Electromagnetic Field

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Lecture - 2

Introduction to Vector

Good morning!

This is the second lecture in the video course on electromagnetic fields aimed at EEE students. Last lecture, I had introduced the course, discussed the syllabus, the textbook and talked about some other conceptual tools you will need. And then I had introduced the basic elements of vectors. So this lecture I am going to continue that and take you to the point where we can use these. We can use vectors in our equations in electromagnetic theory. one of the most important things we have to do with vectors is to is to write them in coordinate systems. So let us take a look at what that implies.

I have a vector. Remember what I said last time- vector is a direction along with the magnitude. This is any vector- let's say it's a force vector F. As I mentioned last time, if I draw a line underneath this symbol it means it's a vector. But if I have three dimensional space I can imagine drawing many different vectors in it. They can be vectors coming out of the board, they can be vectors going into the board, to the left and to the left and to the right. one of the standard results of vector spaces which you would have learnt in your math course is is that for three dimensional space you can have only three independent vectors. An example is the unit vector along X , the unit vector along Y and the unit vector along Z, and we know that any vector can be built out of these three vectors.

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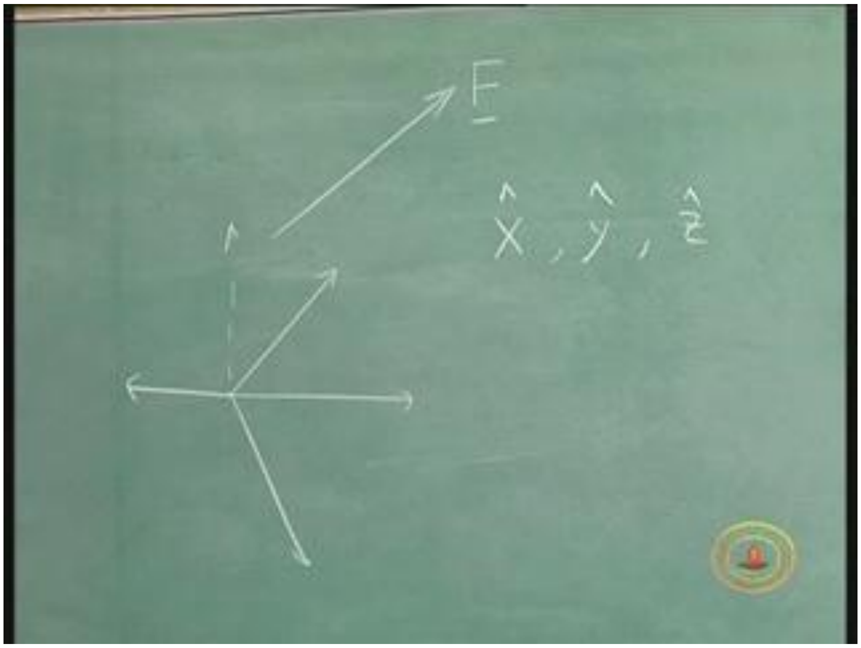

For example, I can build a vector called four units along x plus three units along y plus two units along z and any vector at all can be constructed using these three vectors. But there is no requirement that we should use these particular vectors. I could have used some other set of vectors. For example, supposing I define vectors the vector u which is nothing but unit vector along x, a vector v which is the sum of the unit vectors x and y, and a vector w which is sum of unit vectors x and z.

So if I had to draw them, this is what it will look like. I have a coordinate system- this is x, this is y, this is z. My vector u is this, my vector v is actually this way and my vector w, I have to draw a square this way. So I have three vectors of this type. u is actually along x, v is in the x-y plane, it’s midway between x and y and w is midway between x and z, it’s in the x-z plane. Now it turns out that I can use u, v and w to construct any vectors I like. Just like I could construct using x, y and z, I can construct any vector at all using u, v and w. Let’s try it out on this vector itself. I am going to call it z. How will I do it?

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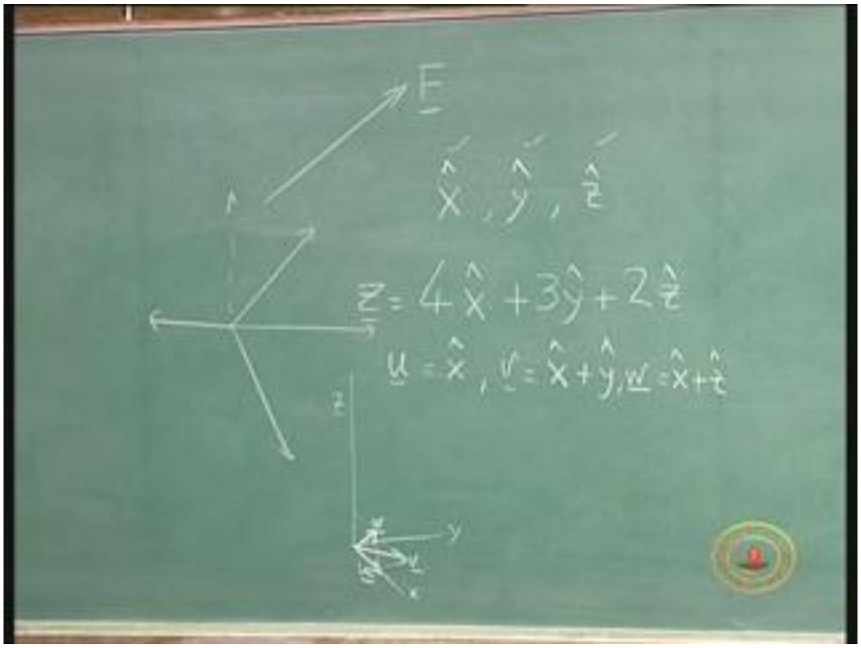

Well, what I do is I try to figure out what these three vectors, unit vector along x, unit vector along y and unit vector along z, what these are in terms of u, v and w. So what are they? Well x is easy, x hat is nothing but u itself. I have got it there, but what about y hat? y hat, well y hat appears only in v. It is equal to v minus x hat from this equation, but x hat itself is u. I have already got that equation there so it is equal to v minus u. Similarly z hat is equal to w minus x hat which is equal to w minus u.

It is a particularly a simple example because i shows it that way. So I can write down what x, unit vector along x, unit vector along y and unit vector along z are in terms of u, v and w.

So then I will try to write what z is. Let me write what it was earlier. z was four x hat plus three y hat plus two z hat, but I have expressions for what these are. So it is four u plus three times v minus u plus two w minus u, all right. All I have done is I have taken these expressions and substituted for x hat, y hat and z hat. I can collect terms I can collect terms because vector addition is associative and commutative. You can read about that in the textbook. Those are just mathematical ideas which say that we can shuffle together any such expression.

So I am going to write it as four u minus three u minus two u plus three v plus two w. So I have taken the minus three u, brought it here, minus two u and brought it here, what does that give us? It gives us that z is equal to minus u plus three v plus two w. It is a rather strange result- I have a nice straight-forward vector if I was plotting. It is four units four units along x, it is three units along y, it is two units along z. So I would just draw a rectangle, complete it, make a cube out of it and this point an arrow drawn from the origin to that point is my z. For some strange reason that z actually says it requires minus u. To start with, we wanted four units of u but once we have taken care of v and w we want actually minus one unit of u. The reason why this strange result happens is because when we look at the elements we used to construct are z, u, v and w, what you find is that to construct w and v you not only moved in the directions y and z you also moved in the direction x.

