Electromagnetic Fields
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Lecture – 23

Electro Magnetic Field for EEE Students
Ampere's Law

Good Morning. Today I am going to continue on with what I had done last time and establish the magnetic properties of materials. Last time what we had done was we had talked about the magnetic field due to a small loop.

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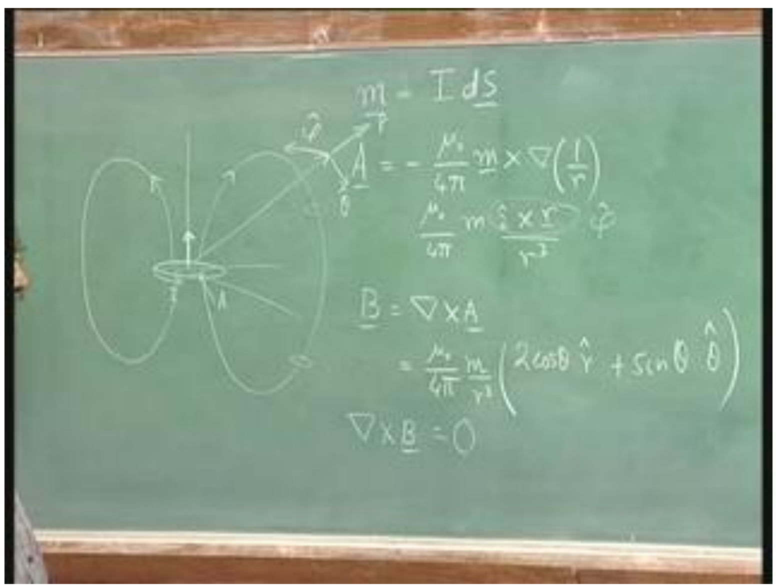

Let us say the loop is sitting at the origin. So if you go far away and you want to know what the magnetic field is there well this loop has an area a, and there is a circulating current i. We defined something called the magnetic moment which is current times the surface area of this loop. So d s points upwards, that is it is normal to the plane of the loop and whether it is upwards or downwards depends on you right hand rule. If you take a mentally take a screw driver and tighten it in the direction of the current and see which way the screw driver will move.

That way is the direction of d s. So you have for any loop a magnetic moment m and we worked out last time that the vector potential is going to be minus m minus mu naught over four pi m cross gradient of one over r or equivalently mu naught over 4 pi m times the z hat cross r over r cubed. This minus gradient of one over r is nothing but r over r cubed. Clearly z hat cross r z is in the vertical direction, r is this direction.

So the cross product is nothing but in the direction into the board. So a has only the phi component and when you derive from a b b is nothing but curl of a you find that it is equal to mu naught over 4 pi times m divided by r cubed times twice cos theta r hat plus sine theta theta hat. That is the magnetic field has a theta component and a r component this is in spherical coordinates. So in a spherical coordinate system r is in this direction r hat theta hat is in this direction and phi hat is in this direction. So there is a component in the r direction and a component in the theta direction. This looks exactly like the dipole field.

One over r cubed times twice cos theta r hat plus sine theta theta hat. So basically the magnetic fields are doing this. This is a dipole magnetic field. It is a very important result and it is the building block of everything we do with magnetic materials. But before I get to magnetic materials let me use this in another way at the end of last class I showed you that if you take the curl of this you can take the curl you get 0. What that means is if you take this expression for b and you apply the curl anywhere that is you take a small loop there and work out b dot d l you work a small loop here work out b dot d l as long as you are away from this loop the curl is 0.

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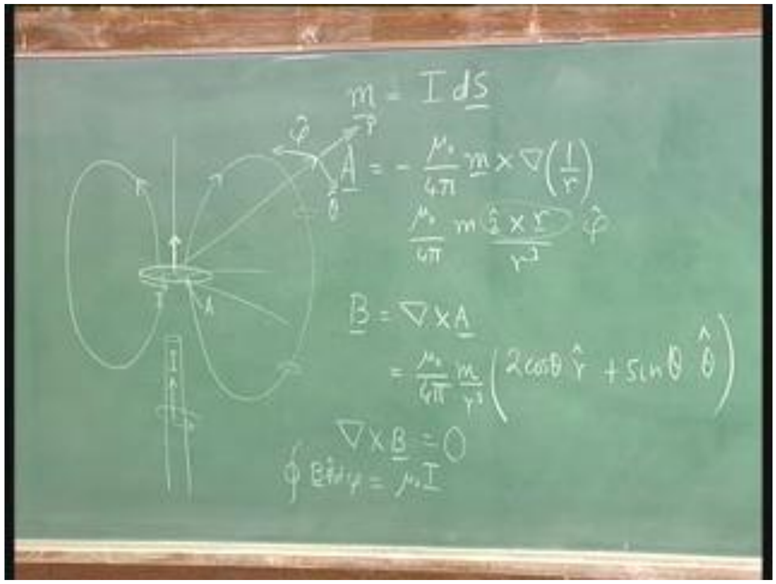

We also know something else. We know that if you take a line current and you take a loop around the line well the loop the b is always in the phi direction going round. So loop integral b dot phi hat d phi is non-zero and if you work this out you find that this is equal to mu naught i. This you can work out directly. I mean it is just application of biot savart law will give you this result. Now if you take these two pieces of information, then we can combine it into one rule.

