Digital Integrated Circuits
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Lecture-3
Modelling of BJTs

Last two classes we were discussing the p n junction diode and now we move on to the next device in terms of complication, BJT the bipolar junction transistor. The model which we shall discuss for the bipolar junction transistor is the Ebers Moll model which is simple yet very useful model and it can be applied in most cases and we shall be applying it also in our discussion on integrated circuits. So Ebers Moll model of the bipolar junction transistor considers the bipolar junction transistor which you know is a three terminal device consisting of 2 p n junctions as two diodes connected back to back.

If you have an n p n transistor, you have one p n junction and the other p n junction like this, in addition you have two current sources. These current sources gives us the coupling between the two junctions. So this is the emitter terminal say this is the collector terminal and this is the base terminal. If we call this current $I_{F.}$ which is the forward current which is given by $I_{ES.}$ which is the same diode relation which we are writing now and you have the other current $I_{R.}$.

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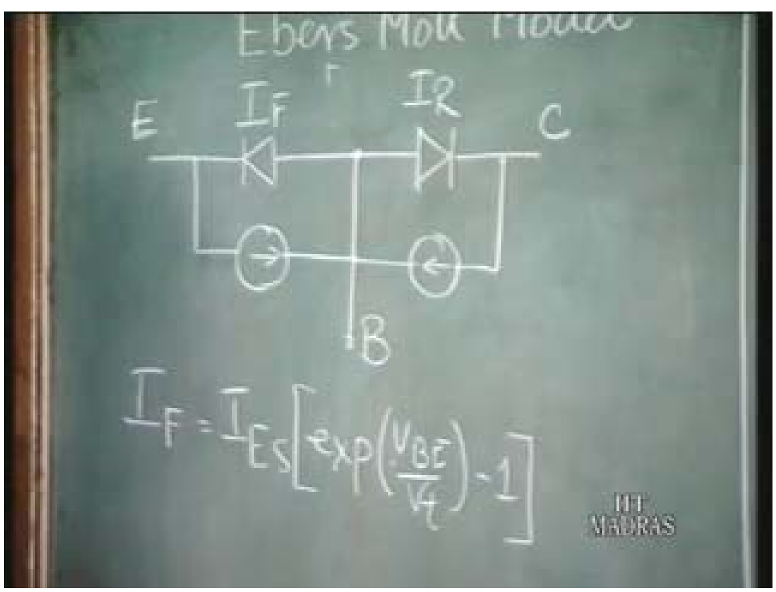

So this is the base collector junction, this depends on the base collector voltage $I_{R.}$ is equal to another constant and these current sources are actually the coupling between the two junctions. So this is given by a constant into $I_{F.}$, the constant is called alpha$_{F.}$

and this current source is given by alpha$_{R}$ I_{R} where alpha$_{R}$ is another constant. So the idea of this model is that if you know the applied voltage between the junctions.
For example here you have two voltages emitter base and base collector, you should be able to evaluate the different terminal currents. That is the emitter current which in normal n p n transistor will be flowing in this way I_{E}, you have the collector current which flows this way and you have the base current which is this way. So we should be able to evaluate the three terminal currents in terms of the two voltages that is the emitter base and the base collector voltages.

