Electromagnetic Field

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Lecture - 1
Introduction to Vector

Good morning! This is a course on electromagnetic theory, which is aimed at EEE students; that is, to say students who are doing power electronics, machines, power systems and other subjects. I will first discuss the syllabus that we will follow in this course, give some details about the syllabus of the first part of the semester, discuss the textbook and then go into some review topics, so that I can bring everyone to a same state of readiness. So, let us first look at the syllabus of this course. The course consists of electrostatics, magnetostatics, time-dependent fields and waves. These are the four main areas.

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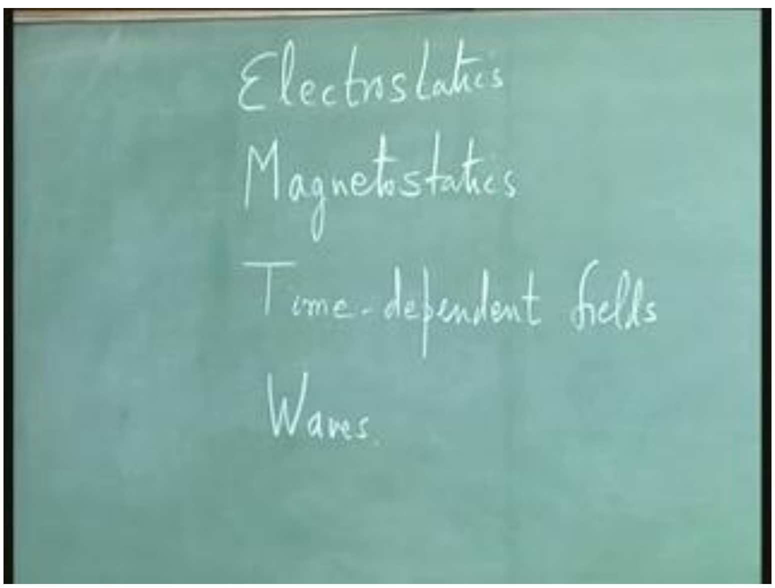

Electrostatics introduces the electric field. It also introduces the electric potential and it introduces an important equation namely Poisson's equation. So in this, this is the first part of the course. About 8-10 weeks, we will spend in this. We will basically understand capacitance problems. The second part has to do with the magnetic field and we will study how currents produce magnetic fields, that is, Ampere's law and we will will also study, combining these two equations, how particles respond to electric and magnetic fields. The third part deals with Faraday's law. This is the crucial part of the course for EEE students, because it is a combination of Ampere's law and Faraday's law that makes all electric machines, generators, and almost all power generation systems to work. Once we have finished Faraday's law, we introduce the last law, which is the generalization of Ampere's law, called Maxwell's equations and from that we will derive waves.

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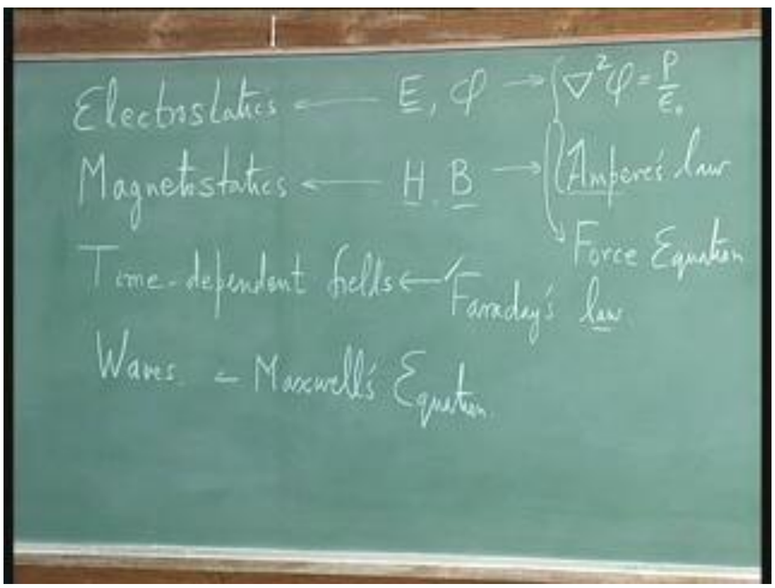

So, this is the basic syllabus we have. We will do electrostatics, then we will do magnetostatics. We will do the first time-dependent field, namely, Faraday's law and then we will do Maxwell's equations. Now inside electrostatics itself, there are several topics and I plan to cover them as follows. First we will do review and introduce vectors, that will take probably two lectures. Then I will introduce Coulomb's law and the electric field and this seems a little excessive but I plan to take around two lectures to pin down all the concepts.

Once we have done that, I will introduce Divergence and then Gauss's law and that should take two more lectures. Once we have done Gauss's law, we should be able to go from electric field to electrostatic potential. And from electrostatic potential we should get to Poisson's equation in another two lectures.

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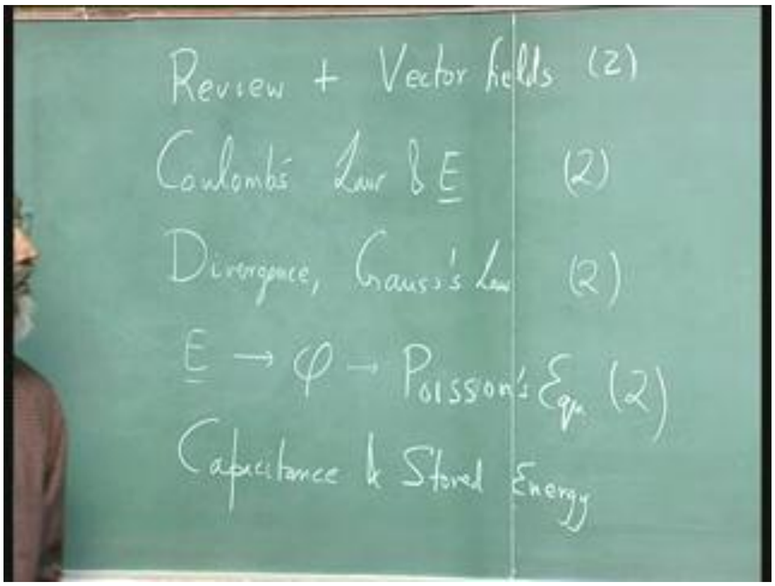

Once we have finished these fundamental ideas, then we will start applying them. First of all, what is the notion of capacitance and stored energy? I will talk about dielectrics and I will introduce at this point, an important idea which will keep coming back, which is, Boundary Conditions, again two lectures, and finally solution of Laplace's and Poisson's equation and application to capacitors, two lectures.

