

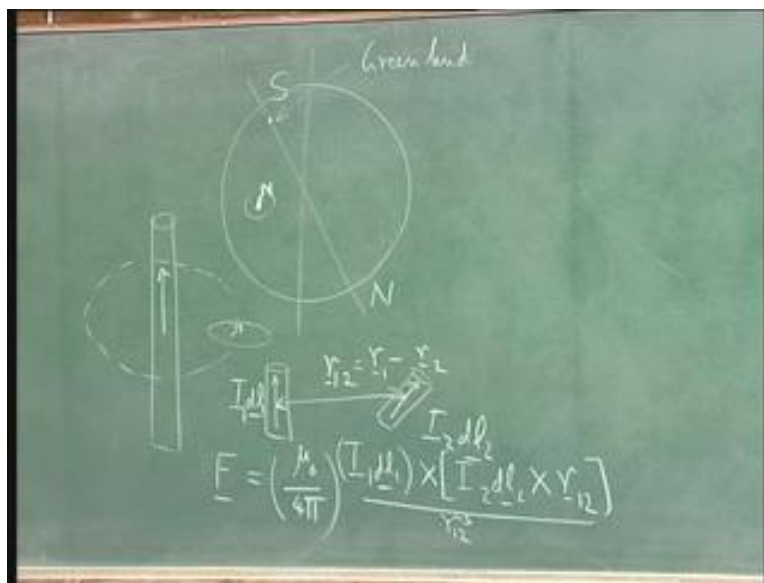
Electromagnetic Fields
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Lecture – 19
Magnetic Field – 2

Good Morning. Last time we had started talking about the magnetic field and as you know, I have told you the magnetic field is nothing but a modification of coulomb's law. So, actually if you did relativity properly, you know need the magnetic field its just part of the electric field itself. However most of engineering is actually done at low speeds, so the velocity of charges, the velocity of objects, they are all very slow compare to speed of light.

Therefore, we do not want to use a complicated equation of relativity in order to engineer simple machines that we designed in electrical engineering. So, what we have done is give try to abstract from what we learned last time and come up with the new field which we call the magnetic field. So, let me review.

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Earlier on people knew that the earth had some kind of axis. Let us say that it came out of Greenland and the southern part came out somewhere south of Australia and any compass placed anywhere on the earth, tended to point towards this point coming out of Greenland. And the part of the compass that pointed to Greenland was called the north-pole of the compass and the part that pointed south was called the south-pole. So north came from pointing geographically north.

Now, we understand better, we know that actually the earth itself is a gigantic magnet. And in fact what is that north-pole is a south-pole of the south magnetic-pole of the earth and what is of the south-pole is the north magnetic-pole of the earth. So the north-pole of compass is attractive to a south-pole of the magnet and the south-pole of compass is attractive to a north-pole of the earth magnet. So this is what we know. What we know is there is some attractive force between the north-pole of the magnet and south-pole of another magnet.

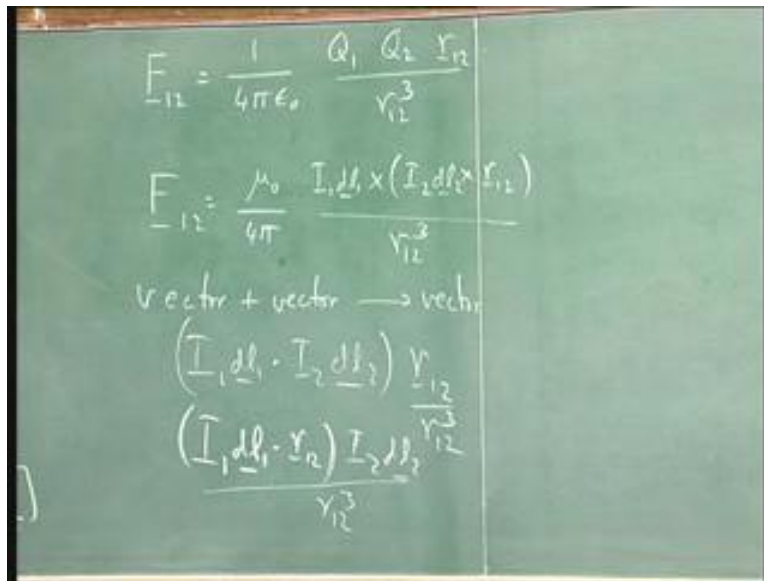
Now, we also know there if I put a current carrying wire and take the same compass and show it near the current carrying wire, let this compass again deflects it no longer points north in state it points in a direction which is tangential this wire. So apparently, whatever this wire is doing is the same thing that the earth is doing except is doing it in a direction which is very strange. Instead of making the compass points towards itself, it is making the compass point at right angles to it.

So, these two pieces of information the first obtain. Then, people started studying what happens when current elements have placed in each other. So, you suppose that there is a current element I_1 and the current element I_2 . It has a length dl_1 and the other one has a length dl_2 and dl_1 , dl_2 , are vectors because it is a direction. You can draw the vector connecting these two elements and I am going to call that r . Sorry. Let me put the arrow other way $r_{1,2}$; $r_{1,2}$ means r_1 minus r_2 .

Then in terms of these this picture, you can write down the force there is exerted on the first element due to the second element. And the force that you get is like this. F which is

a vector is equal to some constant times $I_1 dl_1$ for vector. Vector cross product $I_2 dl_2$ cross r_{12} and the whole thing divided by r_{12} cubed. And the constant comes from r units. The constant is μ_0 not over 4π . I will talk about the constant later. It is just some constant. Now I want to write down the coulomb's law as well, so that you can see the similarities and differences.