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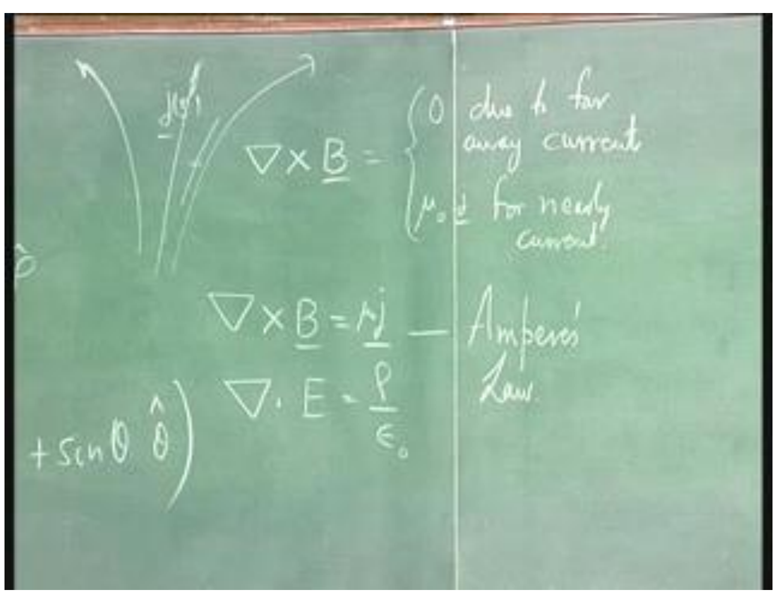

Supposing we have some general current, this is j, this is a function of r prime. I want to know what the magnetic field is doing at any point. I can take this j and I can build it out of lots of little loops as I discussed last time. So, what it does mean is the magnetic field is not responding to the j far away because the curl due to a loop gives me 0. However whatever current is going right through that point, I know that if i do d l dot-, b dot d l. I get a non-zero answer. So if I work out curl of b for any general current distribution it is equal to 0 due to far away currents and it is equal to mu naught j for nearby currents.

But of course b is actually due to all these currents. So we can write down finally this answer. That curl of b does not respond to far away currents at all. They all give it zero curl of b only responds to the current where your instrument is. Therefore it is equal to mu naught j. The corresponding equation in electrostatics is divergence of e is equal to row over epsilon naught. The electric field is due to charge density everywhere but the divergence of electric field only reacts to charge density at the point where the instrument is measuring b is due to currents everywhere but curl of b only sees the local current. This is called ampere's law.

Now there are elegant mathematical ways of deriving this result. One of them is given in your text book but ultimately the mathematics is not very satisfactory. You do the integration by parts and you throw away terms saying they go to 0 at infinity. They do not explain what is happening. The only satisfactory explanation that I have been able to find is that you can always build up any current distribution due to loops and we have see that infinitesimal loops have 0 curl, far away.

So, if there is a curl of b plus n it must be due to loops very close by and if you are talking about loops very close by then, we have a separate result which says that loop integral of b dot d l is equal to enclosed current. We derived it for a straight current line but it can be applied when you are considering only currents very close by. So this is a very general result and it is an extremely important one. Let us use it immediately to do a trivial problem.

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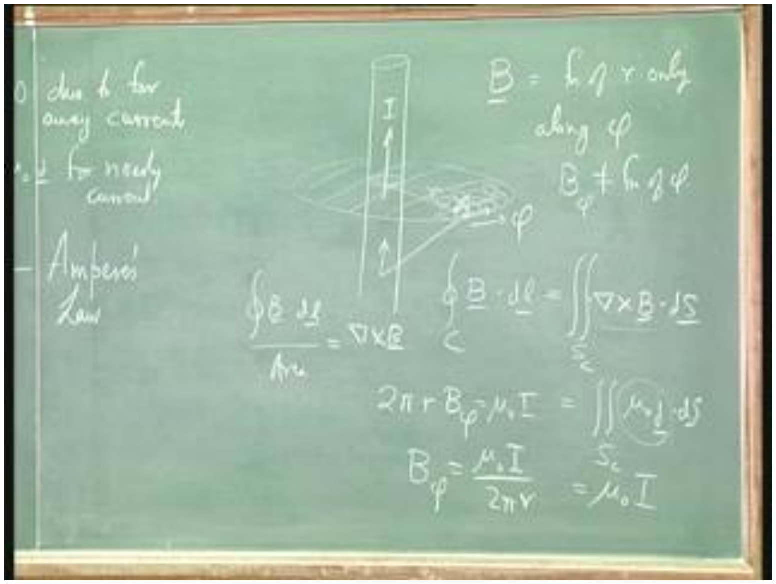

Supposing, I have a long wire carrying a current i... Well, if I want to calculate the magnetic field, what I do is, I take a loop whose radius is r centered about the wire. Now, whichever angle I look at the result should not change because this problem does not depend on the angle about which I am measuring. So, my instrument measures the same b here, as here, as here. So, b must be a function of r only. It cannot be a function of theta and it cannot be a function of z.

