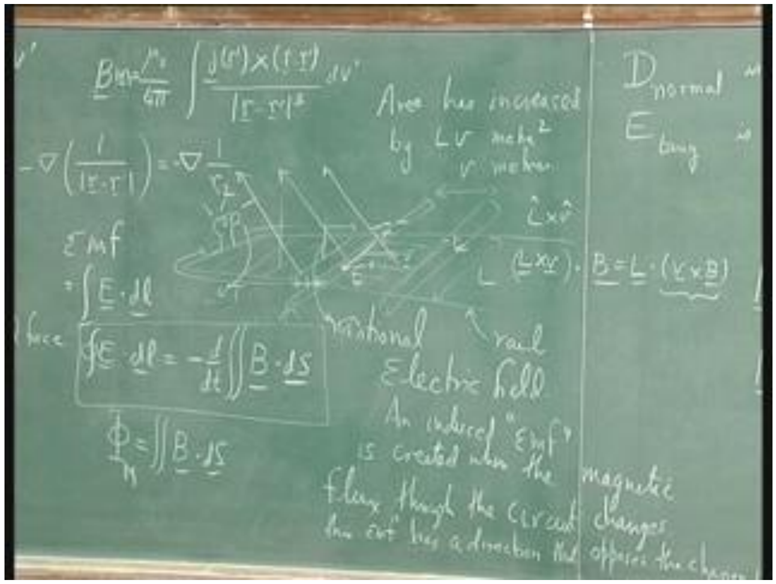

Loop integral e dot d l is equal to minus d d t of surface integral b dot d s. Now I know everything about loop integral of something dot d l because we have already encountered it in ampere's law. This piece is nothing but surface integral. Let me call this c. This is the surface connecting c of curl of e dot d s. Now that should give us a warning. We have already had a proof that curl of e is 0. Why because, we had e was equal to minus gradient of phi which implied curl of e is identically zero and yet a loop integral of e dot d l is nothing but surface integral of curl of e d dot d s. Now on the right hand side there is a d d t of a small integral. For the moment let us keep that surface constant. So, we have got a circuit a battery resistors etcetera and it is just a stationary circuit. So, the surface is a constant surface. In that case this time derivative is not talking about how the surface is changing. Its talking about how the field is changing.

It is quite an important point because you people are EEE students. So, you will be doing a lot of cases where it is the surface that is changing rather than magnetic field. For example, if you have inside a machine when the rotor moves surfaces move, so the motional e m f is very important to you. But let us look at the case where the surface is stationary in which case the surface is not a function of time. Then this time derivative cannot act on s. It can only act on b. So, it becomes minus integral over the surface time derivative of b. Now, this quantity is a function of time. So, it makes sense to write d d t of that quantity. But, b is a function of x y z and t. So, I cannot write d d t here because if I write d d t here what does d d t mean? When I write something like d b d t what I really mean is move along some x of t y of t z of t and compute d b d t. But that is not what I am going to do because x of t y of t z of t do not exist. This is a, they are dummy variables. They are what II used to construct my d s. So, these are all zeroes. I mean they are constants.

So, what it means is that in this dependence x, y and z are kept stationary, when I do this time derivative I only take the derivative with respect to t. That means I have to do a partial derivative with respect to b. I have a surface integral on both sides. So, I want to combine them. So I get surface integral curl of e plus del b del t dot d s is equal to 0. Then this whole derivation I have not made any assumption about what c is. C could be anything, c could be large, c could be a square a circle anything and as I told you before, if i have any kind of integral of this type arbitrary surface v dot d s is equal to 0, it has to imply that v is zero.

Because if it’s not true, then I can go to wherever v is not 0 and do a small surface in only that part where v is not zero and I will get a non-zero answer. It must be true that v is zero everywhere which means this relationship which is actually an integral over a surface is true at every point of the integrant. So, I can write down my final equation which is faraday’s law. Curl of e is equal to minus del b del t.