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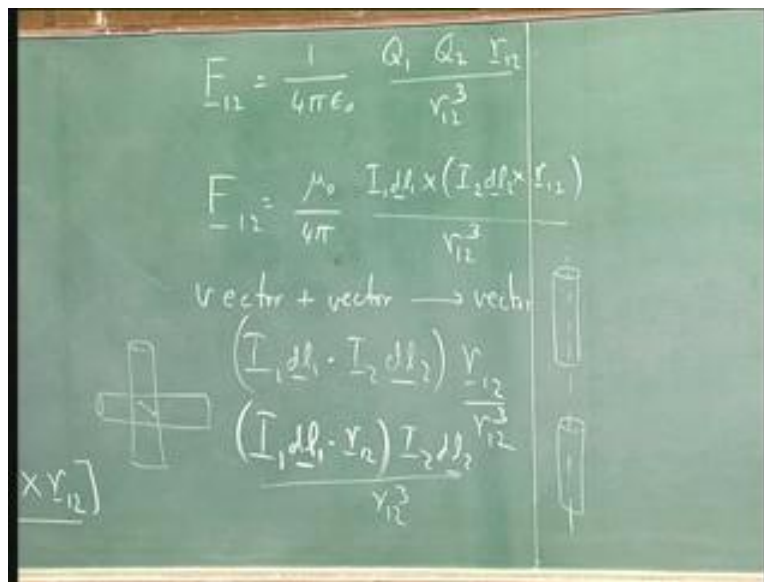
For coulomb's law a force F_{12} was equal to 1 over $4\pi\epsilon_0$ $Q_1 Q_2$ r_{12} divided by r_{12} cubed. And the force magnetic F_{12} is μ_0 not over 4π $I_1 dl_1$ cross $I_2 dl_2$ cross r_{12} divided by r_{12} cubed. As you can see, there is actually very close similarity between these two expressions. They both involved a product of sources of the force. In electrostatics it is the charge, in magneto statics it is the current. So, it is $I_1 dl_1$ and $I_2 dl_2$ instead of $Q_1 Q_2$ and in both cases does not r_{12} vector divided by r_{12} cubed.

So there is scaling is the same it both of them involve product of the sources and both of them involve the direction between the true sources. However here Q_1 and Q_2 are scalars and the force is in the direction r_{12} . Whereas, here $I_1 dl_1$ and $I_2 dl_2$ are vector

themselves. Force is also a vector so we needed to start with vector and another vector and we needed to get to a vector which is the force.

Now there are many ways in which you can start with two vectors and end of with the vectors. One way for example could be $I_1 dl_1 \cdot I_2 dl_2$ multiplied by r_{12} over r_{12} cubed. So it is entirely possible magnetism followed this or you could have had $I_1 dl_1 \cdot r_{12}$ multiplied by $I_2 dl_2$ over r_{12} cubed. These all also create vectors. So, why do we have to use this particular form which is after all rather complicated? Well, in the end we do not have any choice the force equation is what we measured. So what do we measure?

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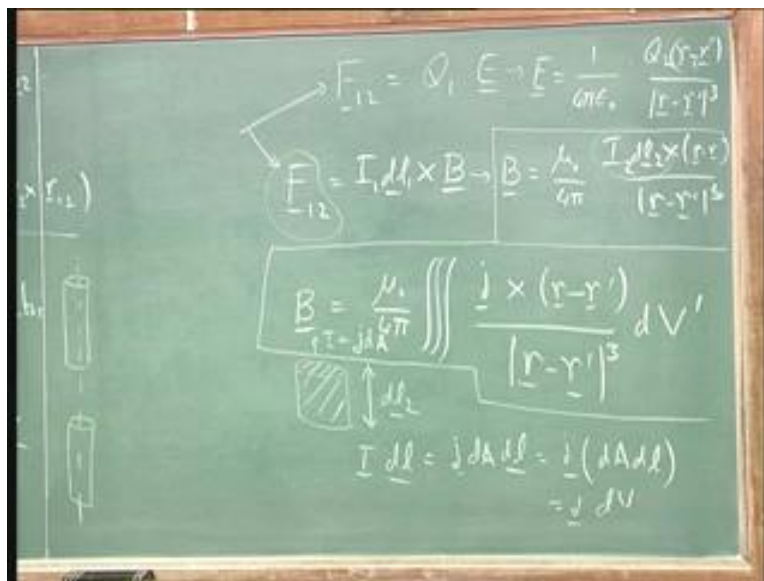
We will look at a wire and we actually measure the amount of force that is exerted when we have current flowing through this wire acting on another wire. And when we do that we find something very interesting, two wires that are pointing along the same way and along the same axis line exert no force on each other. That is, there is no force if both of these are along the same z axes. There is also no force if one wire is vertical and the other wire is horizontal but displaced.

So, it is like I have one wire here and another wire here and I keep the wire is displaced. If they are in 90 degrees with respect to each other, then again there is no force. Now if you look at this other kinds of definitions, they do not satisfy that. But this definition does. And after observation the pin down that, this is infact what the force is doing. As I told you; you can derive this from relativity. But that is like saying you can write much more complicated equation and derive this simple equation.

From the engineering point of view, we treat this as the starting point. We say that this force equation which takes two current elements and the distance between them it is just like coulomb's law except instead of dot products of multiplication sign we have cross products. Now, it is an important to realize this statement about no forces. When wires are in parallel there is lot of force. Wires are perpendicular, there is no force.

This is very important because when you do wiring and you want to relay telephone cables near power cables, for example the rule is always make them go 90 degrees to the other cable, you want to minimize pick up its coming out of the same ideas. That is when magnetic field does not talk, does not cause cross talk when the wires cross at 90 degrees okay. So, we have this force and just as earlier in coulomb's law.

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We said the force F_{12} was equal to Q_1 times E . In other words we define the electric field was equal to $\frac{1}{4\pi\epsilon_0} \frac{Q_2}{r^2} \hat{r}$ divided by r minus r' prime cubed. I would not use, r^2 notation once I am going to a field. Similarly, to this I know have F_{12} from magnetic field is $I_1 dl_1 \times$ some other field and I am going to call it B which gives me a definition that B is equal to $\frac{\mu_0}{4\pi} \frac{I_2 dl_2 \times \hat{r}}{r^2}$.

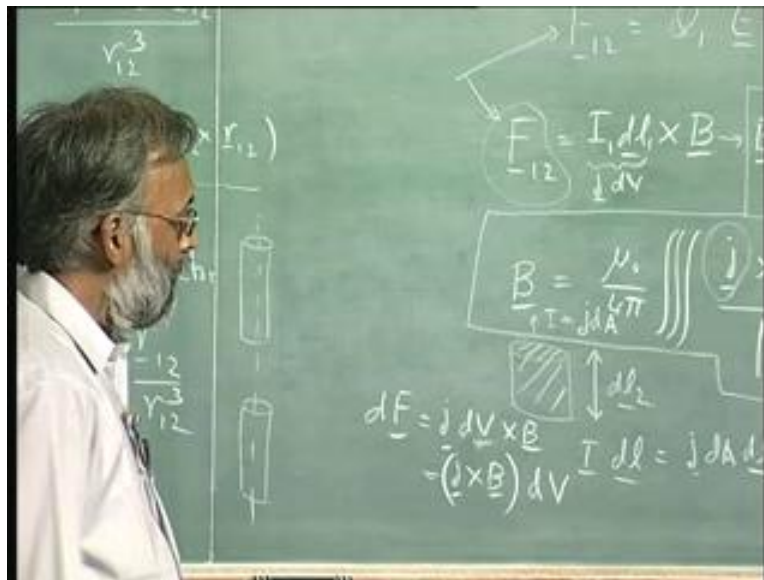
Now, you can see that the bias servo law I mentioned last class is nothing but this. But the bias servo law is just a formula. Ultimate, the only thing that any engineer knows is what we can measure and only thing we can measure this force. Because you have to put a meter in there and measure a deflection, any effect on an instrument has to involved exertion of force. So some kind of force has to be exerted and it is the force that we measure. This is the only real thing the magnetic field is something we have imagined.

