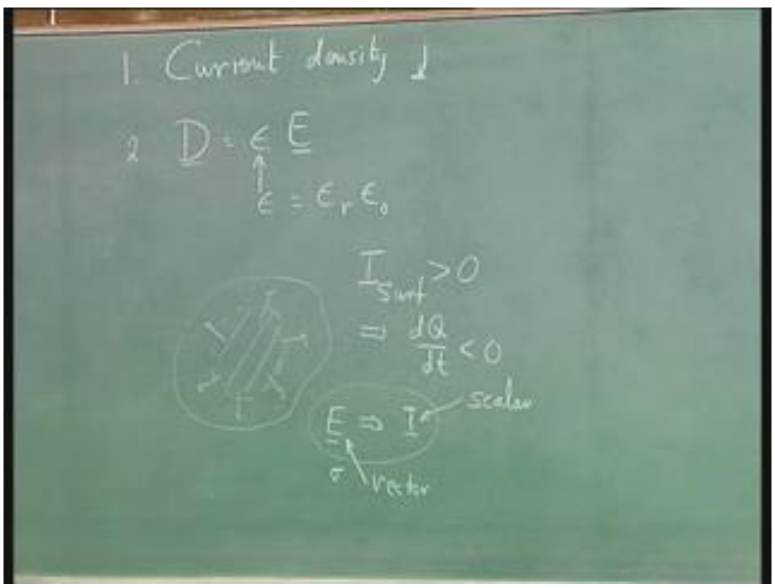
Electromagnetic Fields
Prof. Dr. Harishankar Ramachandran
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 12
Fields in Material Bodies

Good morning. Last time I had started talking about what fields do when they enter materials? Today I am going to continue that topic. The first part of the lecture I am going to talk more about currents and current density. And in the second part I am going to come back to what I called the dielectric constitute.

(Refer Slide Time: 01:30)

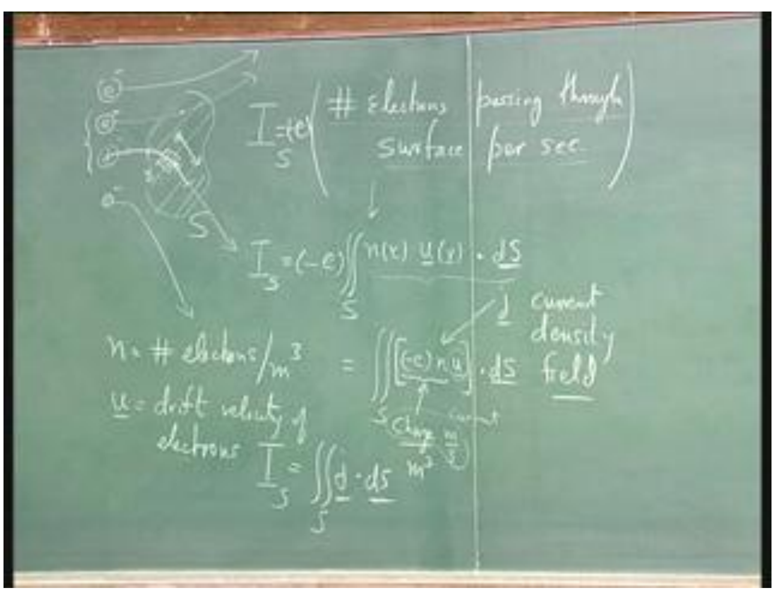


So two topics, one is current density j. It is very important topic. It will become form one important once we talked about magnetic fields. Second topic will have to do with D equals, epsilon E. So the dielectric constant epsilon; let us go back. The first topic some time, I go ahead talked about why it is that inside a conductor you can not have radially diverging currents in steady state. I said that if you make some imaginary surface like a Gaussian surface current is going out everywhere. Which means, charge is leaking through the surface at every point.

At no point, is charge entering and if charge is not entering. But is continuously leaving it must mean that the charge content inside this contain by the surface must be continuously reducing. So I was saying that I surface greater than 0 implies let dQ dt is less than 0 and this is correct. Now what I want to do is to make this idea more mathematical, more useful. This is correct as a qualitative statement. But it is not very useful. You cannot put in to an equation and calculate anything. So let us see what we can do with this idea and make it go further.

And other thing that I did was, I connected up the presence of electric field and current. And I say that if you have an electric field and you have a conductivity sigma the electric field implies current. That is one problem about all these statements. Specifically this one out here electric field is a vector. But current is a scalar you can see that from this relationship. The current is a related to dQ dt. But form both of these by this drawing arrow is representing current and by this relationship. You can see that you really want a vector quantity hole current. You know want a scalar quantity hole, current the current that comes into ohms law is a scalar by the current we want to put into this things is a vector. So we need to be more precise what we meant by current. So let us look at that problem.

(Refer Slide Time: 05:12)



Supposing I have some surface, I am going to call it S and supposing I have electrons, these are all minus charge and these electrons are flowing. As these electrons flow some of them pass through the surface and others missing. So for example these two pass through the surface cutting through it a two points. Whereas these two just go away from the surface. So if I want to know how much current pass through this surface. What I need to do? I need to say that I sub S the current through the S is equal to number of electrons passing through surface per second.

That is why it is not a vector current is a scalar, it does not know direction. But how do I calculate this quantity? I have to count up. Electrons will come with the number density. N is the number density of electrons number of electrons per meter cubed. And at every point I will have a velocity vector u which is drift velocity of electrons. See if I want to know this is actually after multiply this by the charge of the electron of course minus e. So if I want to know this bracket, I would like to say this bracket is equal to a surface integral over this surface.

The number density of electrons at some point on the surface, the flow velocity of electrons at that point, so if I take some region here whether the number density and there may be a flow velocity. But this flow velocity would not be in the direction normal to the surface. So just as in Gauss's law we must take a dot product, dot dS. And if I integrate this quantity through this surface then I would get the number of electrons that pass through this surface per second. Now is that obvious. Now let us look that how I got that statement. Let us say an electron drifts this for in one second. In next second it is drifted this part third, second, it drifted this part. Fourth, second it is drifted this part. Fifth, second and so on and so forth.