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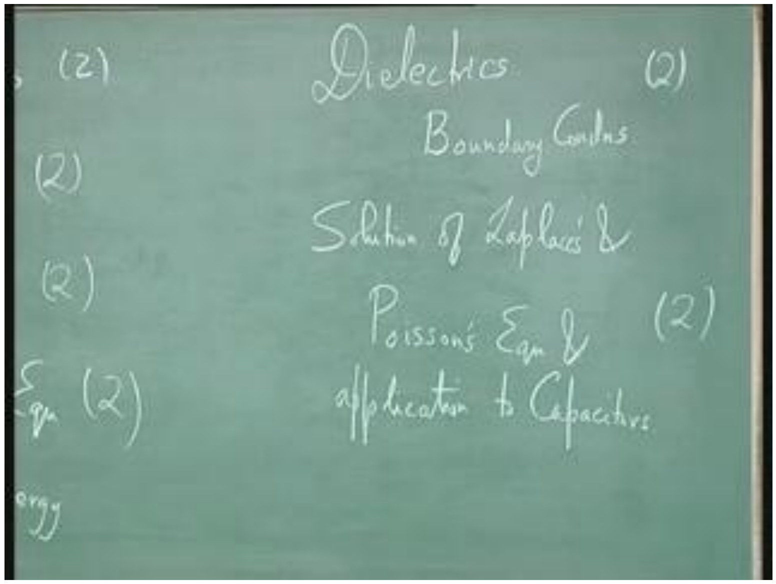

So the first part of our course is going to be basically like this. So you can see that its going to go rather slowly. 12 lectures I plan to take. I may compress one or two parts and expand one or two parts but this is more or less the speed at which I will teach the material. After that we will move on to magnetostatics and after that, introduce Faraday’s law and finally finish up with electromagnetic waves.

Okay. So as I said I first would like to start with a little bit of review. Before that let me also introduce the textbook for the course. The textbook is this book. It is called “Engineering Electromagnetics”. The authors are W. H. Hayt, Jr and John. A. Buck.

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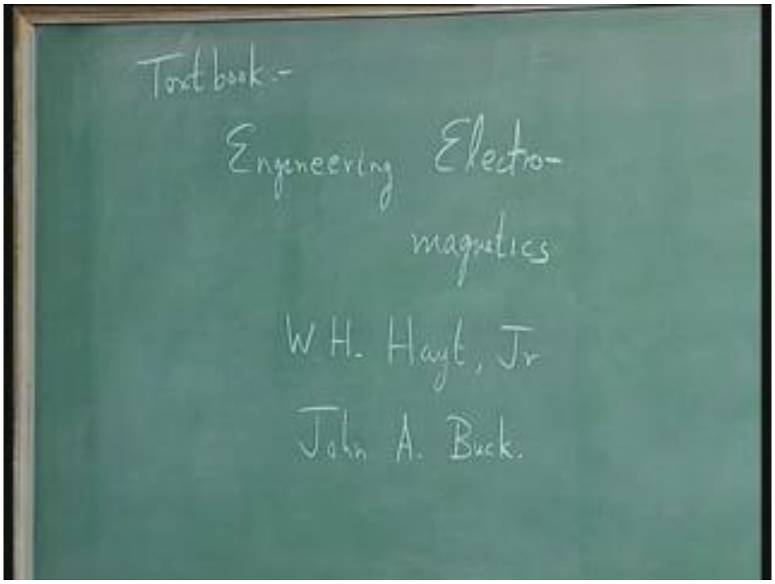

Professor Hayt is one of the most famous authors of Electrical Engineering books. Hayt and Kimberly and many other books that he has been an author of are now classics. Unfortunately he passed away quite some years ago and his books have been continued by John Buck. This book is a very elementary book. There are more advanced books available but I think it is also the best introductory textbook for Electromagnetics. Anyone who wants to go further into the subject, there are many excellent books and I will mention them as I mention various topics but as a self-contained textbook this is one of the finest books around.

Okay. Now one of the problems that students have with Electromagnetics is that they have already learnt it in school. They have already learnt it in Physics. So they do not know what is left to learn and they are scared actually to learn to find out what they have left to learn. So let me review what you already know so that we can put a context to this course. From your first course in electricity and magnetism you already know that there is something called Coulomb's law. You know that if you have two charges, q 1 and q 2, and if these charges are separated by a distance d, that there is a force that is proportional to the amount of charge q 1, the amount of charge q 2 but is inversely proportional to the square of the distance. That is, the further apart these charges are the less they exert force on each other. You also know that the smallest charge you can find is the charge of the electron and the proton. The proton is positive, the electron is negative. The amount of charge that an electron or a proton has is 1.6 into 10 to the minus 19 Coulombs. The unit of charge has been named after Professor Coulomb himself.

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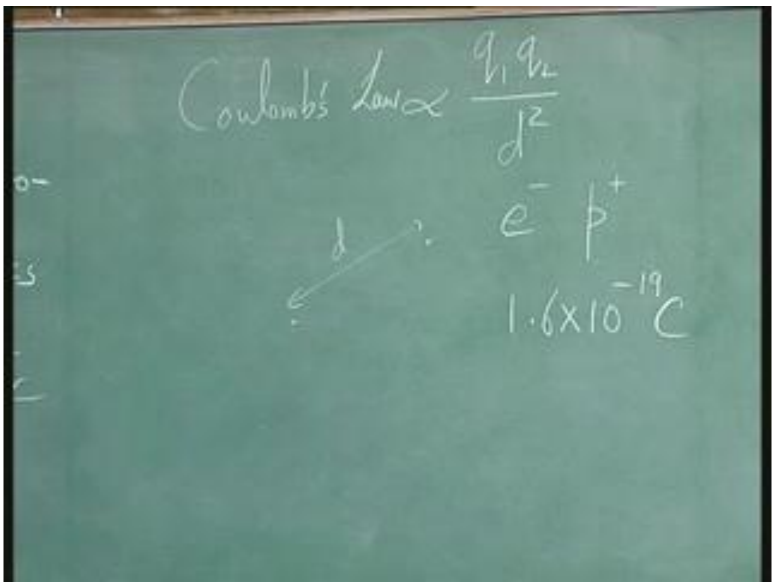

Okay now you also know quite a few other things about electricity and magnetism which you have learnt in school, you have learnt in college. One of the things you know is that, if you have currents, if you have a wire and the wire is carrying current, then a magnetic field develops and the magnetic field goes around this wire. It doesn't radiate away from the wire. It circles the wire. And there is a rule for how it circles the wire. You are supposed to take your right hand, point your thumb along the current and the direction in which your fingers will curl will tell you the direction in which the magnetic field will go around, all right?

