Electromagnetic Fields
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Lecture – 11
Fields in Materials

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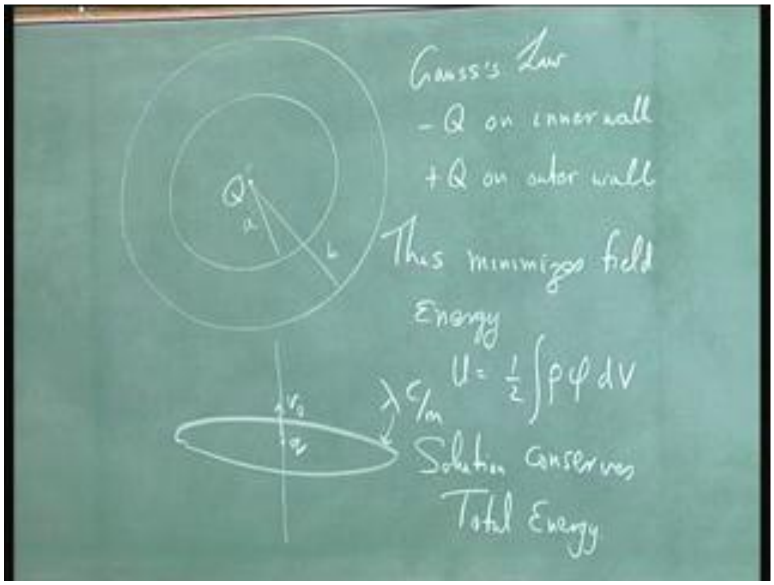

Good morning. Last time I did a set of examples, 2 very important examples there I did, were done by 2 very different approaches. Let me remind you. First I did a problem whereas you have a charge Q enclosed within a spherical shell. The shell had a inner radius a and outer radius b. And in this case we use Gauss's law and what we found was minus Q on inner wall, plus Q on outer wall. And in addition I did a derivation which showed this minimizes field energy. If you do not remember all this, go back and look at your notes, to sparely simple to derive. This was one example we did. The second example there I did, I took a ring.

The ring had lambda coulombs per meter and then I put an electron in the middle, the charge Q and give it a starting velocity v not. And the charge moves up and down and oscillates. And in that case, I said solution conserves energy. Now these two are very different statements. In this problem the charge moves exchanges kenotic energy from potential energy turns around comes back and bounces back and forth. It is like a pendulum. And when you do the solution, you are conserving the sum of kenotic and potential energy.

In this problem, I directly go and say I am minimizing potential energy. Field energy is nothing but potential energy. So how can I use different approaches for different problems? I mean, tomorrow if I give you another problem, how I suppose to know which think you are supposed to conserve? I supposed to minimize field energy. I supposed to conserve total energy. So, the answer must be contained within the problem. And to give your first answer to this, I am not going to continue with this problem and try and reach something like this probe.

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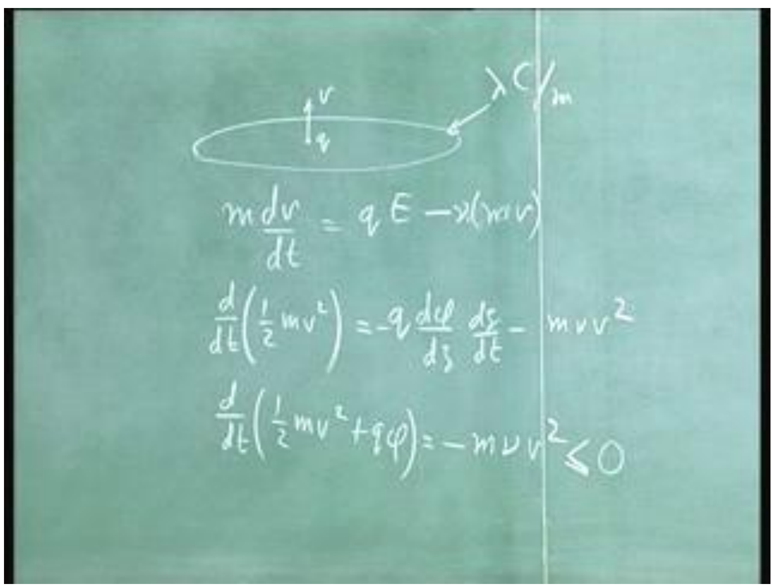

We have ring lambda coulombs per meter. Here we have a charge q, velocity v. And if you remember we wrote on a force equations, m dv dt equals qE. I am dropping the vector notation. It is understood that v is vz and E is Ez. Then I multiplied both sides by v and I got ddt of one half m v squared was equal to q minus q d pie dz times dz dt. Aand this is nothing but ddt of minus q pie where I am using the chain rule. So that is why I have got conservation energy. I said ddt of one half mv squared plus q pie equals 0. Now let us go back to this equation. This is the force equation. It says, rate of change of momentum is equal to the applied force. And applied force this clearly and electro static force which is qE. But supposing the charges moving in a viscous medium, then they would be friction. So would write minus m new v. That is, there is a frictional drag on mv; mv is the momentum.

So the rate of change of momentum is increased by the applied force. But it is decaying as minus new. So I should write this as minus new times, mv. Now if I take this equation and multiply by v, this term remains what it is, this term remains what it is. But I get minus m new v squared. So after I combine these 2 terms again ddt of mv squared over 2 plus q pie. But the right hand side is not 0 anymore. It is minus m new v squared. Now m is mass of the electron, is positive. New is the constant of friction, is positive. Now vz can be either positive or negative. But v squared it is clearly positive. So the right hand side is less than or equal to 0. It is equal to 0, if v is 0. Now this is a very different kind of answer.

