Electromagnetic Fields
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Lecture – 24

Examples of Ampere's Law

Good Morning. Last time we had done a bit of work on what are magnetic materials. But today I am going to take a step back and solve a few problems involving coils and towards the end of the lecture we may come back and talk about magnetic materials again. What I am going to look at is the following problem.

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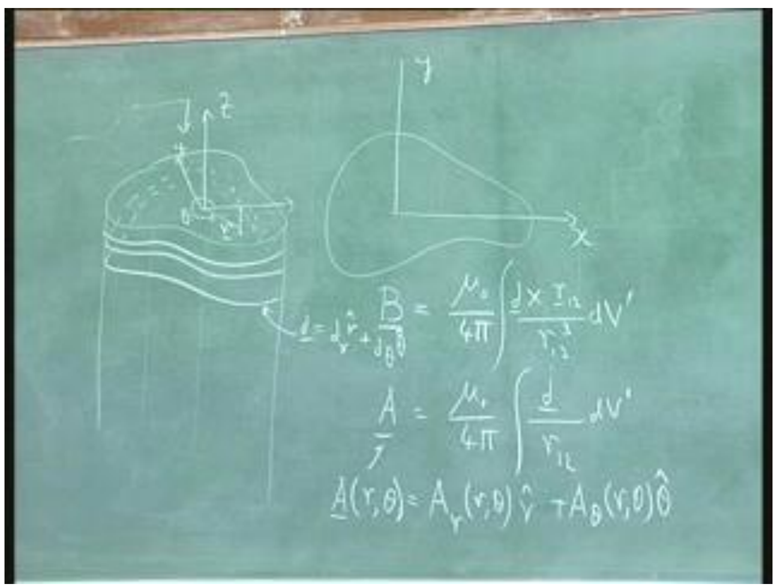

I have some arbitrarily shaped cylinder on which I have wound a coil. I have n turns per meter and I want to know if I take this arbitrarily shaped cylinder and find out the magnetic field inside what does the magnetic field look like. If I look from above, what I am going to see is some irregular shaped cross section. Let us say this is x this is y and this is z. So, if i take this kind of system what kind of magnetic field is possible? Then after having looked at this problem, we will go back to the, interesting one which is the cylindrical or square cross section coil, okay?

If I look at any point r i know that let us say this is filled with air. So, I am not talking about any complicated magnetic field inside. I know that the magnetic field b is mu naught over 4 pi volume integral j cross r 1 2 over r 1 2 cubed over d v prime. As I said, that is an extremely complicated operator. So, instead I will work with the vector potential mu naught over 4 pi volume integral j over r 1 2 d v prime.

Now let us look at this what this, a can depend on. Supposing I move one centimeter or one meter in the z direction, well this coil is infinite in the z direction. So, if I move up by one centimeter, my magnetic field is not going to change. My vector potential is not going to change because there is as much current above as below. So, this a cane depend on r. It can depend on theta where this is r and this is theta. That is the radial distance from the z axis the angle it makes with the x axis it can depend on those things but it does not depend on z.

Furthermore a is built out of j and if you look at this coil, what is j? j has directions along r and along theta j has no z component. It is going round and round in the x y plane. So, this, a has a same direction as j which means it is a r which is a function of r and theta along the r direction plus a theta which is a function of a and theta in the theta direction. I have not actually solved anything. I am just saying since the currents that are creating my magnetic field are in x y plane or parallel to the x y plane my vector potential must also be in the x y plane. It cannot have a z component. And because by shifting in z, there is no change in my calculation. The integration is identical. So, my a, and more importantly b cannot depend on z, all right?

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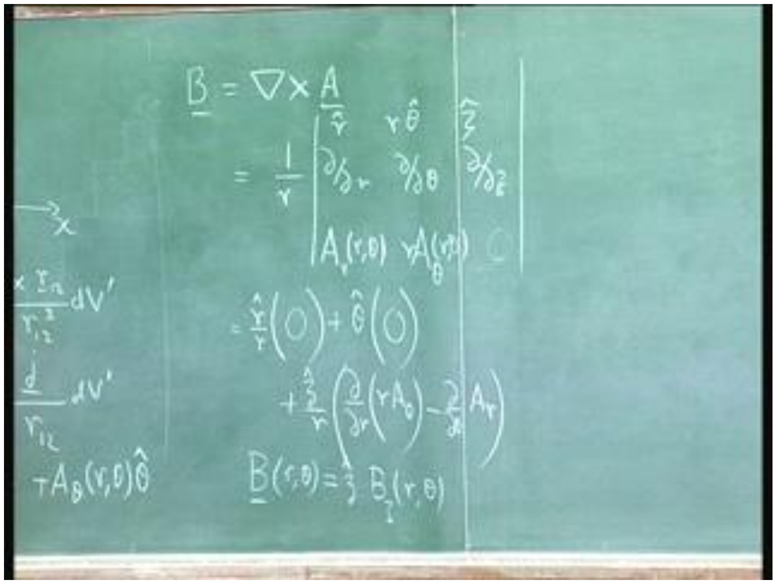

So now what do we get from this. I am going to work out the curl. My magnetic field b is the curl of a. I already have what a, is. So, let me write out let me remind you curl in cylindrical coordinates is one over h 1 h 2 h 3; h 1 is 1 h 3 is 1 but h 2 is r. Determinant of r hat r theta hat z hat del del r del del theta del del z a r which is a function of r and theta a theta which is a function of a and theta and 0. This is what I want to solve to get b. I know that vector potential has only r and theta components and those components depend only on r and theta. So, what is the answer? The r hat component is r hat over r, then I can take the derivatives del del theta of 0 minus del del z of a theta.

