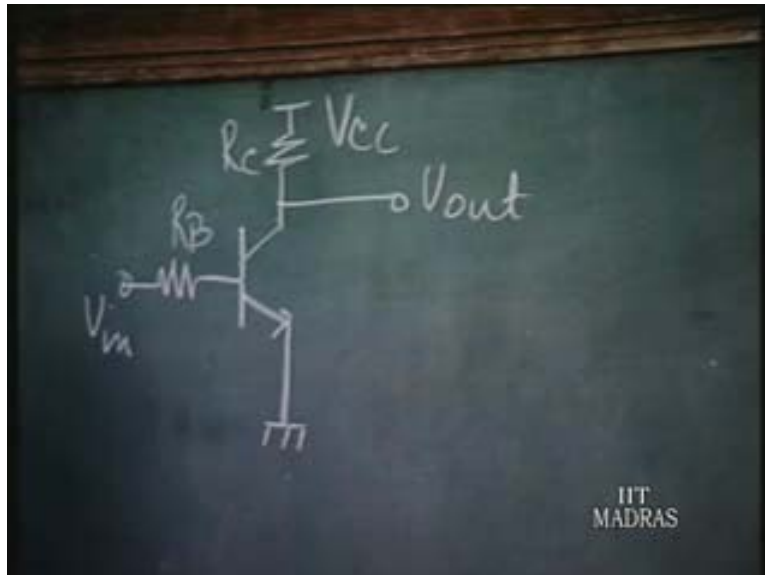


Digital Integrated Circuits
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Lecture-5
BJT Inverters DC and Switching Characteristics

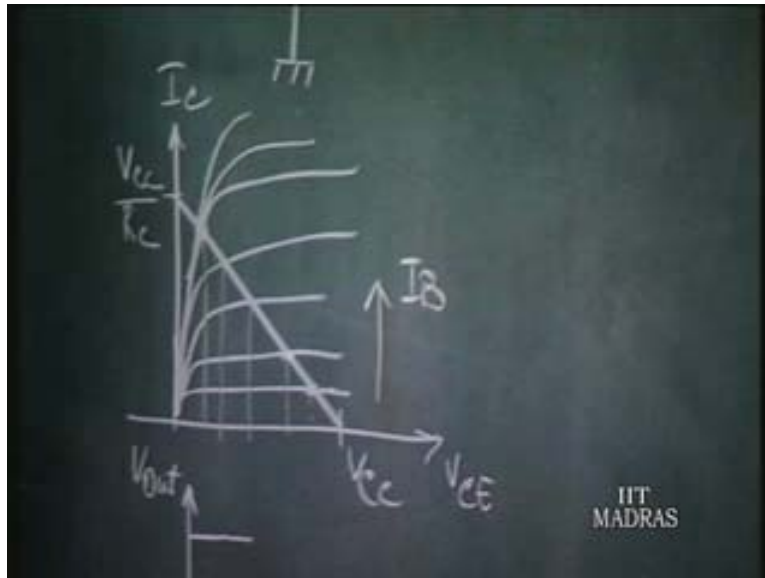
So last class we were discussing the circuit where the inverter circuit with the bipolar transistor and as I said it is very important to understand the working of the circuit because this forms the basis of almost all bipolar logic circuits. So we shall just do a brief recapitulation. What we had done is we have seen that if you have a circuit like this with a bipolar transistor and base resistance, collector resistance and a supply voltage and the input voltage here and the output voltage given at the collector.

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Now for this circuit how do you analyze? You have to draw the load line, so you take the output characteristics of the transistor and then you draw the load line and you know how to draw the load line. This is going to be a straight line and the points of intersection at the axis is given by V_{CC} on the V_{CE} axis and V_{CC} by R_C on the current axis. Now this is for different values of base current so we had seen that. Now if you plot the output voltage versus the input voltage, when the input voltage is low the base current is almost zero. The output voltage is given by V_{CC} and once it starts conducting then what happens is as you go on increasing the base current the collector current increases with the result, the output voltage drops. The operating point is given by the points of intersection of this output characteristics and the load line. So V_{CE} drops, output voltage drops till finally it comes to this point where afterwards if you go on increasing the base current the collector current is almost a constant and so we say that the collector current has saturated.