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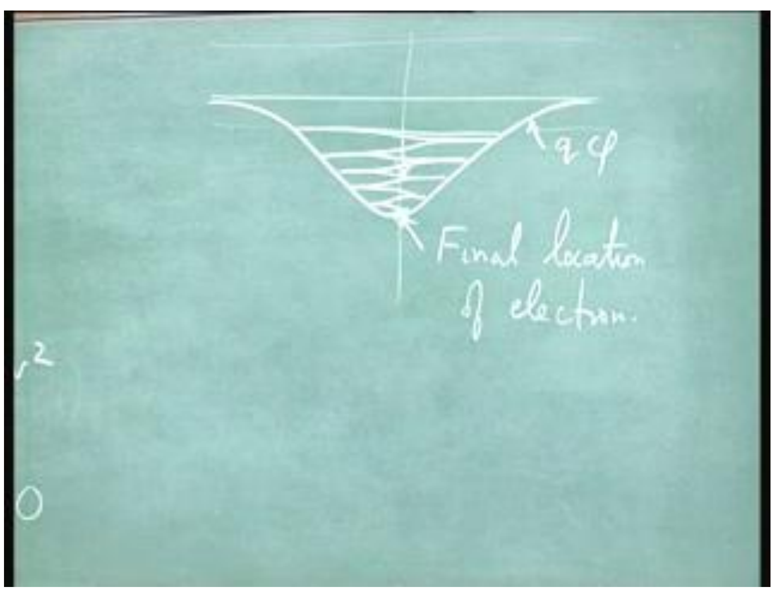

Let us draw the potential graph again. This was the potential energy q pie. And I said that if an electron had this much total energy, then you cannot confine it. It is moving so fast that, despites the potential energy, just leaves. And other hand if it had a total energy of this level, it will oscillate. But now you have a new thing happening. We have the rate of change of total energy is not 0. It is actually some negative number. So what is going to happen is, in time the rate of change of total energy is going to look like this. The rate of change is largest when the velocity is largest which means is largest near z equal 0.

So the slope is very flat near that point where velocity was to 0. It is still that moves nears that equal to 0, it is always reducing. So the total energy is continuously dropping. And after enough oscillation, electron lands up at z equal 0. Now, what happens at z equal 0 at z equal 0; the electron has no velocity left. Because it has no velocity left v squared is 0, which means we go back to ddt of total energies constant. So the electron just remains that. Now let us look what we have achieved by doing this. In the absence of friction, the electron was just oscillating back and forth. It remained oscillating forever. That meant that, if you went back to this picture.

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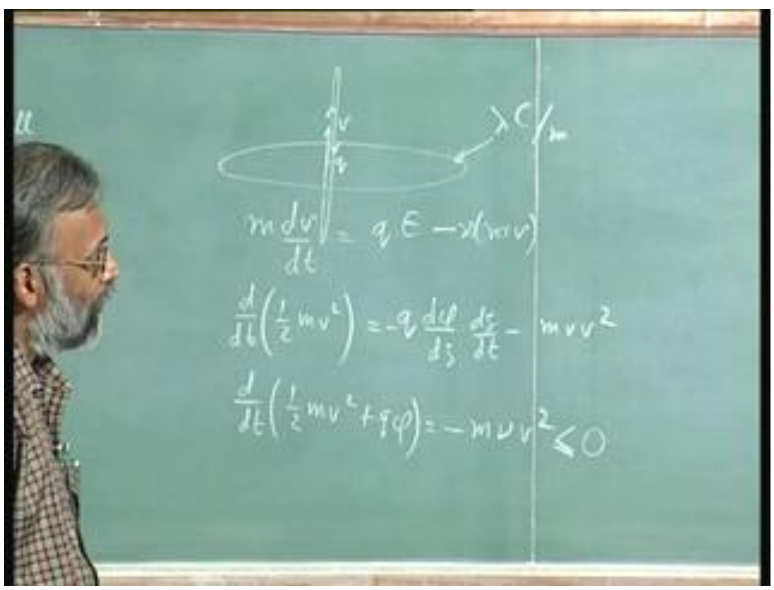

The electrons, moved up, turn around, went down and came up. But, due to the presence of friction, the excursion of this oscillation starts shrinking. So, the electron no longer goes up like that. Next round it goes less, next round it goes less, and finally lands at z equal to 0. And z equals 0, interestingly is also the location where potential energy is minimum. This is of course what Gauss's law told us. If you go back to this problem, the original problem, Gauss's law said.

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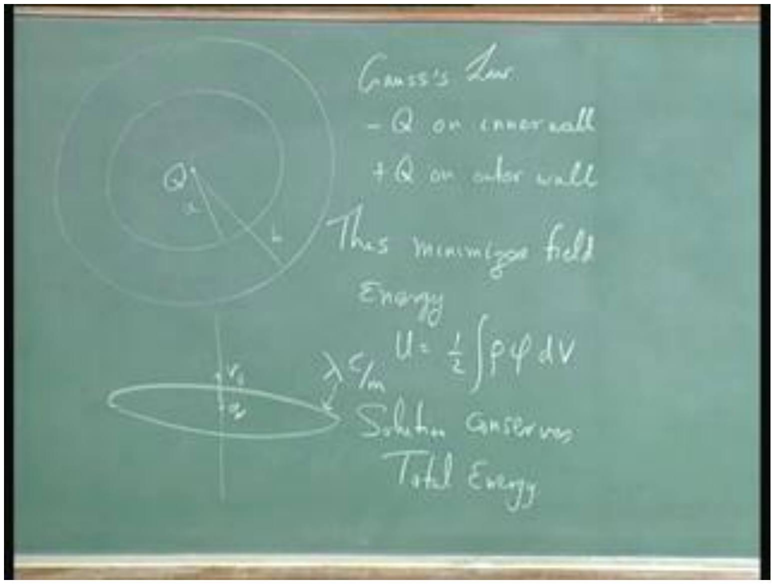

If you take a conductor and wait for a friction to do, all it can do. Then the final answer is got by minimizing field energy. And that minimum answer is all the negative charge equal to minus Q is on inner wall and corresponding positive charges on the outer wall. Just to sort of understand what does happen, see lets look at the way in which it happens at t equal 0.