A theta does not depend on z. So 0 plus r cancels out theta hat times de del z of a r minus del del r of 0 del del r of 0 is 0 a r does not depend on z. So again, 0 plus z hat over r. There should b a r here, z hat over r of these two terms which is del del r of r a theta minus del del theta of a r. This is very interesting without having solved anything at all. I have the result that magnetic field b is a function of r and theta but not of z is only along the z direction. This is important enough that let me just go through the argument again.

I take an arbitrary shaped cylinder. I am not actually as yet told you what shape it is. It is any kind of shape I wind the coil very tightly around it. When I do that, I have currents which are in the r direction and currents which are in the theta direction. Theta is my angle with respect to the x axis. The magnetic field is got from the biot savart law, but it is much easier to work with the vector potential. So we will work with vector potential, a is an integral of j divided by the distance from observation point.

Now, the j is in only r and theta and further more the answer cannot depend on z. This is an infinite cylinder. I cannot have my magnetic field growing forever in the z direction. So, I have an answer for vector potential without solving anything. I can just write down the form. It has a r component. It has a theta component because those are the only components of j present. The r and theta components depend on r and theta. They do not depend on z.

So, just using this form, if I put it into curl, what do I get? I find that the r component goes away the theta component goes away only the z component remains. So, if I take any infinitely long cylinder, I am going to get b is always in the z direction. Now, let us take this idea and apply it to the interesting case of a symmetric solenoid.

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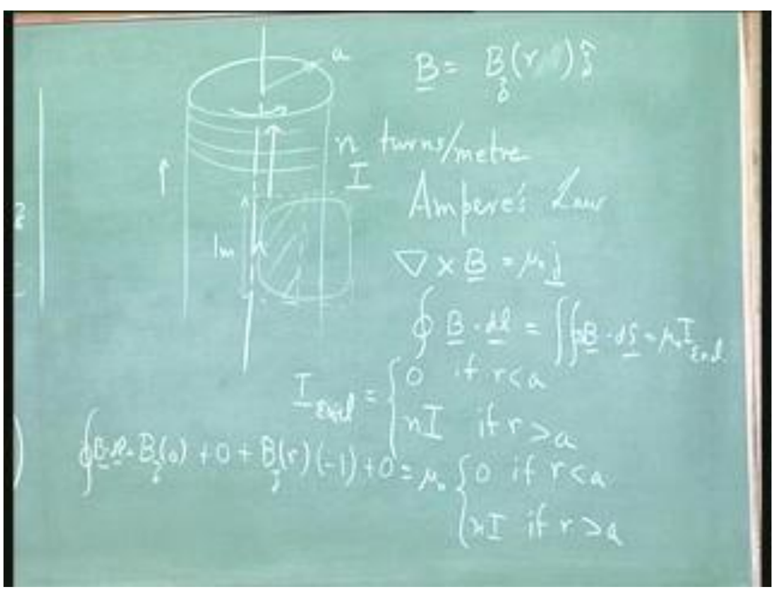

So, solenoid is a circular cross section. It has the radius a. It has n turns and each turn carries current i. This is n turns per meter. Now this is also an example of the previous case. So, I know without solving anything, I know that the magnetic field everywhere whether inside or outside is in the direction. So, I know that b is equal to b z which may be a function of r and theta along the z direction. Now, because of the symmetry, it is a completely symmetric solenoid in theta. If I rotate the solenoid by say 30 degrees the picture does not change which means that my calculation of b should not change either.

So, b even though the previous case depended on theta this current case depends only on r. So, it is b z which is a sum function of r times the z unit vector, okay? So now that you have established this much we have got enough symmetry and we can now start using ampere's law. What does ampere's law tell us? Ampere's law tells us that curl of b is equal to mu naught j or alternately loop integral b dot d l is equal to surface integral b dot d s curl b dot d s is equal to mu naught I enclosed l. Supposing this is my symmetry axis and I take a loop one side of which is on the axis. The other side is at some r. So, what do I get?

The current enclosed. I enclosed is equal to 0 if r is less than a, and let us say this loop has a length in z of one meter. So this I enclosed is 0 if r is less than a, but if r is greater than a then the amount of current that is enclosed depends on how much current there is per meter in the solenoid. We will assume that the current is flowing this way. So, the current is going into the board and it is n turns per meter times I if r is greater than a. Now I do b dot d l loop integral. I know b is along z. I proved that earlier. I did not have to solve anything to get that result.

So that means b dot d l cannot exist on the horizontal portions of this loop. It can only exist on the vertical portions. So let us say I am taking the loop this way. So, I will say b z at r equals zero times one meter plus 0 plus b z at r times minus 1 because; I am going in the opposite direction plus 0. This is b dot d l. This is loop integral b dot d l. What is it equal to? It is equal to mu naught. I enclosed into 0, if r is less than a, and I if r is greater than a. So, what that means writing it out is that I will take the b r that side and I will take the mu and this side. So I get b z of 0 minus mu n, I enclosed is equal to b z of r. We can plot this.