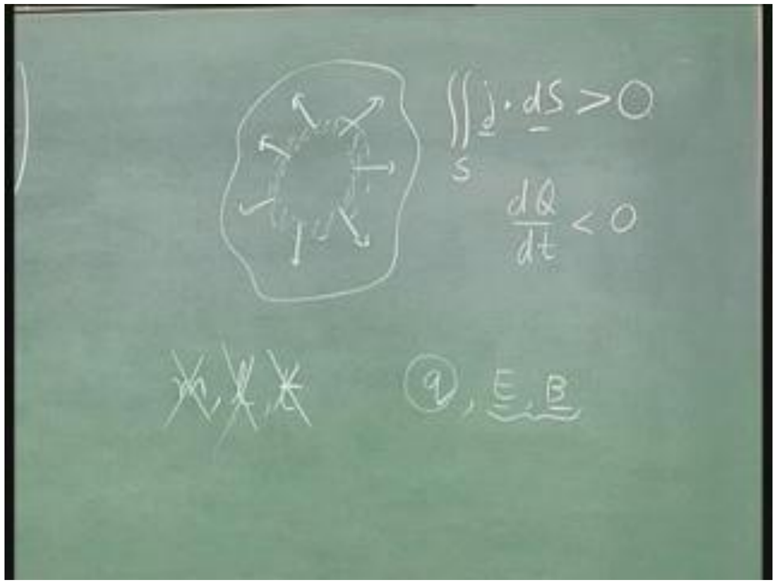
Now it penetrates this surface at sometime t. Now I want to know between t and t plus one second how many electrons pass through. Well what all do as a construct a small spear small cylinder and make that cylinder one second long. What would I mean one second long? As I will make it actually at distance this length equal to the flow velocity of the electrons because, in one second electrons move u meters u is the velocity which is meters per second. So in one second you move u meters, so I have take a cylinder which is u meters long and count up all the electrons in it that corresponds to the number of electrons at will pass through this region.

So this quantity represents number of electrons passing through the small piece dS per second and then I integrated. So is the surface current is minus e times this quantity. Let me put it as a proper integral surface integral S of a quantity which is minus e times n u dot dS. This is true for any surface. I choose the current through that surface is going to be these square bracket quantity dot dS. Now if you look at this square bracket e times n is a charge per meter cubed, u is a velocity.

So if you take charge times velocity, this is current this is a current density. It does the mentioned of the current density and so we infact give it a name. We call this j which is the current density field. Now what are the properties of this quantity? First of all if you integrate j over any surface, you get the current through that surface. That is, I through a surface is equal to surface integral j dot dS. So very useful result which a way of, building up currents for arbitrary shape surfaces, the second thing that you can see is j is a vector because u is a vector.

Now as you will recall when you discussed earlier the problem is coming up against towards I was using vector concepts. But using a scalar to define those vector things like I said a vector quantity electric field was inducing as scalar quantity current and I was drawing to represent those currents. If I draw arrows, I should draw a vector if I mean a scalar, I shown draw the error. So that arrows that I have drawn here are really representing current density. Now, if I want to use current density and I want to redefine my earlier statement. Here is what it is.

(Refer Slide Time: 13:14)



I have a conductor and I draw some imaginary surface closed surface 3D closed surface and let us say that I have current flowing out. So what do I mean? Now I mean surface integral j dot dS is greater than 0 because current flowing out is current through the surface in current through the surface is nothing but integral over that surface j dot dS. Now current is moving charge and this one of those very important conserved quantities charge. When Einstein started thinking over that relativity in the beginning of the 20 of century he had a choice. He could throw out Newton's laws or he could throw out Maxwell's laws.

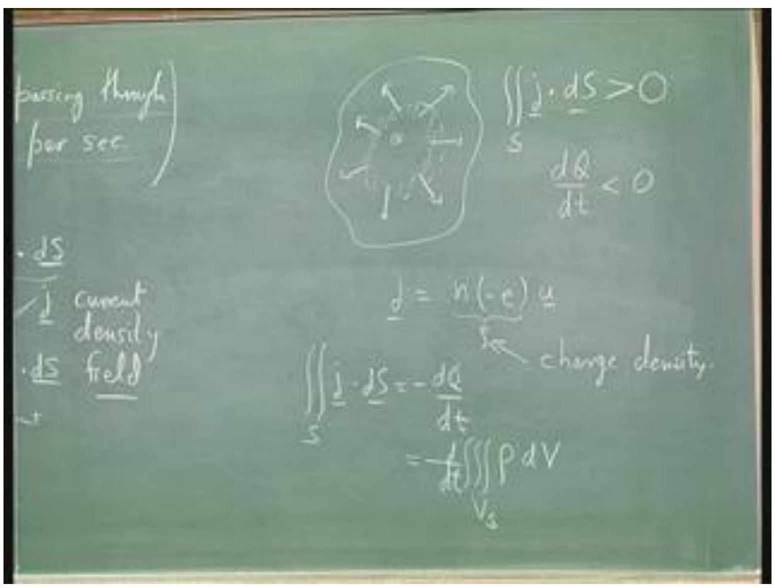
And what he did, if you look at it is the following:
He had a choice he could say,

- He could talk about mass.
- He could talk about charge.
- He could talk about length.
- He could talk about electric, magnetic fields.
- He could talk about time.

And he talked about it and he said Maxwell's equations are perfectly correct. So he kept e and b in all the equation e and b correct. He said charge is perfectly conserved. Even if you go in to science fiction novel and start moving a speed of nearly light, q will be still q. But mass is not constant, length is not constant, time is not constant. So the conservation of charges are very fundamental thing and in modern physics which one of the very important conserved quantities. Now what is that mean it means?

If they had this conserved quantity so much of it, say 10 coulombs of it inside this surface. Now because of this current, I can imagine drawing another surface that represents how much of the charge move out in one second. So, all this charge was lost after one second. So clearly what is left is less. Charge is conserved I have got reader some charge therefore I have less charge left. So if j dot dS is greater than 0 it must imply dQ dt. Can I make that more precise? Well, yes I can.