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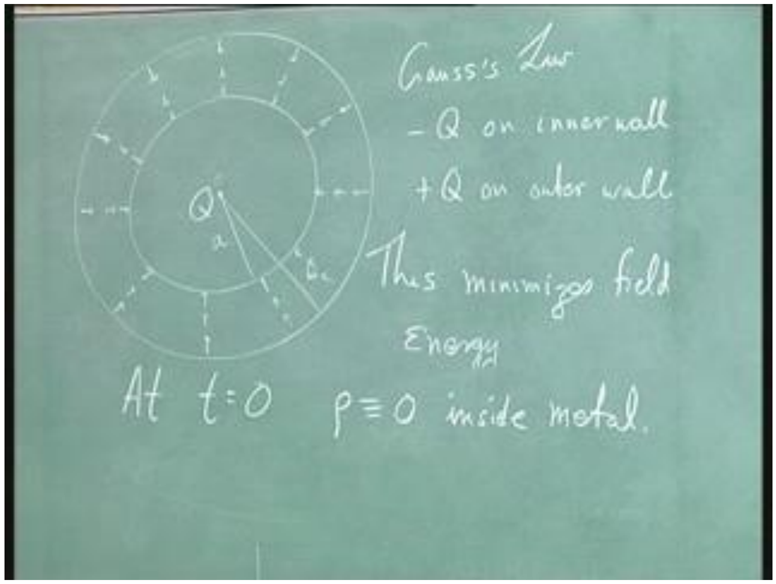

At t equal 0, row is identically 0 inside metal. So there is no charge on the inner wall, there is no charge on the outer wall. So it is imaginary situation. It is not actually possible. But you started the system like that. Now you want to know what the charges do? The atoms, the nuclear of the metal cannot move, so the electrons are going to move. Well how do the electrons move? There attracted by this charge Q. So all the electron starts moving inwards. But that something you should realize. It is not just the outside the electron that is moving; the middle electron is also moving and so is the inner here.

All of them move under the field. If there was no, there were no collisions, this was the perfectly friction less medium, then what will happen is the electron would came great amount of velocity and then they would meet the inner wall. And add the inner wall, they would just come out of the surface experience, the work function turn right round and go bouncing back. And that would be like the electron oscillating forever round the ring charge. But in state what happen is that, the electron has moved inwards keep colliding. So they forget that they came from there. They think, they came from here and then start over again and they start over again and they start over again. As a result at every instant in time there is some new potential. Potential due to the presence of charges everywhere in the metal and the same equation holds for every electron.

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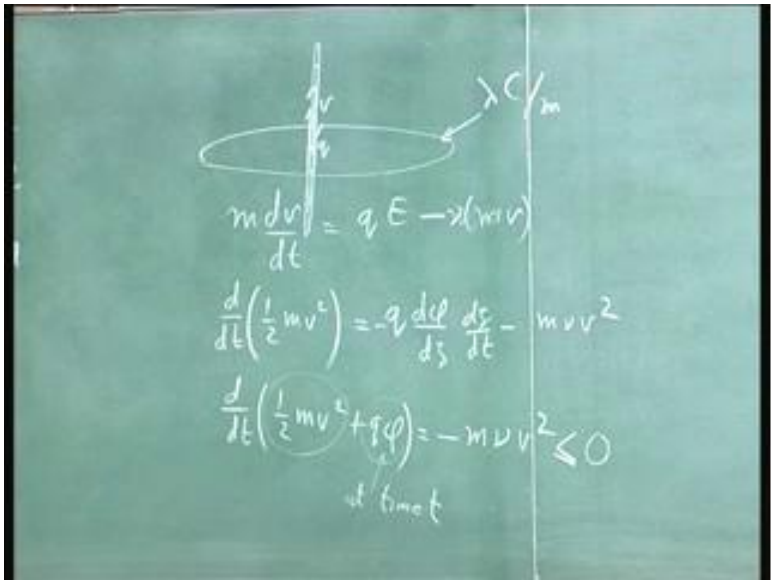

Every electron you take, the sum of it is kenotic energy, and the sum of it is potential energy at time t. So q is not, pie is not a function of position alone. It is also a function of time. And you take the time rate of change of total energy. That electron it is total energy drops by due to friction. And it is truth for every electron in the conducting media. So all the electrons are continuously losing energy and in the process, they lose all the collective kenotic energy and they left only with potential energy. So they reach a stage whether they have only potential energy and they have no velocity.

Now what is stage came that be inside the material, if they have no velocity they missed have no electric field and that is nothing but Gauss's laws result. Namely that there is no field inside the metal all the charge is sitting on the inner or outer wall. This concept is very important. You have to understand that when we talk about metals not having electric fields, when we talk about metals pushing the charges to the surface something like that, this is something that happened in the initial stages. On after that, when steady state has happened, that is in Gauss's law, is applicable.

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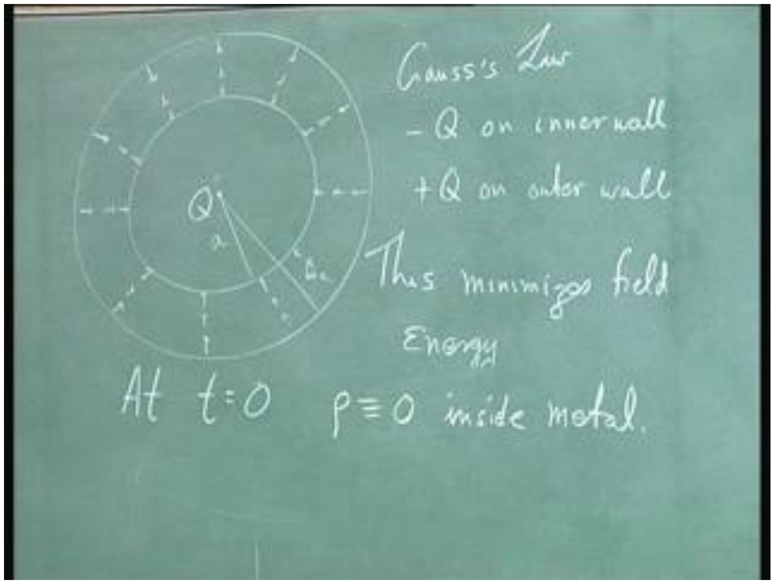

Now I have mentioned this earlier, but let us review it again.