It cannot be a function of z because if I move slightly up, it is still the same problem. There is infinite amount of wire above infinite amount of wire below. Just from looking at the geometry, I also know it is along phi. Phi is this angle. So, I now apply my Stoke's theorem. I say loop integral b dot d l, it must be equal to surface linking this loop curl of b dot d s which must be equal to from the previous ampere's law surface integral mu naught j dot d s.

But what is mu naught j dot d s? It is the total amount of current that is penetrating the surface and as it turns out this surface is penetrated only at the wire by any current. That is equal to mu naught i and b is a function of r only and it is along phi. So, I can write this as 2 pi r b phi is equal to mu naught i or b phi is equal to mu naught i divided by 2 pi r. It is a very standard result. I am sure you have it done it in your electricity and magnetism. But let us see where, what we did to get there. First we said the magnetic field is symmetric in phi.

Secondly, we said because the current is in the z direction the line connecting the point and the wire is in the r-z plane. Therefore j cross r must be in the phi direction. If j cross r is in the phi direction then b is along phi, b is also a function of r only because it does not matter where along the loop you measure, your b it will be the same value and it does not depend on z either.

So, b phi is not a function of phi, then I take this loop integral b dot d l. Now b dot d l can be built up out of a whole lot of loops. This is what we did in Stoke's theorem. You build this surface out of a whole lot of loops. Internally each loop has field one way and field the other way. So, the field contributions cancel. The only contribution left is the field along the edge. But each little loop represents nothing but curl of b dot d s because that is any little loop small loop b dot d l divided by area is equal to curl of b. That is the definition.

Therefore this big loop b dot d l is nothing but the sum of whole lot of little loops, each of which is a curl of b dot d s integrated over the surface area. But now ampere's law says curl of b is mu naught j, the local value of curl of b depends only on the local current and mu naught j integrated over the surface is nothing but current enclosed. So let is just summarize. I always like to make the comparison to electrostatics.

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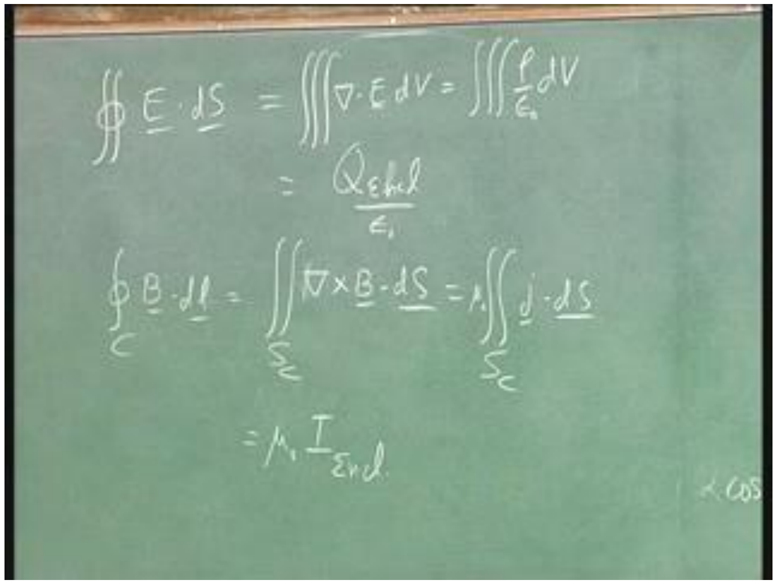

So, in electrostatics we had that surface integral e dot d s, we used divergence theorem. We said volume integral divergence of e d v which was volume integral row over epsilon naught d v which was equal to charge enclosed divided by epsilon naught. That was the divergence theorem. Now here we have loop integral b dot d l which is equal to surface enclosed by that loop curl of b dot d s which is equal to surface j dot d s and a mu naught which is therefore equal to mu naught i enclosed.

So you can see the symmetry. The differences have only to do with the fact that electrostatics works with gradient and divergence. Magnetostatics works with curl. But, otherwise there is an extra ordinary symmetry between these two results. And so the two results are really saying very similar things and you should expect to see many similarities in the kinds of conclusion we get from electrostatics and the kinds of things we derive from magnetostatics.