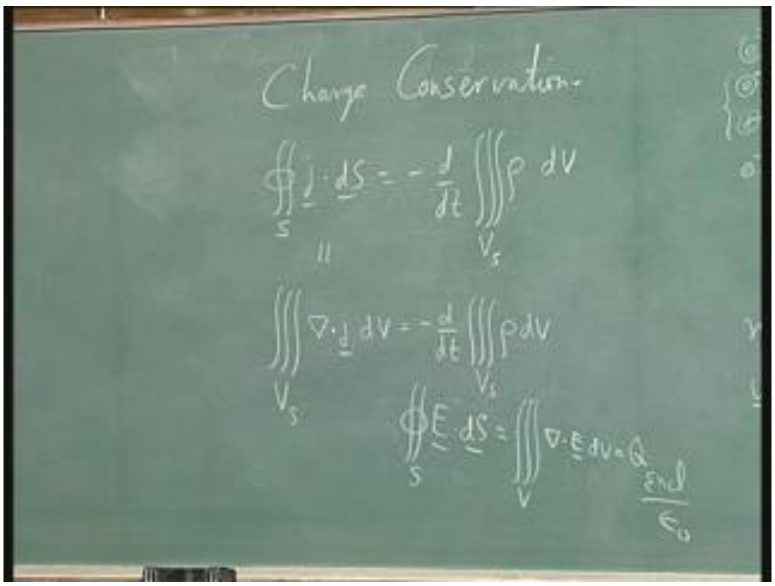
(Refer Slide Time: 18:40)



This j I have already said is n times minus e the charge times u. This is the rate at which charge is going out. This j times in area is infact what we are talking about here. Now this e times, n is nothing but charge density. I can rewrite this equation as see surface integral j dot dS which is the rate at charges. Leaking out is equivalent to minus dQ dt the rate at which charge is reducing inside. But what is dQ dt is minus integral over the volume correspond surrounded by S of row dV.

Let me interpreted again j dot dS is nothing but the charge that is leaking out row dV is the charge occupied by any small volume inside the ddt inside any small volume or this surrounded volume. So if I integrate over all the volume row dV have counted up all the charge inside the surface. So charge linking on the surface must be equal to the amount by which the charge contained is reducing in time. This is called charge conservation.

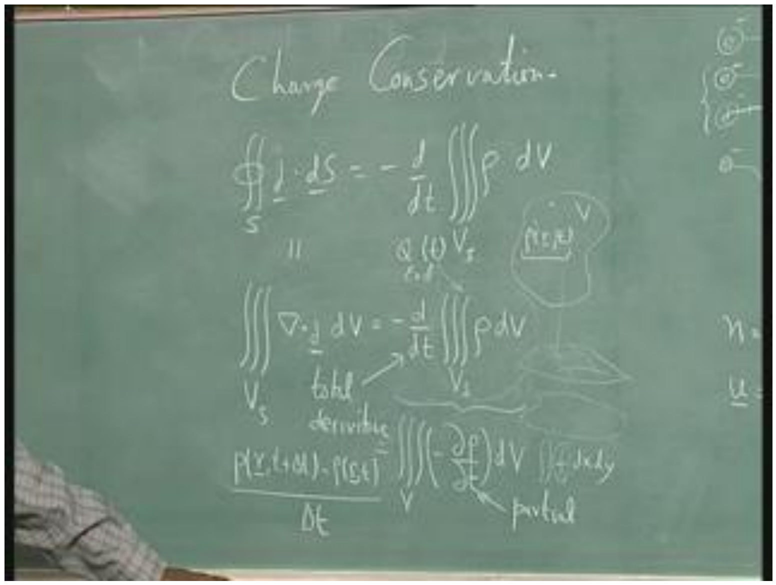
(Refer Slide Time: 18:57)



Let me rewrite the equation surface integral over closed surface j dot dS is equal to minus ddt volume integral volume enclosed by that surface row dV. Now I know the theorem that converts a surface integral to a volume integral, so that divergence theorem. So I can write let this surface integral is also volume integral over the enclosed volume divergence of j dV this is Gauss's law. If seen it before when we derived it for electric field and there we had surface integral over a surface E dot dS is equal to volume integral divergence of E dV which was equal to charge enclosed divided by epsilon not.

It is a same thing j dot dS instead of E dot dS is equal to divergence j dV is divergence E dV. So divergence j dV over the volume is equal to minus ddt over the same volume row dV. As you know integral equations are very nice, but we like to make them into differential equation. So I would like to convert this equation into a differential equation of the type that you already been using.

(Refer Slide Time: 21:01)



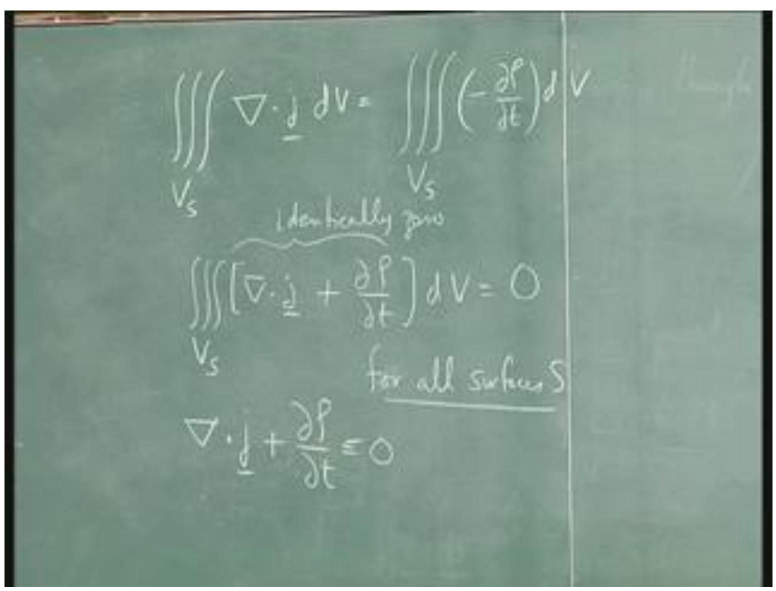
Let us look at this term it says rate of change negative the rate of change of charge enclosed. If I have a volume V and I have charge everywhere. So row of r row is also function of t when I integrate this row over V and I asked what is the change of this integral with time. The weighted thing of it is, let us imagine I have some surface on that surface. I defined a function and I want to know how the area of this integral of this function over this area changes in time.

Either the area changes in time how the height of the function changes. So if I do the double integral here f dx dy either the limit change or f changes in time. Now we are considering a stationary volume because it is an imaginary volume. In any way so we are not moving that volume. So the only way that this integral you can change is a row depends on t. So really what I mean is integral over the volume of minus time derivative of row dV. Now how do I write that time derivative? You see I want to know how row of r t changes in time.

So what do I want I want row of r t plus delta t minus row of r t over delta t. So that look at this you can see I am keeping r constant I am not varying r I am only varying t. So what goes in here is what we called a partial derivative. So we went from a total derivative to a partial derivative as a very important thing to realize outside the integral this quantity is Q enclosed and Q enclosed is the function only of time it does not have any dependence on r. All the dependence r was integrated over. So you will get four coulombs "4.1" coulombs.

You would not going a get "4.1" coulombs at such and such a place because all are space information as got integrated a way. But once you are working inside this is charge density I am saying four coulombs per meter cubed I do I have to say where as well as when. So row is a function of r and t bit only the t derivative is taken. So this becomes a partial derivative. This is called a total derivative and this is called a partial derivative which you have been using.