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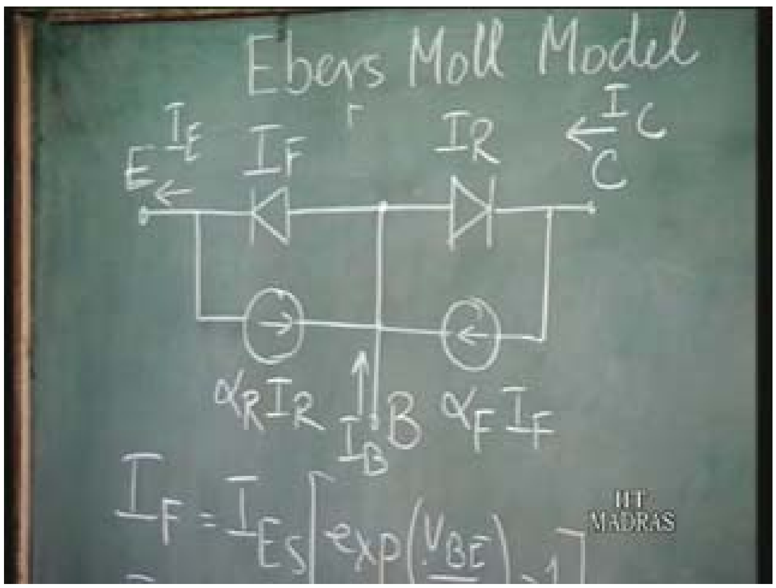

In order to do that now we see that we have 4 constants which are to be or 4 parameters which have to be known beforehand that is I_{ES}, I_{CS}, alpha$_{F}$ and alpha$_{R}$. If you know these parameters you will be able to evaluate the terminal currents. So now let us see what these parameters depend on, we shall go ahead in developing this model. Just before we go ahead, if we take for example $V_{BC} = 0$ in this transistor, what does it imply? I_{R} is equal to zero, if v_{BC} is equal to zero I_{R} is equal to zero. Then what do we have?

We have I_{C} is equal to alpha$_{F}$ I_{F} because this current is zero, I_{R} is equal to zero so this current source is zero and I_{B} is equal to 1- alpha$_{F}$ I_{F} which gives us that if you take the ratio of the collector current to the emitter current, it is equal to alpha$_{F}$ which is called the common base current gain and if you take the ratio of the base current to the collector current, you have this I_{B} will be equal to alpha$_{F}$ by 1- alpha$_{F}$.

So I shall write it here, I_{C} by I_{B} is equal to alpha$_{F}$ by 1- alpha$_{F}$ which is equal to beta$_{F}$ which is called the forward current gain. So in this condition when I_{R} is equal to zero or v_{BC} is equal to zero all these terms. We are in the forward motion of the operation and we have the β_F or the common emitter current gain as ratio of collector current to the base current.

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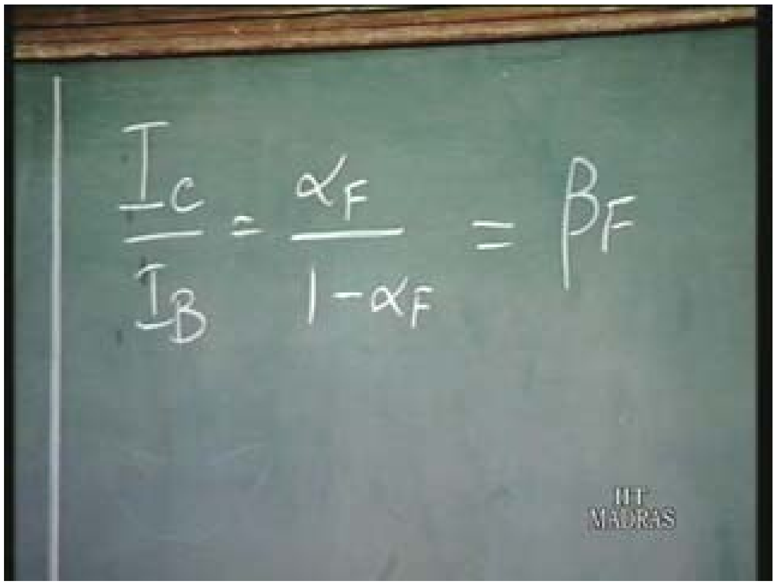

You can also have the reverse mode of operation in which case in reverse mode if V_{BE} is equal to zero, you have I_F is equal to zero and similarly all I_F if you put as equal to zero, you are operating in the reverse direction so I_E by I_B will be equal to β_R. So this is called the reverse beta of the transistor.