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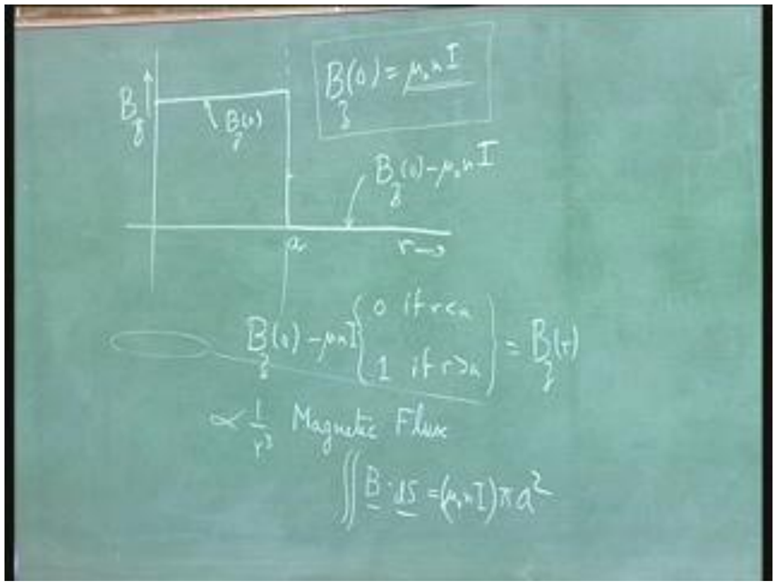

This is the radial direction, this is a, and I am plotting the magnetic field b r sorry b z, b z of 0 is some value. We have not fixed what that value is. If and let me write down the result so that I do not have to jump boards. What we got was b z of 0 minus mu n times 0, if r less than a 1 if r greater than a is equal to b z of r. So up to r equals a, this term is 0. So, it says b z of 0 is equal to b z of r, field is uniform. Then for r greater than a, it is b z of 0 minus mu n i. So, there is some jump and then it is again constant. So, this is b z of 0. This is b z of 0 minus mu naught n i.

Now, very far away we know that the current loop when you go far away gives us a magnetic field that is proportional to one over r cubed. We did that and even if you have current loops that from minus infinity to plus infinity, the one over r cubed is going to dominate. So, the field outside as you go very far away cannot be non-zero. So, the only reasonable curve we can draw is to say the field outside is zero.

It comes from the fact that a current loop is like a dipole. Therefore its field goes like one over r cubed and even if you have an infinite cylinder of it the field will therefore go away as one over r square had it been a line charge the field would have gone away is one over r. But, in any case as you far away the field must go to zero and the only way to make the field go to zero is to make these two terms cancel. So what does that mean? It means the b z of zero is equal to mu naught n i and this is of course a very standard result. You learnt it in school. You probably did not go through the justifications we did. What we have shown is the fact that the magnetic field is along z is not an assumption.

We can prove it. It just comes from having an infinite symmetrical cylinder. Symmetrical, I mean it does not change its shape in z and if in addition it was circular we can prove that there is no magnetic field outside the solenoid and therefore the magnetic field inside. The solenoid is given by mu n i and since the field is constant inside up to a, this, mu n i is the value not only at the center. But it is the value at all points inside the solenoid. It is a very important result.

Therefore you can also work out that what you call magnetic flux which is the equivalent of electric flux the magnetic equivalent of that then namely it is the surface integral of b dot d s. It is going to be equal to mu naught n i times the area cross section which is pi a square. Now we will come back to this concept the concept of magnetic flux. It is an extremely important concept, but it does not make sense till you have done Faraday's law.

So, once you have done faraday's law you can tackle what an inductor is and an inductor makes sense only when you talk about magnetic flux. I want to reiterate that by using ampere's law what we have been able to do we established first from general arguments that b is along z. Having established that we then managed to use ampere's law and get the answer. Once again I will make this statement and it is a very important statement that the vector potential is a much simpler quantity than the magnetic field, even though both are vectors.

So, in the case of electrostatics the scalar potential is a scalar electric field is a vector. So, we said it is simpler because it is a scalar. Vector potential is also a vector, but the vector potential is a much easier concept to calculate, much easier field to calculate than the magnetic field. We could simply not have made these easy conclusions if we had not had the vector potential to play with.

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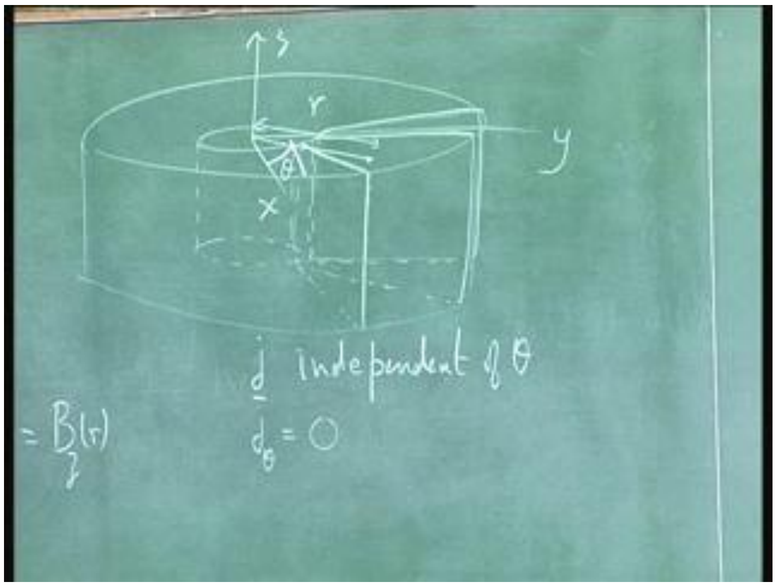