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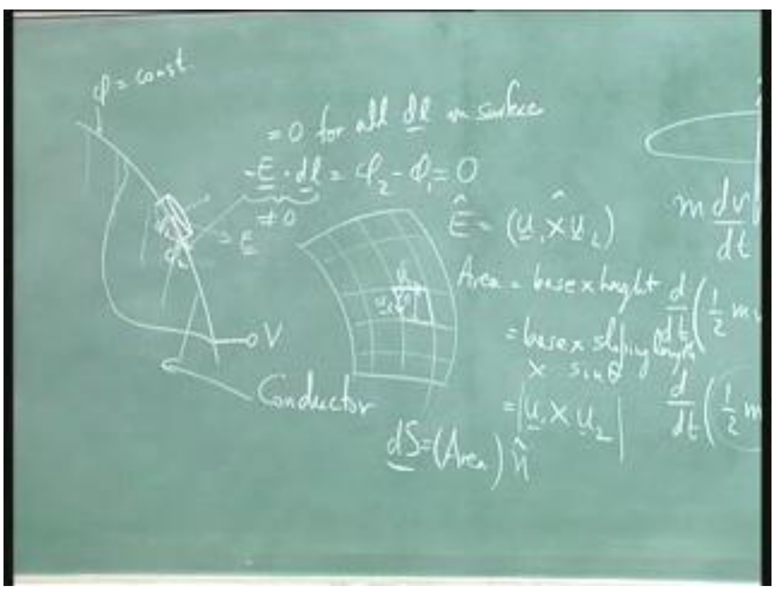

Supposing I have a conductor and supposing I look very close to the conductor. At some point that is extremely close to the surface but outside the conductor. I would like to know what is the potential at that point? The conductor itself is maintains at some voltage V. Last time we worked out that by taking any path within the conductor and using the fact that E is identically 0 on that path, the potential at all points of the conductor are equal. So the question is potential is constant at every point on the conductor. But what is the potential a little for away from the conductor?

Gauss's law comes to our help again and this time we draw a little cylinder. Part of the cylinder is inside the material, part of the cylinder is outside. Now since potential is constant, if you have any electric field at this point, the electric field must be pointing away from the surface. Supposing it is end, let us say the electric field this way. Then what it means is that the potential cannot be constant along this line. Because E dot dl is equal to this amount is equal to pie 2 minus pie 1. If I take 2 points, see this is pie 1 and this is pie 2, minus E dot dl will be equal to pie 2 minus pie 1. But this is dl and this is E, this is not 0. Which means pie 1 and pie 2 are not equal.

But you already figured out that every point inside the conductor is that the same potential. So we already know that this is 0. So we get a contradictory answer. So what it must mean is E dot dl is equal to 0 for all dl in surface. Let you think of any surface, you can imagine lines that mapped the surface. You can draw a kind of atlas. So at this point you can draw a vector and along one other lines and another vector along the other line. Clearly E dot dl is 0 for this vector; E dot dl is 0 for that vector. Because both vectors live in the surface. So, we want a vector which is 90 degree to this vector as well as to this vector. And we know very convenient operation that tells us what that is namely it is a crossed product.

So this is a vector u 1. This is a vector u 2 then the direction of E is u 1 cross u 2 unit vector along that. Now as I told you this before, this is one other reason why the concept of surface has been given a direction. Because, if you take the area of this little piece, area depends on this angle. Because of this parallelogram is equal to the base times the height. So the area is equal to base times height which is equal to base times the sloping length, times sin theta. Which is nothing but u one cross u 2 magnitude. So we have a concept of area whose direction is a direction of u 1 cross u 2, the magnitude of whose area is u 1 cross u 2.

So we might as well take u 1 cross u 2 as area itself. And so this is why we have this concept that, dS is equal to area of a little part of the surface in to the normal vector. A normal vector is nothing but the direction of any two lines inside the surface cross each other okay. This was the discretion had already covered this before. But I think this is one concept with this we cannot cover too often. So we have that an electric field must be normal.

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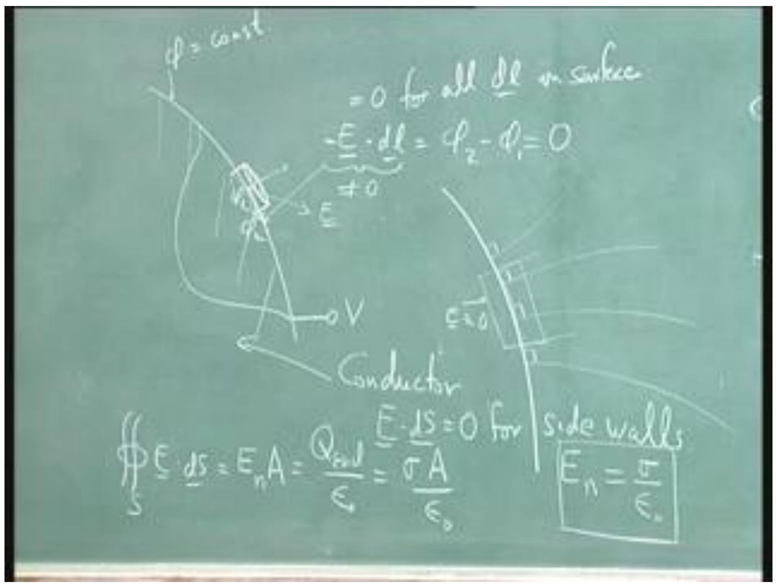

So I draw a cylinder a Gaussian cylinder who in imaginary surface whose top surface is parallel to the surface to the metallic surface, whose bottom surface is inside the metallic surface and whose side walls or parallel to the normal vector. So what will happen? Let me draw few lines. So this is my metallic surface, so you might, electric field is normal to the surface itself and then it bends.

I have now put a cylinder like this. Now you see the electric field is tangential to the sloping walls. So, E dot dS equal 0 for side walls. E is identically 0 here because it is inside the metal. So we take this cylinder and apply Gauss's law what do you get? You get the surface integral over S E dot dS is equal to the normal component of E. There is one, the flat top play plate times the area of that top plate is equal to charge enclosed divided by epsilon not. But what is the charge enclosed? Well we know that there is no charge inside the metal.