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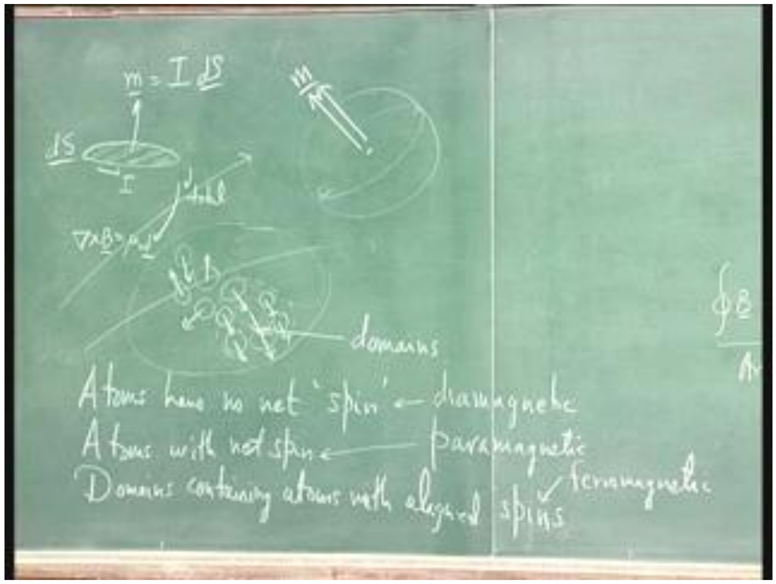

Now we have already got this result that if you have a small loop of area d s with the current i. We can define a magnetic moment m which is equal to i d s and this m generates magnetic field and it generates vector potential. Now, when we look at what is happening in material bodies supposing you imagine that there is some material some object may be a plastic may be a glass may be a lump of iron and let us suppose there is a magnetic field.

Now we know that curl of b is equal to mu naught j. This is true everywhere. There is only one problem and that is this j this j total. What happens is in the presence of a magnetic field if there are atoms inside this material the electron surrounding that nucleus they are going round and round, if you look at any atom you will find that there are electrons which are tending to go round this way.

There are electrons tending to go round this way. There are electrons tending to go round in all possible ways. These different ways in which they go round are what we call the electron orbitals. Usually if you take up all the different directions in which electrons move their net tendency of going round cancels out. So, if you look at any atom by itself and you ask what is the direction of the rotating field, the rotating electrons produce even though every electron is going round and round and its orbit encloses an area.

So, each electron is creating a magnetic field. But the set of all the electrons in the atom create no net magnetic field. That is for every electron that is tending to create a magnetic field. This way the remaining electrons tend to create a magnetic field this way, so that at every point the magnetic roughly cancels out. Now there are a few atoms where this is not true. There are atoms where the when you take all the electrons there is a net tendency of the electrons to spin in one way.

So, if you take such an electron such an atom you find that there is some direction for the net current which means that the atom has a net magnetic moment. Now in a material like this what will happen is different atoms will have different magnetic moments. So you will have one atom like this and another atom like this third atom like this fourth atom. So you can see that the different atoms would have spins in different directions and because they have spins in different directions, when you average over the entire material even though each atom produces magnetic field, the entire material as a whole does not tend to produce magnetic field. So it is the same story as the earlier case.

There are atoms where the set of electrons making up the atom cancel put when they produce magnetic field then there are atoms where this cancellation does not happen. But in a material the atom point in different directions and so the material as a whole is magnetically neutral and then there are materials where this magnetic moment is so strong that if an atom points this way it forces the next atom to point this way.

The next atom forces the third atom to point this way so much, so that you tend to have regions where all the electrons all the magnetic moments are lambda. Such materials tend to have different areas inside them areas that are quite small may be about sub millimeter in size where things are aligned up and these are called domains. Once again different domains point in random directions. So, you have three different types of materials which we can talk about, one atoms have no net spin.

What I mean by saying no net spin is that there is no net current that I can observe the atom have not. Then I have atoms with spin net spin and then I have domains containing atoms with aligned spins. So, there are three cases. This case where there is no net spin top begin with is what is called a diamagnetic material. Atoms with spin are called paramagnetic and atoms with domains. I mean materials with domains of atoms with aligned spin come into a class there is not one class.

There are many subclasses to this and I would call them ferromagnetic. Now there is a question that should be worrying you. It is this, when we talked about electric materials dielectric materials we only talked about dielectric. We did not talk about paraelectric. We did not talk about ferroelectric. We did not talk about domains. But when we talk about magnetic field and magnetic material suddenly we start having many, many categories.

Why is it that magnetic materials have so many categories. But when you talk about the electrical properties of the materials there are very few properties. The reason is the following: In fact it is the same reason that I had been mentioning all along. The electric field is a very strong field. It is such a strong field that it cannot remain unbalanced. If you actually have electric forces that continuously add up, then the amount of energy that will be stored by the system, the amount of force that will be exerted by that system will be so much that it will crush the material itself.