Now I am going to take another problem which is the following. Supposing we have a torus, this is a square cross section torus and you have coils wound and so on and so forth. So, it is a very standard solenoid that we use in electrical engineering square cross section. Square cross section is only to simplify out mathematics. Now what I would like to know is what is the magnetic field inside this and how do I go about calculating it.

Now, there are certain things we can say immediately. We can say the coil j is independent of theta. Here let me define my coordinates. My coordinates are this is z, so this is x y some general direction this distance is r and the angle between the x axis and r is theta. So, the current in the coil does not depend on theta. So, j is independent of theta further more j theta is equal to 0, j is r and z only. Now what I would like to do is to figure out what components of vector potential are present. Now this is not quite as easy as the previous case. The reason is, when I say j theta is 0, I cannot conclude a theta is 0.

The reason is, when I write a is equal to m naught over 4 phi volume integral j over r 1 2 d v, this j is not a function of the coordinates at which a is being measured. It is a function of other coordinates namely r prime theta prime z prime. So j at theta prime is 0. But what does that mean? Let me draw a graph and you can see what the problem is.

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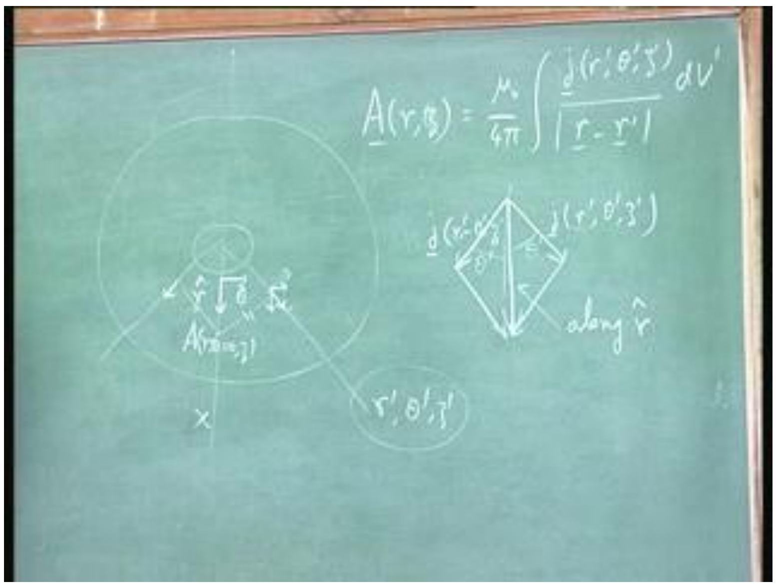

I am looking from above and let us say this is the x axis and I have rotated things. I am looking at some point here. I want to know a, at r theta equals 0 z. Now, the a is due to currents let us say from here. I am looking at a little bit of current coming from there. Now this current has its own r prime theta prime z prime and in this coordinate system there is j r and there is j z. There is no j theta. But this is no j theta in the theta prime coordinate which means there is no j in this direction. There is only j in the radial and z directions.

But you can see that this direction can be broken up into this x and y directions. So, there is a little bit of y component to the j. But at theta equals 0, the y component is at theta direction because this is half and this is theta. So, the problem is in a torus when you take currents even though the currents are wound in the r z direction. Because, we are talking about other coils else where these currents actually provide a theta component as well as a r component. How do we escape this problem or is it true?

Well, the answer is you do not look at one of these j's, you look at two of them. You take the symmetric one this way and corresponding to this j there is a j this way also. Now, if you look at the expression for a, a which is a function of r not of theta and z it is equal to mu naught over four pi integral over space j which is a function of r prime theta prime z prime divided by r minus r prime d v prime. If I take two of these which are symmetrically placed one is at plus theta other is at minus theta, then the distance this distance and this distance are the same. Since they are equal r minus r prime is the same, So, I can just add up vectorially the j's.

Let me draw that itself. This is j at r prime minus theta prime z prime. This is j at r prime theta prime z prime. Equal in magnitude and this is angle theta this is angle theta, theta prime. So, if I add them up vectorially, I am going to get a vector that is along r. The y component of this j of r prime theta prime z prime exactly cancels this j. These two components cancel out. The only component that adds is the component along r.

So now let us go back to this toroidal picture. I can always rotate my measuring instrument. So that it is sitting along the x axis because the answer does not depend on theta. If I sit anywhere in this plane, let us say at some point here. I choose currents along plus theta and along minus theta. Now, the currents that are flowing on these two legs, they are flowing in r the r direction. By that argument they give me a radial component.