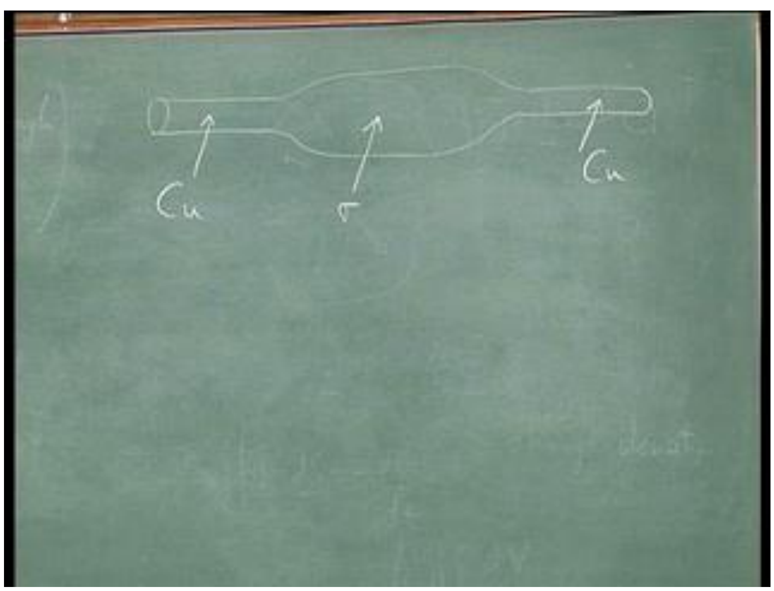
(Refer Slide Time: 24:30)



So to write it all down here is what we have. We have volume integral volume enclosed by surface S divergence of j dV is equal to volume integral volume enclosed by surface S minus del row del t dV. I can combine this, two integrals. So I will just make it one integral I will say integral over the volume Vs of divergence j plus del row del t dV equals 0. Now this s there I started with is completely arbitrary it is an imaginary surface. So I can choose any surface I like and this is true for all surfaces. When something integrated over an arbitrary surface gives me 0 the answer must be that this is identically 0. Why?

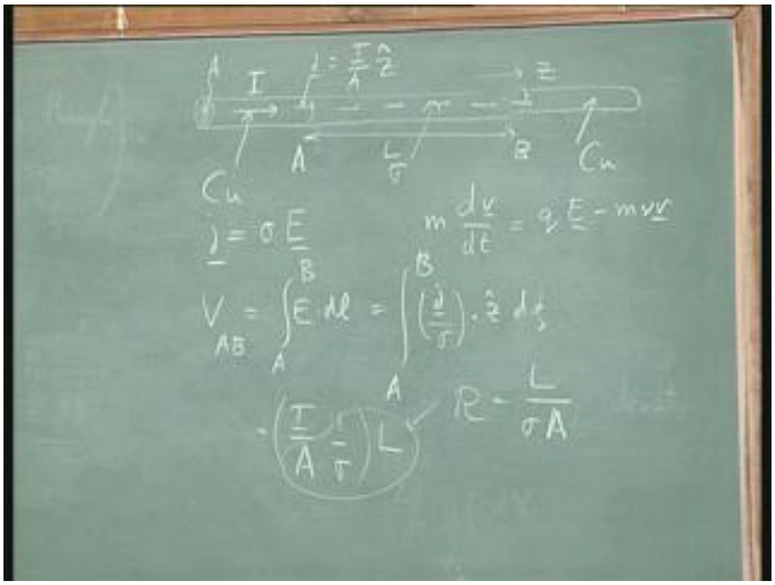
Because supposing it will little positive somewhere or it will negative somewhere I will choose a surface just covering that little positive region and I will get a positive answer or covering just the little negative region I will get a negative answer. If I get 0 for every possible surface, it can only be that this quantity itself is identically 0. So divergence j plus del row del t is identically 0. And this is called current continuity or charge conservation various name. But basically what it means is when charge tends to flow out of the point with charge at that point tends to decrease, saying nothing and more than conservation of charge. Now this is a very useful equation. Let us try and use it in a simple problem.

(Refer Slide Time: 27:18)



Let us suppose I have a wire and the wire is connected to a resistor and comes out. So this is copper this is copper and this is some resistive material so, it as a sigma that much lower than copper. Now this is little more difficult to solve so I am going to pertain that resistor is not fat.

(Refer Slide Time 27:53)



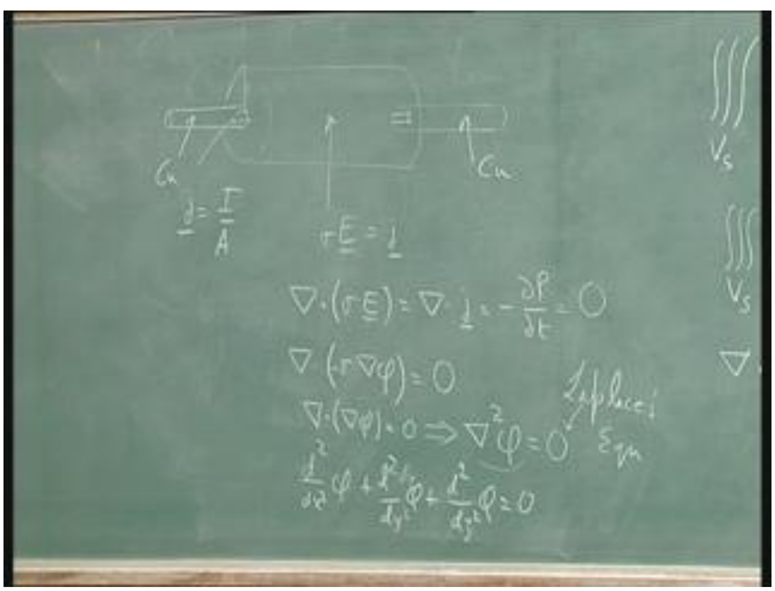
Assume that actually it's a same cross section as the copper wire. But it has some conductivity sigma. A current I is flowing and the cross section here is A. So the current density j is equal to I over A. But I have also to give it a direction. Let us say this direction z, so its set cut. So the current density is a vector it has the unit of current per unit area and it has a direction z cut. Now you have worked at last time may be two lectures ago that if you have an electric field E and you have a you have a current as a result at the electric field they are related by j equals this sigma times E.