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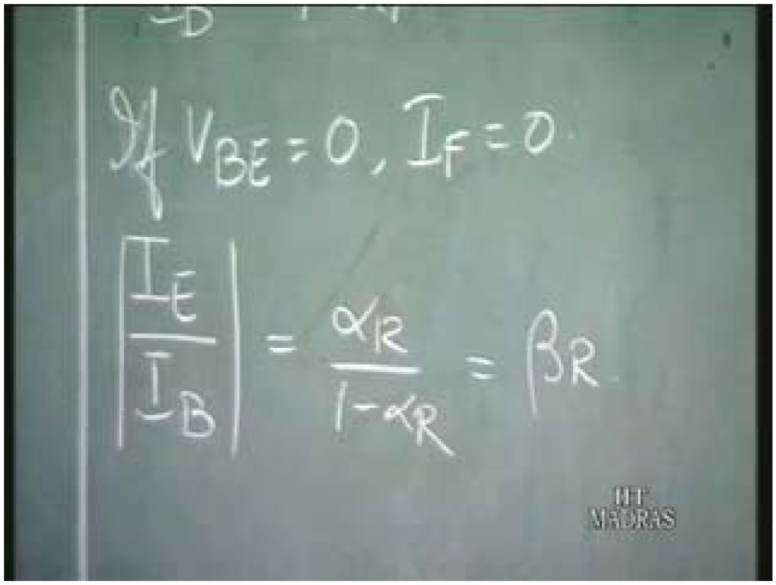

So you can have forward mode of conduction and the reverse mode and if you consider that both the forward and the reverse current exists so the normal mode that is V_{BE} is not equal to zero, V_{BC} is not equal to zero.
What is I_C equal to? Alpha$_F$ I_F – I_R. What is I_E equal to? I_F – alpha$_R$ I_R and I_B will of course be equal to this. So these are the different currents as given by this model.

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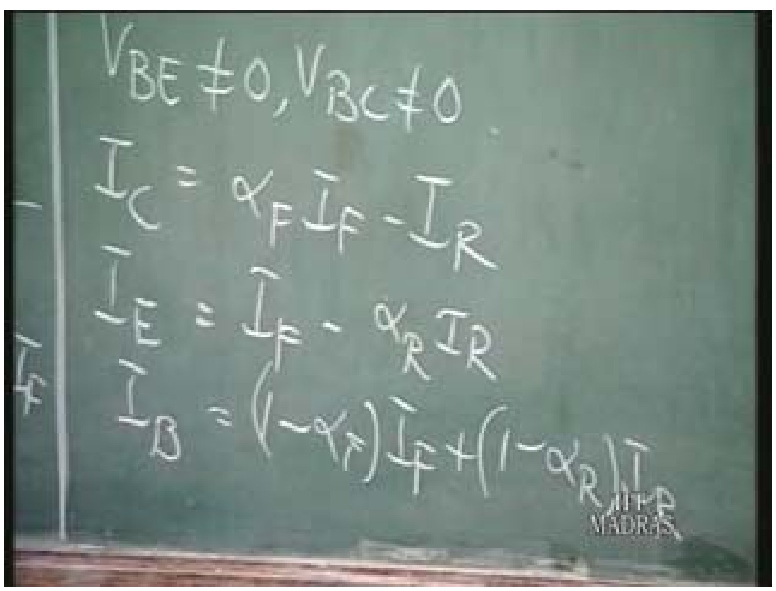

Now what we shall do is we try to evaluate these currents and try to get a simplified model. Basically what is done is we shall be able to see that instead of four parameters which we now have, we shall be able to eliminate one of them and write the equations in terms of three parameters. To do that now let us again take the transistor so this is the emitter, this is the base and this is the collector. Suppose this is n p n transistor so suppose we have forward biased this emitter base junction, now let us look at the charge profiles in the device. Now if you have forward biased this junction so this is the depletion region. How will the charge profile look like? This is the excess charge profile so draw it like this.