For instance, I think I derived the case where if you look at just a few grams of a material and you assume that they are completely separated out electrons and protons both and you ask what is the force between them that force is enough to lift the earth. We are talking about enormous forces. Therefore you cannot have such forces out of balance. Such force must always be in very delicate balance.

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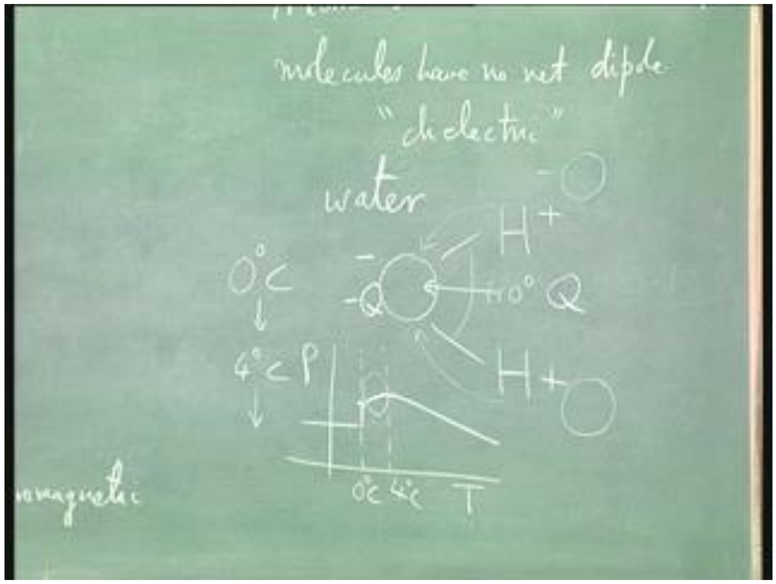

So much so atoms have no net charge. Now this is not strictly true. You do have molecules where atoms have charge but the molecules together have no net charge. Furthermore not only do you not have net charge atoms and molecules have no net dipole. The reasons are the same as I mentioned before, if you start having net dipole and net charge the system forces would overwhelm the material. So the result is when you do not have intrinsic dipoles and when you do not have intrinsic charge, you have only dielectric properties.

Now there are exceptions to this and surprisingly the biggest exception is the most common material we know. If you look at water it does not satisfy this, If you look at the structure of water, there is an oxygen atom and there are two hydrogen atoms and the angle here is something like 110 degrees. Now oxygen is a covalent element and it needs two electrons to complete its electron shell. So, what it does is it borrows the electron that hydrogen has or it tries to share it and so half the time the electron of the hydrogen is sitting on the oxygen. This allows the oxygen to become stable.

But the result is the hydrogen; do not get their full share electrons. So there is a net negative charge here and there is a net positive charge on the hydrogen. The result of this is that there is a net dipole moment. I am not sure which way the arrow is put but there is a q and there is a minus q. Now the fact that water has a huge dipole moment means that water has some very special properties and these special properties do appear.

For example, the dielectric constant of water is very, very high in the order of eighty and this is one of the reasons why when water freezes to become ice. Because of these very strong dipole moments the water molecule cannot close pack. The water molecule must arrange itself. So that, next to this hydrogen is oxygen. So that these charges cancel out and when you put such requirements the structure of ice becomes very does not become very dense. It becomes very loose. The result is the density of ice is much lower than the density of water which is why ice floats on water.

What is also true is, when ice just melts if you take water at zero degrees centigrade this water is ready to freeze. This water has most of the closed structure of ice. If you look up to say 5 to 10 angstroms, that is up to 1 nanometer, you will find the structures of the molecules of water at 0 degrees kelvin 0 degree centigrade are pretty much the structure of ice. But, then the structure slips. As you increase the temperature up to 4 degrees centigrade, this ice like structure slowly gives away and it becomes more water like and beyond 4 degrees it has reached its fully fluid structure.

So, the result is that if you plot the density of water versus temperature you find that up to zero degrees centigrade, it is 0.9 grams per c c. Then it will suddenly jump, this is the density of water. Then the density continues to rise till reaches 4 degrees centigrade and then it starts to drop. This is the usual behavior of liquid. As we increase the thermal energy of liquids they do not like to be close packed.

The kinetic energy causes them to exert pressure and become less dense. But, actually this is a region where water is becoming more dense even as we increase its temperature. It is all because of the very unusual properties of water because it has a permanent electric moment. But such materials are rare and their properties are very special. If you talk about the general material, the electrical systems do not have dipole moment and therefore they have only induced properties and that is why we call them dielectric.