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Once it reaches this point, this value is called V_{CEsat} and this is the saturation current. What happens is if you plot the corresponding input output characteristics you get something like this. This is the plot which we have seen, so this is the characteristics of an inverter. When the input voltage is low the output voltage is high and when the input voltage is high, the output voltage is low so this is the characteristics of an inverter. This is something which we have already done. Now let us discuss it a slightly more quantitatively. When the transistor has gone to saturation the collector current is given by I_{Csat} which is equal to V_{CC} minus the collector emitter drop V_{CEsat} divided by the collector resistance.

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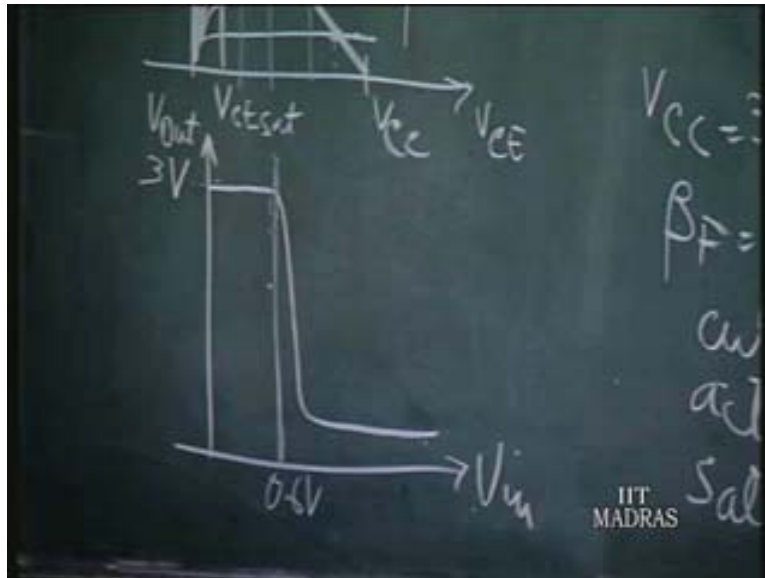
$$I_B > \frac{I_{Csat}}{\beta_F}$$

$$I_B > \frac{V_{CC} - V_{CEsat}}{\beta_F R_C}$$

Now we also know that when the transistor is in the active region the collector current is related to the base current by a factor β_{F} , forward beta of the transistor but once when it goes to saturation then the ratio of the collector current to the base current becomes less than β_{F} . We can say that for the transistor to go to saturation I_B must be greater than I_{Csat} by β_{F} for the transistor to go to saturation which means that I_B must be greater than V_{CC} minus V_{CEsat} by $\beta_{F} \cdot R_C$. Now we can actually put some values and see for example let us suppose for this circuit V_{CC} is equal to say 3 volts, say R_C is equal to 1 kilo ohm. Let us put R_B is equal to 0.5 kilo ohm and say β_{F} is equal to 50.

Now in addition some conditions we had already discussed last class that for example just for the sake of analysis let us assume that when the transistor is cut off V_{BE} is equal to less than 0.6 volts, in the active region V_{BE} lies between 0.6 and 0.7 volts and for the transistor to be in the saturation region V_{BE} is equal to 0.7 volts and also let us assume that V_{CEsat} is equal to 0.2 volts. Now with this data we can actually find out the different points in this characteristic. When V_{in} is zero volts what is the value of V_{out} ? V_{in} is zero means, I_B is zero and so I_C is also zero, so V_{out} is equal to V_{CC} . This voltage here this characteristics, this is going to be equal to 3 volts that is equal to V_{CC} . Now the next point is what is the value of the input at which this output starts to fall? This value that is when the transistor is in cut off the output is equal to V_{CC} and when the transistor starts conducting the output is going to fall, so that voltage is equal to 0.6 volts. After 0.6 volts when V_{in} exceeds 0.6 volts then that is going to be the base current and the corresponding collector current with the result the output is going to fall. So from 0.6 to 0.7 volts the transistor is in the active region and it behaves as an amplifier and you know that this amplifier has a negative gain.

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So you know that a sort of common emitter amplifier, the output is going to fall as the input increases. Now the next thing is this point. At this point the transistor has gone to saturation.