We have already seen for a diode, if you have a forward biased diode how the excess charge profile looks like. It is basically the same thing, only thing we have considered the profile to be of short base diode. That is the profile changes linearly which is true in all modern day devices because the actual size of the device in all modern day integrated circuits would be much less than 10 microns even much less in some cases, whereas the diffusion lengths are much larger. So we can always assume that these profiles are linear. So this is the charge profile if you forward bias the emitter base junction and this is the emitter base voltage V_{BE} and then suppose this base collector junction is also forward biased then what happens? You will have a similar situation let me draw this like this. So this charge let me call this Q_E, this is Q_{BF} the F actually stands for the forward case that is when you have the emitter base junction applied voltage, we have seen that you have two cases when you have the emitter base voltage applied and the base collector is zero we call that the forward case, forward mode of operation and the reverse mode when you have these charges, this is q c that is the collector charge and this component this is the base charge in the reverse direction.
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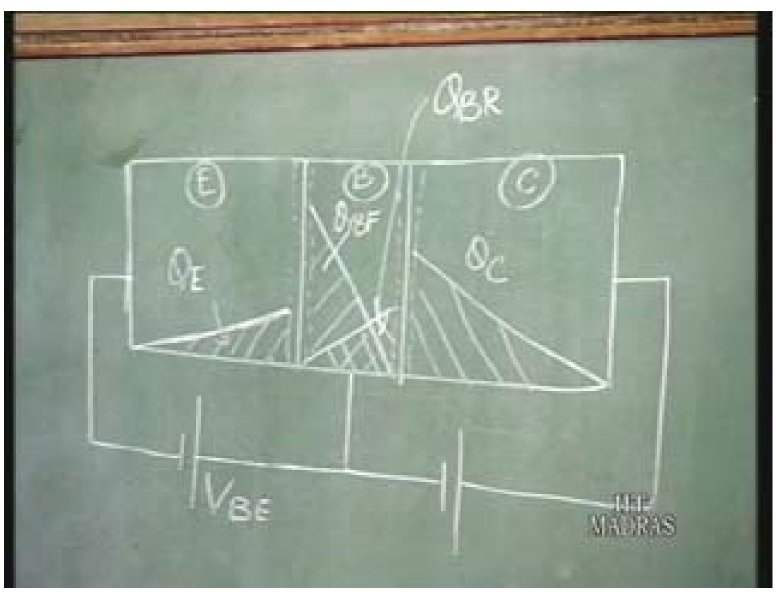

So you have the base charge for the forward case given by this, reverse case like this. Now if V_{BC} is equal to zero then this reverse charge in the base will not be existing, you just have the forward component and also if V_{BC} is negative that is the base collector is reverse biased this also would not be existing because you know that the charge at this end is equal to the minority carrier concentration thermal equilibrium value into exponential V_{BC} by V_T. V_{BC} is negative this is actually going to be negative so this is not going to be existing. So if you have both the cases that is both the junctions are forward biased, the total charge will be the sum of these two charges and basically you have a charge something like this.

So now just let us take the first case that is V_{BE} not equal to zero, V_{BC} is equal to zero. We can write I_C is equal to, what is I_C equal to? It is the current which is flowing here. So we can write which is electron current, this is diffusion current and is given by the slope of this. This is almost a straight line which implies that there is very little recombination in the base. So this is the equation for diffusion current and this is the slope of the electron concentration in the base. So what is D_N D_X here? It is the slope so it is this height divided by the base width. What is this height equal to? This height is equal to I can write n_i square by N_A exponential V_{BE} by V_T -1 divided by the base width. So let me write, so this is the collector current which I can write as I_S where I_S equal to qD_n by n_i square by N_A.