There is no charge outside the metal. So, all the charges on the surface of the metal, so it is equal to the surface charge density times the area sigma times area divided by epsilon not. Area can be cancelled. Area cancels are because if I take a bigger radius I collect more electric flux. But I collect more charge as well. So it is propositional. So again an answer that says for a conductor only for a conductor, I get the normal electric field at the surface of the conductor is equal to the surface charge density divided by epsilon not. Just to remind you what is surface charge density.

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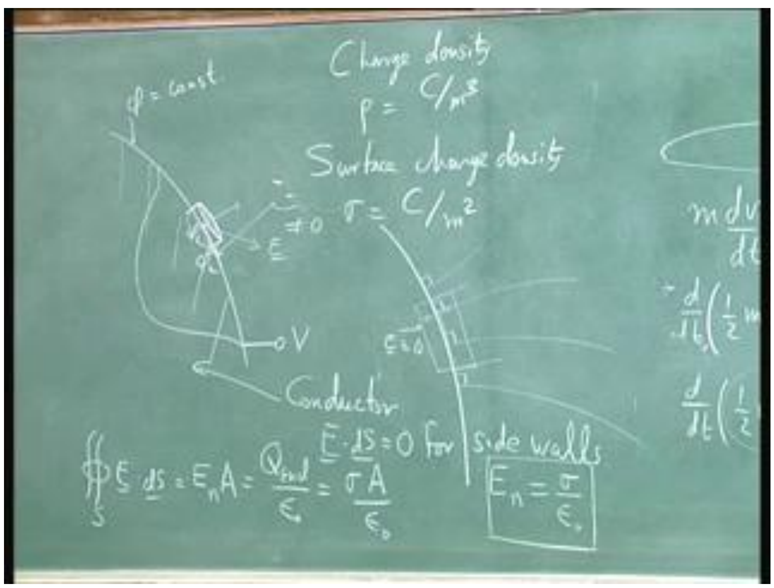

Charge density which I called row is coulombs per meter cubed. That is if I take a charge distribution and I take a very small box, then I calculate the charge inside the box and divide with the volume of the box that is row. Surface charge density, sigma is column per meter squared. So this is the number of amount of charge per meter squared per unit area of the surface. There is no third dimension. Because a charge is exactly on the surface it is not inside the metal it is not in the air. This is an approximation because if you look at ant material, supposing I have put one column of charge on a metal metallic object, so what will happen?

What will happen is, this one column will have to arrange itself on the surface. And for every timing region on the surface you would require to put a lot of charge. But that charge has to come from the electric charge that can be present on the surface. For example we are talking out positive charge. It means remove electrons from the surface. There are so many electrons you can remove from one layer of metallic atoms. Because you know your semiconductor physics, you know that you have a conduction band and a valance band and you have in metals on overlap.

So, one electron per atom may be mobile. So if you manage to put so much charge that more than one electron per atom needs to be removed, then it is not fair to say all the charges on the surface. Actually the second mono layers of charge also start to form. In metals, this will never happen amount of charge available is so large that long before this happens. You will find that you put so much charge that you overcome your work function and electrons just start walking away from the metal. However if you are talking about weekly conducting materials like semiconductors, you will find this can happen.

In other words it is not. It is only an approximation to say that all the charges on the surface. More accurate it is to say the materials separate in to two regions. One region is a conducted with no charge. Other region is a kind of a depletion region. Region I have only charge, but zero conductivity. This kind of concept, you will come back in your solid state physics. So I would not go any further into it here. Now let us go into another kind of material which is extremely important to electrostatics. Up to now we have been talking about free charges. We will be talking about charges which are given in existing location.

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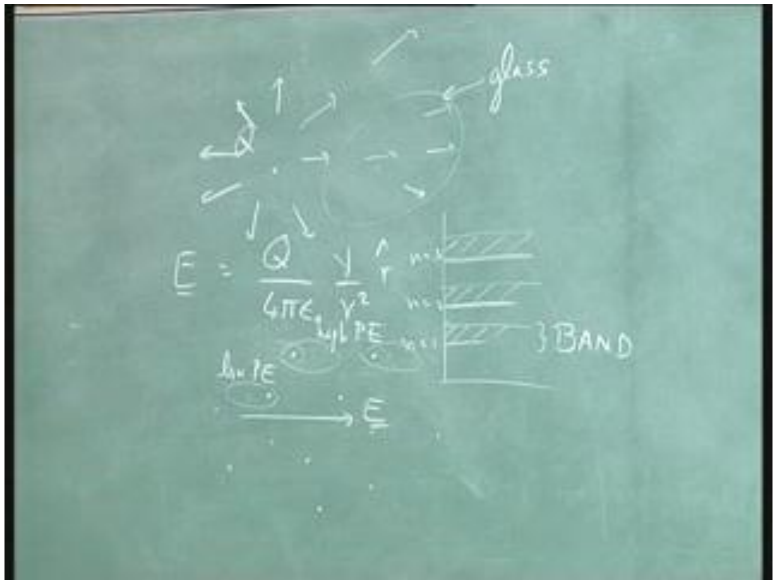

So we got a charge Q and we know that the electric field is Q over 4 pi epsilon 0 1 over r squared r hat. However what happens if you put a material lets a glass near the charge. When electricity and magnetism was first discovered, infact it was experiment like this. They did it. They brought a glass rod near a charge. They rubbed amber, they rubbed ebony and they found the different materials had different static properties. Some of them seem to be positively charged, some of them seems to be negatively charged.

And it was an effort to understand those phenomena that led to the entire field. But now what exactly happens inside such a material? Now the electric field due to this charge is all round it. So there is no reason to believe there is no any electric field in the region. There is an electric field there. Now the presence of this electric field does something to the molecules that of making up this material. Now for that we need to go back to some more physics. You know that if you look at any material and by material here I mean an insulator.