We know that the force exists. But to say that the magnetic field is exists in all space that is mathematics. We have mathematics is got so sophisticated that we cannot to believe in electric and magnetic fields and not only that in electrostatic potential as well but the only real measurable thing is force. Still it turns out of very useful to talk about fields and so we do that. This is for one charge current element if I have many current elements, I will write that the magnetic field is equal to $\frac{\mu_0}{4\pi}$ volume integral over all current.

Now how did I go from there to here, well, what I did was I took this piece $I dl$. Now what is if I draw it out $I dl$ is a length dl and the total current I . Now, this total current I is, equal to the current per unit area current per centimeter squared or current per meter squared times the area of this wire. So, this I can be written as $j dA$ alright. The total current is nothing but the current per unit area times the area. So this $I dl$ can be written as j times dA times dl to the vector sign.

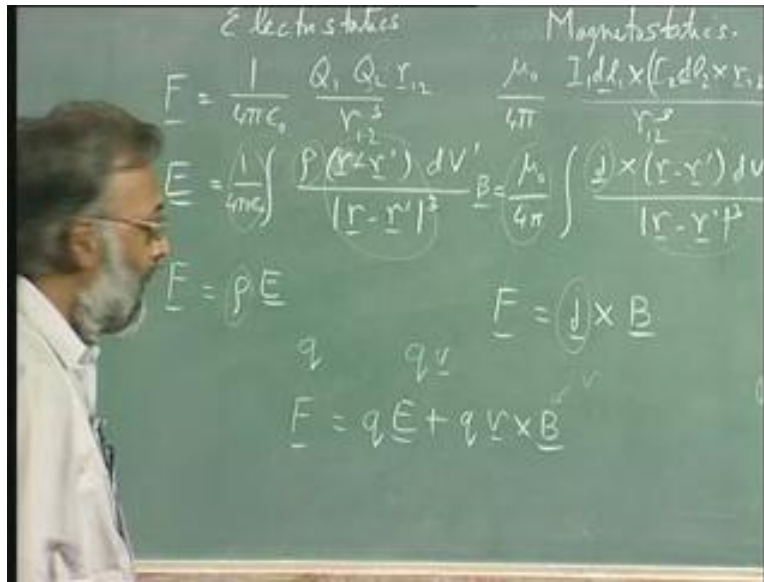
Now I will remove the vector sign from $d\mathbf{l}$ and put it on the \mathbf{j} because the direction I want to associate with current and not with volume. So, I call it \mathbf{j} times $dA dl$. But what is $dA dl$ is nothing but volume of the cylinder which is $\mathbf{j} dV$. So, that is what I have $\mathbf{j} dV$ times cross cross product of \mathbf{r} over r cubed. This formula is a staring formula in your textbook and that is alright in fact the text book talks about \mathbf{H} rather than \mathbf{B} . But that is a cosmetic difference. But it is not very satisfactory from understanding which is why I have come from force and brought you to \mathbf{B} . And therefore it is necessary to understand having this \mathbf{B} what is the use of having it? The use of having it is if I have the current density, I can ask what is the force some the current density.

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The force on current density is again which can be written as $\mathbf{j} dV$. So, the $d\mathbf{F}$ is equal to $\mathbf{j} dV$ cross \mathbf{B} or its equal to \mathbf{j} cross $\mathbf{B} dV$. So \mathbf{j} cross \mathbf{B} is the force per unit volume and if I have tiny amount of \mathbf{j} in tiny volume dV the force on that tiny amount of \mathbf{j} is \mathbf{j} cross $\mathbf{B} dV$. So, I have now let me write down those equations. It is a kind of summary.

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I have 2 equations. I will out electrostatics here and magnetostatics. So, I have force is equal to $\frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2 \underline{r}_{12}}{r_{12}^3}$. Here it is equal to $\frac{\mu_0}{4\pi} \frac{I_1 dl_1 \times I_2 dl_2 \times \underline{r}_{12}}{r_{12}^3}$. The brackets are important because it does the cross product does not really come yet, okay? So that force from force we define electric field which was volume integral $\frac{\rho(\underline{r}-\underline{r}') dV'}{|\underline{r}-\underline{r}'|^3}$ $\frac{1}{4\pi\epsilon_0}$ and here you have $\frac{\mu_0}{4\pi}$ volume integral $\underline{j} \times (\underline{r}-\underline{r}') dV'$ over $|\underline{r}-\underline{r}'|^3$ cubed.

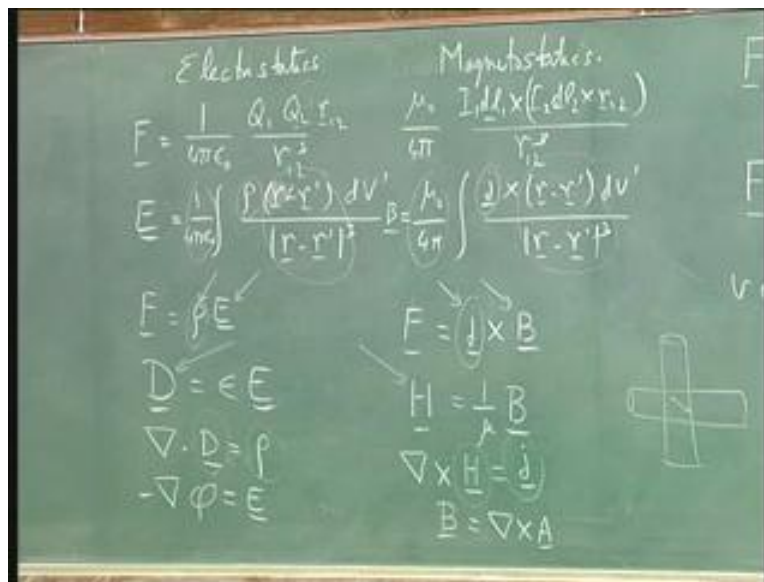
You can see very great symmetric I have a constant which is dependent on my units. I have the source of electric field source of magnetic field and I have the same operator $\frac{1}{r^3}$ in both cases. In one case the electric field is directly in the direction \underline{r} over r^3 . The second case I have two vectors and I have to get a vector out of it. So it is the cross product of those two directions, in terms of \underline{E} and \underline{B} this is \underline{B} equals \underline{F} is equal to $q\underline{v} \times \underline{B}$ that is $d\underline{F}$ is equal to $q\underline{v} \times \underline{B}$.