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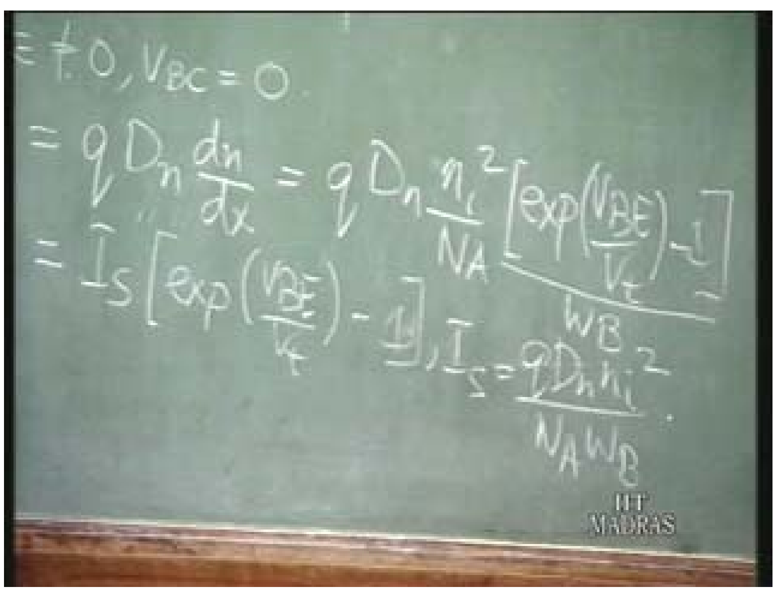

So this is the collector current. Now what is the base current? I will just write here so this is equal to, what is this equal to? In this model what is the collector current equal to when V_{BC} is equal to zero, this is equal to alpha$_F$ I_F. What is I_B equal to? The I_B or the base current in the bipolar transistor here in this case would consist of two components when v_{BC} is equal to zero. What are the two components? One is the hole current which is injected from the base into the emitter and the other is due to the recombination in the base so holes are flowing into the emitter. So it has to be flowing in from this base terminal and also there is recombination. that is electrons which flow from emitter to the collector part of it gets recombined and recombined with holes so to compensate for that also holes must flow in. So basically the base current consists of two components.

Now what is the current which flows into the emitter q D_p this is in the emitter D_p plus I should say q in the base. That is the charge divided by the lifetime that is the total recombination current in the base. So this q D_p dp dx what is that? It is this charge so this is the slope here of this line, this charge divided by the emitter width by w_e which we call let us call the emitter width w_e. So this is w_e, this is w_b this is w_c. So again what do you get n_i square by N_{DE} in the emitter plus this component. What is the total charge in the base? It is given by this area of this triangle. So what is the area of this triangle? Half base into altitude so here you have w_e. So half, what is the base? W_b into again there will be a charge, this altitude is again of course you can write it in a better fashion by taking the exponential v_{BE} by v_t -1is common.

Now the interesting thing is this is equal to 1 – $alpha_f$ into I_f, I_b is equal to 1- alpha into I_F. Now if you take $beta_F$ is equal to I_C by I_B which is $alpha_F$ by 1 – $alpha_F$. What do you get? I_C by I_B, so you have to divide this by this expression. Now obviously that exponential v_{BE} by v_t -1 that gets cancelled out on both sides, so what you are left with is if you write it in this fashion one by q D_n n_i square by N_A just write down the expression, you can do that first to get this expression.
Now this expression tells us a lot of things, so for beta to be high what are the conditions? What is D_n tou_n that is L_n square, that is the square of the diffusion length.