The currents that are flowing straight down, they are in the z component and the z direction is the same in all rotated frames. The current that is coming back is again in the minus r prime directions. So, once again the same argument gives me a minus r component and finally there is a two z components which add up. So the answer after doing all these additions is that this a which is a function of r theta z has a r component a r, which is a function of r and z but not of theta along the r direction plus a z which is a function of r and z which is along z component.

The arguments I have made have been basically general. I have not actually calculated any integral. I am just using angles and symmetry to get this answer. But what does this answer get us? Once again we have got to do the curl. We should stop being afraid of the curl because actually when you try it out you find that it simplifies rather than complicate things.

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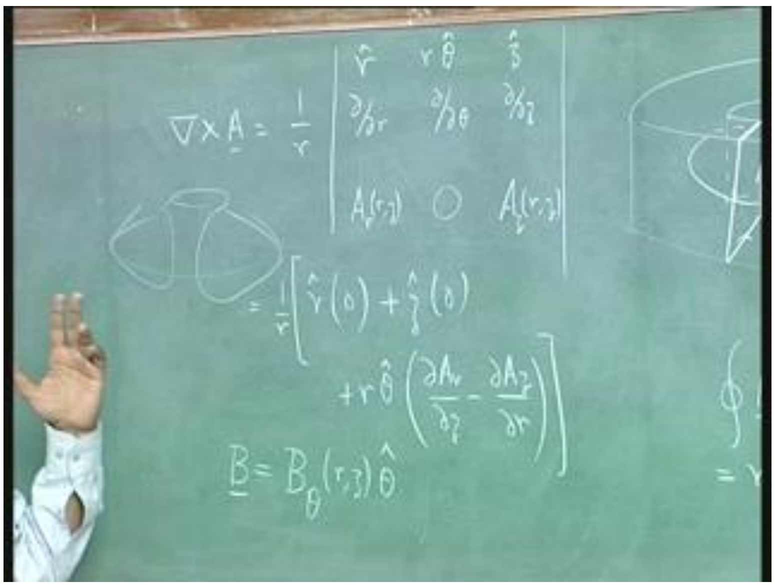

So, curl of a, which is b is now going to be equal to one over r r hat r theta hat z hat del del r del del z there is no de del theta. Even though I will write it down you know that things are not a function of theta, a r which is a function of r and z no a theta a z which is a function of r and z. Once again let us do it. 1over r times r hat del del theta a z r z minus del del z of 0 del del z of 0 is 0 del del theta of this. This does not depend on theta. So this is also 0 r hat times 0.

Let us look at this z hat component del del r of 0 minus del del theta of a r of r z; a r does not depend on theta. So, del del theta is 0. Then the theta hat component which is plus r theta hat del del z of a r minus del del r of a z. It tells us something very interesting. It says b without having solved anything yet is only along the theta direction; b is only along the theta direction.

Why because, if I have symmetry in theta and a theta is missing, these two things alone will tell me that I have only a theta component present. Basically these two zeros here effectively prevent me from having any minor that is coming out of r hat and coming out of z hat. So I must choose to expand along theta. So, that is r theta hat times this place. But that is very interesting because if b is a function of theta only that immediately makes me think of ampere's law. What does ampere law tell me?

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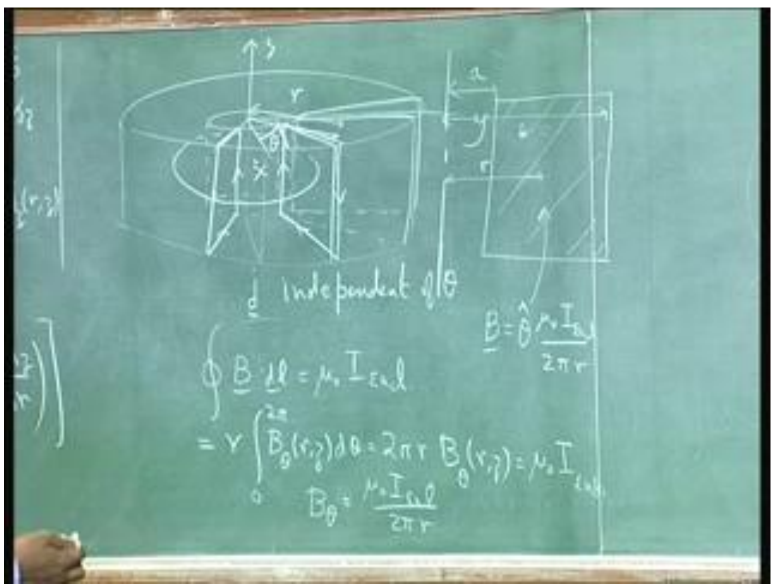

Ampere's law tells me that if I take any loop which is linking some current, when I have loop integral b dot d l is equal to mu naught, I enclosed, well I know that my b is only b theta. So, I will choose a circle. Furthermore I know that my b theta is not a function of theta. It is a function of r it is a function of z but it is not a function of theta. So I get r times integral 0 to 2 pi b theta of r z d theta, r d theta is d l. But b theta not being a function of theta I can pull it out. So, it is equal to 2 pi r b theta r z is equal to mu naught, I enclosed or b theta is equal to mu naught I enclosed divided by 2 pi r.