Now there is another thing that you know, which is, if you have a magnetic field like this and you put a proton, this proton will feel the force, if it is moving. So if you have this proton, it's got a velocity v, this proton will feel a force, in fact, the general force law that you have is, if there is a force, it equals the charge times the electric field plus the charge times the v cross B of the magnetic field.

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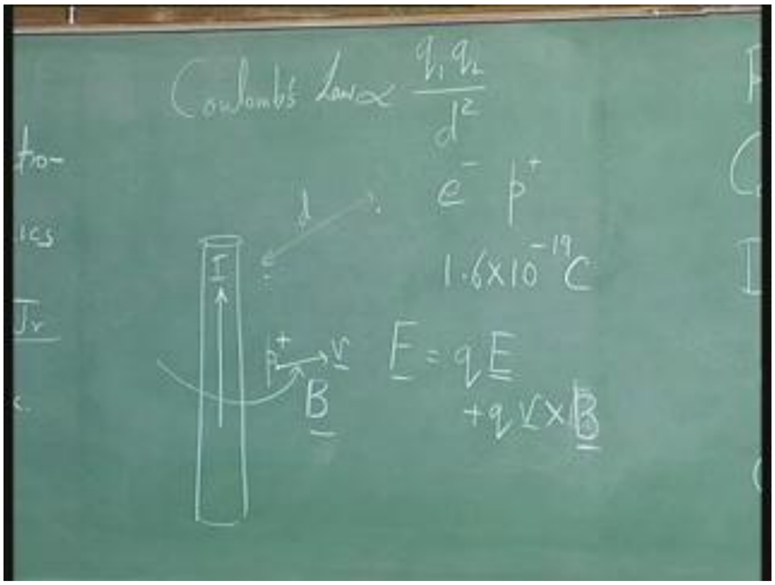

When I put lines underneath these variables I mean they are vectors. This is just one kind of notation. The other kind of notation you could have written is, q E with an arrow above, plus q v, with an arrow above, cross B, with an arrow above. Both are used and both are all right. Since I am comfortable with this notation, and it is easier to draw, so I will be using it. But they mean the same thing, alright?

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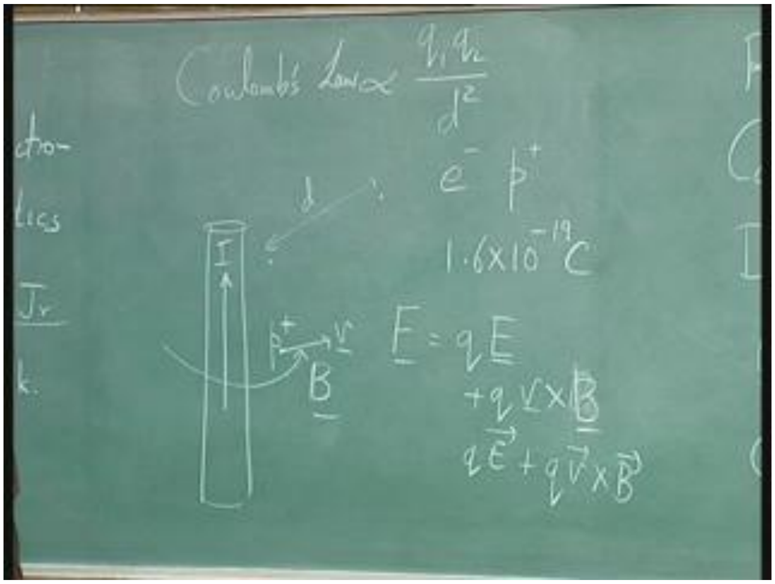

Okay now what else do we know about electricity and magnetism? We know that there are things called capacitors. You can go out to the electric electronics shop and you can buy them. These capacitors are made of silver foil and are separated by some insulator. Symbolically we draw them like this. So you have aluminum-silver-copper plates which are separated by a very small distance. When these plates are brought very close to each other and then you apply a charge, you put a charge on the top plate and a minus charge in the bottom plate, you find that there is stored energy inside here and this leads to a concept we call Capacitance. And you know that the capacitance of a plate capacitor like this is equal to epsilon A by d where d is this distance and A is the area. So this is something you would have learnt in school.

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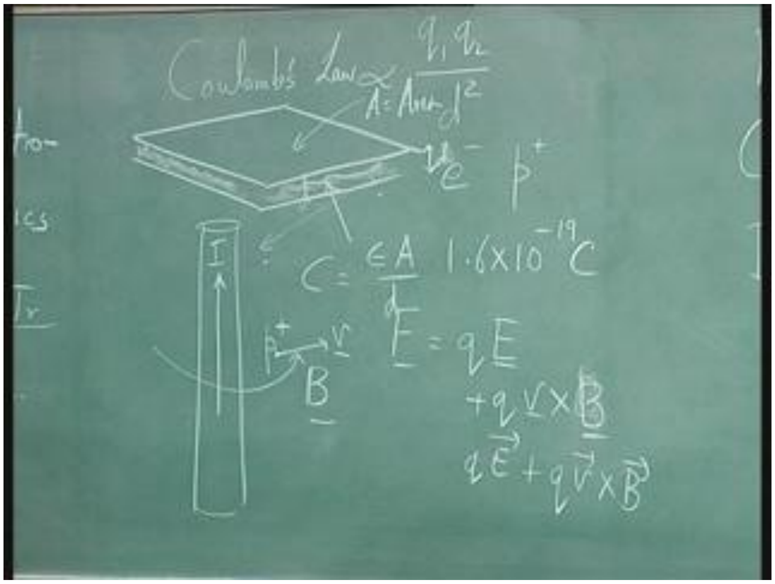

Now similarly you have learnt of magnetic stored energy. Magnetic stored energy is comes in the form of inductors they are crucially important for machines because all machines in generators are basically inductors. Transformers they are all inductors.

So what is an inductor? An inductor is a coil in which current is flowing. This current, as it flows, creates magnetic field, and as we will show later, using these same arguments here, the inductance the the current in the coil creates a magnetic field that threads all these coils all these turns. This magnetic field represents stored energy. Again it is a different kind of stored energy called magnetic stored energy.