So if you look at a glass what it consists of is a whole lot of molecules. So, 2 molecules in random locations which are roughly known distance apart a few Armstrong. Now these molecules are consists of nuclear the silicon, the oxygen and electrons are around there at every silicon di oxide molecule has a certain arrangement of electrons. Now I am not going to teach you quantum mechanics. Here that is not. This is not a right subject for it. However you know from whatever you have read that electrons do not just move around electrons, exist in energy levels. So you have seen a energy diagram where you told that you have the 1 is 0 and then 2 is 0 and then 2p is and then 3 has and then 3p and so on and so forth.

And you also know from here semiconductor physics that when atoms are brought in very close approximate to each other, each of these levels the n equals 1 n equals 2 n equals 3, what we called the different electron shells become bands. They become bands because neighboring atoms start influencing the electron positions. So if this nuclear had an electron which was sitting more to one side, then the other side that one had the electrons that also like this. These 2 electrons are pushed again each other. The result the particular energy level corresponding to this particular electron is actually higher because of the preserve the other electrons.

If in other hand the electrons decided to do, this it will be different. And if in the third hand the electron did, this it will be different again. So, depending on the different electronic states of the electron around each of this nuclear, you get slightly different energy levels. And when you take to the 23 different electrons and you put them in the huge number of combinations, you can have that is what research in a band this is called an energy band. Now you take such a material. And now you impose on it and electric field up to this point. The electrons were arranging themselves, whatever direction they wished subject to available energy. Where is the energy coming from? The energy comes from temperature. So every electron has half ktm amount of energy.

So it is rarely around going jumping from one energy state, when other sometimes it has the when it jumps to this state the other one jumps to that states it is come down in energy. Other times they are opposing each other. So it is gone up in energy. And because it has, this random energy associated with temperature, the electrons are moving up and down inside each of these bands and they are filling them up. However once you put an electric field things change, the electrons prefer to be in the direction that is opposite to the electric field. That is because they want to move opposite to the direction on electric field. So this becomes high energy and this becomes low energy. So result the electrons start making transition which make them move to this kind of energy distribution, probably it is distributed. What does that mean?

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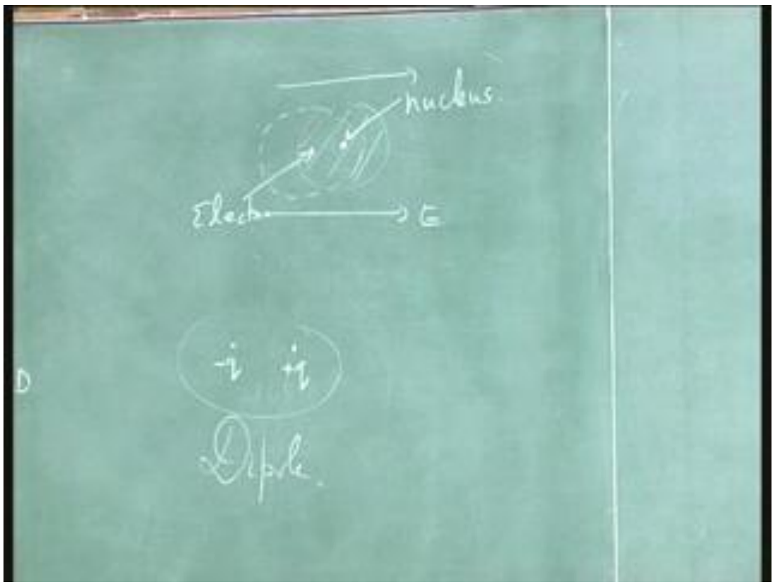

It means you had an atom. Normally an average the electron is entered around it, which means that if you took the mean location of the electron, it is right on top of the atom. So use Gauss’s law. It is all spherically symmetric. Electric field is due to total charge enclosed. Total charge enclosed to 0. No electric. However in the presence of electric, external electric field, this symmetric distribution of electron does not happen. Electrons prefer to be this start so much. So if you ask where is the electron, the electron is shifted. The nuclear is here actually the nucleus plus most of the electrons are here. The outer electron has shift. What is the result?

The result is instead of having electron and nucleus sitting in the top of each other, there shifted it respect each other. In other words you have a plus q here and a minus q. Now you know all about this. This is noting but a dipole you looked at it earlier twice. One time when looked at it was to explain why the electric field, even though it is so strong compare to the gravitation field does not dominant. At that time I worked out for you what the dipole field would be. Secondly we did we computed the potential due to a dipole. Let us complete that calculation.

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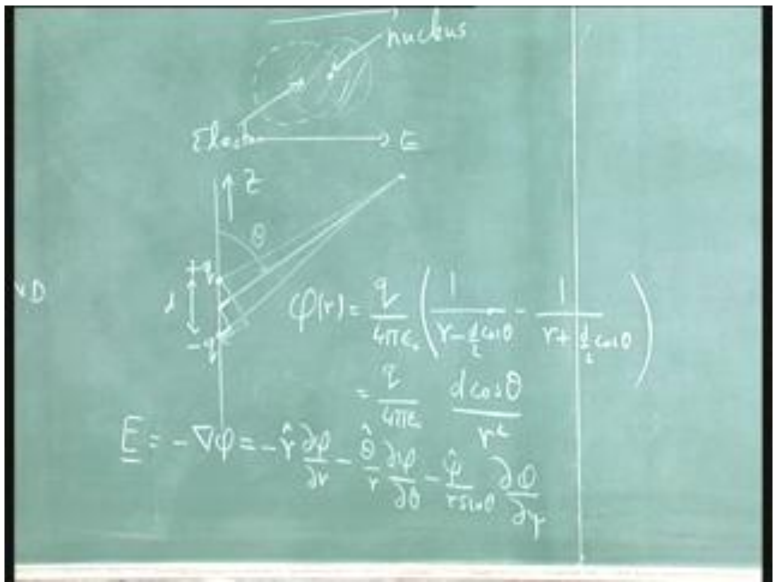

Let say, this is a z direction. I have plus q and minus q. So they represent the nucleus and the electron. This is the mid point between the two. I am talking about the field far away. I am in spherical pole of coordinate. So this is the angle I am looking at theta. See if I ask, what is the difference in distance between these 2 lines, we can do the same calculation. We did earlier this is the difference in distance. So the potential pie of r, this is the origin is equal to q over 4 pi epsilon not times. Let this be r 1 over r minus. I am going to call this distance d; d over 2 cos theta minus 1 over r plus d over 2 cos theta, d over 2 because I am taking this as the origin.