Whereas, here \underline{F} is equal to $q\underline{v} \times \underline{B}$. Now if I have actually had a single charge and it was moving around then I can combine these two that single charge has a charge it also

has little bit of current. What is the charge? The charge is q . What is the current? The current is nothing but qv . So, the force in general on any charge is qE plus qv cross B . And as I told you last time, these make sense because the magnetic field itself is due to another current.

So, there is another factor of v in there. So, we get a factor of v squared and if you wrote this in cgs unit or other than SI unit you would have actually have one over c 's in all these places so that you could clearly see where v squared over c squared is coming and you will see that this is just nothing but it is the only effect of relativity that you see even in every day life alright.

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So, we have an expression for force you have an expression for magnetic field. Now you can see the force depends on electric field the force depends on magnetic field. But in electrostatics we added one more effect, effect we added was is that there is something called displacement vector and we said the divergence of D is equal to row. Now we need something similar in the magnetic field side and the something similar I am going to tell you before I derive it. T

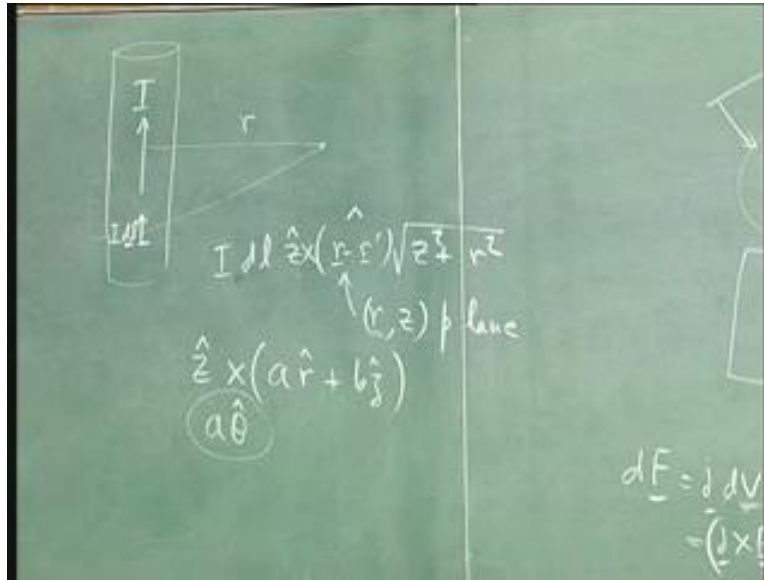
he something similar is called H and you can say that ϵ is in the denominator here μ is in the numerator here not surprisingly you are expect that H will have a one over μ B . It is simply because of the way we defined our terms. So H equals 1 over μ B and you do out here everything we are doing involves gradients and divergence. So that also $\text{grad } \phi$ minus $\text{grad } \phi$ is equal to E . So everything here involves directly the vector divergence gradient.

Everything here involves curl cross product. So it is not surprising that you will find that the appropriate equation to be used is $\text{curl of } H$ is equal to j . What is similar about these two equations is that the displacement vector depends only on charge. The magnetic field intensity defines only on current. It does not depend on a medium. Just as displacement current does not depend on medium and anticipating there is something called a potential that potential can be given us B is equal to $\text{curl of } A$.

Now let me just show you point out again the symmetries. The fundamental quantity is always force and if you look at what force involves force involves charges and currents and it involves electric and magnetic fields. So, E and B are the fundamental quantity they are the quantity that should be understood thoroughly, from this quantity we have derived or we will derive two more quantities. One is the displacement vector, the other is H . These two vectors satisfied two more equations and those two equations are divergence D equals row $\text{curl } H$ equals j .

And finally these two fundamental vectors vector fields E and B can be derived from potentials. One potential is the electrostatic potential which says E equals minus $\text{grad } \phi$ other is what is called a vector potential which is B equals $\text{curl of } A$. So, this is where we are trying to go and this is where I want to reach. So let us see how far we can get in this particular class. Now, the first thing I actually want to establish is this equation, however getting here certain amount of mathematics, so what I am going to do is I am going to put down some equations and get down to working out of few examples. Then we will come back and derived all this relations.

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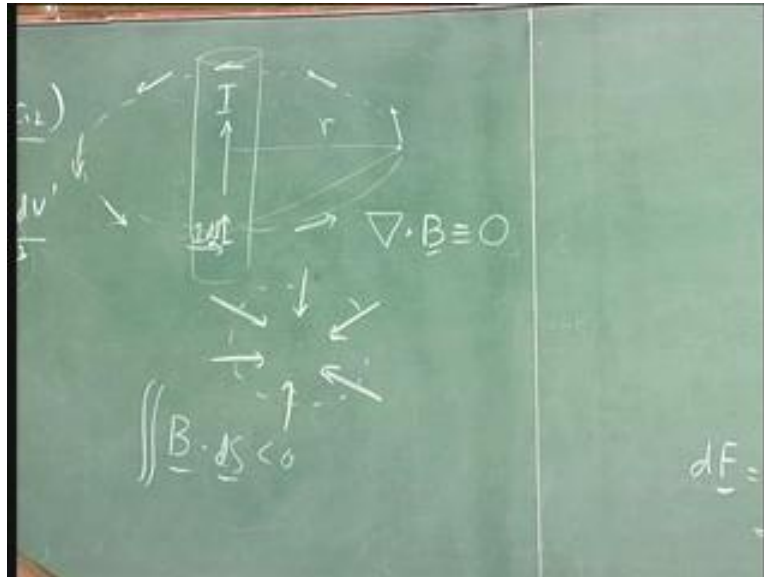
So first of all we have that if you have a conductor long conductor carrying a current I . We already look at that in the seen that when you go at any distance r from this conductor there is no magnetic field pointing up. There is no magnetic field in the r direction, there is no magnetic fields all in the theta direction. The reason is we take any piece $I dl$ and you connect that by a line to this point. And you know there is $I dl$ cross r I is along z so it is $I dl z$ hat this line is r minus r prime hat times the length.

The length is nothing but z squared plus r squared and we want a cross product. So, we will look at this this vector is in the $r z$ plane. You can see that it is because there is a z component and on r component to that line, since this is in the $r z$ plane and this is in z plane if take the cross product you will get z hat cross lets say $a r$ hat plus $b z$ hat some general form, z hat cross r hat is θ hat. So, $a \theta$ hat z hat cross z hat is 0 . So, you will always have the magnetic field in the theta direction.