This is not an approximation. I have actually got the correct answer the exact answer. So, it says that anywhere across the cross section of this device, if I look at it and I draw a plane through it. I will have a cross section that looks like this. There is a minimum radius, call it a. There is a maximum radius, call it b. So throughout this cross section the magnetic field b is along the theta direction and it is equal to mu naught I enclosed divided by 2 pi r where r is this distance. It is quite remarkable how much you can do by just observing the directions of currents and using the vector potential. The vector potential is so powerful a technique that you should not get scared of the fact that it involves a vector integral.

It actually should be looked as an opportunity because the moment you can do a vector integral you can look for cancellations. Now, if you look at the kind of argument we did and you see what all we said, you can see that the cancellation of j theta did not depend on being a square cross section. Any j that was in r z plane but independent of theta would have given me the same result.

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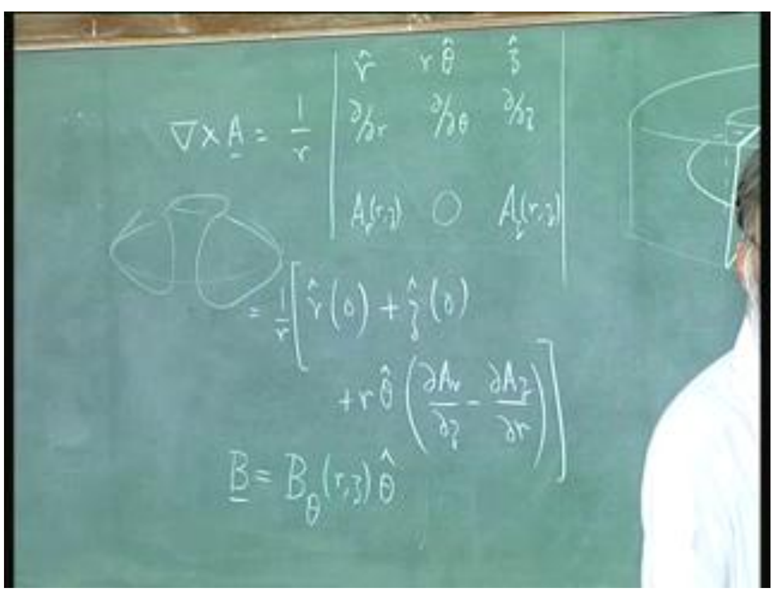

Which means, even if I had a torus that looked something like this, it has a triangular shape, it is got a peculiar kind of coil structure, even such a torus provided it has surface of revolution. That is it does not depend on theta. It is the same cross section for all thetas. We will still again have a b that is only in theta direction and once again within that cross section, if you apply your ampere's law you will once again get that b is equal to theta hat times mu naught I enclosed divided by 2 pi r. So it is a powerful kind of way of thinking about magnetic fields and you should always be using it.

You would not always be lucky to get simple geometry. But the solenoid is such an important geometry that every electrical engineer should know how to manipulate solenoids. So, any solenoid with any cross section will have a theta direction b field but the solenoid should be a surface of revolution, if the solenoid in one place and square cross section somewhere else that will not work. If the solenoid is not fully uniform in theta and has an air gap it is filled with a ferrite, but has an air gap that will break the symmetry. But as long as it is symmetric in theta this kind of calculation is valid, all right?

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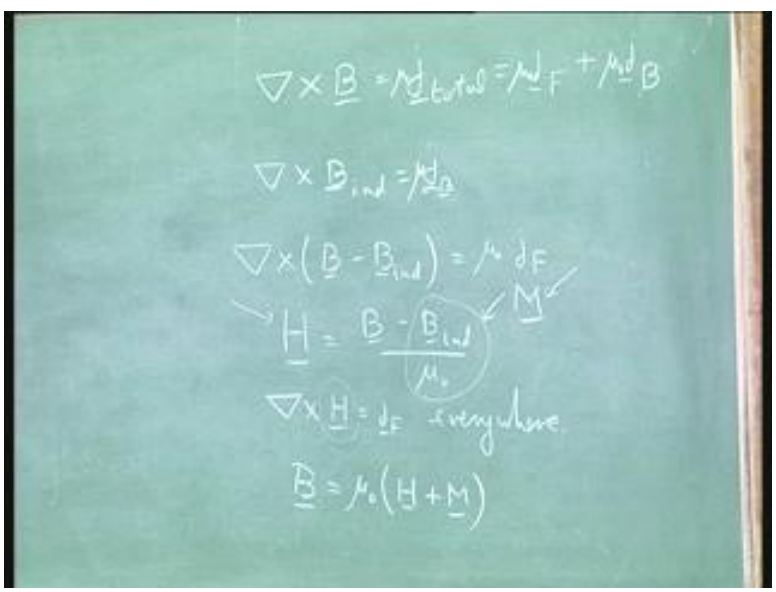

Now last time I had been talking about deriving a new vector from the magnetic field. What I had said was that you can obtain the magnetic field b from the total current present. Now this total current is equal to the current we know about what we call j free plus the current that materials when they react to this applied magnetic induce, so that is j bound. Now we can talk about another field. The magnetic field due to just j bound and it you could talk about a curl b induced which is mu naught j bound. If you subtract these two, then you get curl of b minus b induced is equal to mu naught j free and sow e define a new vector a vector, we call h which is b minus b induced divided by mu naught and if you have such a vector then we have the curl of h is equal to j free everywhere.