This word dia means that it is a induced property that is not present when the external field is applied and is in a direction which tries to suppress that eternal field. Similarly, diamagnetic tries to suppress the magnetic field. Whereas, paramagnetic actually tries to enhance the magnetic field and ferromagnetic materials are materials where the magnetic field can be generated can exist without an applied field. We will come back to these concepts later. But let us try and understand what are the consequences for ampere law.

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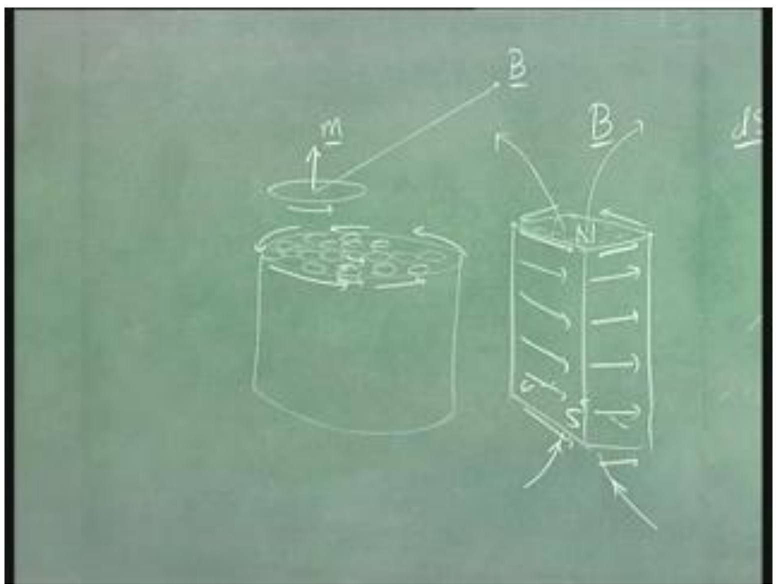

We have already seen that if you had a loop of current, you can generate a magnetic field. You can generate vector potential as well as magnetic field and you know that the amount of magnetic field you generate has to do with the magnetic moment. Now if you had a cylinder let us say of magnetic material and supposing all the atoms inside had their spins aligned up, so you can tile them all up, then obviously a stoke's theorem would come in because, if you have current like this, then the current here results in a current this way, which cancels the current in the next loop.

So, all these little spins in different atoms will internally cancel out and so what you will be left with is a net current due to internal atomic spins. This current if it is present in the absence of an applied magnetic field is what we call ferromagnetic and that is what is found in a bar magnet. If you have a bar magnet, what you are really having is a whole lot of these loops present through the entire material of the magnet and their internal currents all cancel out. So, you have net current like this and this net current causes a magnetic field.

Similarly, you have the same net current at the bottom which causes the magnetic field to enter. So, this is where the concept of a loop of current causing magnetic field comes from. I mean, leads us to, because once you have this idea and once you have a idea that a atom or a molecule, can have net current electron current around it. Then you can build up larger currents out of the response of these materials and that is where permanent magnets come from.

So, this picture of atomic currents is what unifies magnetostatics because, up to this point we said magnetism started by looking at permanent magnets. But then we forgot all about permanent magnets and talked about biot savart law. But if we still have to explain what a permanent magnet is and the way to explain a permanent magnet is to take all of these little, little loops associate them with atoms with molecules and then tightly pack these spins and see that that results in a surface current. The surface current is there all through the surface and this surface current causes the magnetic field to become intense and come out at the north-pole and go in at the south-pole, okay?

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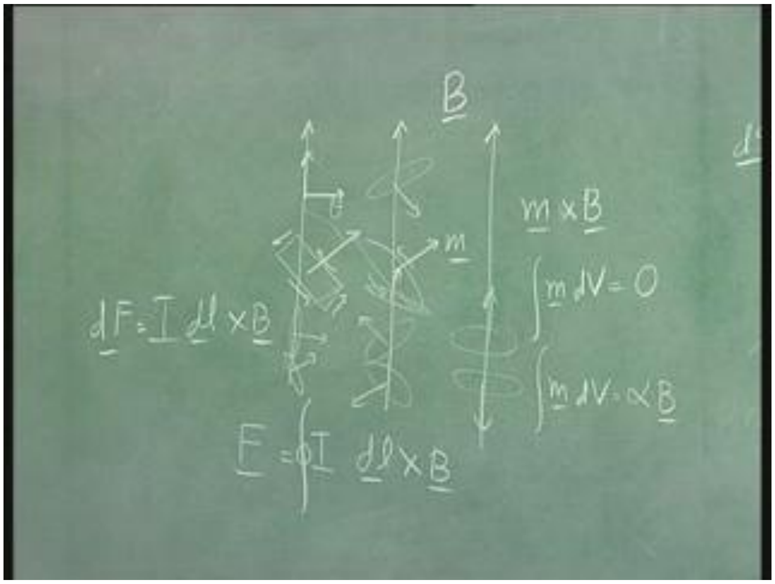

Now, let us take this one step further. Let us suppose we had a material, the material has atoms with net spin and in the presence of a magnetic field, what does such a spin do? For that we have to look at how a loop of current responds to a magnetic field. So, we have a loop as a magnetic moment m, it is equal to the current times the area of the loop, the loop itself has current flowing this way going round. Now, we worked this put couple of lectures earlier that, if you have a loop then in fact we took a square loop and you have a magnetic field there is no net force because the current.