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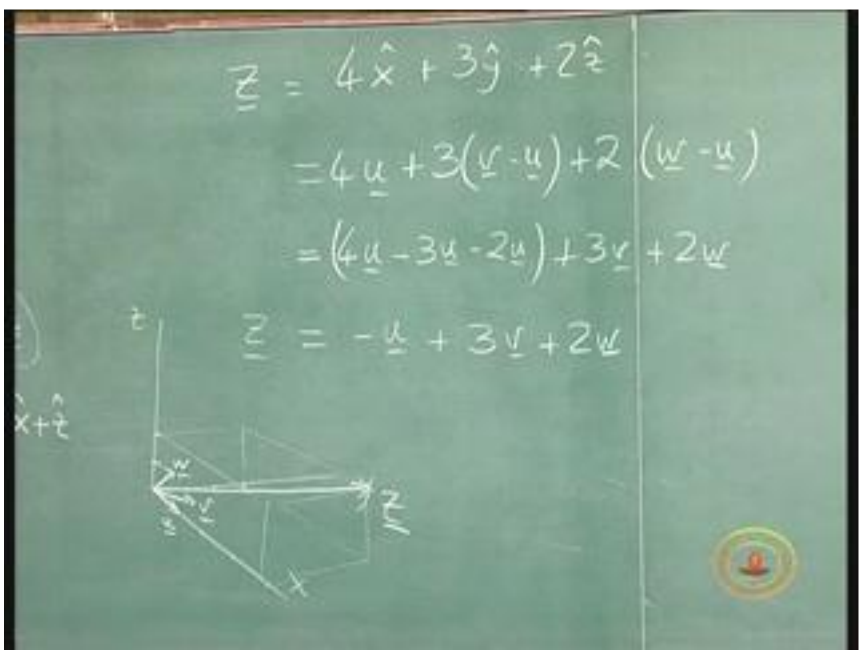

So z actually has four units along x, three units along y and two units along z. To take care of the y and z, you have to move along v and w. But when you move along v and w you end up moving along x as well. You move so much along x, you actually move five units along x. So by the time you have taken care of this distance and this distance, you have already gone beyond in x, so you needed to correct it by coming back. So that is why you ended up with a minus u. But what this means is that if you had such a strange coordinate system like u, v, w, you couldn't predict ahead of time how much u, how much v and how much w there is in this vector z. Straight- forward vector but by the time you have finished with figuring out its coordinates, you are getting strange answers.

So this is why people have come up with some simpler coordinate systems- some useful coordinate systems which don't confuse us- these coordinate systems are called orthonormal coordinate systems. So what is an orthonormal coordinate system? For any vector z, I can always write my minus u plus three v plus two w. Supposing I now want to take the dot product of this z along some direction. So I say I want to know how much u there is. So I do dot product, z dot u. Well z is already given to me, so I can write minus u plus three v plus two w dot u. Now this dot product is distributed. That means if I have a sum of vectors dot u, that sum can be pulled out, so it can be written as minus u dot u plus three v dot u plus two w dot u. And immediately there is a problem because this v dot u and w dot u, what are they actually? v is unit vector along x plus unit vector along y dot unit vector along x, it is equal to one w. Dot u is unit vector along x plus unit vector along z. Dot x is also equal to one which is why when you take z dot u and add it all up you get minus one plus three plus two equals four which is not surprising at all because we started with z equals four x hat plus three y hat plus two z hat, so we had to get four out of it. But you can see that this is a very confusing situation.

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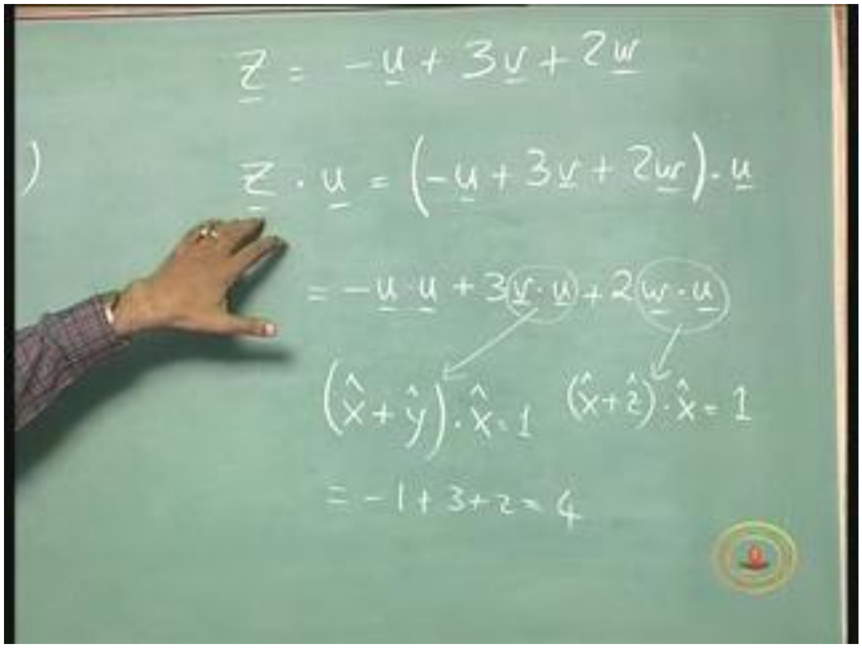

I want a simple dot product with building block vector and the answer doesn't turn out to be minus one- answer turns out to be something else. If the answer is to turn out to be minus one, I would require, this would be zero and this would be zero. If they were both zero then the answer would come out directly from here. Now that happens in the case of Cartesian coordinate systems. Let's verify that. Take the same vector z and I want to do z dot y hat. So then I would say it is equal to four unit vector along x, three unit vector along y, two unit vector along z, dot y hat, which will be equal to four x hat dot y hat plus three y hat dot y hat plus two z hat dot y hat. But here we know our coordinates- this is x, this is y, this is z. We know that if a vector is along x and a vector is along y there is ninety degrees between them. x dot y is zero. If there is a vector along z and a vector along y, there is ninety degrees between them. y dot z is zero. So this becomes three y dot y and since the length of y is one, y dot y also becomes one, is equal to three. So for the Cartesian coordinate system if you take the dot product with one of the building block vectors, you get the coefficient that is used to build up your vector. It's a very useful property of the coordinate system because that way you can figure out how to form this vector z given the building block vectors. All you have to do now is say that my vector z is equal to z dot x hat along x hat plus z dot y hat along y hat plus z dot- along it's a different z from this, batch-wise permutation. This is true in coordinate systems like Cartesian system. It is not true in bad coordinate systems like the one I used- u, v, w. It's a major difference and it is so major that nobody uses non- orthonormal coordinate systems if they can avoid it.

So what are the properties of an orthonormal coordinate system? We know that x hat dot x hat is equal to one, x hat dot y hat equals zero, x hat dot z hat equals zero. Similarly x hat dot y hat equals zero, y hat dot y hat equals one, y hat dot z hat is equal to zero. Thirdly we take x hat dot z hat. We get you should get a unit in metrics. This was done upside down.