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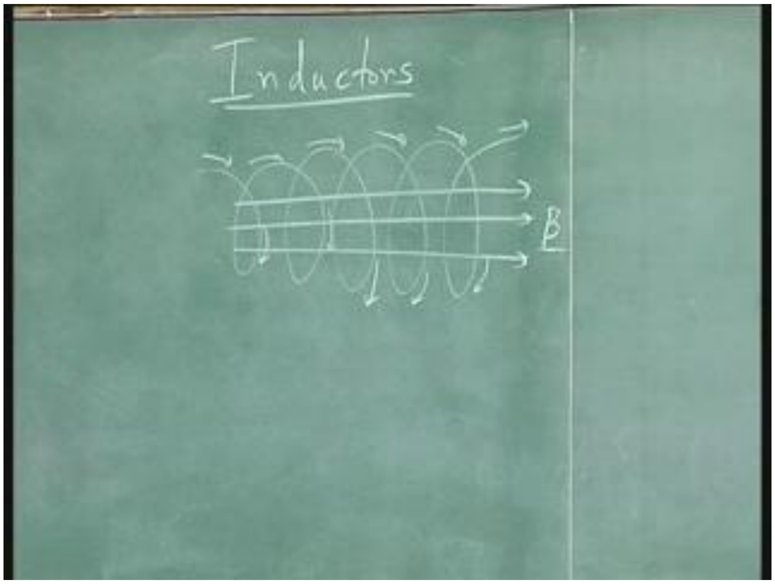

Okay so the these are the kinds of concepts we have already met. We know of Coulomb's law. We know that currents can create magnetic fields. We know magnetic fields and electric fields can create force. We know that if you bring plates very close to each other you can create something called Capacitors which store electric energy. If you twist turns of wire very close to each other you can create magnetic fields which are stored magnetic energy.

So these are all the concepts you already know. So what is there left to learn? Basically in this course what we are going to learn is how to develop flexible ways of thinking about electric and magnetic fields and how to develop powerful techniques to analyze these electric and magnetic fields.

Supposing the geometry of this capacitor was not simple, what would we do? Well we have techniques for calculating capacitance of anything. So that kind of topic is what we will be attacking in this course. What are the tools that you will require? Clearly, electromagnetic theory is a very mathematical subject.

The mathematical subjects that you will need to be good reasonably good at include vectors- we will review vectors- but still you should have learnt them. You need to understand differential equations. More than vectors you need to understand what vectors can do in the presence of differential equations, so this is called vector calculus and you need a different form of differential equations called partial differential equations. Up to about fifty years ago that's all you required but since the advent of computers the entire field of electromagnetics has completely changed and a new tool has appeared and this new tool is now universally used in this subject and this is the tool of computer modeling and simulation.

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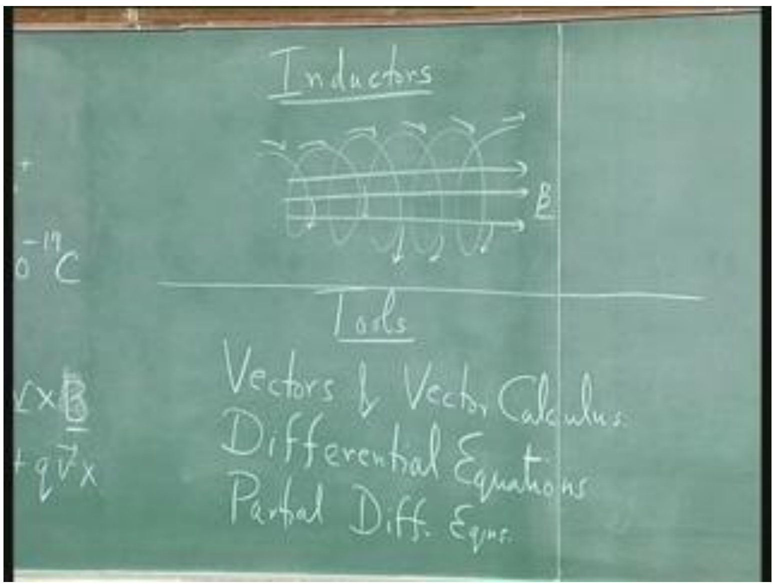

So if you look at the four topics, you need to understand what vectors are, you need to understand vector calculus. We will be covering this area subject in some detail in this course, the textbook does quite a bit of treatment on it. Differential equations, you should have learnt by now. We will basically assume that you know how to solve differential equations. Partial differential equations- well, the techniques for solving them we will learn in this course, but you will have to you will have to have already learnt some amount or else you will find this part rather difficult. As far as computer modeling and simulation is concerned the techniques that are required will be introduced in the course itself. Okay so that's as far as what the course is going to be, what the course requires and what we plan to cover in the first twelve weeks.

Now for the rest of the lecture I am going to talk about some other preliminaries, namely, vector fields.

Okay so when we talk about electricity and magnetism we deal with certain kinds of quantities, for example, we say that if a charge Q is present and another charge q is also present then wherever we keep this small charge q there is a force on it. If the charge q is kept here, the force is in this direction. But if I took the same charge and put it down here the force is in this direction. That's what Coulomb's law tells us. But we can pretend even before the charge was placed here the force was waiting to happen. So that when the charge is placed the force is actually felt. The concept of a force defined by every point in space is called a force field. So force field or a field in general is a quantity which is defined for all positions.

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So a quantity like q 1 is not a field, q 1 is a single value, q 2 is again a single value, but when we talk about variable distance d, this force which is a function of d- d is a variable- that force becomes a field. So field is a quantity and is a physical quantity that takes on different values at different locations. So the commonest example is the force field. There are plenty of other examples. For example, supposing this was the earth and there is an object suspended above the earth, this object feels a force. The force is the force due to the gravity. Now there is a there is a gravitational field, namely, a force waiting to happen, which is surrounding the earth. In all directions it is trying to pull things towards the earth. When the object is here it gets pulled down this way. When the object comes here it gets pulled down this way. If the object comes all the way here it gets pulled down this way.

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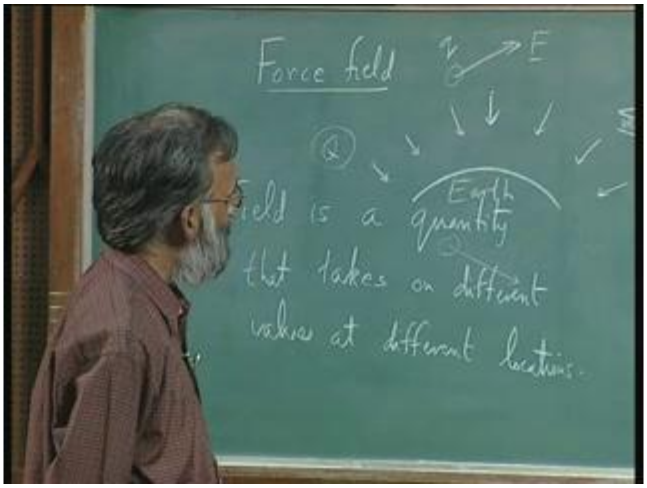

So this is another field and it is called gravitational field. Similarly you can have a field which is simply the location of a point. Every point has coordinates x and y and if it has height it has a coordinate z. Now if we define some point, let's say the centre of the earth, as our origin, then we can draw a line from the centre of the earth to any point and that point would have an x, a y, and a z.