This just comes out of applying Newton's laws of motion comes out of looking at dv dt times the mass is equal to q times the field minus m new v. And you work it all out you get a relationship between current density and electric field. So now we have a relation between j and E and this as a length L. So there is a the current is uniform which means E is uniform which means the drop in voltage between the point A and B. V AB is equal to integral E dot dl A to B and this integral is nothing but j over sigma dot z dl dz integral from A to B. So that becomes what I have written here which is j itself I over A, then one over sigma times integral A to B dz.

A to B dz is nothing but L so I can replace that by, so we have that actually I am sure I have gone ulta somewhere you will have to help me out. I would not continue this example because it seems to be releaving to up site down answer. But you can see that resistance is coming out of the relationship. So what do you have j is equal sigma E applied to this resistive material. When you work out what the voltage drop from here to here is you get it is, I into an expressions which gives you the resistance.

Now the resistance must be propositional to L inversely propositional to A and inversely propositional to sigma. So we can write R equals L over sigma A. You can confirm whether I have made any mistakes in the normalization. But you can see that a concept which talks about current density in a wire naturally gives us ohm’s law. And that is a trivial example. But it is actually telling you what you can do in a more general case. The more general case would be as follows.

(Refer Slide Time: 32:55)



Supposing I had the lead adder cylinder of the material and then I have a lead going out again. These are copper and out here entering in to the resistor and out here leaving the resistor there is a current density j is equal to I over A. Inside this material what do I have? Well inside the material I have that electric field times sigma is equal to j which means divergence of sigma E is equal to divergence of j is equal to minus del row del t is equal to 0. Because I am in steady state so no charge collecting no charges vanishing.

So I have divergence of sigma E is 0 but E itself can be written as minus grad pie. Let us assume that sigma is a constant that is this resistance does not have different types of conductivity at different points it is just one material. I can pull the sigma out. So I get divergence of gradient of pie equals 0 or del squared pie is 0. This is again the same formula we got earlier when we solved when we took combine the divergence theorem and grad pie. So this is nothing but what is called Laplace's equation.

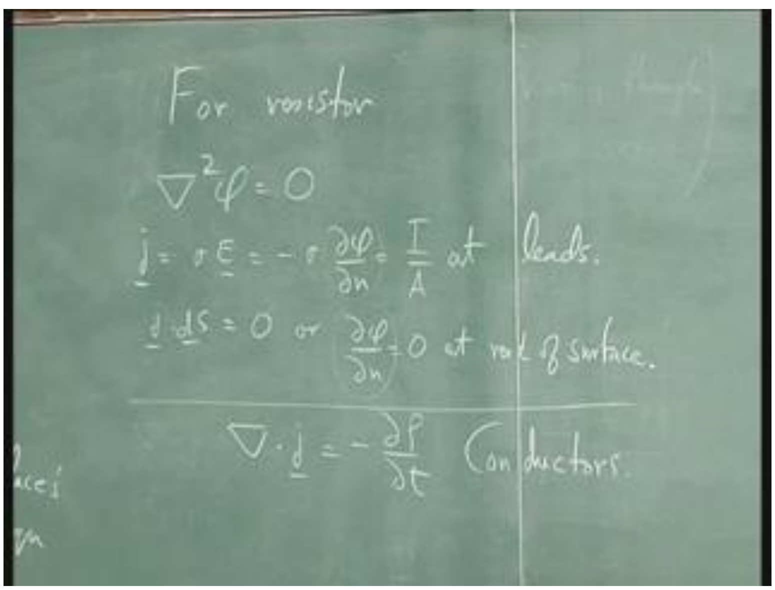
Now Laplace's equation is a differential equation. So if we want to solve Laplace's equation, it is a second order equation. You can write it as s in Cartesian d squared dx squared of pie plus d squared dy squared of pie plus d squared dz squared of pie equals 0. So you can see second derivative in each of the directions. And we know from are here we have differential equations that it if you have two derivatives in an equation in any direction you need to conditions to pin down the answers. So you need what I called boundary conditions.

Now typically if you are doing problems on electrostatics the boundary conditions are potentials. So you would like to be a say potential is two volts here its ground here its ground there. But there is no such boundary condition. Here the boundary condition we have the only one we have is current being injected is I over A. So how can we solve this equation? Because this equation we have to solve. Well there is the condition is a fairly important condition. Supposing me, imagine the drawing current lines, so what will happen to the current?

Well, be current coming which will go straight through. But if you look at current slightly half center that current is going to go ballooning out. The reason is, there is more space. So there is no reason for it to stay right in the middle. We will try to fill up all the volume of this resistor. That is why the factor resistor has lower resistance. But you will never find current doing this. This is not allowed. That is, current cannot leave the resistor current can fill the resistor. But it must always stay within the resistor.

So what is what is the mathematical way of saying, this will the mathematical way of saying. This is at the surface if you look at what the current is the current must be tangential. It cannot have a component like this. For that matter it cannot have a component like this either. Because if it did, then charge would start the pleating at this point. So you have a condition and the condition is that j normal or j dot dS is equal to 0. So the boundary conditions under which you must solve this equation becomes as follows.

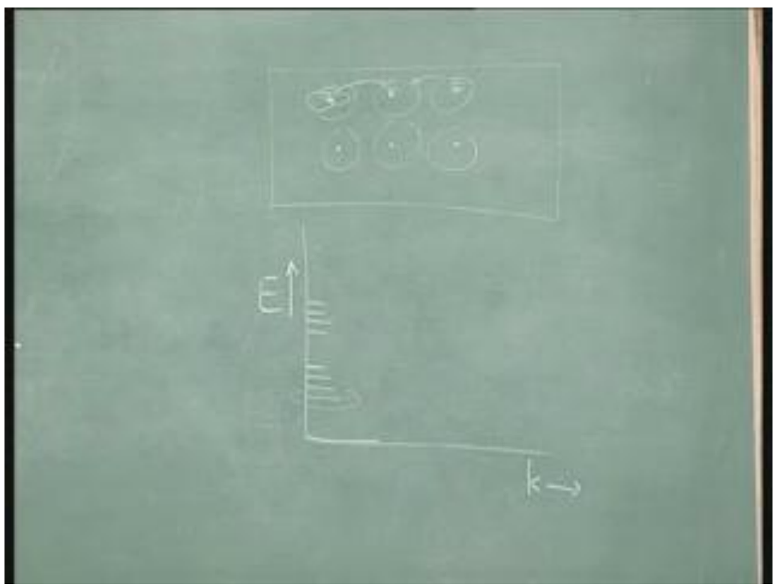
(Refer Slide Time 38:33)



For resistor del squared pie is equal to 0 j which is equal to sigma E which is equal to minus sigma del pie del n is equal to I over A at leads. And j dot dS equals 0 or del pie del n equal 0 at rest of surface. This is the kind of problem that you end up having to solve. It is quiet similar to the electrostatics problem except instead of specifying potential we are specifying del pie del n. Later on we will see how to solve such problems, right?