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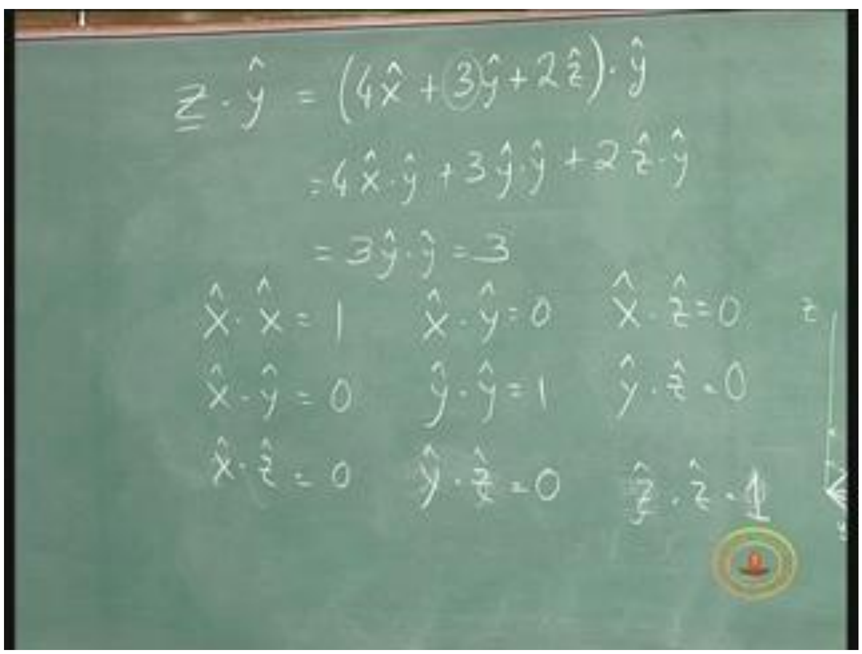

This is the general property of an orthonormal system. So if I have a system consisting of vectors e i and I want to say they are orthonormal, so I will go from some one to let us say n, then I will say if I take any e i, dot it with any e j, the answer will be one if i equals

j. one equals i equals j, i equals j, i equals j, answer is one. It is zero, if i not equal to j. and you can see half diagonal everything is zero. Let us look at some examples.

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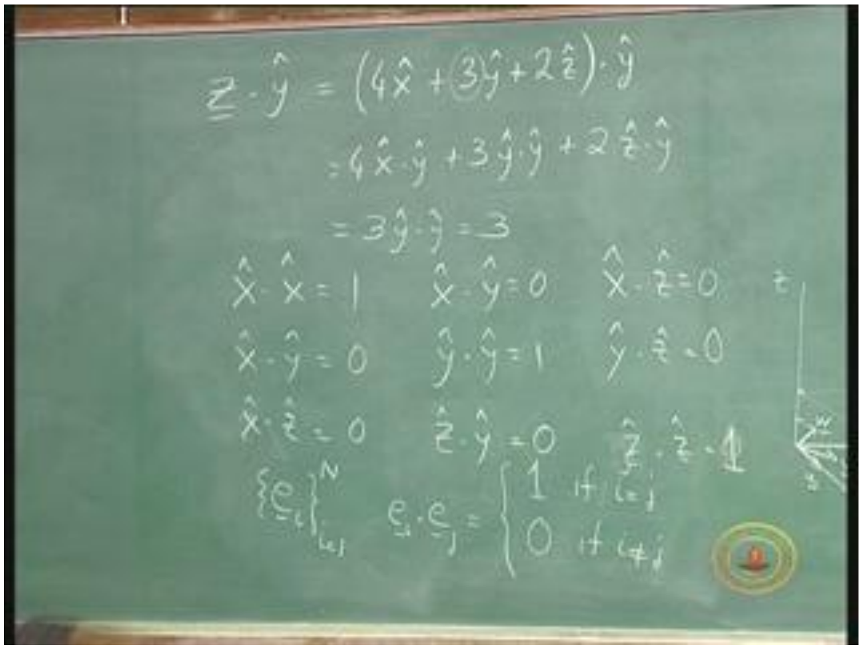

What For orthonormal systems, they include all the common coordinate systems, obviously, the Cartesian coordinate system. We have just seen that it is orthonormal. But here is another coordinate system that is orthonormal. This coordinate system looks like this. It is the x- y- z. The z hat is a unit vector along z. x hat plus y hat divided by the root two is a unit distance along x and y. So I go on the diagonal of the square formed by x and y but I divide my root two. I don’t go the full distance. Then x hat minus y hat- I have to go backwards, I go on this diagonal, again I don’t go the full distance; I go seventy percent of the distance. Now I know that each of these angles is forty-five degrees, so obviously this vector dot this vector is zero and both of these are ninety degrees from the z vector, so they are all orthogonal to each other.

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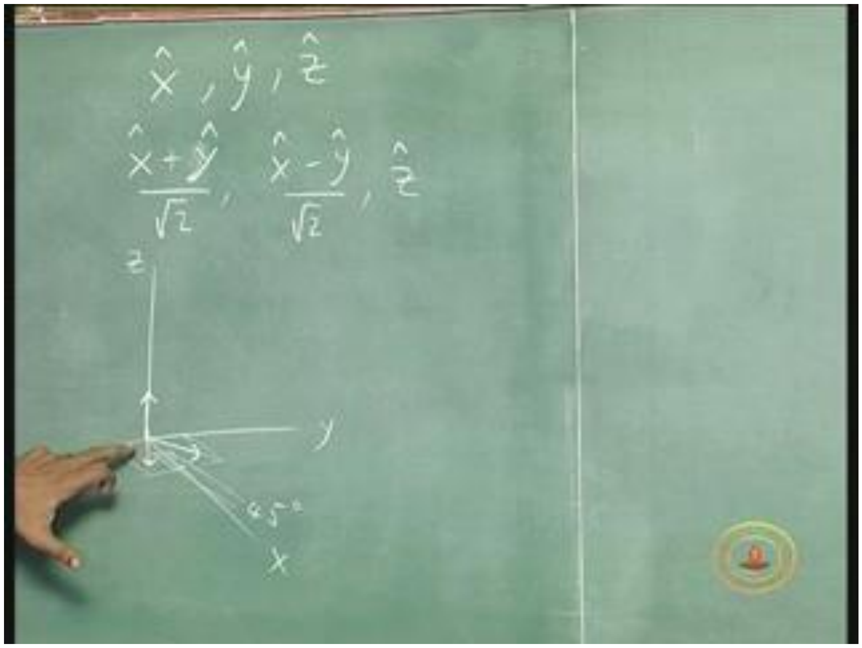

Let us just verify that. x hat plus y hat over root two dot x hat minus y hat over root two is equal to- well there are four terms in this. x x hat dot x hat which is one, x hat dot minus y hat- zero, y hat dot x hat- zero. y hat dot minus y hat- minus one, divided by two which is zero.

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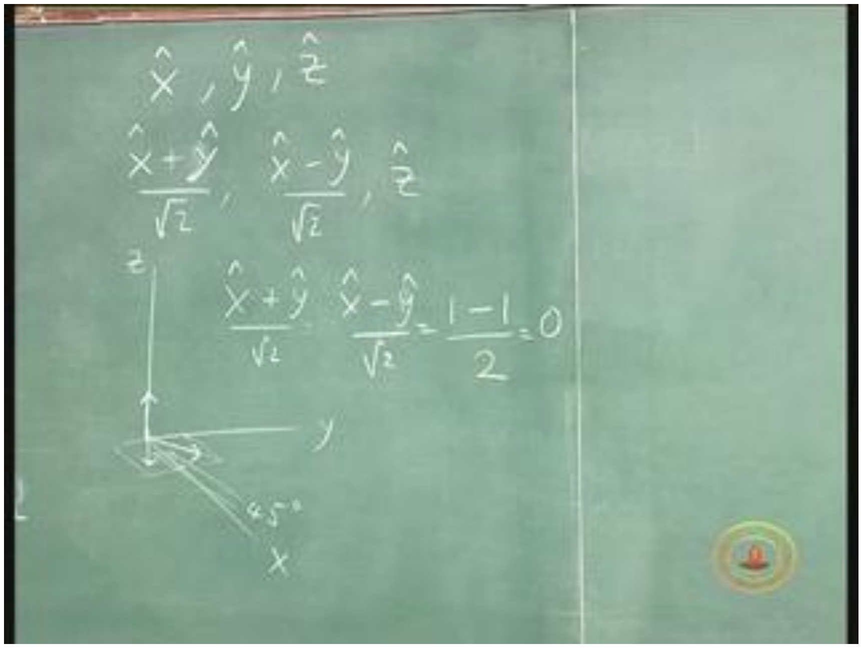