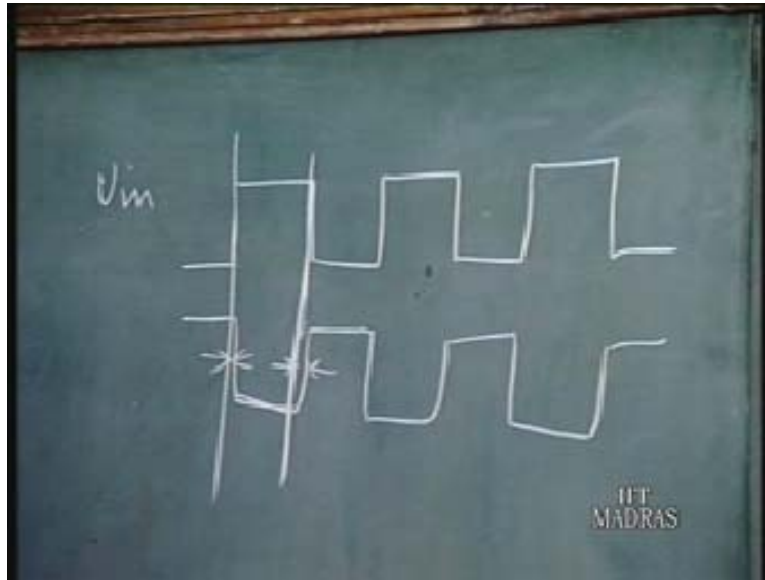
Once it has gone to saturation what is the output voltage equal to? 0.2 volts and what is the input voltage at which the transistor has gone to saturation? How do you calculate that? You have to know first what is the base current at which the transistor goes to saturation? What is the base current, how do you calculate? You can calculate from the relation given here that I_B must be equal to V_{CC} minus V_{CEsat} by $\beta_F R_C$. That is the relation, now we know all these values. So we can plug in these values and find out the base current at which the transistor goes to saturation. So what is it? $3 - 0.2$ divided by 50 into 1 k . If you do that I think it should come to 56 micro amperes and so that is the base current when the transistor has gone to saturation.

Now we have to find out the input voltage at which this goes to 0.2 volts that is easy because we know this voltage is 0.7 volts when the transistor has gone to saturation plus I_B into R_B . That is 56 micro amperes into 0.5 kilo ohms that will give you the drop here, so that is the input voltage when the transistor goes to saturation. This value if you do it, I think it works out to 0.728 volts. This actually gives us the total characteristics so that is how you can do it quantitatively. Finally we see that if the input voltage is low that is below 0.6 volts, output voltage is equal to 3 volts and when the input voltage exceeds 0.728 volts in this case that is the input is high, the output voltage is going to be low which is 0.2 volts in this case.

This is the inverter characteristics, so for this inverter circuit when the input voltage is low the output voltage is high and when the input voltage is high, the output voltage is low, so that is one aspect of a inverter. If you have a input waveform that is an input waveform like this what do you expect? When the input is low, the output is high; when the input is high, output is low. You expect something like this so is an inverted waveform. So that is what is important for us in digital circuits but actually there is

another aspect to this. One is of course that you should get something like this in a digital circuit but the another important aspect is when this input changes from low to high, how much time does it take for the output to react, that is to change from high to low?

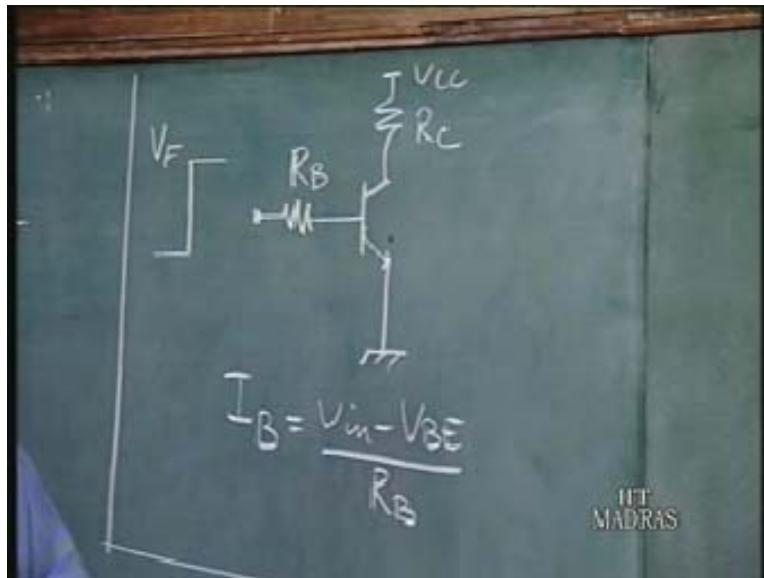
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That is going to be a small displacement so that is the delay. That is also very important aspect of the circuit and one must really design in such a way that this delays is going to be minimum. The other delay is when the input goes from high to low, the output changes from low to high and so you have something like this and these delays need not be the same. So we shall now take up this aspect, the same inverter circuit from another consideration that is the switching consideration. That is how much time does it take from the switching point of view to react when the input changes. So for that let us assume initially that for the same circuit which we have drawn, I am just going to draw it again. It's the same inverter circuit is very important to understand this.