And other way of seeing this is it must be perpendicular to z must be perpendicular to r minus r prime. So, which perpendicular to r minus r prime it must be in this plane. It is perpendicular to z it must be in this plane. So, which got to be perpendicular to both then

it must be the intersection of those two circles and one obvious direction is the direction out of the page that is the theta direction.

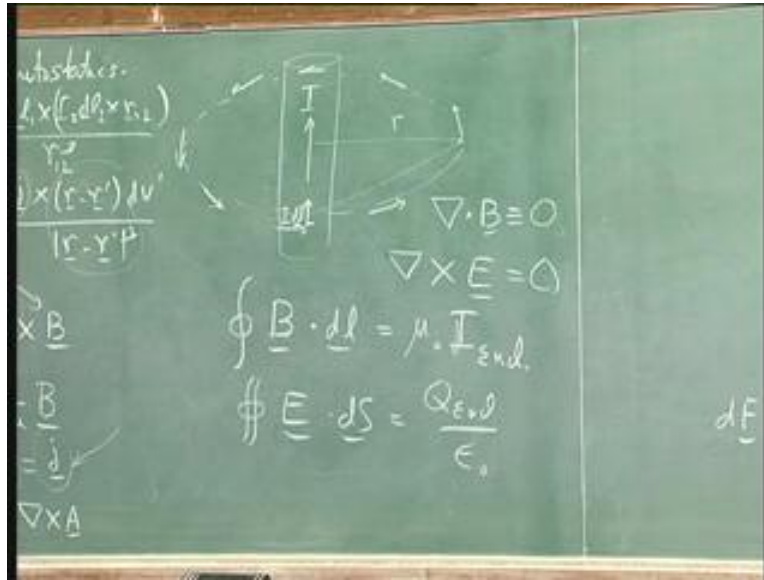
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So, the magnetic field is along theta and we know that the magnetic field is basically independent of theta. So it is a same magnetic field here, here, here, here, here, so it is a uniform magnetic field. Let us going around in a circle, it is a general characteristic of magnetic fields that they do not do things like they do not converge to a point. This convergence to a point or lack of convergence to a point is exactly what gauss's law checks out.

Because, if magnetic field converge to a point then if integrate over a surface surrounding that point you will find the magnetic fields entering everywhere. So, you can do a, surface integral, $B \cdot dS$ and you find $B \cdot dS$ would be less than 0. However you can show it and we will show it later. That magnetic field does not have such properties. So, in fact you can show the divergence of B is identically 0. It is never different from 0.

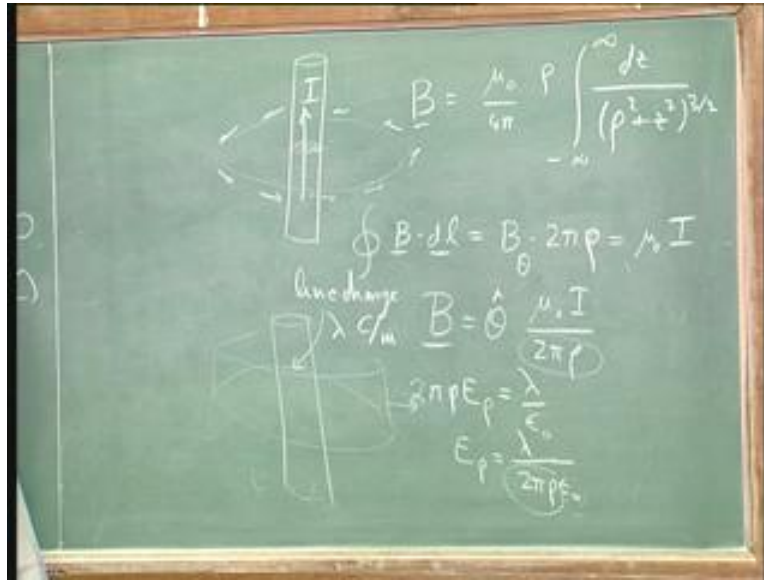
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Now the formula you should compare these two is a formula. We have already derived which is curl of E is also equal to 0. You remember that we did it that if you take any close loop that the integral E dot dl is equal to 0 and the relationship between E dot dl and curl of E I will define later when I introduce stokes theorem. But this is saying the same thing. Now the statement of this equivalent of the divergence theorem for electrostatics that magnetic fields are making it is the following to be derived later. It says, if I take let me not use H I will use B B dot dl on any loop I choose then it is equal to mew not times I enclosed.

This is exactly to be compare to surface integral well not D but E dot dS is equal to charge enclosed divided by epsilon naught. This is a magnetics, this is an electrostatics. It is the same idea if you got a field if you want to look at how much field is leaving it is that is the electrostatics it is telling how much charge there is for magnetic field everything is that 90 degrees. So, you cannot talk about field leaving you talk about field going round and round and if you talk about field going round and round the tendency of field to go round and round is connected to the enclosed current. Mathematically we will derive these things but I want to use them directly.

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Let us see where it takes us. Last time I had stop half way after deriving what the magnetic field should be if I have a current in a wire. If I remember right I got that B was equal to μ_0 not over 4π rho divided integral minus infinity to infinity dz divided by rho squared plus z squared plus power of three halves. There may be some factor is missing but something like that. However the reason why I did not finish doing this derivation was because there was simpler way of doing.

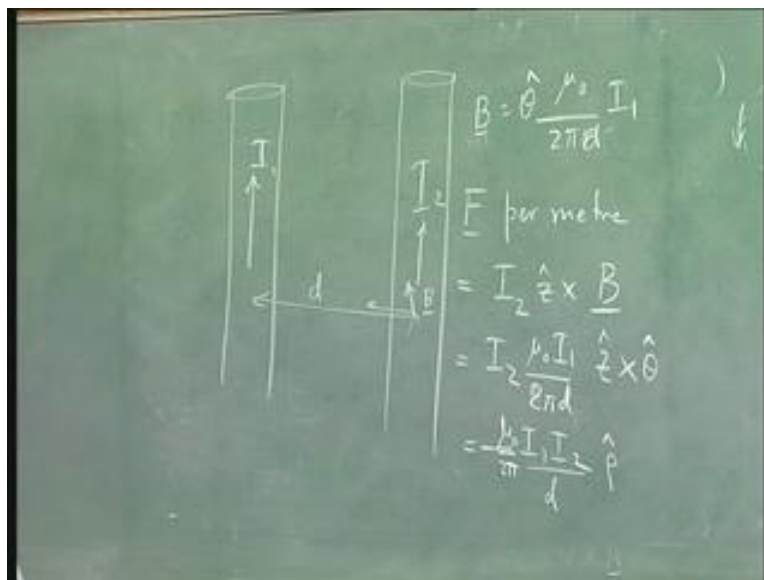
What I am going to do is I am going to apply the same formula on a circle centered on this wire. You already know that the magnetic field is uniform on this circle. Since the magnetic field is uniform and circle if I do a loop B dot dl B is constant. So, I just call it B theta integral dl is nothing but the circumference of the circle. So it is two pi r or two pi rho and its equal to μ_0 not I, the enclosed current is whatever current there is in this wire that immediately gives me an answer which is magnetic field is in the theta direction times μ_0 not I over 2π rho it is precisely because it is so easy to derive it this way that I did not continue there.