Now this is a very useful thing to do. That is why it is done. Otherwise we could always work with the magnetic field b and take into account j bound especially for magnetic systems it turns out to be indispensable to talk about h and we also talk about this quantity which is b induced over mu naught. So, we tend to talk about something called magnetization.

Magnetization is the induced magnetic field due to the applied magnetic field. So we can immediately write in relation which is b equal to mu naught h plus m. I will come back to how we use them. But in fact I do not have to come back to it because probably half a dozen other courses, you will do will talk about magnetic circuits and will talk about transformer design and all those interesting topics where the magnetization as well as the h get discussed. For this course we will only touch about upon these ideas. The main purpose of this course is more to talk about fields in vacuum.

That is, a complicated enough subject that we do not really to go into the complexities of materials. But, you should be aware that there is a lot of rich engineering and physics inside the magnetic response of materials and when you do designs of transformers when you do designs of any ferrite, you will find that a material responds to a magnetic field is a very complex thing. And as I mentioned last time, it is complex precisely because it is weak. The electrostatic forces are so strong that the any imbalance will actually destroy the material. So, if you are going to get a solid electrostatic material like sodium chloride, it means that the electrostatic forces are in very delicate balance.

Otherwise the solid could not have formed. So, you would not have any extra energy left over to make structures such as these. But the magnetic field even when it is not in balance cannot really create disastrous forces. Because of that you have a very rich variety of magnetic phenomena magnetic materials exhibiting different phenomena, all right. I now want to go over to one other topic and let us yes, it is what is called the scalar magnetic potential.

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We have already seen that we have divergence of b is equal to 0. This came out of the it is a conclusion out of the fact that b is equal to mu naught over 4 pi volume integral j total cross r 1 2 over r 1 2 cubed. If you just take this formula, you can prove this and since it is valid even if you took the detailed bound plus free currents. It means that this is true even in the presence of materials. So it is always true. Now we had a second equation which was curl of b equals mu naught j and we know that this is now going to be replaced. It is going to be replaced by curl of h is equal to j free.

This is also always true provided you put total. But since we do not know j total, it is not a useful equation. Here you see even though this conclusion comes from taking total current the conclusion does not involve knowing the total current, divergence b equal 0 is a usable equation but curl of b equals mu naught j total is not useful. So, we have gone to a new equation. Curl of h equals j free. Now in many cases for example a solenoid there are regions within the area we are studying where current is 0. For example we have coils and you want to find something about the magnetic fields away from the coils. So, you have that j free is 0.

If j free is 0, that what does this mean? It means curl of h is equal to 0 and if we remember our definition of curl of h it means loop integral h dot d l is equal to 0. Now, this is true provided we take a loop and curl of h, is 0 everywhere within the loop. Provided this is true, that is curl of h is equal to 0, this conclusion comes true and if that is true, then I can take any point call it one another point call it 2 and I can say that integrating from 1 to 2 an integrating from 2 to 1 and adding them up gives me 0 or integral 1 to 2 h dot d l is equal to 0 is equal to function. Now just look back at what we did with the electric field. We use the same argument. I will write it here.

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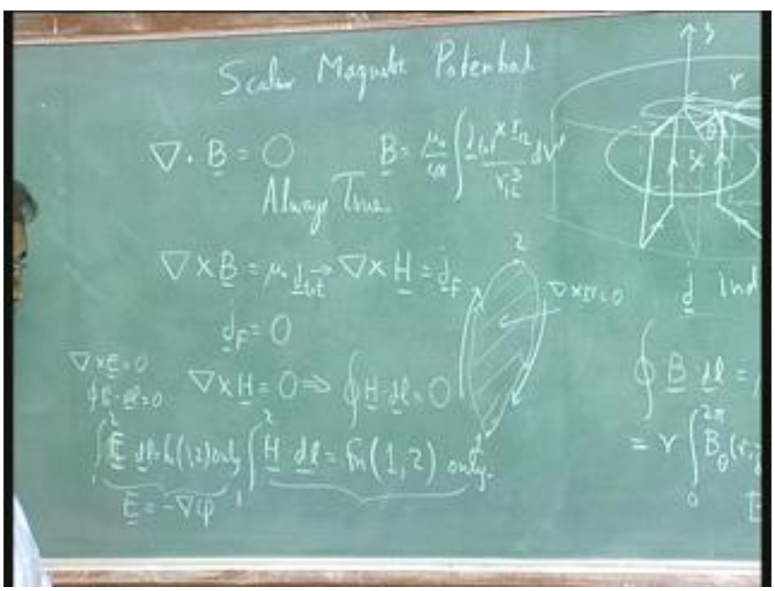

We said curl of e is equal to 0 which implied loop integral e dot d l was equal to 0 which implied 1 to 2 e dot d l was equal to function of 1 2 only. It did not depend on the path connecting 1 and 2. It depended only on the n line n points. There is only one difference between these two. The difference is that the electric field intrinsically satisfies curl of e equals 0. This is true everywhere. This statement curl of h, is 0 is only true where there is no current. So, as long as you do not get near current coils it is true but if you do get current coils it is not true.