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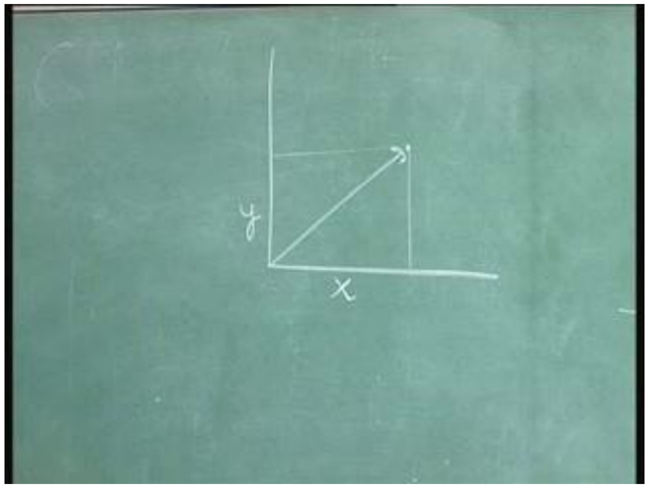

Now supposing we attach to x, the direction along x, and I note denote it this way, I put a little carat on top of the quantity. What I mean by that is, take this direction, give it magnitude one so it has no magnitude, its magnitude is just one but it has direction. The magnitude is sitting inside x. The direction is sitting in x carat. Similarly, magnitude inside y, direction inside y carat. Magnitude inside z, direction inside z carat.

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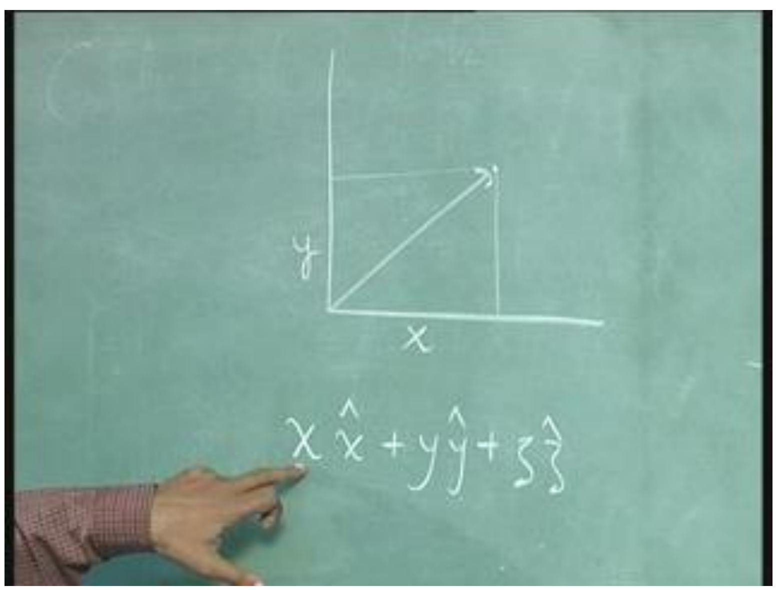

This defines a particular direction. It is the distance from the centre of the earth to wherever we are measuring. Now this too, is a vector field because it depends on where you are measuring. You go to another point, that point has a different value for x, y and z. And because it has a different value for x, y and z, it is a different vector. So this too is a vector field. Not only can you have vector field, you can also have scalar fields. For example, if you look at the newspaper it will show you the map of India and it will tell you what the temperature is at different points in the country. That map showing you isotherms saying that this is thirty four degrees, thirty two degrees, this is thirty degrees, this is actually defining a scalar field. It is saying that at every point in this subcontinent, you can associate a temperature. The scalar field is the temperature. At this point, the temperature is thirty four degrees. At this point, the temperature is thirty two degrees and at some point on this line the temperature is thirty degrees.

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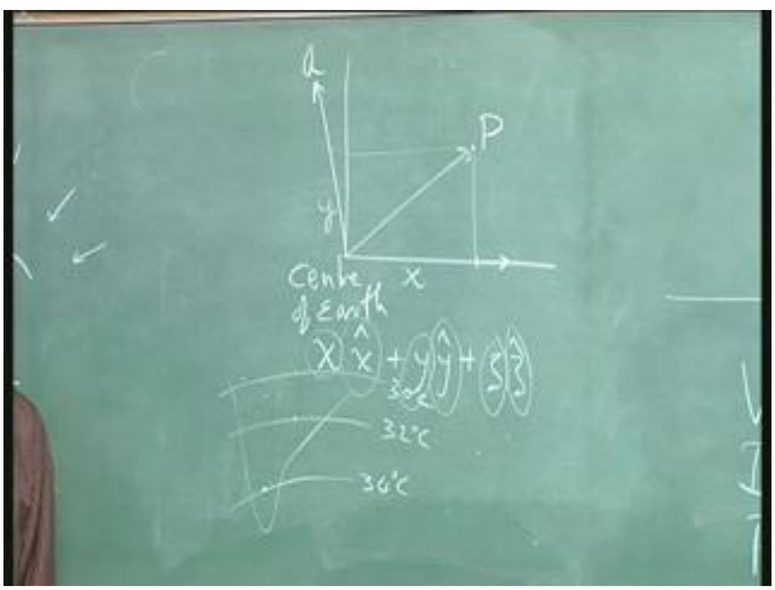

Like temperature, you can have many other kinds of scalar fields, for example, pressure; temperature itself, and a well known scalar field that we will encounter, which is electrostatic potential. These are all scalar fields. The difference between scalar fields and vector fields is only one thing. Scalar fields have magnitude. They have a number but they don’t have a direction. Vector fields not only have a number they also have a direction. The all of these- whether it is scalar fields or vector fields- the quantities we are looking at, we are describing, must be physical quantities. So we cannot say that the colour of a car is going to be a scalar field. That is a very arbitrary kind of quantity, so it doesn’t come under this kind of description. Okay so if it is a physical quantity we are describing, that qualifies to be a field, and we can write equations for it, and we can solve those equations and predict what these fields will do.

Now I have started talking about vector fields but I have not said what a vector is. That’s because you people are supposed to have learnt all about vectors earlier, but let me revise. We can draw coordinate system, that is, x, y, z and a vector is something that has a magnitude, that is the length and a direction that is, it makes angles with all three axes. So a vector is is an arrow. The length of the arrow gives you the magnitude, the number associated with the vector; the direction in which arrow is pointing determines its direction.