But you can also derive it from the integral. Now what as this saying and how can we understand. Let look at the electrostatics equivalent supposing I have charge a line charge

with the charge which is lambda coulombs per meter alright. Then, we used Gauss's law and we obtain that $2\pi r \epsilon_0 E$ was equal to total charge enclosed that is lambda divided by epsilon naught or E was equal to lambda divided by $2\pi r \epsilon_0$. Epsilon naught is now replaced to the mu not in the numerator.

The lambda coulombs per meter is replaced by I amps and the $2\pi r$ is unchanged that difference is the electric field is in the radial direction. Magnetic field is in the theta direction that is expected because the magnetic field is always at ninety degrees to where you expect fields to point. So, this is having the exactly similar dependents the electrostatics has electrostatics lines create 1 over $2\pi r$ magneto statics currents also create 1 over $2\pi r$ fields.

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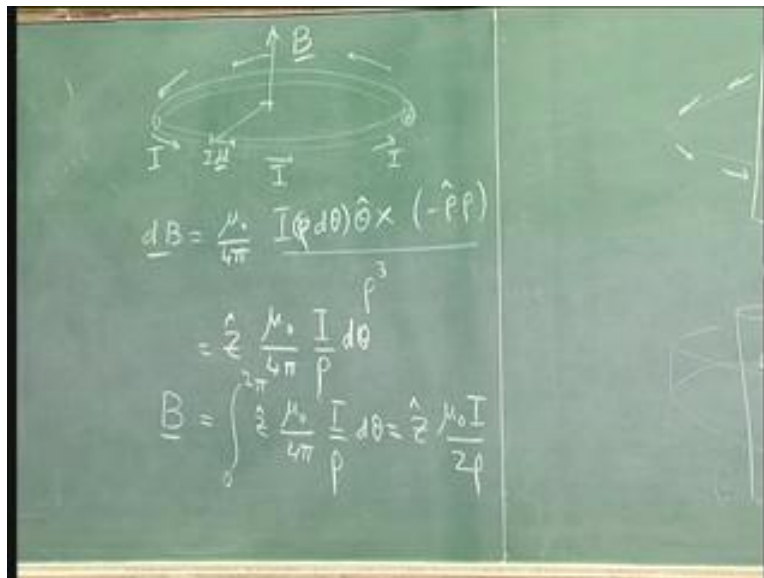
If I now have two wires current I_1 , current I_2 , that the distance d and they are parallel that due to this current there is a magnetic field at this wire. Magnetic field along the theta direction is equal to theta hat times μ_0 over $2\pi r$ or $2\pi d$ times I_1 this is the magnetic field at the wire at the second wire. The force per unit length on the second wire is going to be equal to I_2 times dl_2 which is one meter cross B . So this current dl cross

B is the force, but B itself is given here. So it is equal to I_2 times μ_0 not I_1 over $2\pi d$ times \hat{z} cross $\hat{\theta}$.

Now if you look at \hat{z} cross $\hat{\theta}$ \hat{z} is in the upward direction $\hat{\theta}$ is in to the board. So if you use a right hand rule \hat{z} cross $\hat{\theta}$ \hat{z} is minus radial direction. So it is equal to minus \hat{r} and let me put the μ_0 not over 2π out scale. So it is a negative μ_0 not over 2π $I_1 I_2$ it is one over d and it is in the radial \hat{r} . So this is a force the first wire creates the magnetic field that is going round it this magnetic field at it is in the second wire and causes the force to pull it towards the first wire.

Now, if you remember last, this is the exactly the starting point of my derivation I said there is a current electrons are moving atoms are stationary. The moving electrons get slightly squeezed due to the relativity. So, there is more negative charge less positive charge and that causes the force of attractions between these two wires. That is the force of attraction that that we see here. It is propositional the velocity of electrons in the first wire times the velocity of electrons in the second wire.

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We can use the Biot-Savart law itself to also calculate certain magnetic fields. Supposing I take a loop; loop like this drawing the cross section here and I want to know what the field is at the center of the loop I have a current going through this loop. Now, because of Kirchhoff's current law, I know that current through the wire must be the same at all point in that wire. So there is a circulating current in this loop it cannot be I here and two I here and minus I here current must be uniform.

Now, if I look here and take any portion, there is an, $I dl$ and there is a radial direction and now I want to know the magnetic field there. So the magnetic field dB is equal to $\frac{\mu_0}{4\pi} \frac{I dl \times \hat{r}}{r^2}$. What is dl ? This is a radial distance r , so it is $r d\theta$ in the θ direction cross the distance. The distance vector between the current on that point, so that vector so it is equal to minus \hat{r} times r lets keep call this r divided by r^3 .

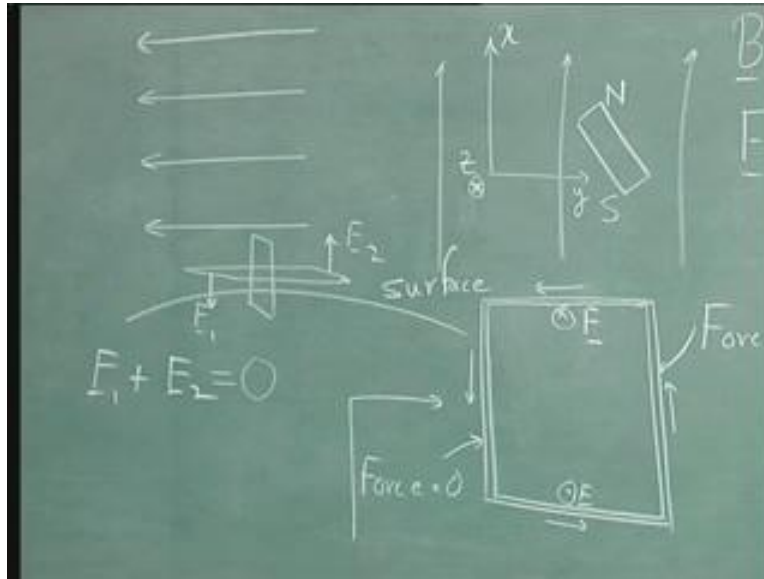
So it is $I dl \times \hat{r} / r^2$ divided by r^3 $\hat{\theta} \times \hat{r}$ is minus \hat{z} the minus sign becomes \hat{z} . So, this is magnetic field is in the \hat{z} direction that is to say this current cross this distance, I use the right hand rule field must be up times $\frac{\mu_0}{4\pi}$ there are 2 r 's and one r^3 here so one over r and I have $I d\theta$ all the other terms are taken care of $\frac{\mu_0}{4\pi}$ has come here I has come here $r d\theta$ is come here $\hat{\theta} \times \hat{r}$ that is minus sign.