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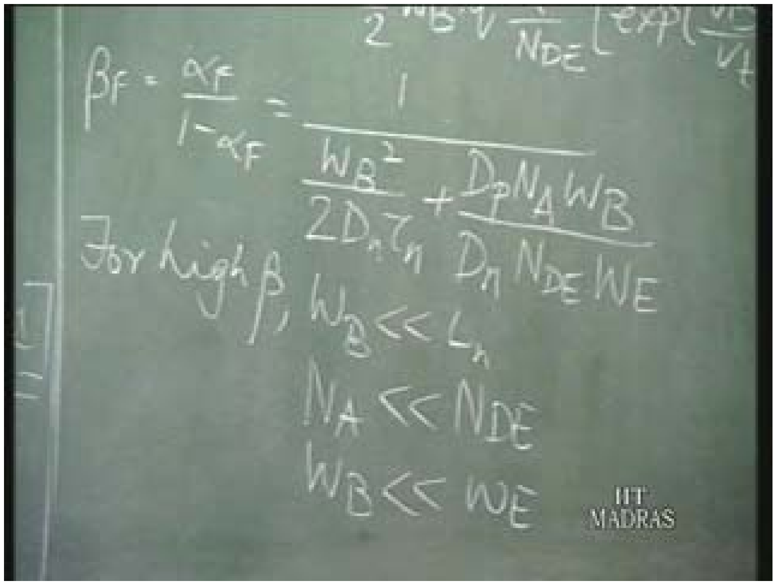

So for high beta w_{b} must be very much less than L_{n} then N_{A} must be very much less than N_{DE} and the base width must be very much less than the diffusion length of the carriers that is quite understandable because if the base width is quite large compared to the diffusion length lot of recombination will take place, the beta will be low. The acceptor impurity concentration in the base must be less than the donor concentration in the emitter because the base current is proportional to the, if you look at the base current here it is inversely proportional to the donor concentration in the emitter whereas the collector current is inversely proportional to the acceptor concentration.

For large beta the emitter concentration must be larger than the base concentration and of course the base width must be very much less than the emitter width because that determines the slope. So if the emitter length is smaller the slope is going to be higher and you have larger base current flowing in. So that also tells us, we always questioned that although it's a n p n transistor why we cannot interchange the collector and emitter terminals because the collector doping concentration is much less than the doping concentration. This has to be made because you are applying a reverse bias across that. normally you apply reverse bias across the base collector junction and also because of breakdown reasons and all you have to make the collector doping much less and so that the depletion region for the base collector junction goes on the collector side not into the base side so that you don't have punch through condition. For this reason collector doping concentration is much less than the base doping concentration.

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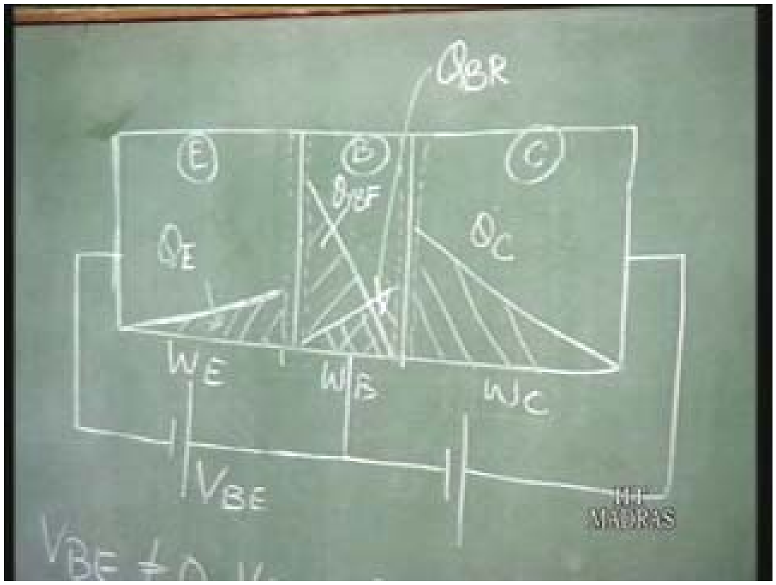

The collector length is also larger compared to them, of course that is not a problem. So these are the conditions one must satisfy anyway. So we shall go ahead, we have derived a relation for I_S here. Now suppose we have the other conditions that is v_{BC} is equal to zero, I just remove this. Suppose v_{BC} is not equal to zero and v_{BE} is equal to zero then what is it going to be like? The situation if you calculate I_E, which is you can take the modulus because the signs will be different, this is given by now you have v_{BC} not equal to zero, v_{BE} is equal to zero, so this forward charges are missing so you have a situation like this and the emitter current is actually the current injected from the collector into the base which flows into the emitter.