So these are orthogonal. Are they orthonormal? Because I need also this condition- e i dot e i is equal to one. So if I just dot product this by itself, what do I get? x hat plus y hat over root two- dot well x hat dot x hat is one, x hat dot y hat is zero, y hat dot x hat, zero and y hat dot y hat is one- over square root of two multiplied by square root of two- so square root of two square, obviously square root of two squared is nothing but two and the numerator is also two. So it is equal to one. This is a general statement actually, if I take the x- y- z coordinate system and I rotate it anywhere I like, I can rotate it about the z axis that is what I did here, I can rotate it in general in 3- D, it will still be an orthonormal coordinate system.

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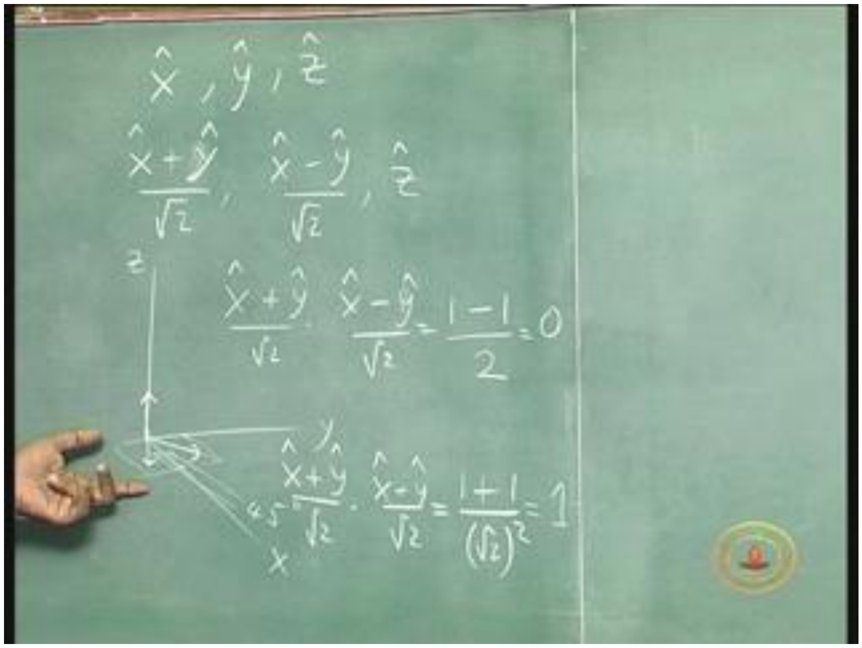

What about some other systems? Here is one that you should be familiar with. This is what is called the cylindrical coordinate system- this is a system where I have the z- axis, but instead of x and y, I now choose to think in terms of circles. Any point I say has a radius, r and then angle with the x axis, theta. So any point in three- dimensional space can be captured by specifying r theta and z. We know we can do that because x is equal to r cos theta and y is equal to r sin theta. So if I can specify a point in x- y- z, I can specify it in r theta z. But what are these quantities? The unit vector along z is nothing but z hat. The unit vector along r means keep theta constant, keep z constant, change r.

So I am now going to draw a general point, where on a cylinder, it comes to some height and we have come to some angle theta. At this point, the z direction is vertically up. This is z hat. The r direction means keep that z fixed, keep that theta fixed, increase r. So you go on that line. This is called the r direction. The theta direction means keep z fixed, keep fixed, but vary theta.

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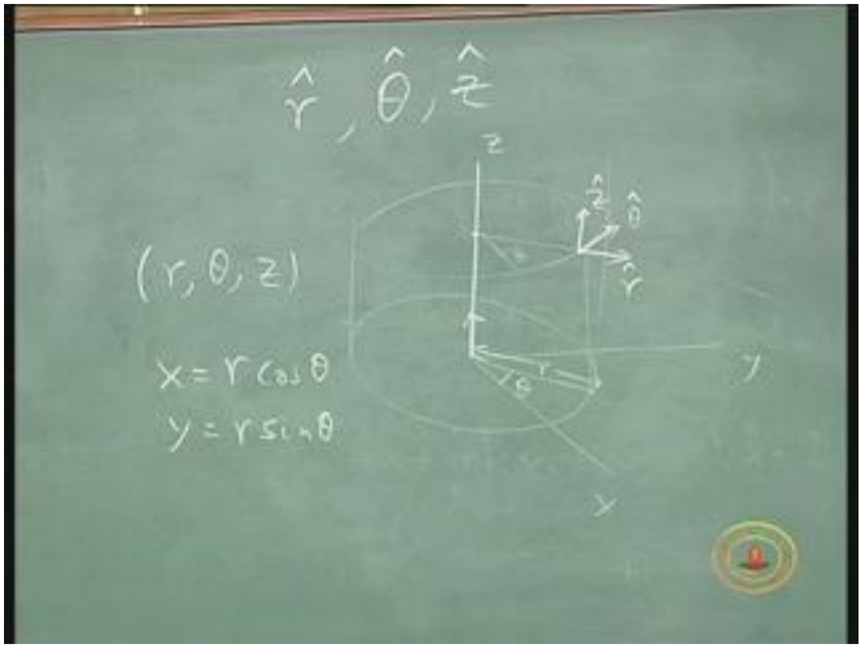

So we vary theta. You are moving along the circle and the direction along the circle is this. Now once again r and theta unit vectors are clearly orthogonal to z. But what about between themselves? Well if you look from above, it is a circle. This is the x- axis and you are here somewhere. So the r is along the radius, theta is along the tangent. So one is tangential and the other is normal. It is easy to show these two are ninety degrees. So this is again in orthonormal set but is a strange orthonormal set. Very useful, but very strange. It is strange because supposing I change my angle, I go to a point there, so I have got a new theta now. My z has not z hat has not changed, but my unit vector along r no longer points in this direction it is pointing in that direction. And my unit vector along theta is now pointing this one.

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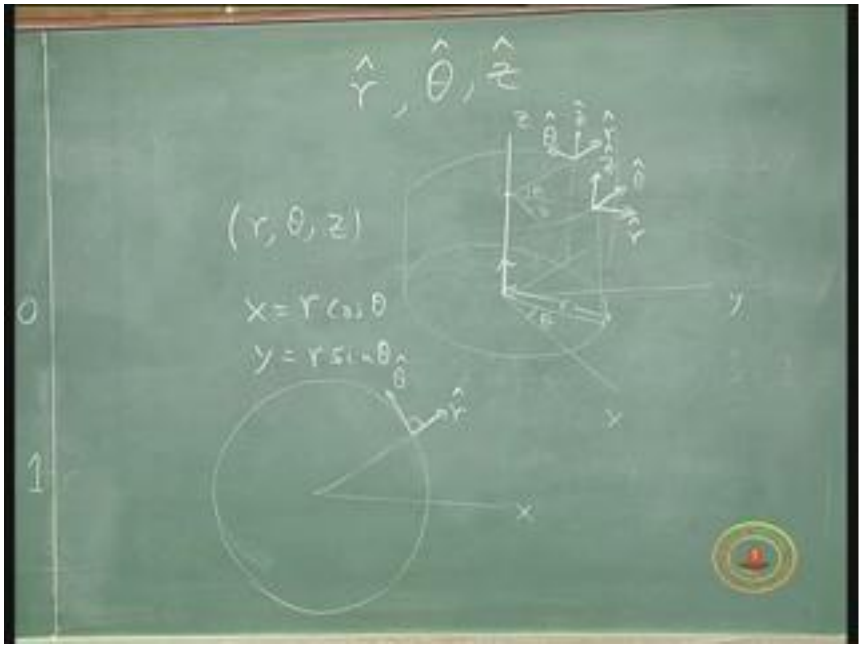