Come here r^2 over r^3 is one over r . So what is the magnetic field? Magnetic field is equal to the integral over θ 0 to 2π of $\hat{z} \frac{\mu_0}{4\pi} \frac{I}{r} d\theta$. But, integral 0 to 2π $d\theta$ is 2π . So, it is equal to $\hat{z} \frac{\mu_0}{4\pi} \frac{I}{r}$ over 2π . You can always check out the errors by comparing with the text book which has derived all this things. With a physical sense in which this is correct is that take your right hand and try to twist a screw driver in the direction of the current.

So, if you try to twist a screw driver this way, it is opposite to the direction of the current try to twist a screw driver this way. It is in the sense of the current. That means the direction in which this screw will go in this direction is the direction in which the

magnetic field will be current. Now, as I said there is a mathematical preliminary is that you have to get through to handle curls and loop integrals. But I am going to skip those for the moment. What I want to go back to is to look at why we define this magnetic field on the first fields.

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If you look at the earth the earth magnetic field is so big that where we are it looks basically uniform. So, we can say that at the surface of the earth over distances may be mile or so the magnetic field is the uniform magnetic field it is not changing its pointing in some direction and luckily when you if you are in India its pointing roughly horizontal, if you are in Greenland it will be pointing straight down in to the ground. But, in the tropics this magnetic field is basically horizontal.

Now you take this magnetic field and you put a magnet in. Magnet has north-pole and it has a south-pole and we know because all of you have done this experiment that if you put a magnet like this what it will do is, it will spin back and forth back and forth slowing down until finally it will point along the magnetic field. The question is why does it do that? Now, answer why does that we need to go back a step and ask what happens if I took this current loop and put in a magnetic fields okay.

Now I am not going to use this circular loop even though I have done the calculation I am going to use a square loop. First thing I am going to do is I am going to say supposing I have a square loop carrying a current I and I put it parallel to the magnetic field. That is the magnetic field is pointing like this side ways and I am putting the loop also side-ways. So, what does it mean? It means that the magnetic field is along the wire in these two lines sections and it is going right through the wire in these two sections.

The force is equal to $I \, dl \times B$ that came from the original definition force is $I \int dl \times B$ cross $\frac{1}{r^2} dl \times r$ over r^3 . So force is $I \, dl \times B$. Now $I \, dl$ is in this direction and so is B . So, force equals 0. Similarly, here $I \, dl$ is in this direction B is in this direction both are parallel actually anti parallel. So, force is 0. In this case I have that the current let me give coordinates lets say this is x and this is y .

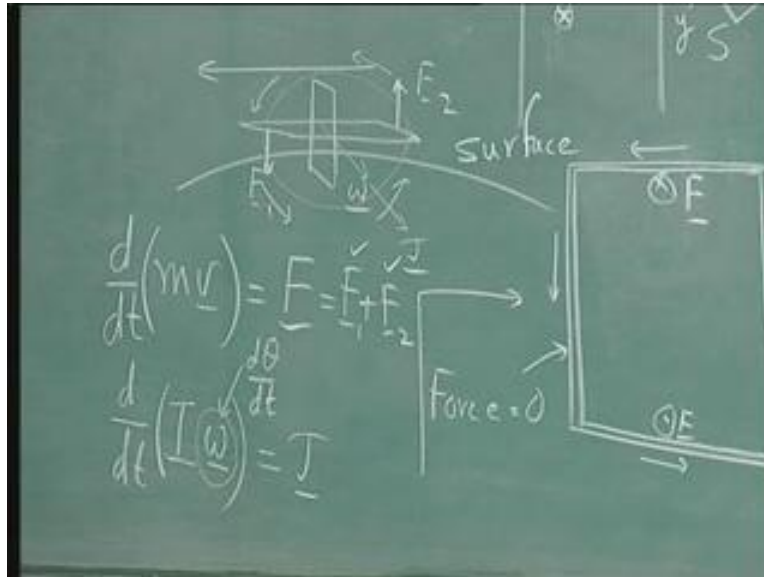
So the current is along minus y and the magnetic field is along x so its $y \hat{y} \times x \hat{x}$ which means if you do your right hand rule z is into the board, $y \hat{y} \times z \hat{z} \times x \hat{x}$ hat sorry is minus $z \hat{z}$. So, the force is out of the board. You can work that out again from just looking at $I \, dl \times B$ and fairly show you have looked at somewhere you can check up the force should be in to the board its minus $y \hat{y} \times x \hat{x}$. So, it is in to the board $z \hat{z}$ hat here the current is along y magnetic field is along x so the force is out of the board.

So, what is happening? I have my loop the bottom I am having a push there is pushing it up the top I am having a force at pushing it down. So, this force is pushing it down. This force is pushing it up. So the loop is going to turn. What will happen to this loop? If I look at its side ways is I have a magnetic field that uniform. This is the earth surface and I have this loop and this loop will start turning and from this position it will go to this position, because there is a force upwards there and a force downwards there.

When forces are present on an object I can do two things I can push that object or I can make it rotate. Now, the total amount of force acting on the object that is this force and this force if you add it up this net force is what makes this coil move basically from one

place to another. But we have an equivalent and opposite forces. So, F_1 plus F_2 is 0. So, this is coil does not actually move anywhere it stays where it is.