So this is plus d over 2, this is minus d over 2. So each of this is half, this distance, these are 2 distances. I am looking at this is a lesser distance than r. So minus d over 2 cos theta the greater distance than r. So plus d over 2 cos theta, d over 2 cos theta is small. So I expand and I get q over 4 pi epsilon not times d cos theta divided by r squared other said we have done this derivation already, right? Remember right we did it four lectures ago. Now having got potential we can get electric field the electric field is equal to minus gradient of potential.

But we need the gradient in spherical pole of coordinates, so what is that? It is equal to minus unit vector along r dell, pie dell, r minus unit vector along theta 1 over r dell pie dell theta and minus unit vector along sie over r sin theta dell pie dell sie have derived this earlier. So go back your notes and look at it. It just these 2 r and r sin theta. I just represent the radius of the circle on which you do d theta.

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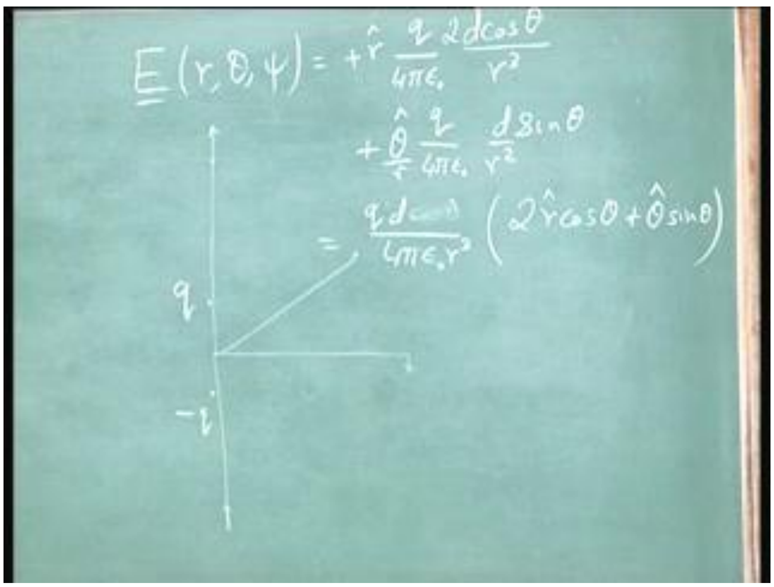

So you get for E minus r hat after differentiate with respect to r. So q over 4 pi epsilon not d cos theta divided by r cubed with the minus sign. So the minus becomes plus, then I will have a theta hat q over 4 pi epsilon not d over r squared derivative of cos theta again minus goes away sin theta. When I differentiate r squared I should get a factor of 2 ddt one over r squared is minus 2 one over r cubed. So you can now see let the answer is mostly common you can pull out. All the common factors qd cos theta over four pi epsilon not r cubed. Now where it I so this also one over r for the theta r cubed times 2 r hat cos theta plus theta hat sin theta. I have made a mistake here.

Can we understand the result q minus q I am going to some point there, if theta is 0, so I am going vertical direction, I expect a lot of r electric field I expect 0 theta electric field. So cos theta is one sine theta is 0. If I go in the perpendicular direction, I do not expect any r field, but I do expect a field in the theta direction. So I get sin theta of one cos theta of 0. If I go in a negative sign direction, I expect an electric field that actually pointing in words. Because the negative charges closer than the positive charge, cos theta is minus sin theta is 0. So the field expression is more or less as we expect. Now what is this going to do? For us every molecule in a material, as this kind of dipole response to any electric field.

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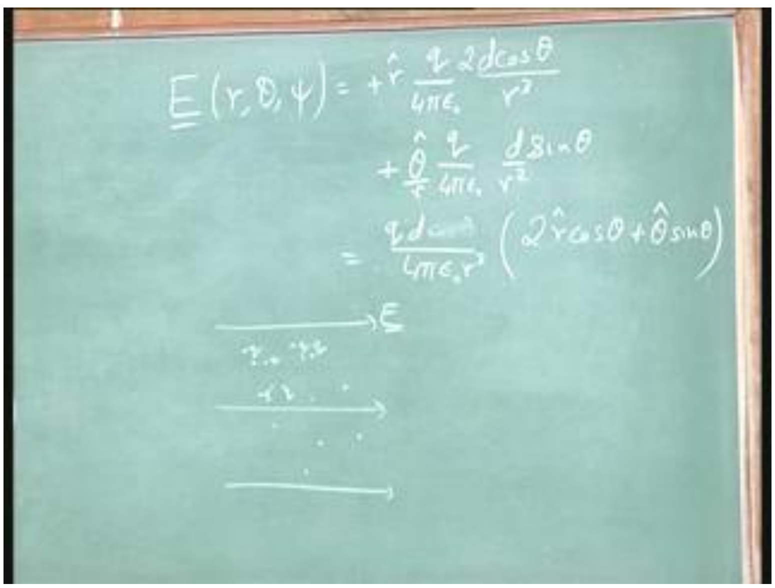

So I have an electric field and the molecules in the material or all of them creating an electric field like this. Why? Because first of all this electric field intern creates a q minus q q minus q q minus q. So every atom has now split into little, little dipoles. Each of this dipole is now creating an electric field. That looks like this. Now if you take a whole lot of these diploes, if you take a sheet of dipole and ask what is the potential due to a sheet of dipole, you get an interesting result.