So this is a coordinate system that depends on the point where you are looking. If you look at this point, r points this way. If you are looking at that point, r points there. If you look at this point, r will point away from the origin in that direction. But even so this is an orthonormal coordinate system, which means that if you take these three you will still get e i dot e j is equal to 1, if i equals j; 0 if i not equal to j. Similar to cylindrical coordinates there is a spherical coordinate system. I will introduce that later. It is introduced in the book and you can read it up, but there is no new concept there. It is the same idea that if you move along one of the coordinates keeping the other two fixed, you get a direction and that direction defines a unit vector.

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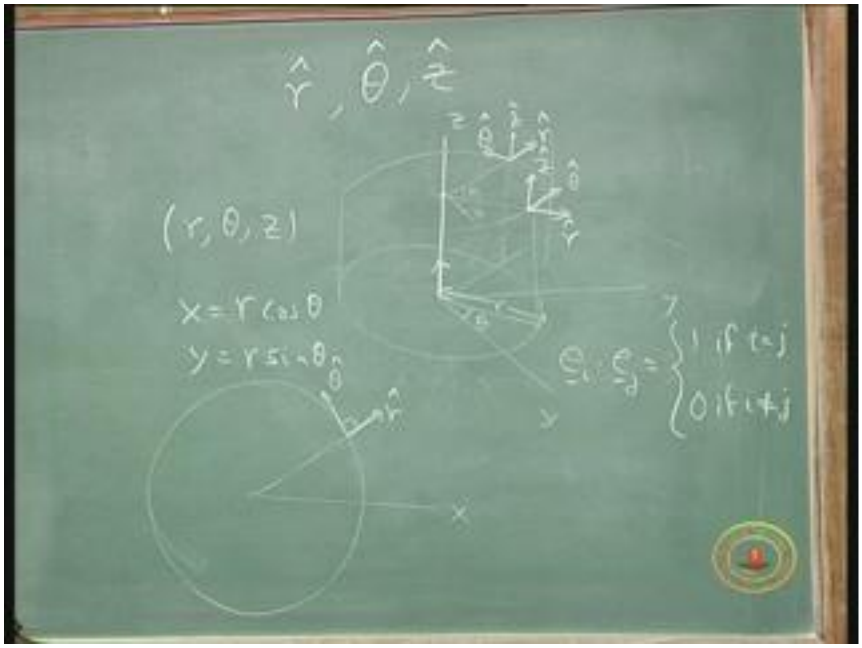

Now let us put the concepts you have got so far to work. Supposing I have coordinate system and let us say a satellite is following some trajectory. The satellite is experiencing gravity at all points of its orbit as it is falling towards earth. I want to know how much kinetic energy does the satellite gain as it moves. I am going to take a simplest ideal case- I should really be using spherical coordinate system- I will pretend that the earth is flat. So the satellite starts at a position r 1, goes to a position r 2, r 3, r 4 and finally r N. What do I mean by these r 1- r 2? I mean I can draw a line from origin to that point and its coordinates are r 1. r 2 would be a line joining the second point. r 3, the third, r 4 to the fourth. So every point in space defines an arrow- an arrow going from origin to that point that has a distance and it has a direction. So if I look at r 1, it is really x 1. So it is this distance x 1, this distance is y 1, this distance is z 1. So it is x 1 along x direction plus y 1 along y direction and z 1 along z direction. Similarly r 2 is x 2 along x direction, y 2 along y direction plus z 2 along z direction and so on. Now the distance between r 1 and r 2 is this vector. I will call that r 1 2 and this vector which is the distance between r 2 and r 3, I will call r 2 3. Distance between r 3 and r 4, I call r 3 4. From a vector addition I know that r 1 plus r 1 2 is equal to r 2, because if I go to r 1 and go further by an amount r 2, I reach a point which is nothing but r 2. So this is vector addition.

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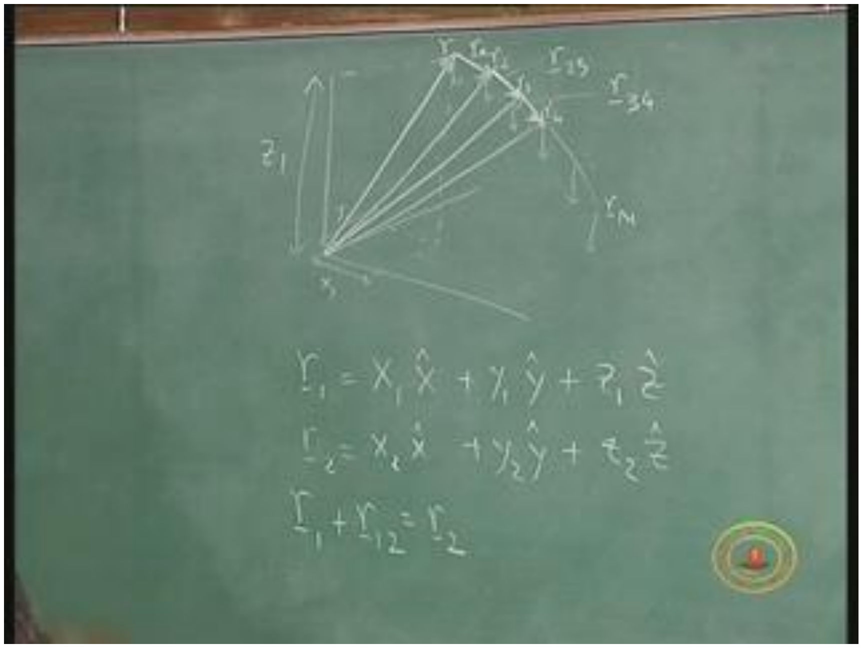

Similarly I can write r 2 plus r 2 3 equals r 3, that is I go to r 2, then go a distance distance direction along r 2 3, I will reach r 3, and similarly r 3 plus r 3 4 equals r 4 and so on.