Now I just want to formulate the problem. So you can see that there is a lot of use in being able to write this equation divergence of j is equal to minus del row del t. And this is the governing equation if you are talking about conductors. Now the question is what happens if you are working with an insulator? Now last time I introduced and discussed two problems and both those problems I think need to be kept in mind. And we talk about dielectrics.

(Refer Slide Time: 40:43)



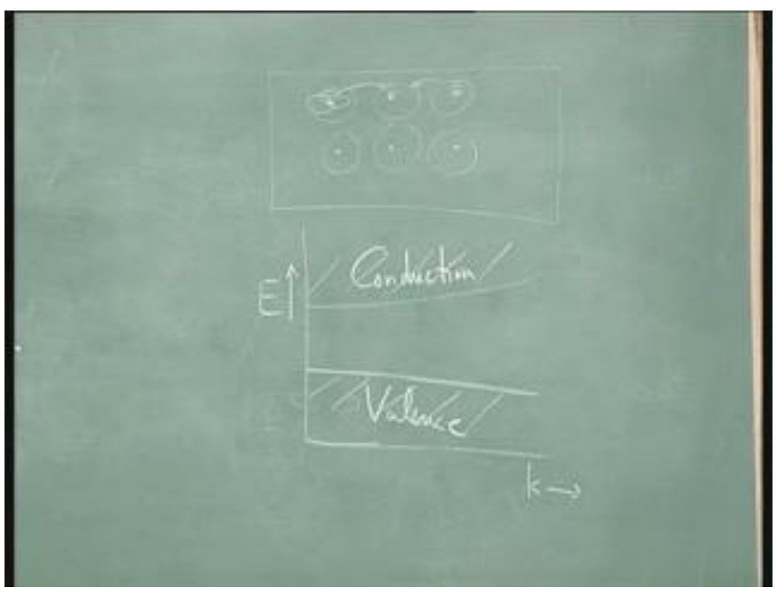
The idea for dielectric is you have lot of nuclei and nuclei have an electron clouds around there in other story is different for conductors, for semiconductors and for insulators. Here I am going to discuss insulators. Now what is an insulator? I hope you have done sound solid state physics. Because, the answer is going to be given interms of what we called an energy band diagram.

If you look at an atom and you ask what happens in an atom, you know that we have quantum mechanics. Because of quantum mechanics we have a ground state then you have levels more levels and so on. These levels are the energy state of electrons possible energy states of electron. So the hydrogen atom would have something like this and these are called limen, bowmen, pastern, etcetera, these spectra are very famous and they mean study for centuries.

Now that is the simplest kind of system when an atom as more than one electron already this picture gets confused. Because the electron starts fighting each other and it is no longer this clear storing the equation picture is what is called multi electron system. Each other electrons inside the atom attempt to get as far away from each other is possible and so this original energy level that are there including the equation get modified.

But still you have some energy levels modified but again what we call line spectra. Once you go to a material a solid or a liquid material then what happens is energy levels in one atom modified the energy levels of neighboring atoms. So much so depending on whether the electron here is in this state or in this state, the energy levels of this atom also change.

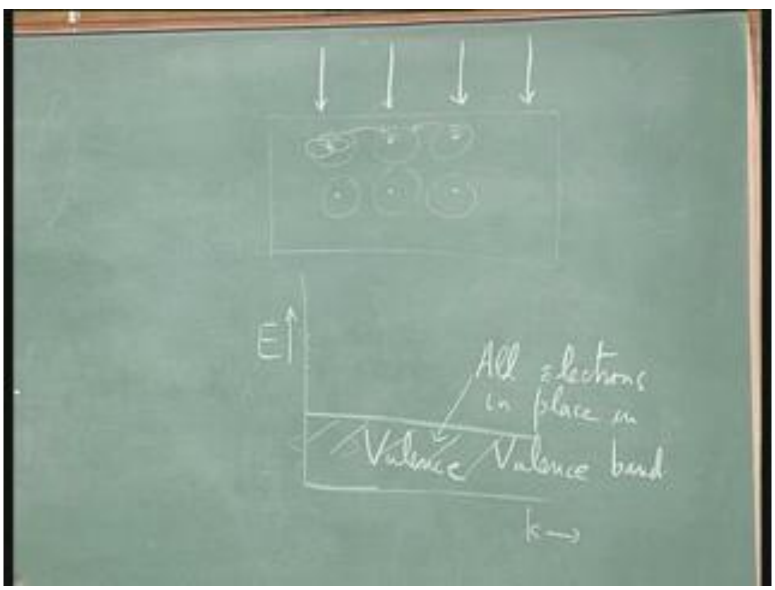
(Refer Slide Time: 43:35)



The result is that instead of having these lines clear energy levels in materials in solids you tend to have only show the last two bands. There are D bands inside the there is a conduction band and a valence band. Now here is where we distinguish between conductors, semiconductors and insulators. In conductors that conduction band actually overlaps the valence band they touch. Because it touch valence band electrons can jump into conduction band levels.

And that is how they are able to move in semiconductor the gap is very small. It is so small that is comparable to thermal energy. Once it is comparable, when I say comparable it is still ten times larger. Then we its possible there is very few electrons will be able to jump up to the conduction band and move around. And insulator has a very large band gap. Because it is as a very large band gap, electrons never jump to the conduction band.