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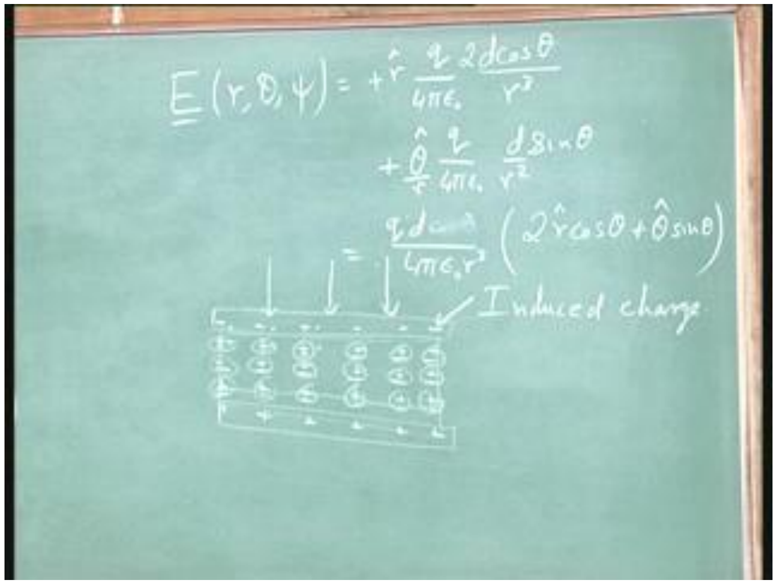

Imagine that I have a block of glass and have put an electric field is pointing downwards onto the glass. The result I am going to have negative positive, these are induced on the surface. But as the electric field is experienced inside I will again get minus plus minus plus and then next layer will again be the same and so on and so forth. Now what actually happens is the plus out here gets cancelled in the minus out here, plus out here gets cancelled in the minus out here. So each dipole layer cancels the dipole layer next to it. So these cancel out, these cancel out, these cancel out. I am going to short in the length of the material.

Let us say just this long. So this keep canceling out except for the final plus that happens at the bottom. So you get a whole lot of cancellations plus some charge at the top plus some charge at the bottom. So the effect of these microscopic dipoles is actually to create some, what is called induced charge at the top surface as well as at the bottom surface. In the middle, this no charge at all because whatever positive charge that induced is cancelled by negative charge as it is induced. So I can replace this whole model picture by saying, I do not know exactly how much it is.

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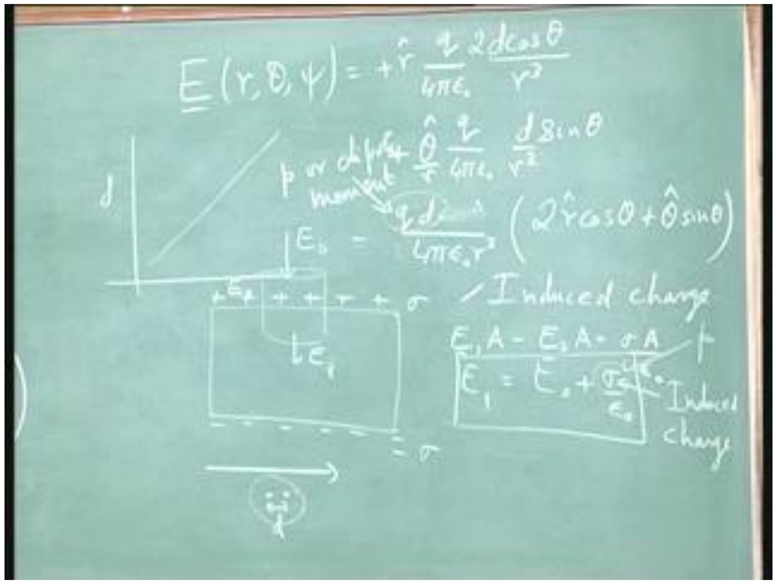

But there is some amount of positive charge sigma and then equal amount of negative charge minus sigma. If I now apply Gauss's law on the surface, I have a known electric field coming here E not. I have an unknown electric field going out here which I called E 1. E 1 times area minus E not times area is equal to sigma times area divided by epsilon not. That is what Gauss's law tells us. So what it means is that, E 1 is equal to E not plus sigma over epsilon not. The electric field inside the material is different from the electric field outside the material. So, very strange result in individual atom in the presence of electric field gives you dipole, that dipole looks like this.

It is an electric field is a function of angle. But when you put it on material then what you are getting is dipole sheets. The dipole sheet all canceled out in the middle you have charge of the top charge of the bottom. When you try to use your Gauss's law and obtain, what is the charge inside, you find if charge the electric field inside is different from the applied electric field, which different by how much different by the induced charge. So you can see, there is something happening here, that due to the material properties, there is some amount of induced charges. This induced charge must depend on what we called qd.

Because depends on how the amount of charge there is available to be shifted and the distance by which is shift. This keeps you the dipole everything else is spatially spatial information. But all the information about the material is in qd. So this quantity qd this called p or dipole moment. So this sigma is clearly related to the dipole moment per unit volume. So it is a material property and based on this material property, the electric field inside the material is different from the electric field outside the material.

Since the electric field is tending to try and cancel the since the sigma is trying to cancel the electric field invariably you will find this sigma and the electric field are opposite direction. So much so that E 1 will be less than E not. Now in principle, all of this is covered by coulomb's law. All of this is covered by the equation that we have already written. So you could write an equation for all the dipole moment. You can write the susceptibility of the material. That is the amount of dipole moment and you can solve exactly. So you can write a Gauss's law, you can write your Poisson's equation, you can write a coulomb's law according to the known amount of induced shift in the electron position.

However that is a very complicated process. And it is so complicated you never get any job done if you did that. So in state, what people do? What engineers do is to say that we have a way of connecting sigma top the dipole moment. Wherever we have connecting p back to the applied electric field, now there you have to do experiments that and find out. Because it really has to do with you apply an electric field, you have a nucleus, you have a electron cloud and the d that we are talking about this d. So it has to do with the quantum property is of the material. But if you did enough work you believed that you can work out d for given electric field.

In experiments we have been done and when you do the experiment what you find is that if you change the applied electric field E not and you look at what d is it a straight line. It is not obvious that is a straight line. And in fact so for very strong electric field, it is not a straight line. Once the electric field becomes very strong, that line begins to bend, which means that the amount of induced charge is not propositional to the electric field. But for small amount it is and the next lecture, we will use this fact that is linear to derive extra conclusions.