Suppose the input goes from low to high, suppose this was low it went to high. This low means the transistor was in cut off, there was no base current suddenly it has gone to this value where the input voltage is V_{in} . Then what happens, what is going to be the base current? There is going to be a base current, isn't it? This base current is going to be equal to V_{in} minus whatever is the V_{BE} divided by R_B . So I can write I_B is equal to $V_{in} - V_{BE}$ by R_B and you know in a forward bias this base emitter junction is obviously going to be forward biased and this V_{BE} is usually in this type of circuit where you have a limiting resistance, this voltage maximum it can be 0.7 or may be slightly higher.

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If V_{in} or this V_F basically at this point in time when you consider this input has gone high to V_F , V_F is much greater than V_{BE} . We can say I_B is nearly equal to V_F by R_B if V_F is very much greater than V_{BE} . So you have an almost constant base current. Now the next thing to see is what happens, when you have a constant base current flowing? Initially there is a sequence of events which takes place actually. Now if you look at the base of the transistor, when this base voltage was low the input voltage was low, the transistor was cut off, what was the charge stored in the base? There was no excess base stored charge. Now as the base current flows in, a constant base current is flowing in what is happening?

The base charge begins to build up, so now for that let us consider a transistor. Let us go back a little bit, we have already discussed the transistor, the base current of a transistor. It consists of the forward conduction mode of a transistor, it consists of basically two components, if you remember. What are those two components? One is the injection of carriers from the base to the emitter and the other is the recombination of the charges in the base. Now if the doping concentration ratio width of the emitter and the base is very large that is the emitter doping concentration is very much larger than the base doping concentration, we can for all practical purposes neglect the current component that is the injection of carriers from the base to the emitter and we can say that the base current is almost totally made up of the recombination in the base.

So let us take a transistor where the emitter doping concentration is very much greater than the base doping concentration and the base current is totally composed of the recombination in the base. For that type of transistor we can write this relation that the base current is equal to Q_B by this is for an n p n transistor τ_{0N} plus this relation we have already seen when we discussed the diode.

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$$I_B = \frac{Q_B}{\tau_n} + \frac{dQ_B}{dt}$$

So basically if you understood a diode is very easy to understand the operation of a transistor. This is the recombination in the base and that is proportional to that is base charge divided by excess base charge divided by lifetime. So that is going to be the recombination part and so if the base current is more than this, what happens? The base charge is going to build up that is charges flowing in. So partly compensates for recombination and whatever is excess, it increases the stored charge in the base. As the stored charge is increased this Q_B term goes up so the recombination goes up, the rate of charge goes down. Finally what is going to happen because its building up, the rate is going down but the amount of charge is building up, finally this goes to zero, I_B becomes equal to Q_B by τ_n , you reach a steady state.

Basically for that particular amount of base current you will reach a steady state where I_B is equal to Q_B by τ_n but you understand right now that it is going to take some time. It is not an instantaneous process, it takes some time so what we are interested is how much time. So basically you have to solve this differential equation for the Q_B . If you solve this equation you will get, I am just writing the result you can very well solve it yourselves. You take the initial condition that Q_B is equal to zero, you will get $Q_B(t) = I_B \tau_n (1 - e^{-t/\tau_n})$, τ_n is of course the minority carrier lifetime in the base.

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$$I_B = \frac{Q_B}{\tau_n} + \frac{dQ_B}{dt}$$

$$Q_B(t) = I_B \tau_n (1 - e^{-t/\tau_n})$$

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That is electron volt, this you see that when t is equal to zero $Q_B(t)$ is zero $Q_B(t)$ reaches $I_B \tau_n$ only after an infinitely long time. That is you can visualize it from this equation itself so that is the relation. Now the question is we have put this equation, so basically what is going to happen is the total base charge is going to increase. Now I will just draw this figure, this is the base of an n p n transistor. Now the initial charge was zero so the charge is building up, so the charge profile is going to be something like this, with time it goes on like this, with increasing time this is the emitter side that is the collector side obviously. So the charge builds up like this, this is minority charge carrier profile in the base. So as with time it goes on increasing like this, you see that with time that is increasing.