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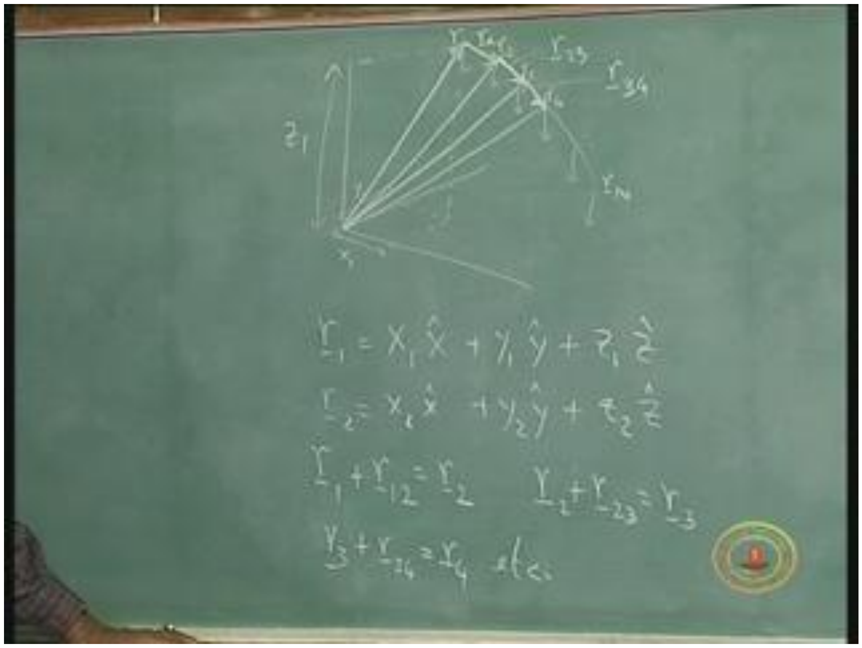

Supposing I want to write r 4 in terms of r 1, what do I do? Well I will write r 4 is equal to r 3 plus r 3 4 but r 3 itself can be written as equal to r 2 plus r 2 3 plus r 3 4. But I can substitute for r 2. So when I do that I get r 4 is equal to r 3 plus r 3 4, r 2 plus r 2 3 plus r 3 4, r 1 plus r 1 2 plus r 2 3 plus r 3 4. I have put brackets because that is how I have defined these quantities. But this is where some rules about vector addition come in. The brackets do not matter.

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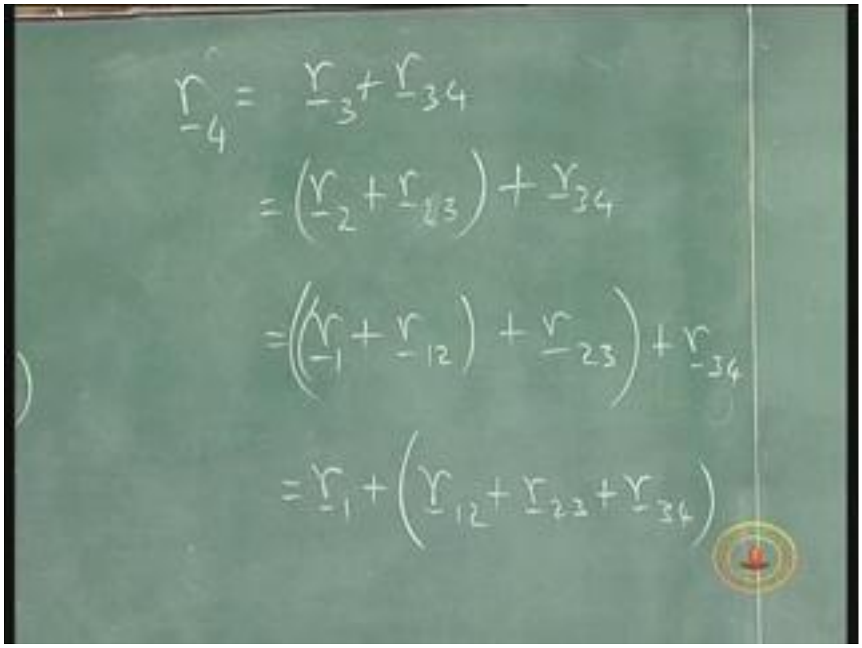

So I can as well write this as r 1 plus r 1 2 plus r 2 3 plus r 3 4. So what it means is to get to r 4, I could have gone to r 3 and gone to r 4, but I could equally well have gone to r 1, gone r 1 2, gone r 2 3, gone r 3 4, is the same thing. It is obvious that it is the same thing, but mathematically what it means is these brackets can be dispensed with when we do vector addition. What is the use? Well we would like to know how much kinetic energy changed. Change in kinetic energy is equal to work done on satellite. Well there is a force due to gravity and the work done is force dot distance travelled. Well what is the distance travelled, well it is r 1 2 plus r 2 3 plus r 3 4 because if I took r 4 minus r 1 this is all that remains.

Now I decompose this r 1 2, r 2 3, r 3 4 into components. And what do I get? I get the force due to gravity is minus mg along the z axis- it is pointed downwards- dot r 1 2 in the x direction, x hat plus r 1 2 in the y direction, y hat plus r 1 2 in the z direction, z hat plus etcetera. Large number of components.

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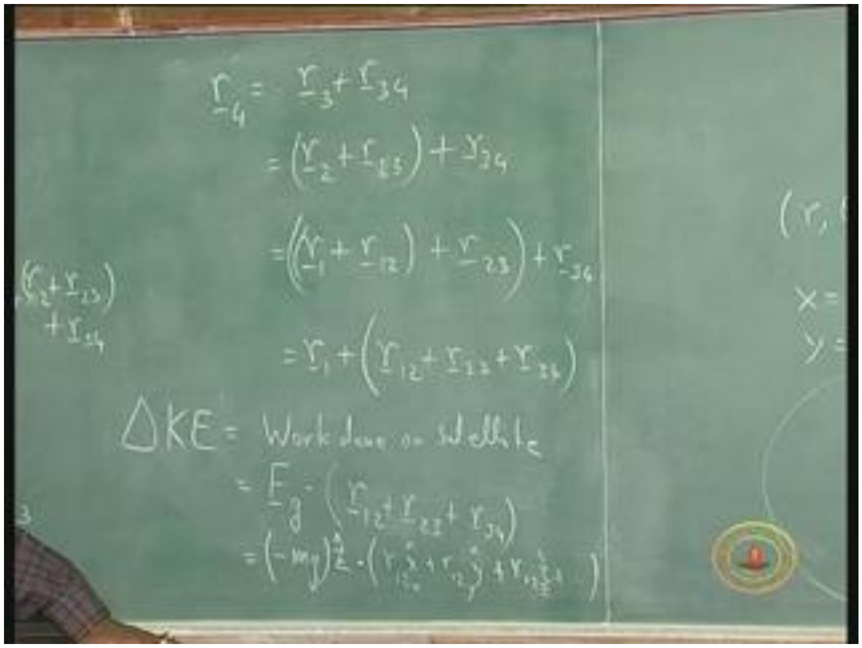

But now I know x dot z is 0, y dot z is 0 and only z dot z remains and z dot z is 1. So I need only to keep the z components. So what I get is that the change in the kinetic energy is equal to minus mg times r 1 2 z plus r 2 3 z plus r 3 4 z plus r n minus 1 n along z. Now each of these, if you look at this, is just the amount of distance by which the trajectory dropped in z. So it can be written as minus mg z 2 minus z 1 plus z 3 minus z 2, many other terms, plus z n minus z n minus 1. And you can see that the z 2 will cancel out and the z 3 will cancel out and so on and so forth. So you are left with minus mg into z n minus z 1, which tells us what we already knew, which is that when an object falls some distance the gain in kinetic energy is nothing but the distance mg times the distance it falls. So it is not as if vector algebra gives you anything new, that is not the purpose; the purpose here is to show that something familiar that we already know is consistent with whatever vector algebra is saying.