This current cross the magnetic field causes a j cross b which is in the outward direction, this causes a j cross b this way. This causes j cross b this way and this causes j cross be this way. So, they are all in the plane. So, if the plane of this loop is along the magnetic field, there is no effect of the magnetic field. But, if the plane is oblique that is this is the normal, there is an angle theta which is not zero. Then what happens is that this current i cross b gives a force that tends to push this wire in that direction and the corresponding current here tends to push the wire in this direction. Therefore, there is a torque and the torque tends to rotate the loop till it becomes flat.

Now, this torque we worked it out, then it was r cross f and f itself is i d l cross b. But, if you work it out in terms of magnetic moment that same torque tau is equal to m cross b. This you can verify, m when it is along b is 0, if there is no torque. When m is perpendicular to b there is maximum torque. Now what is the effect of this torque? The effect of this torque can be seen if we consider what is happening in a material. In a material we actually have many atoms and each atom has its own spin. So, some atoms are aligned with their spins along the magnetic field.

Some are aligned with their spins opposite to the magnetic field, still others sideways. Then there will be those which have oblique spins. Now, the effect of all these spins put together is that there is no net spin. But once we apply a magnetic field what happens is each of these little spins. These atoms experiences a torque this same torque and what this torque does is it tries to turn the spin of these atoms a little bit. But the atom is in a material, it cannot just change its spin arbitrarily.

So, even as it tries to change its spin the neighboring atoms resist because when this atom changes its spin it is actually pushing against other atoms. So, the result is each spin does not align only with the magnetic field. It changes a little bit this spin becomes a little closer to vertical. So, does this. This becomes closer to horizontal. This bends up a little bit and when you add up all these spins what you find is that there is now a net amount of spin pointing along the magnetic field.

So, initially if you took the average of m, you took m d v. The volume average of m you would have got zero because the spin is pointed in all possible directions and there was no net spin. But once you have a magnetic spin present what happens is the m d v is equal to some fraction of b itself. So there is a one percent or half percent tilt towards b. What is happening is a spin that is like this is, now like this. A spin that is like this has become like this. So, everything is tilted a little bit towards b. The result being when you average it over there is a residue. It is no longer zero. It is a little bit-, the amount that is pointing towards b is proportional to the strength of the b. The reason being that is the force is proportional to b. What is the consequence?

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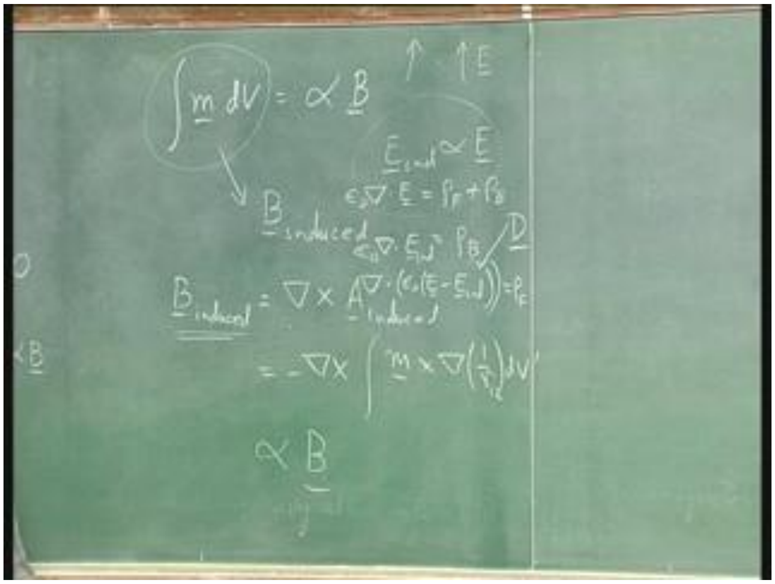

Now you have that the volume average of the internal spins are proportional to b. Then what is going to happen is that that, these magnetic moment are going to b induced because each of them is capable of creating a magnetic field. Earlier the b induced was equal to curl of a induced which is equal to minus curl of integral of this m cross gradient of one over r. This is the expression minus m cross gradient of one over, r is induced due to one loop. I am integrating over all the loops in a volume. Now if this integral m d v is proportional to b obviously the b induced is also proportional to b.