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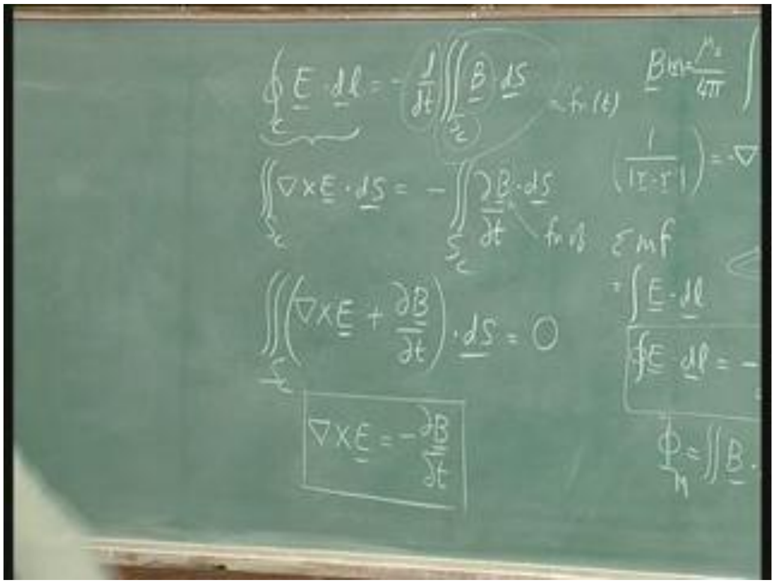

So, this is the integral version of this equation and this is the differential version of this equation. So, let us see, let us sort of summarize and see where we have reached because this is quite a packed lecture. What we have done is we started with a set of equations divergence of e is equal to row over epsilon naught, curl of h curl of b equals mu naught j. Curl of e is equal to zero. Divergence of b is equal to zero. This is where we started and what we have found is that this equation is not correct because if you look at this equation the correct form of this equation is curl of e is equal to minus del b del t. It is not surprising we did not find it before because up till now we were not taking into account time rate of change of things.

So, it is only when we started looking at how things change in time that we started noticing an error in these equations. I can see that it makes a big difference because it was only this equation that allowed us to say therefore electric field is equal to minus grad phi. Why is that? Because if you take loop integral of b dot d l, it is equal to surface integral over the same surface over a surface connecting this loop curl of e dot d s. And if curl of e was zero it gives me zero and from this idea we had that if you have any point one nay other point two if you take any two different ways of going from one to two. You could do an integral one to two plus an integral two to one of e dot d l and it will give you zero.

It gives you zero because loop integral of e dot d l is zero which meant integral from 0.1 to 0.2 of e dot d l did not depend on how you got them. It was completely independent and because of that you were able to define a function that depended only on the beginning point and on the ending point and then we would take this beginning point and put it at infinity and say we defined a function that is a function of position. So, everything depended on this equation being correct. Our entire electrostatics hinged on this and now electrostatics is wrong.

Now if you do this problem you can do integral 1 to 2 and 2 to 1. It is equal to loop integral e dot d l which is not anymore equal to 0. It is equal to minus d d t of surface integral b dot d s. The electric field is no longer derivable from a potential and that is the main difference. That is what is changed everything here yet and I would like to repeat this again and again. Faraday's law comes out of coulomb's law. Faraday's law is not a new law.

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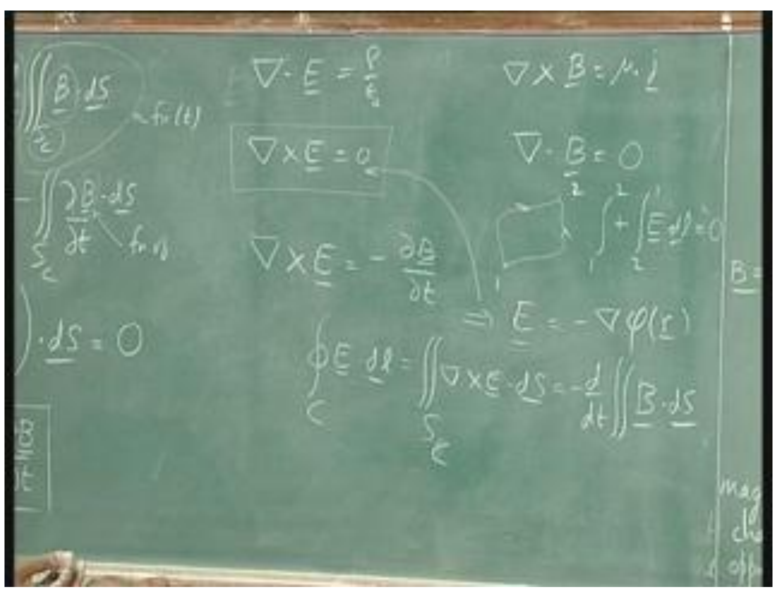

It is not an overstatement to say that if you know coulomb’s law and if you know relativity all the other equations are derived, magnetic field can be got from relativity. Faraday’s law comes from shift invariance. Similarly, when we generalize ampere’s law it also comes from the same kind of symmetries. Coulomb’s law requires that we generalize it and some how we have to add something new to get an improved definition of electric field. We will do that in the next class and complete our understanding of faraday’s law.