So, while statement is very strong this statement is not quite, so strong. I can write a very strong statement that electric field is minus the gradient of potential and I can build a potential out of the electric field and solve the problem everywhere. In this problem I can say h is equal to minus grad phi magnetic. But I have to say that this phi magnetic is a local definition because the moment I get near a coil curl of h was not 0, therefore it is no longer a function only of n points. It is also a function of how I got there. Let us see an example of where this happens.

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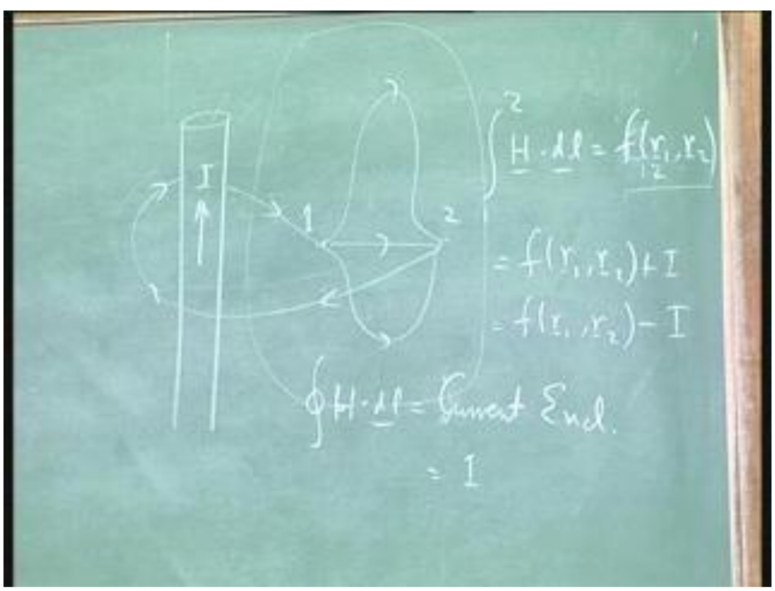

Supposing I have a wire, the wire is carrying current. As long as I am looking at points 1 and 2 away from this wire, however I connect them up, I can write integral from 1 to 2 h dot d l is equal to some constant which depends only on 1 and 2. So, it is some function of r 1 and r 2. But supposing I went round the coil and went to 2. Well now it is a different matter because if I did this loop and then came back this way, this is a full loop. So, loop integral h dot d l is equal to current enclosed which is equal to i, suddenly it is no longer 0.

So, the result is that it is only certain paths that give me this answer. Other parts give me f of r one comma r 2 plus i. Still others give me f of r 1 comma r 2 minus i and i can if i circle, this wire twice and come here it will become plus 2 i. If the circle the wire in the opposite direction to I, it will become minus 2i. So, the presence of current means that my statement that the potential exists is limited to a region where there is no current. So, if I say let me define this part of the region only my path is not allowed to leave it. Then, within that region any path will give me this answer. But, if I ever allow a path to go and link the wire then I am going to get a different answer. This is not as big a difference as you would think.

Actually once we generalize the electric field, then we will find that the electric field has the same problem. But in electrostatics it is a very general statement we are able to make whereas, in magnetostatics, already we are not able to make such a powerful statement. This idea of being able to talk about a magnetic potential is quite useful and you would already have used it because, when you do a transformer problem.

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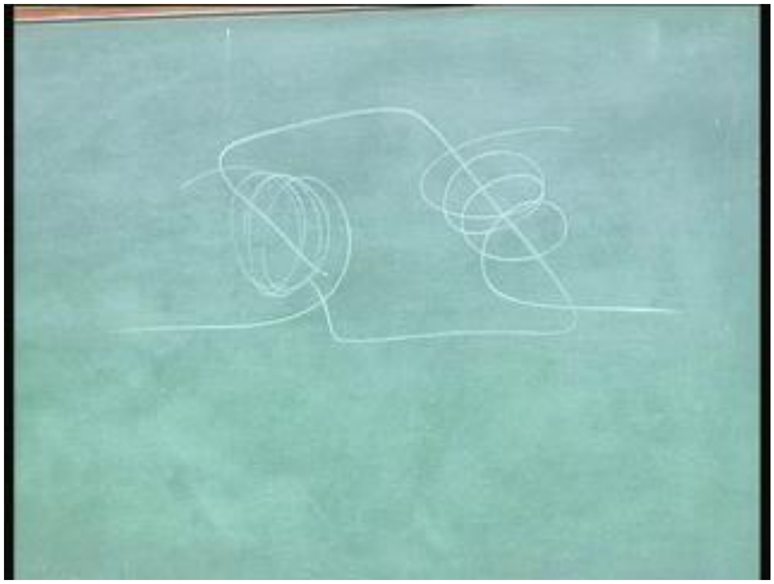

And you have a magnetic circuit where you have a ferrite core. You have a primary you have a secondary, you are actually using concepts which are very similar to a magnetic potential. The idea there is that inside this core you have no current. Therefore the magnetic field can be derived from a potential in doing this loop you know exactly what currents you are linking. Therefore you can work out what the contribution of those currents is and therefore you give, you can reduce the problem to a scalar. I will probably not go very much into this. But you will learn a lot about it from other courses.

Thank you.