What is happening to the collector current? As the base current is with time collector current is increasing for example here you see it was initially zero then it went here, so the collector current depends on the slope of this profile. So that keeps on increasing and as the collector current keeps on increasing what happens to V_{CE} ? It goes on decreasing and so it goes on increasing V_{CE} goes on decreasing. When V_{CE} becomes equal to V_{BE} what is the condition of V_{BC} ? That is the collector emitter voltage is the same as the base emitter voltage, base collector drop is zero.

Now if the collector current keeps on increasing even further what happens to V_{BC} ? V_{CE} will become less than V_{BE} that is the collector voltage will be less than the base voltage. What does it imply? That the base collector junction has become forward biased that means the transistor goes to saturation. So see that it goes up like this and it will go on increasing you see until the total charge stored here becomes equal to $I_B \tau_n$. Finally it will go up $I_B \tau_n$, the charge will keep on building up although the transistor has reached saturation. It will not stop till you have reached $I_B \tau_n$. Now so what is going to happen is so what we have to know is what time? So basically the idea is when you apply an input voltage, if you look at this figure you have applied an input voltage here.

Now what do you want to do is does the input voltage goes high, you want to drive the transistor to saturation. What is the time taken for the transistor to reach saturation? That is when the input goes high we have seen an inverter, the output goes to 0.2 volts that is when the transistor is reached saturation. So what is the time required for that, how do you do that? In this equation we have to see at what time t that is $Q_B(t)$ equation, at what time t $Q_B(t)$ becomes equal to the value at saturation. What is this value at saturation? How much $Q_B(t)$, what is the value of $Q_B(t)$ at saturation? That is it is going to be equal to the collector current at saturation. We have already seen what is the value I_{Csat} into the forward transit time, we have already seen the definition of the forward transit time of a transistor. It is the ratio of the base charge to the collector current. That is the definition of forward transit time.

So we can write it in terms of the forward transit time so at saturation if you go back to this equation now $Q_B(t) = I_B \tau_{on} (1 - e^{-t/\tau_{on}})$. So you have to write at t is equal to say t_{on} that is the time required when the transistor goes to saturation. At t is equal to t_{on} , $Q_B(t)$ is equal to I_{Csat} into τ_{on} . So i will write in this equation when t is equal to t_{on} that is the t_{on} is the time required for the transistor to go to saturation after you have applied a step. $Q_B(t)$ into τ_{on} , I will write τ_{on} which is the transit time which actually is the forward transit time of a transistor. So τ_{on} is the transit time. So in that equation we have to substitute this expression, so once you do that, you get the value of t_{on} . That is another important relation we have to use, I hope you know this relation.

The beta of the transistor, forward beta of the transistor can be related to the lifetime in the base and the transit time because the collector current is equal to, see the base current is equal to Q_B by τ_{on} and what is the collector current equal to Q_B by τ_{on} . Q_B is equal to I_C into τ_{on} so collector current is Q_B by τ_{on} , so you can write β_F is equal to τ_{on} by τ_{on} because I just write since I_C is equal to, so from this relation you get τ_{on} by τ_{on} . So now if you substitute this (Refer Slide Time: 28:12) in this equation and you take this relation, you will get that t_{on} . I will just write down the equation for t_{on} , you can very well do the calculations.

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$$\beta_F = \frac{\tau_n}{\tau_t} \quad \text{Since } I_C = \frac{Q_B}{\tau_t}$$

$$I_B = \frac{Q_B}{\tau_n}$$

$$t_{ON} = \tau_n \ln \frac{1}{1 - \frac{I_C}{\beta_F I_F}}$$

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I think I don't need to do that, t_{ON} . In I think I called it I_F . I_F is of course the forward base current, the base current I am writing it I_F because later on we will see a reverse base current also so that is why I am writing here I_F . I_F is actually the base current, it is flowing in. now we have seen the condition for saturation. What was the condition for saturation, that $\beta_F I_F$ must be greater than I_{Csat} . **Sorry this is actually I_{Csat} .** (Refer Slide Time: 29:46). So of course if $\beta_F I_F$ is equal to I_{Csat} it is never going to saturate, the transistor is never going to saturate. You can look at this, if I_C $\beta_F I_F$ is greater than I_{Csat} this becomes a fraction and it is going to saturate. Let us take a small example suppose let us say $\beta_F I_F$ is equal to twice I_{Csat} , what is t_{ON} equal to?