Now, this immediately reminds us of the electrostatic situation. Let us look at that case. We had a material, you had an electric field. Because of the electric field the atoms got deformed. They develop dipoles electrostatic dipoles which created net charge inside. The induced charge was proportional to the applied electric field which meant that you got an induced electric field which was proportional to the applied electric field. I say proportional to only in the sense of became is proportional in amplitude. The direction of this was not the direction of the applied electric field.

Similarly the direction of b induced is the not the direction of b. There can be changes in direction, but the stronger the external field the stronger the induced field to handle this problem. What did we do? We said divergence of e and I took the epsilon to the left hand side is equal to row free plus row bound. Then I said epsilon naught divergence of e induced was equal to row bound. Because the e induced is only related to the bound charts and separating the 2, I got divergence of epsilon naught e minus e induced was equal to row free and I identified this new quantity epsilon e minus e induced has a new field which I called the displacement vector. Now we are going to do the same thing here.

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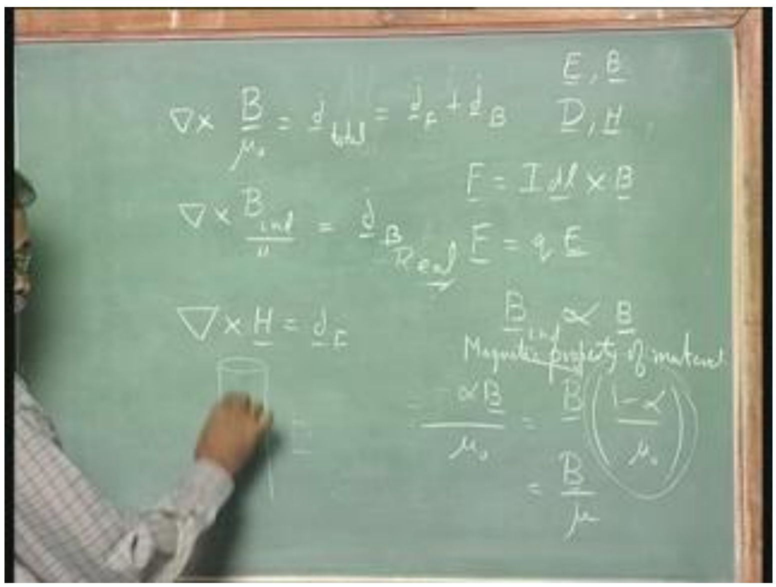

I already have that b over mu naught is equal to j total. This j total is the combination of my known currents and my induced currents what I call bound currents. But I also have curl of volume integral of m d v or I should not put it that way, curl of some b induced divided by mu naught and that part is equal to only j bound. So once again just see the symmetry, I had divergence of epsilon e is row free plus row bound. Divergence of epsilon naught e induced was just row bound and so I defined a new field.

Here I have curl of b over mu naught is j free plus j bound, curl of b induced over row naught is j bound. So, let me take the difference. So I get curl of b minus b induced over mu naught is equal to j free as with electrostatics the right hand side is known, the left hand side is a new field and this new field i call h. I can never remember the official names of b and h. One is called field intensity and the other is called field strength it does not matter. What matters is that the real field is b. This is the… this is the quantity that appears in your force equation, f is equal to i d l cross b. It is not i d l cross h.

So, b is the quantity that is real, h is a construction whereby i can forget about the bound currents and I can only concentrate on free currents. It is the same thing as saying for electrostatics f is q e. It is not q d, d is there only because I do not want to deal with bound charges. So, in electromagnetism e and b are the primary fields and d and h are derived fields. They are fields that exist because material properties make it convenient to have derived quantities.

Now of course different materials have different induced magnetic fields. We have talked about three kinds diamagnetic paramagnetic and ferromagnetic. But for the simplest kind of systems you can say b induced is proportional to b. If b induced is proportional to b then you can imagine that you can pull out a b out of this also. So, this quantity would become b minus alpha b over mu naught and you can write this whole thing as b times one minus alpha over mu naught.

And it depends on the type of material whether the induced b is in the same direction as b in the opposite direction as b etcetera and so you can write this as b over mu. So just as in dielectrics, we use this to define an effective epsilon. We are here using it to define a effective mean. So we have defined a new quantity h and this h is related to b but are we really any further on in this problem. The answer is we are a little forward the reason being that in most materials this quantity is what I had called the magnetic property of the material. It does not depend on how I am using it.

I can tabulate this quantity in a hand book and if I know which particular kind of plastic I am using, which kind of polymer I am using, which kind of alloy I am using, I know my effective mu. Since I know my effective mu, I can now treat h as derived from b in a known way and I get a simpler equation. The simpler equation I get is equal to j free. So, despite the fact that I have magnetic material, I no longer have to deal with the induced currents of the magnetic material. I can only deal with my own applied currents. So, for example if I had a current in a wire and I put a cylinder of some material around that wire. Well the properties of that cylinder will not affect this expression. We will continue this in the next lecture.