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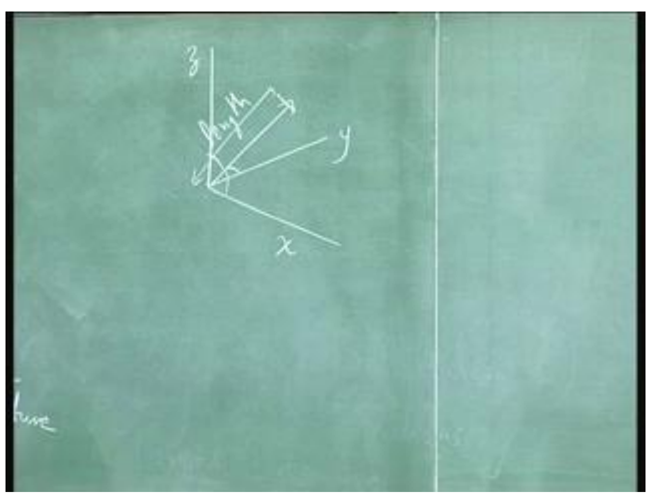

But that is all we can do with vectors. They are not of any use. Most vectors have certain operations you can do with them. For example, vectors can be added. How do you add a vector? We start with the first vector and place the second vector with its foot on the nose of the first vector and then join the tail of the first vector with the nose of the second vector. This dashed line is defined as the sum of this vector and this vector.

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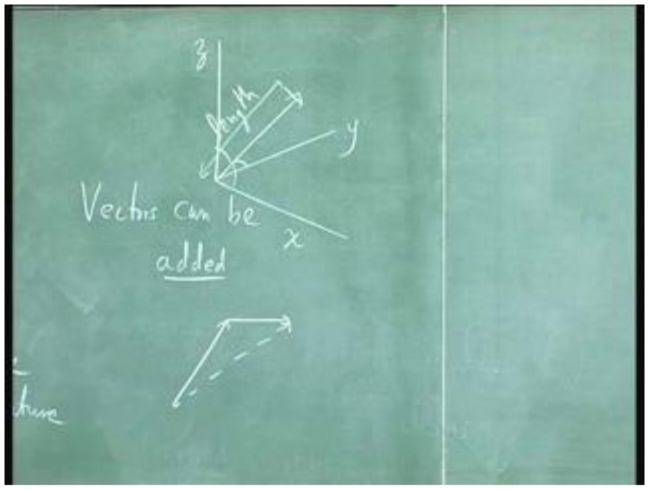

Now in Mathematics, whenever you define operations you like to make classify them. And your textbook classifies them quite nicely. This way of defining addition; it is commutative, it is associative, it has an identity, you can read all that in your first chapter. But for us, it is enough to know that we can place one vector on the top of other vector or we can place the other vector on top of the first. Since these this is a parallelogram whichever way you do it, it will work the same.

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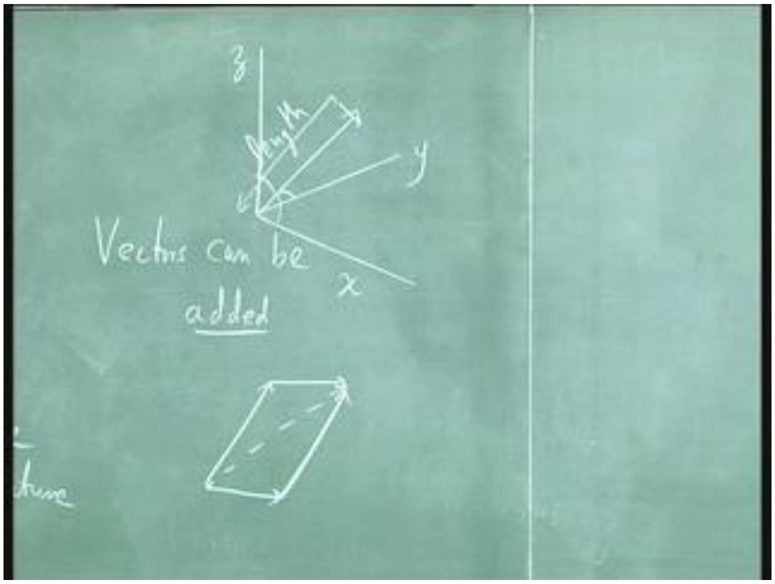

Vectors have another property, which is, they can be scaled. See, if I have a vector, I can also define twice the vector- which is a line that's twice as long- which is a line as twice as long, pointing in the same direction. So I want three times- I draw a line that is three times as long, pointing in the same direction. So vectors can be scaled and they can be added. If you take these two properties and you combine them, you get something which is called a mathematically which is called a field. Mathematical properties of a field are written in your book. It is not an accident that we call the electric field a field. The electric field satisfies certain mathematical properties. These mathematical properties are basically that the vector addition behaves itself as we expect it to and the vector scaling also behaves itself pretty much as we expect it to. They are all common sense properties so I won't go in what those properties are. But now having defined these things coordinate is what are the operations you can do on them, besides adding and scaling? The two important operations you can do which are related to physical quantities are dot product and cross product. I will repeat this next time, but let me introduce these two concepts so you can think about them.

Supposing a ball is rolling down a hill the force of gravity is downwards at every point. Now the ball when it rolls, it is not going straight down, it's going sideways. And we know that, if this were frictionless, that the kinetic energy of the ball, both in if you add up both the rotational energy and the falling energy, the kinetic energy of the ball, when it reaches here, is related to the loss of potential energy. So we know that increase in kinetic energy is equal to m g h. But the ball would have gained the same energy had it fallen straight down also.

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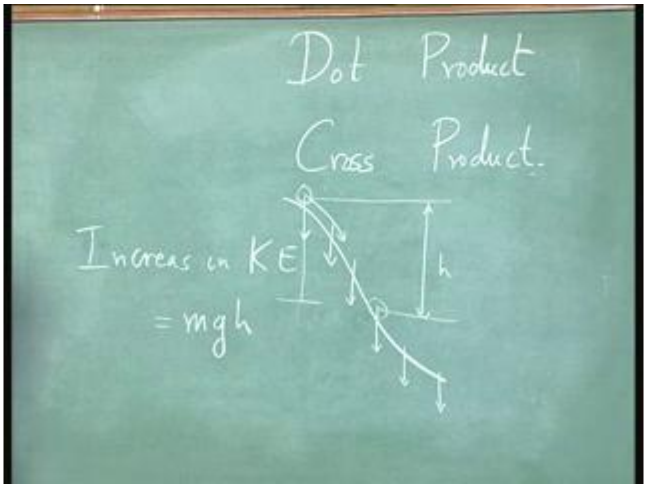

So it seems that how it fell to this point doesn't matter, all that matters is how much it fell. So in other words if the ball fell from this point to this point, straight down, it gained m g h. If it fell like this, it still gained only m g h. The force due to gravity is uniform and downwards.