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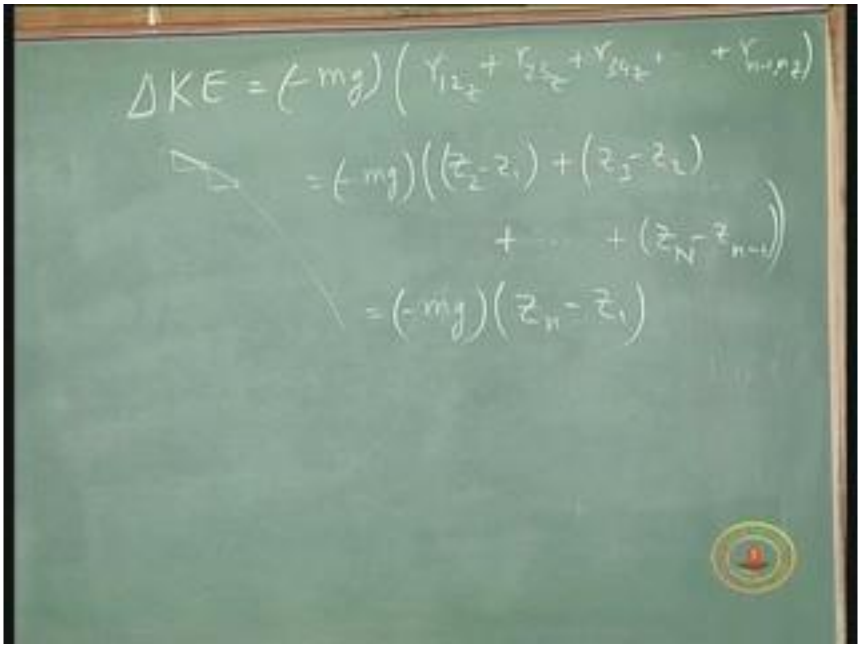

Now here with fifteen minutes more, let us go to another topic which I introduced last time, which is the curl, sorry the cross product. So up till now we have talked about coordinate systems and dot product. There is an equally important operator called the cross product and as I told you last time, the definition of cross product is that if you have two vectors say F and G and if they point if they have an angle theta between them then F cross G points in a direction that is ninety degrees to both vectors, in a direction which you will get by using a right hand rule, that is if you take a screw driver and screw it in so that F comes into G, the direction in which the screw driver will move, that direction is the cross product direction. This would be F cross G. What is the magnitude? This distance is going to be the magnitude of F times magnitude of G times sine of the angle between them. What this sine theta means is that if F and G point along the same direction theta would be 0 and sine theta would go to 0. So you won't have any cross product if two vectors are pointing in the same direction. On the other hand if two vectors are pointing exactly ninety degrees apart you get maximum cross product. This is exactly opposite to dot product; dot product, if two vectors are pointing in the same direction you get maximum, if two vectors are pointing at ninety degrees you get 0.

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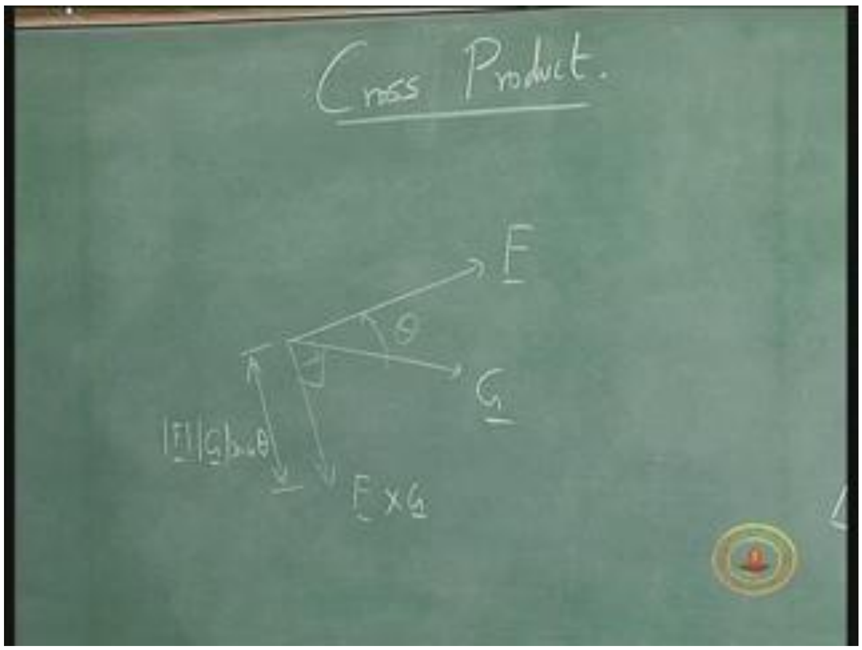

Now supposing I take these two vectors F and I say that this is F x x hat plus F y y hat plus F z z hat let us say G is G x x hat G y y hat G z z hat.

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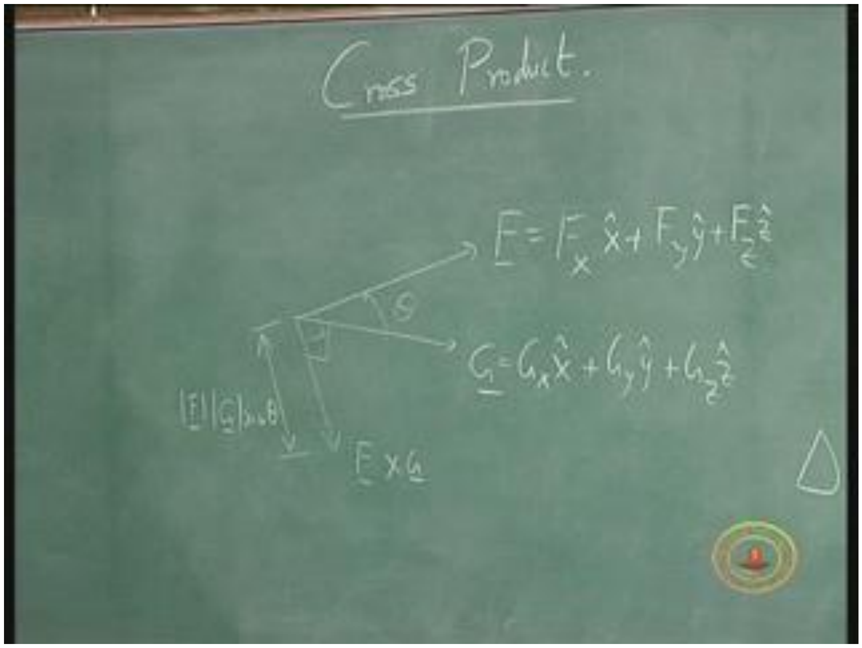

Now it is not totally obvious that what we are going to do is going to work, but I think it is it should be taken on faith at this point this definition of cross product is actually what is called a bilinear operation. It is linear in each of the two vectors. So the result is when you write F cross G, it is actually equal to the representation of each of these, F x x hat F y y hat F z z hat cross G x x hat G y y hat G z z hat. But this cross product, defined this way, it is not all clear that this is correct. But it is actually distributes over the multiplication. So you get F x G x times unit vector along x cross unit vector along x plus F x G y unit vector along x cross unit vector along y plus F x G z unit vector along x cross z hat. Similarly F y G x plus F y G y plus F y G z plus F z G x.