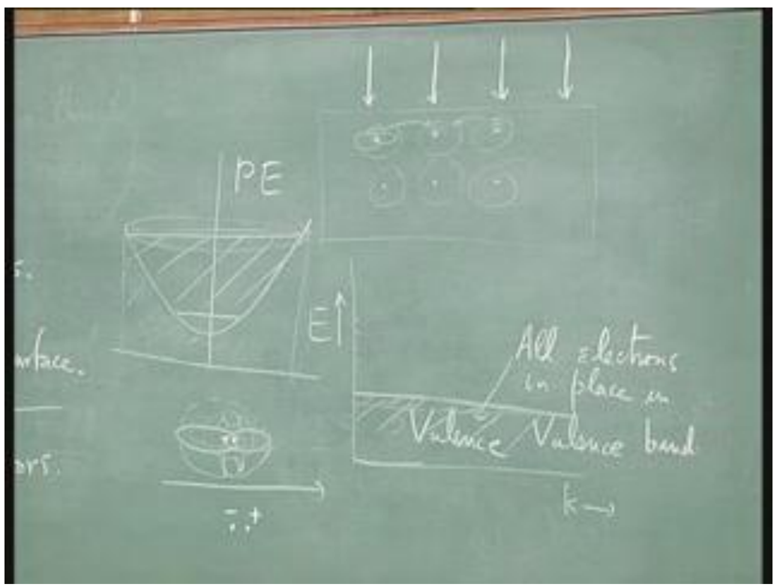
(Refer Slide Time: 45:10)



So the conduction band might as well not be there. So an insulator is a region where electrons all electrons in the places in valence band. There is no room to move we cannot jump shift around. Because there are no gaps, the consequence of that is when I now apply electric fields, the electric field distorter all the electron orbital. So it makes the S orbital slightly distorter likes P orbitals. Slightly distorter, the P orbital in the direction of E is becomes higher on energy.

The P orbital opposite in the direction of the E becomes less energetic. But whatever orbital the electron happens to be in it is struck there, there is no place to move. So what happen is all the orbital get distorted little bit. Not just the outer most orbital, the inner most, next to inner most, next to inner most, every orbital gets slightly distorted. However it is the orbital that outer most that is the most distorted.

(Refer Slide Time: 46:42)

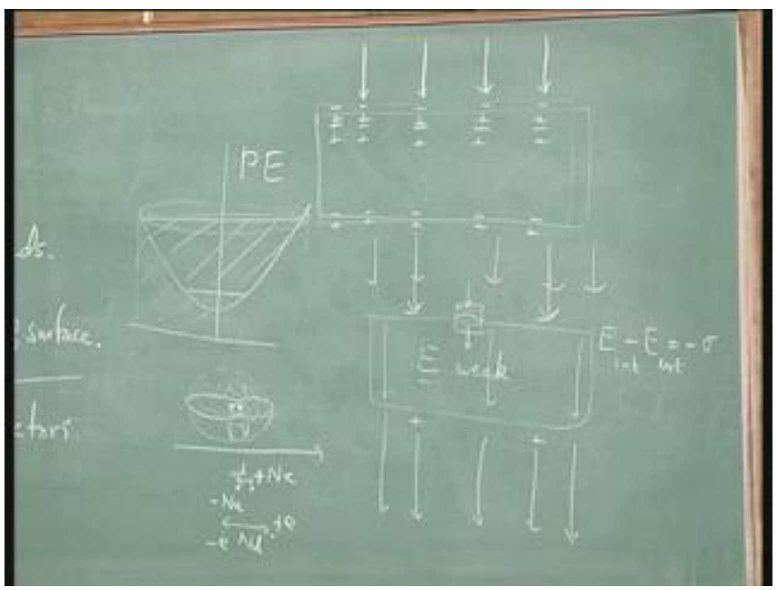


The reason is you can draw potential energy diagram and you can say the electrons I have I have filled up. Sorry not out here. I have filled up. So obviously an electron there is sitting out there has so much so deep in the potential. Well, if the electric field came it does not even notice. This potential energy that the electron may have might be 50v and the electric field would represent very vanishingly small amount of potential energy that it can. Whereas that top energy level it becomes quite aware of the electric field.

So that top energy levels are the once that kept distorted lot. So we always consider only the highest part of the valence band when we calculate how the orbitals are changed. This calculation has been done and what it was done. What they found towards. Let in the presence of an electric field all the orbitals change. So if we take if we take an electric field like this and you have a nucleus the S orbital changes slightly. The earlier P orbital which was like this has now become like this.

The earlier P orbital there was like this has now become stretched out. Earlier P orbital that was like this has also got shifted. All the orbitals get shifted a little bit the opposite direction of the field. So much so it takes them all are you find there is a net change in the location on the electrons. This net change means that you have a plus and a minus center of mass for the electron.

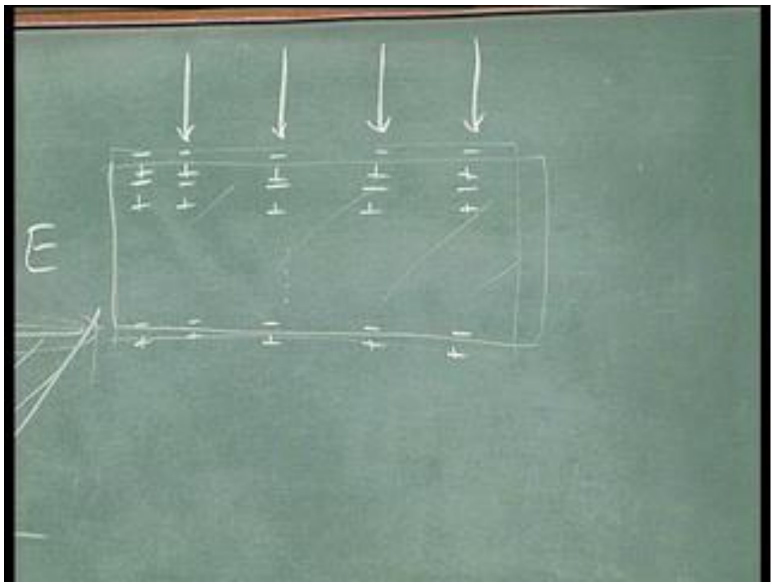
(Refer Slide Time: 49:04)



Now you could put this as plus Ne and minus Ne and you can put a distance d. That is the N represents the number of electrons present in the atom or you could put it as plus e minus e and put Nd. It does not matter both ways you will end up with the same answer. So very often you will use you will see one or the other way of presenting this. Once you have all this in mind.