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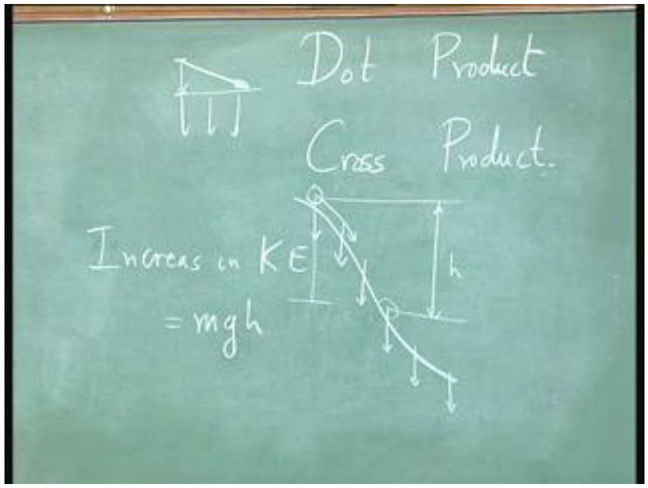

So somehow you need to get a number that is constant, regardless of which way the ball is falling, whether the ball is falling straight or the ball is falling sideways.

So the answer to that is we look at this angle. If you look at that angle theta this length, h you can write a relation. You can say that h is equal to l cos theta. Cos theta is, the base divided by the hypotenuse. And so h must be equal to l cos theta. So what that means is that, if you are moving in any direction, this is the direction you are moving, and this is the direction of force, the amount of energy you gain is not equal to force times the direction distance along the direction, rather it has to do with force times the direction times cos theta. That’s what you get here that is instead of m g l, you get m g h, substituting for h you get m g into l into cos theta.

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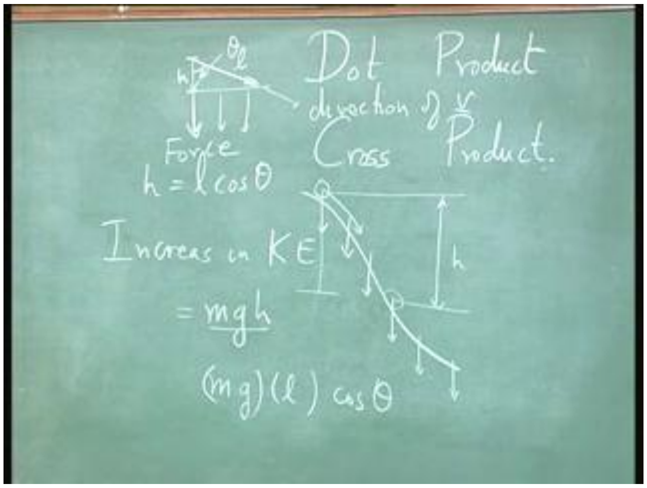

In vector notation the same idea can be written down. The idea is you have the vector m g. I have put a line underneath the g to indicate it is now a vector. This 1 can be given a direction as well, l and then this is cos theta and we take care of cos theta by putting a dot. This dot implies taking the angle between the two vectors finding the cosine and multiplying the magnitudes. This operation is extremely important. It is probably the most important vector operation you know. It is very important in all operations we do regarding work, regarding kinetic energy, potential energy and you would have encountered in many places.

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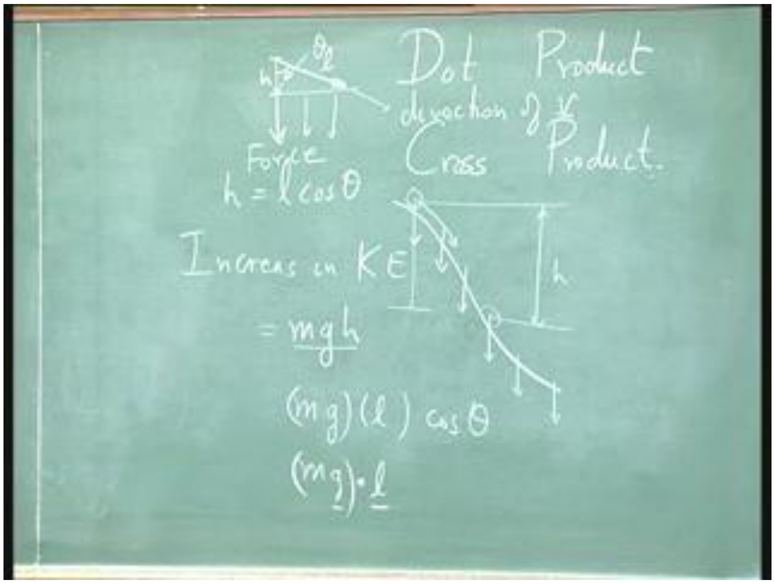

Now let's take a look at another operation which is the brother of the dot product. When a magnetic field is present and you have a proton, and the proton is moving in a certain direction, you find that the proton in the presence of the magnetic field B does not move either, does not change its direction either in the direction B or in the direction v. In fact the proton tries to change its direction in the third direction. It tries to go ninety degrees to B as well as ninety degrees to v. And what is all the more, the amount by which this force happens, this force is proportional to the magnitude, the number associated with v, the length of B into the sine of this angle.

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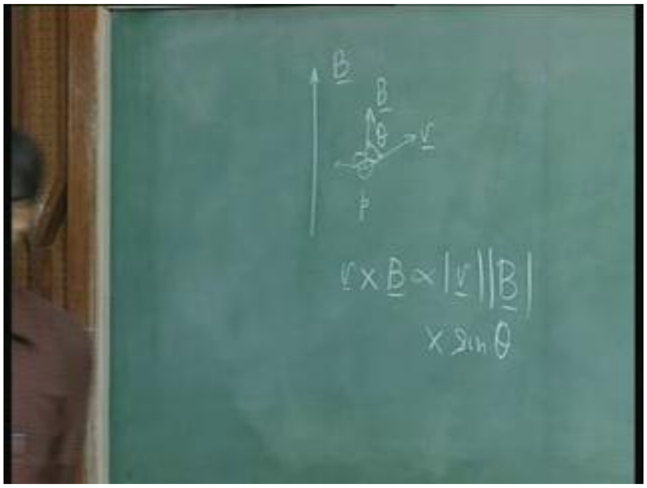

Now in the dot product- the magnitude multiplied by cos theta, in the cross product magnitudes are multiplied by sine theta. But the most important thing of all is the direction that B cross v has. Let me summarize: dot product is a scalar, it has only magnitude- no direction and it has the vector dot another vector is the magnitude of the first vector magnitude of the second vector times cos theta. The cross product, it’s a vector itself. So cross product not only has a number associated with it, it also has a direction associated with it. So if I have a vector v and I find its cross product with a vector w, its value is the length of the vector v, the length of the vector w, times sine theta. And its direction is in the third direction namely normal to v and w. This is a very important place where this keeps appearing.