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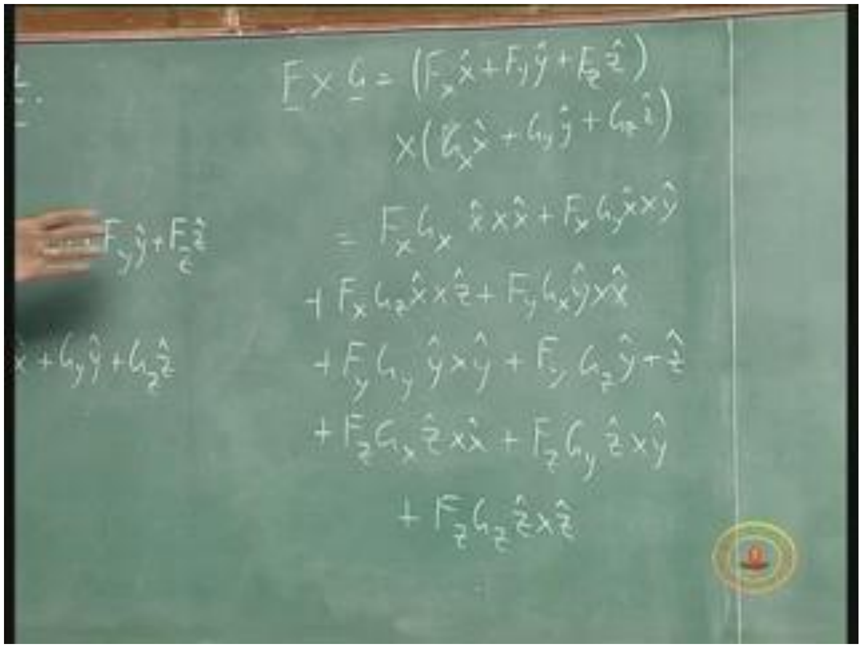

As I said this is not obvious, but it turns out that the definition does in fact distribute. Maybe it is true then there are certain things you can immediately see: x hat cross x hat is 0. y hat cross y hat is 0. z hat cross z hat is 0, because a vector pointing on the same direction as another vector cannot have a cross product. Now what about x hat cross y hat? Well let us draw the coordinate system.

If I use my right hand rule and try to turn a screw driver from x to y, well I point along z; so I have x hat cross y hat equals z hat. Similarly if I do y hat cross x hat, I point in the opposite direction. If you do x hat cross z hat you will see that. And similarly if you do y hat cross z hat you get plus x hat and z hat cross y hat gives you minus x hat. So let us write out these terms. When you write out these terms what you get is F cross G is equal to minus F x G z along the y hat direction. I am sorry I started with a wrong term, F x G x goes away F x G y along z minus F x G z along y hat direction then F y G x y cross x is minus z along the z direction. F y G y- there is no cross product, F y G z y cross z is x, F z G x z cross x is y, F z G y z cross y is minus x and F z G z will go to 0. So we have at least come down to six terms. Now you can collect terms according to each unit vector so what has this become, it becomes x hat times F y G z minus F z G y plus y hat times- there are two terms F z G x minus F x G z plus z term which is F x G y minus F y G x.

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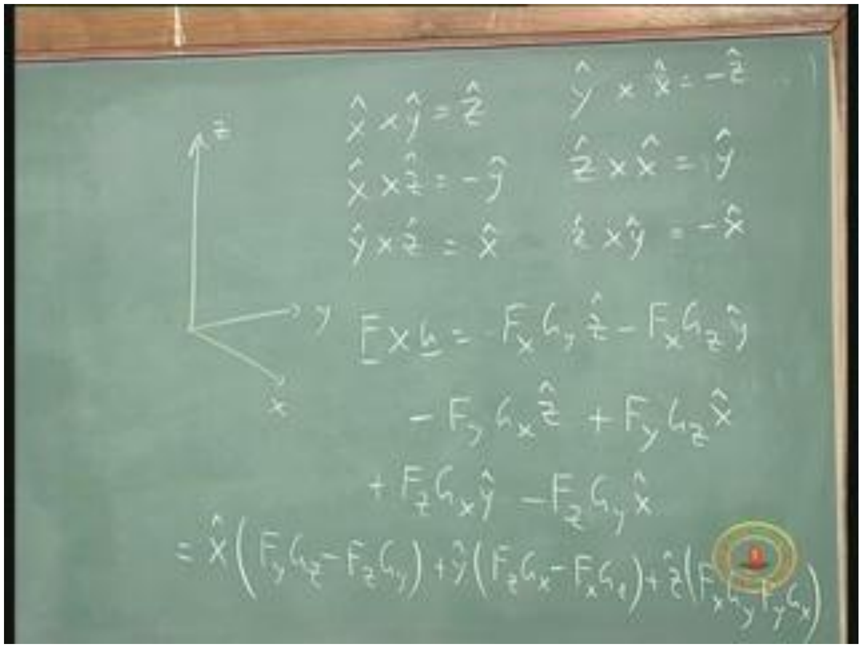

Now all this algebra was in order to get this expression, because when we look at this expression we recognize it immediately. This is nothing but the determinant. So you can write the determinant form of cross product; you can say F cross G is equal to the determinant x hat y hat z hat; F x F y F z; G x G y G z. because when you take the determinant you choose any row let us say we choose the top row then we take sum over all the minors. So x hat times determinant of this minor, which is F y G z minus F z G y plus y hat times this minor, which is F z G x minus G z F x plus z times this minor which is F x G y minus F y G x.

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So you can see that the cross product is intrinsically dealing with a determinant of these two vectors. Now why why is that important? One of the important applications that immediately comes is that if I wanted to take the cross product and dot product it with something else- so i supposing I said F cross G dot H, so what would happen? I would take this expression and I would say that this is equal to the x component of H times F cross G x component plus H y times F cross G y plus H z times F cross G z. But F cross G x is nothing but this piece. So if I simply replaced this determinant and put in H x H y and H z, I would be getting H dot F cross G. We will come back to this later, but this is an extremely important concept because it talks about volume and it is important when we- if we get to it in this course, but it is very important in general when you want to find the volume of different enclosed by different surfaces. Because these vectors in general will not be ninety degrees, so they will point in various different directions and you can draw a three dimensional structure using these three vectors and what this tells you is nothing more than the volume of this parallelepiped.

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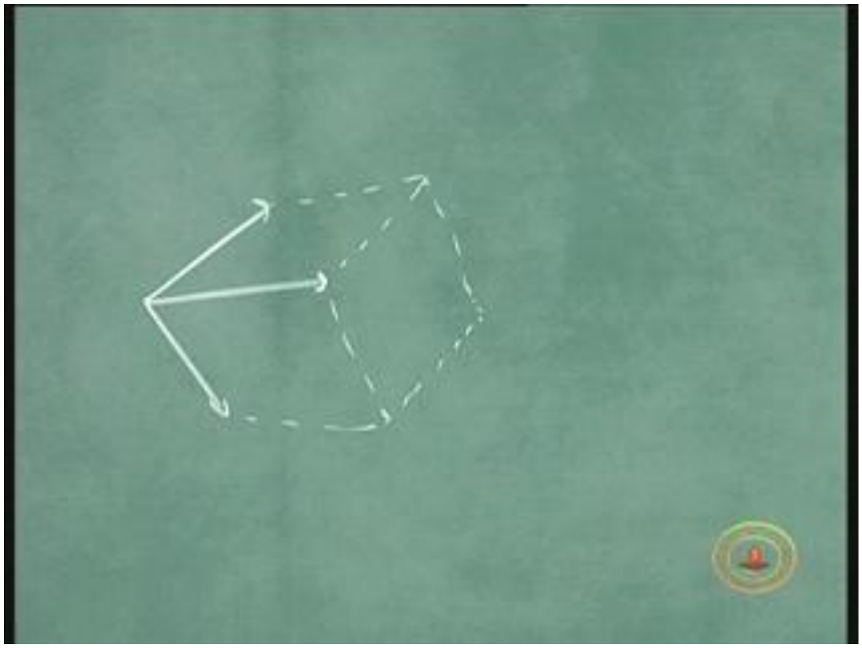

With this I am completing the review of vectors that I had planned to do for this course. You are already supposed to know all of this. So this is more review so that in case you forgot any of it you you will get reminded of it here if you are not comfortable with any of these concepts please do go back to your previous mathematics book and review it. Thank you.