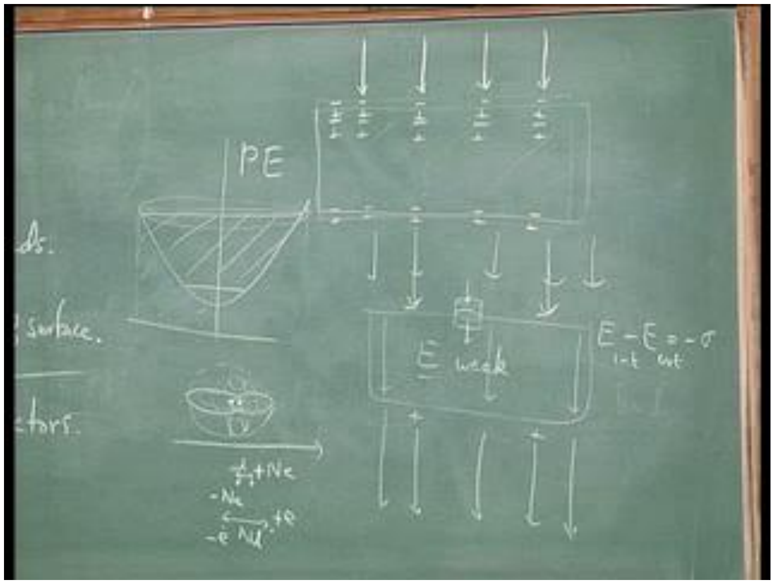
Now what is the consequence? Consequence of having the electric field come on this block as a discussed last time is that you get dipoles induced this picture. These dipoles interned partially cancelled other dipoles and so on. And so forth till the bottom dipole pair command.

(Refer Slide Time: 50:25)



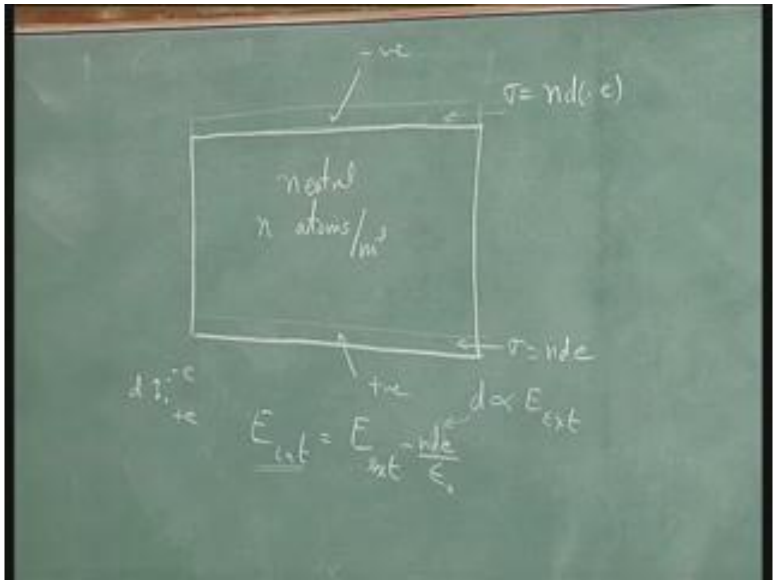
So the internal dipole all neutralized its only true for a specific geometric of drawn. It is not a general truth. But in this case all the dipole cancelled. So what is left? The left with surface charge, the negative surface charge that point of entry of the electric field. The positive surface charge at the point of electric field leaving.

(Refer Slide Time: 50:58)



One way of thinking about this, is to say if I have a block, I have an electric field coming and I have some negative charge, then I have some positive charge. What you can imagine this field line and this field line stop here. Inside these field lines continue, outside all the field lines continue. So just from looking at this picture, you can imagine what is going to happen is electric field is weak compare to electric field here and electric field here that in fact is the case, because if you draw appeal box a Gaussian surface what do you get is the entering electric field is the leaving electric field. The difference between these two is represented by the charge on the surface. So E internal minus E external is equal to minus surface charge. So E internal must be less than E external. Let us put down some numbers for this because without numbers we cannot progress and make equations.

(Refer Slide Time: 52:42)

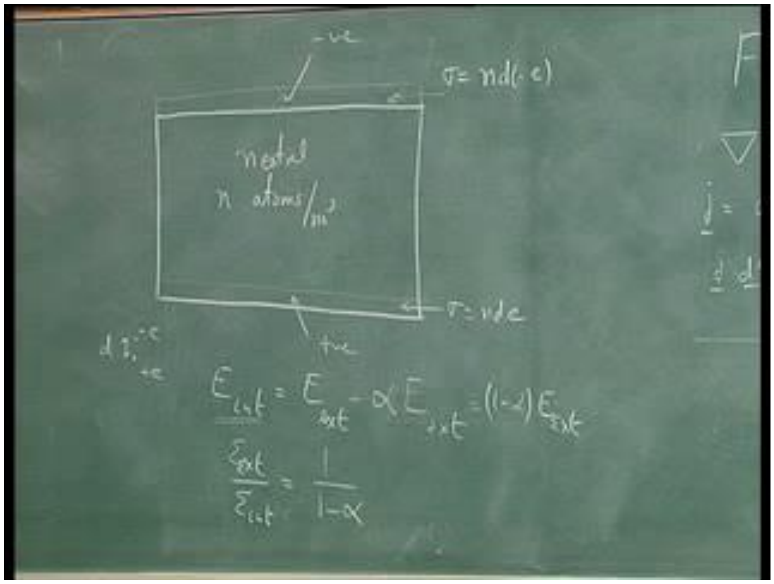


Supposing I had a block and supposing I had n atoms per meter cubed. And later suppose that only one electron per atom was actually moving because of the electric field and that atom move that electron moved by distance d. So for any nucleus there was minus e plus e the rest of the electrons are sitting with the nucleus and a distance d. So what would happen is the n atoms wont shift but the electron would shift by how much they would shift by distance d.

So the positive is the lower rectangle and light color rectangle is the negative charge so this is neutral this is negative this is positive. So how much positive charge would there be? The amount of positive charge that is sigma would be equal to the height times the density nd times the charge minus e. So this sigma is negative. Similarly here sigma is nd time's e. Now if you look at the electric field inside, we just worked out the electric field inside is going to be equal to the electric field outside minus nde divided by epsilon not.

So this cancellation, this reduction in the electric field inside the material compare to the electric field outside the material. Means that analyzing the electric fields inside any dielectric material is a little difficult, so what we do is we look at this expression and if you look at this expression this quantity d is the propositional to external electric field itself. Because, the stronger the external electric field the more the electrons are going to pulled a part from the nucleus. So I can out this propositionally, so I can replace this whole thing.

(Refer Slide Time: 55:45)



I can say this thing is some I am not I am deliberately use the wrong symbol alpha times E external. So the internal electric field is actually equal to one minus alpha times the external electric field. If I look at the fraction E external over E internal, it is one over 1 minus alpha. So this is like the, extends to which cancellation has happened in a medium which is very easily polarizable. You will find most of the electric field has got cancelled in a medium.

That is not easily polarizable easily in induced charge is not easy to happen. Then external and internal electric fields only merely the same. So this quantity has something strongly to say about the dielectric constant. And I will continue in the next lecture to pin down precisely. What it says but you can guess it has to with the displacement vector d.