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Supposing we have two vectors which are at right angles. Then we know that the area of this square is this vector magnitude of the first vector multiplied by magnitude of the second vector. This, this is just ‘multiplied by’. Area of the rectangle is product of the two sides, but supposing you have two vectors which are not at ninety degrees to each other then we know that we have to draw a parallelogram and the parallelogram has an area that’s equal to the rectangle with the same base. So it is this distance which is magnitude. I will call it v. This distance is the magnitude w, but this height is magnitude w sine theta.

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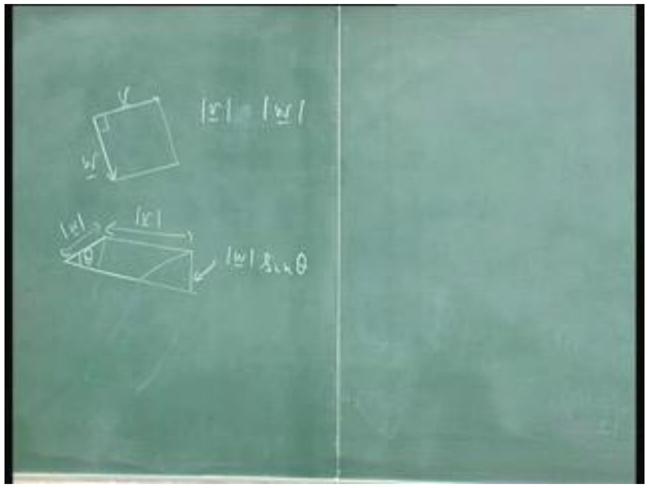

So the area of this parallelogram- it is same as the area of the rectangle- is magnitude v magnitude w sine theta. And this is the same formula that you get for cross product. You can see that cross product deals with sine theta. So the cross product is actually very closely related to finding the area of rectangular objects when you have two vectors which are not at ninety degrees with each other the area is this and when you try to give a direction to an area the only natural direction they give is out of the page. You cannot give the direction of an area as this way or this way. If there is any one direction that defines an area, it is the direction that is pointing away from the area. It's a very strange way of thinking about things but it has proved to be very, very useful in physics.

So if you now take, say a square. A square has some hidden sides also. This area will point outwards like this, this area will point outwards this way, this area will point upwards, that is, areas point away from their surfaces. These areas are also called normals and the direction of a cross product is nothing but the direction of the normal. So if you take any two vectors v and w, the idea of the cross product is, make the parallelogram that is constructed by v and w and then point in the direction of its normal. When you do that you have the direction of the cross product and the area of this parallelogram is the magnitude of the cross product.

As I said it's a very strange kind of definition but, in fact, not only it is useful, half the equations of electro electricity and magnetism use the cross product. They use a complication of the cross product but are basically the cross product. Where does the cross product come? Well the first place where it comes is in Ampere's law where you have a current I and you want the magnetic field B.

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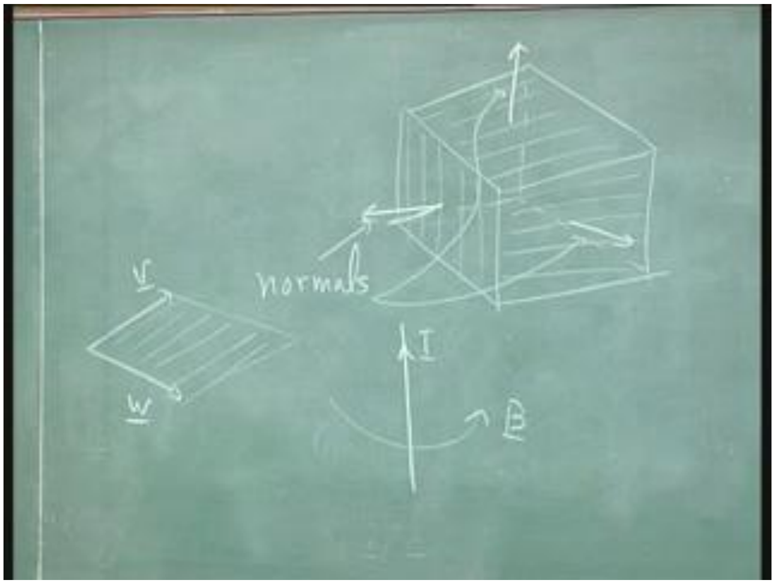

When you want the magnetic field B of a current, you would like to use Ampere's law. Ampere's law tells us that the line joining to this wire to B cross this current has something to do with the magnetic field magnitude and direction. The actual equation involves a differential equation, so I won't put it down right now, we will develop it as we need to. Similarly if you look at the force on a charge- we have already talked about that- it is q v cross B. So if a proton moves in a magnetic field, it feels the force that is perpendicular, that is normal to both v and to B. And so you find that if you have a magnetic field pointing in the z direction and the elec[tron] proton is moving in the x-y plane, the proton is at all times feeling a force that is again in the x-y plane but ninety degrees to the velocity. So what the proton does, it goes in a circle.

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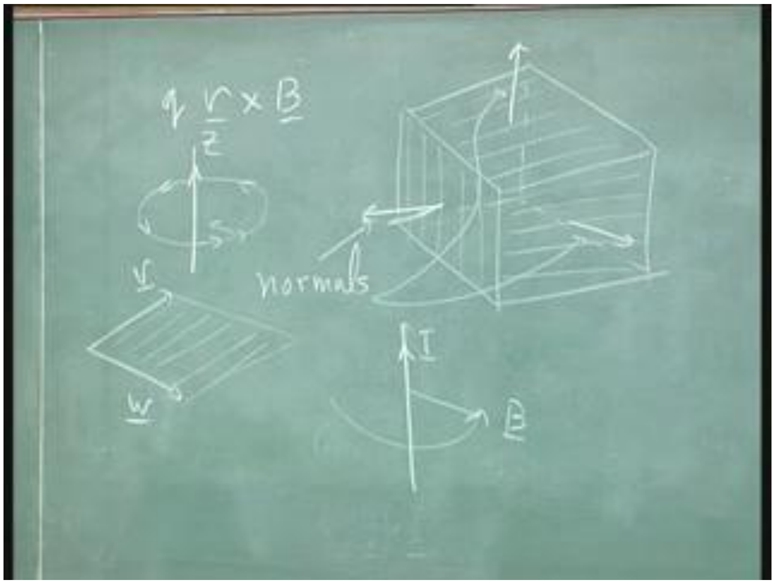

I will come back to this next time. This is basically one of the most common examples of the cross product.