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$$t_{ON} = \tau_n \ln \frac{1}{1 - \frac{I_{Csat}}{\beta_F I_F}}$$

$$\beta_F I_F = 2 I_{Csat}, t_{ON} = \tau_n \ln 2$$

$$\beta_F I_F = 5 I_{Csat}, t_{ON} = \tau_n \ln(1.25).$$

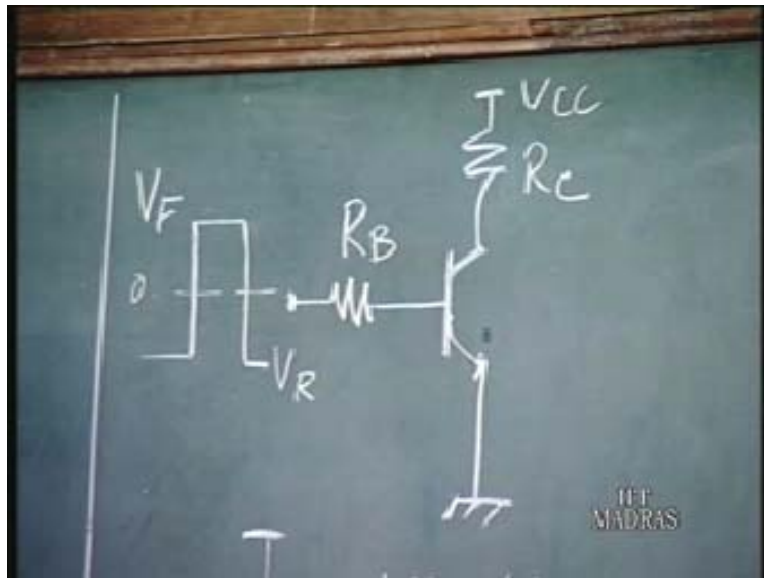
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This becomes half so it becomes $\tau_{on} \ln 2$. Now if $\beta_F I_F$ is equal to say 5 times I_{Csat} what is τ_{on} equal to? So this becomes one fifth so this becomes $\ln 1.25$. So what do we see here that if β_F is a constant for the device for the transistor now I_{Csat} also is a constant you must remember for a particular circuit that depends on the collector resistance also, collector resistance and the supply voltage and all that. So now if for a given circuit, if you increase I_F what happens that is the base current, if we increase the forward base current what happens is the time required for the transistor to saturate goes down. It reaches saturation faster, it is quite obvious because you are pushing in charge at a higher rate.

The increased base current means the rate of flow of charge, current is rate of flow of charge, higher rate of flow of charge, the charge builds up faster you reach saturation faster. So to have a fast switch on, you must have a large base current. It is not only sufficient to have $\beta_F I_F$ greater than I_{Csat} to reach saturation but more the base current the faster is the switching on time. Another thing one must remember is that all this is proportional to τ_{on} which is the lifetime. So if you want to have faster switching on, τ_{on} must be made lifetime must be made small but that also has certain disadvantage because the β_F of the transistor is τ_{on} by τ_{ot} , so if you make τ_{on} small β_F reduces. So τ_{on} must be made small, at the same time τ_{ot} also should be made very small that is the transit time.

The base transit time should be made very smaller. How can you make the base transit time smaller? By reducing the base width, so that is the design of a good switching transistor. You must make τ_{on} small but at the same time you should make τ_{ot} also much smaller that is you must have as small base width as possible. So this is the switching on transient, now we have to look at the other case that is from this you apply a voltage, from V_F say I should say this is zero, say let this be V_R that is the reverse voltage is applied, negative voltage is applied.

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Now what is going to happen once the negative voltage is applied? The same way if you look at again we have to use again the same relation, this is the fundamental relation. So the base current now becomes, another thing what we have discussed in the case of a diode, the base emitter voltage cannot change instantaneously. You have a lot of base charge, now according to this relation the base charge is going to reduce now because this has become negative, dQ_B/dt becomes negative also but it will take some time. So again if you go back to this figure here which gives the charge profile, the charge profile which had built up is going to again go down and as it is going down say it is going to go down, the base emitter voltage is going to reduce but it will take some time. So again see for a long time this base emitter voltage that is the voltage at this junction is going to be positive unless the charge goes down to zero.