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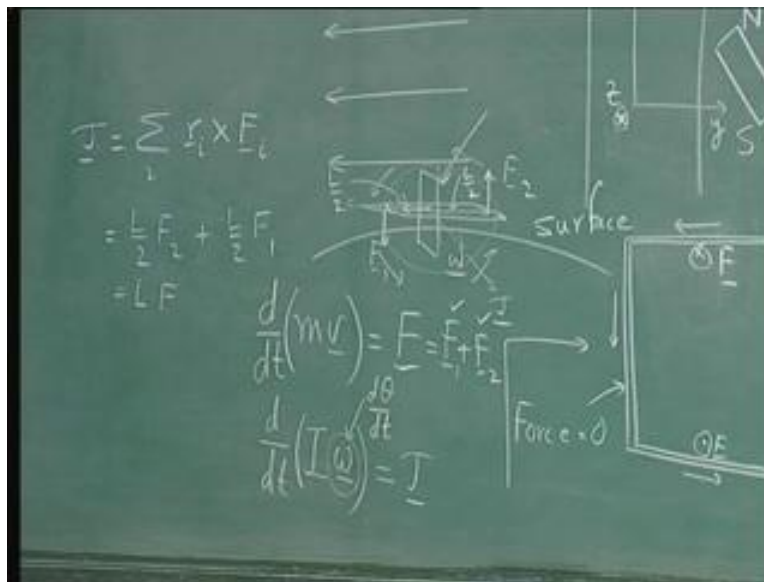


But, as I just said when I have this coil and I am pushing down on one side pushing up on other side this coil is going to rotate. Now, how does it rotate? What is the equation for it? Now let me remind you from your mechanics you know that for linear motion rate of change momentum ddt of mv is equal to applied force this is F_1 plus F_2 . But for rotation we have different equation and usually we do not remember it is so well it is ddt of moment of inertia times angular velocity is equal to applied torque.

So, this moment of inertia this is angular velocity which means $d\theta/dt$ and this is the applied torque and this is all well known to you because you have done mechanics. But when I did mechanics I never understood this part. So, I think it is a worth explaining it in terms of this picture. This coil is going to rotate around this point. So, it is going to around so the question is what is the direction of this rotation? So, the coil rotates in the plane of the board it goes from this position to this position.

But when it rotates what we called omega points out of the board that is omega is rotation in a circle and the convention is because there is no power direction in the board that you can point to omega. Omega actually points out of the board. This is the direction of omega. So, if the coil is stationary omega is 0 as a coil starts rotating omega is not zero. This applied torque is trying to increase omega. So, your applied torque also has to be this same direction because rate of change of omega is related to torque. So, what is this torque? It has to do with F 1 and F 2, but it is not F 1 plus F 2 because of its 0.

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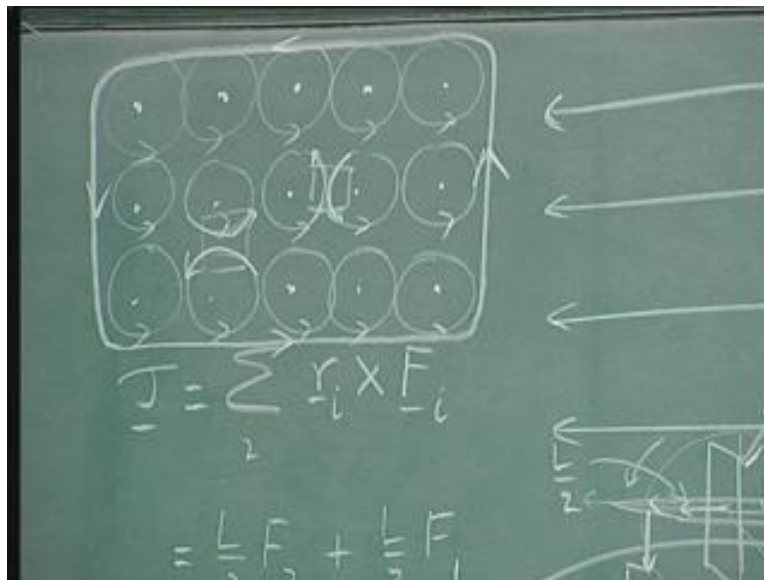
Well, you know the answer to that. The answer is that torque is equal to the summation over all the forces r_i cross F_i . So what do I mean by that? Let us say that my coordinate's assistant's origin is here. So I have one distance this way and another distance this way. Let us say that my length of my coil is L . So, this distance is L by 2 and this distance is also L by 2. So, this torque is going to be L by 2 along this direction times F_2 times the angle between them which is 90 degrees sin of that angle is 1. And it is going to be in direction that going to be r cross F .

So, it comes out of 6, the board now this one is going to be plus L by 2 times F_1 then again when you work out r cross F is r cross F is again going out of the board. Because, if

you have to use a right hand rule and make r become F has to come out of the board. So it is going to be from L times F . What I mean by F ? It is the magnitude of F_1 and F_2 so that torque is not 0, net force is 0, but torque is not 0. And this torque it tends to increase ω which means this coil starts rotating. Once a coil rotates beyond vertical, you can work it out the torque becomes negative. So, it tries to stop this coil. And what the coil will do is, it will start horizontally starts feeding up reach maximum speed keep going beyond come to come to rest and it is horizontal again.

Then, it will turn around and go back and it is going to a pendulum like oscillation. But, there is friction there is air present. So what happens is that this oscillates slower and slower and slower and it finally comes to rest in this vertical position and in this vertical position the coil current as a normal that normal is aligned to the magnetic field. So, any coil with carrying current will oscillate until it reaches the rest position where it is direction is along the magnetic field. Now, in school we have done the same problem with the magnet and we know exactly the same thing happens. The magnet oscillates until the north-pole points along the magnetic field.

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But it is the same thing really because if you look inside on magnet, what a magnet consist of the whole level of atoms in a lattice. Now in a magnet these atoms are special they have electrons which have a preferential spin that is the electrons prefer to go round and round in a particular way. And we are lined up all the electrons are going the same way in a state not only these electrons but also the electrons below and the electrons below them. Now, if you look anywhere inside say here, you have an electron going this way. But you have an electron coming back.

Which means, there is no net current, there is as much current going that way and that much currents going this way. So, if you add up the current it is a 0. If you look here well there is current going this way and there is current going this way. So, anywhere inside there is no net current. But if you look at the surface there is net current infact that the huge current that is going right round, all this atoms. So, this magnet if you look at it this way will seem like a huge current loop which is circling the head of the magnet.

And what does the current loop want to do? It wants to line up with the magnetic field which is what the permanent magnet does. This is a very interesting and very useful. What it means is that, by studying the current loop we are also studying the permanent magnet. So, there is no need to have two theories of magnetism. A theory of magnetism for magnets in the theory of magnetism for electrical currents, it is the same magnetism. It is just that this magnetism is by our electrical circuits. This magnetism is due to electrons going round an atom, but it is the same electrical current.