So basically what is going to happen is the charge is going down to zero and then at this end it will become negative. So for all this time until you have removed all this charge, the base emitter voltage is going to be positive and then it starts going negative. So now with increasing time I think the direction of the arrow is going to reverse. So this is with increasing time, this charge is going to reduce. So what happens is although you have applied a reverse voltage here at the input, this voltage for a long time is going to be slightly positive, going down towards zero. So what happens, again if you go back to this relation now I_B is equal to $V_{in} - V_{BE}$ by R_B , V_{BE} is again small compared to V_{in} . Now V_{in} is negative, again what you have is I_B is going to be, if you look at this relation instead of V_F you are going to have V_R . So I_B will be equal to V_R by R_B now. The reverse voltage minus R_B that is I_B is going to be negative.

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$$I_B = \frac{V_{in} - V_{BE}}{R_B}$$

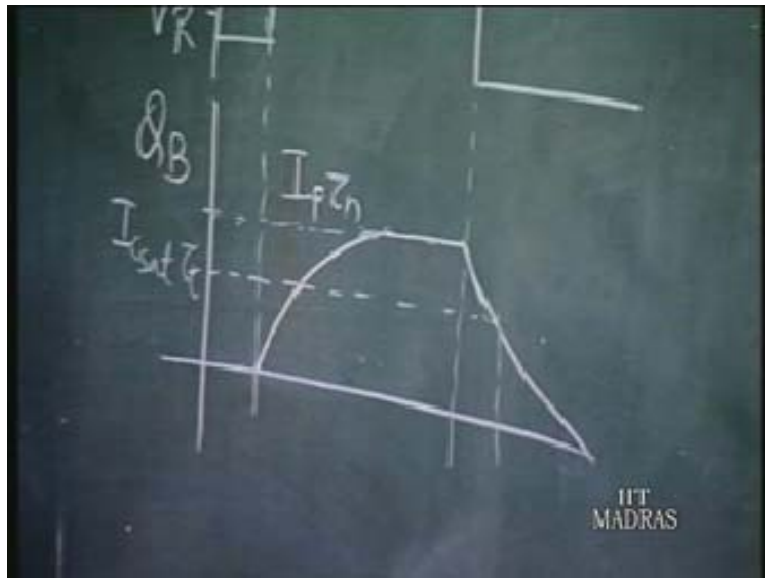
$$I_B \approx \frac{V_R}{R_B}, \text{ if } V_F \gg V_{BE}$$

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You are going to have a large negative current flowing out from the base, reverse base current is going to flow in this direction. Now what is the reverse base current doing? It is actually doing a good thing, it is removing the excess stored charge.

Again if you want to find out how much time now you have applied a negative voltage, so you want the transistor to go from saturation to cut off. So now let us draw another figure I think I will write here. So initially when you applied a voltage you see the input voltage going from negative to positive and then positive to negative this is the input voltage V_{in} , so this is V_F this is the forward voltage and this is the reverse voltage V_R . Now if you plot the charge Q_B what has happened is at this point, it went up exponentially and this value is $I_F \cdot t_{on}$ where I_F is the forward current. Now it reached saturation, sometime $I_F \cdot t_{on}$ is much greater than the charge required for saturation. So suppose if it reaches saturation at this point here that is this is equal to say $I_{Csat} \cdot t_{ut}$. So at this point it reaches saturation but the charge goes up all the way here. Now what happened is so it is going to be something like this, now you apply a reverse base voltage there is going to be a reverse base current and this charge is going to decay, so it goes down like this. We are pulling out charge, charges is going to decay.

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Now up to this point from here to here, although you have applied a reverse voltage the transistor will remain in saturation. So here up to this point because the charge is greater than what is required for the transistor to be in saturation. So up to this point the transistor remains in saturation and after that the current starts to fall and finally it is going to go to zero. So if you now look at the collector current, if I plot the collector current it will go something like this up to this point, the transistor goes to saturation. It is going to remain in saturation, even here it remains in saturation. Current has not fallen and then it is going to fall because only after this it is going to fall, this is the charge at saturation. So once the charge comes down to that then only it is going to decay, so this time for which the transistor remains in saturation even after you have applied a reverse voltage at the input is called the storage delay time, t_{sd} is the symbol which is generally used. So this is the storage delay time, it is very important. Here you have applied a reverse voltage the collector current I_C it will be like this, it remains in saturation even after that is there then it goes down.

So again how to calculate this t_{sd} ? Very simple the same relation here, we have to take this I_B is equal to minus $I_B(r)$ and again what is $I_B(r)$? We know that the input voltage for all the time even when it is saturated or goes down to zero current is slightly positive. So you can take, it is almost equal to V_{in} by R_B . That is minus $I_B(r)$ so minus I_R you can say because forward current we have taken I_F so this is I_R . So this same relation you solve again and there is an initial condition, you solve this same equation that is I_B is equal to Q_B by τ_{on} and the initial condition is $Q_B(0)$ that is base charge at time zero is equal to $I_B \tau_{on}$. Now if you solve this equation, again I am not solving this equation for you; you can very well do that. You get a τ_{on} , I am just writing the result $I_F I_R e$ to the power minus t by τ_{on} minus $I_R \tau_{on}$, so this is the relation.

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$$Q_B(0) = I_B \tau_n$$

$$Q_B(t) = \tau_n (I_F + I_R) e^{-t/\tau_n} - I_R \tau_n$$

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Now how do you find out the storage delay time? At t is equal to t_{sd} , $Q_B(t)$ is equal to $I_{Csat} \tau_{out}$. That is you can see for yourself, if you go back at this curve here at t is equal to t_{sd} , this is equal to zero. So t is equal to t_{sd} , the charge will go down to I_{Csat} into τ_{out} . So if you solve this equation now you will get t_{sd} is equal to, I just again write down the relation $\tau_{out} \ln I_{Csat}$ by β_F plus I_R . This I_R is of course, these are the modulus values.

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$$At + t = t_{sd}, Q_B(t) = I_{Csat} \tau_n$$

$$t_{sd} = \tau_n \ln \left(\frac{I_R + I_F}{\frac{I_{Csat}}{\beta_F} + I_R} \right)$$

So this t_{sd} that is the storage delay time is $t_{ou.n}$ that is the lifetime $\ln I_R$ plus I_F , I_R the modulus the amount of reverse current, we don't take the sign I_R plus I_F divided by I_{Csat} by $\beta_{F.}$ plus I_R . So this is the relation for the storage delay time. Now obviously if I_R becomes very large that is you can neglect I_F with respect to I_R and also I_{Csat} by $\beta_{F.}$ with respect to I_R . What is t_{sd} equal to? It is equal to zero because the $\ln$ one is zero. Obviously if I_R is very large, you are removing the charges faster and so the extreme case storage delay time is zero, I mean it becomes infinitely large but normally you can also see that with I_R is larger t_{sd} will be less.

If you go on increasing the reverse current, the storage delay time will be less because this fraction is going to reduce. What about with I_F , how does it depend on the I_F that is the forward current? If I_F goes up, t_{sd} goes up. Why? Because if i_f was larger than $Q_B(0)$, this initial condition this is I_F into $t_{ou.n}$, that is the forward base current into $t_{ou.n}$. So the initial charge itself was larger, if the forward base current was larger. So to come down to $I_{Csat} t_{ou.t}$ it is going to take more time. If you have larger forward current, you are storing more excess charge in the base so it will take more time to remove that charge. So the delay is going to be larger but you cannot reduce I_F because if I_F is small, the on time is going to be high.

So you actually require a large forward base current to switch on the transistor faster and you will require an even larger reverse current to switch off the transistor faster, I_R must be greater than I_F . In order that t_{sd} is small, storage delay time is small. Then of course finally so if we come back to this curve again so this is the storage delay time and then there is another delay time before the transistor actually goes to cut off. That is what is called the discharge time.

In order to calculate the discharge time its again very simple, you again take this relation here $Q_B(t)$, this relation is for $Q_B(t)$ and you take the value at discharge t_d that is the total time which required for the transistor to totally cut off $Q_B(t)$ is equal to zero. You solve for that and that will give you the total discharge time. I think I will take that up in the next class. That gives us the total picture of the different delays involved in switching the transistor on and off and one must really appreciate this and because if you want to compare between the different circuits, how fast is one circuit compared to the other. That is mostly due to, what are the different mechanisms for removing base charge or creating this base charge in a bipolar transistor. So we shall take up again in the next class.