So it is just like the previous situation of v_{BE} not equal to zero and v_{BC} equal to zero but you have just interchanged the emitter and the collector terminals. So now you have again q D_n dn dx and this one will give you the similar relation. What is dn dx? This is the same n_i square by N_A by w_b exponential v_{BC} by v_t -1 because this voltage now is the base to the collector voltage and what is this term? It is the same I_S, so we can write it as I_S exponential. Now if you look at these expressions α_R I_R is equal to I_S exponential v_{BC} by v_t -1.

Now what is I_R? If you look at the first figure here if you come back here, see I_R is equal to I_{CS} exponential v_{BC} by v_t - 1. Now if you replace I_R there with I_{CS} exponential v_{BC} by v_t - 1 and similarly if you have α_F I_F is equal to I_F exponential v_{BE} by v_t - 1. So if you again in that I_F if you replace I_{ES} exponential v_{BE} by v_t - 1, what do you get? You will get α_F I_{ES} is equal to α_R I_{CS}.

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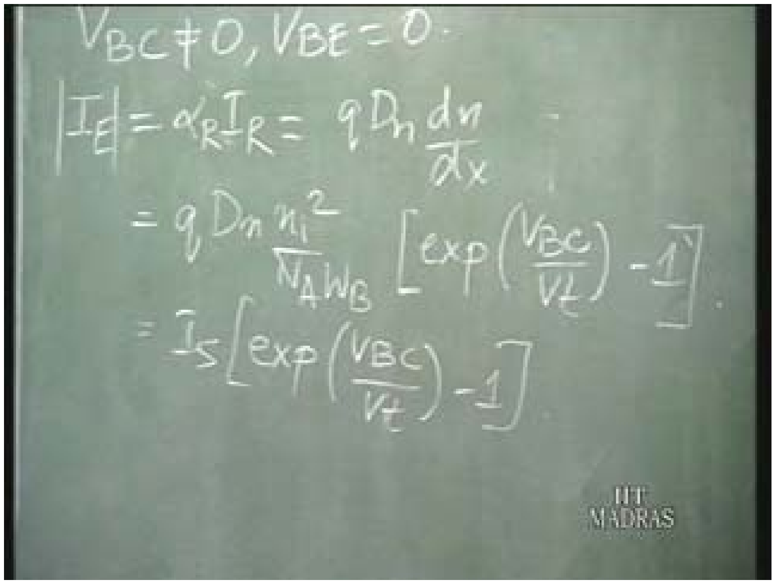

So basically what you have done is you see there is a relation between this I_{ES} and I_{CS} through this $alpha_R$ and $alpha_F$. So basically what it means that you can eliminate one of these four parameters because you can write one of these parameters in terms of the other three and that can be eliminated. Basically you require three parameters. So what we do is we use this parameter I_S and we write the expressions in terms of $alpha_F$, $alpha_R$ and I_S and I_S is related through this expression $alpha_R$ I_R is equal to I_S exponential v_{BC} by v_t -1 and similarly $alpha_F$ I_F is equal to I_S exponential v_{BE} by v_t -1. So that is the thing.

Now the situation where you have both the voltages applied that is the most common situation that is both the emitter base voltage as well as the base collector voltage is applied then what happens is the charge in the base or in the different regions is considered to be a superposition of the two individual cases. So you have this charge plus the reverse charge which is the sum is going to be something like this. What happens to the slope here? The slope has decreased so what do you think that what should happen to the collector current? The collector current should decrease, isn't it? So if you are forward biasing the collector base junction, this end the charges are going to go up so the slope is going to reduce.

The collector current reduces, also the total charge stored in the base is going to increase which is going to increase the base current, so the beta is going to fall. So what we have done now is individually the two cases, you have calculated the collector current and the different currents you can do that. I_C is equal to I_S exponential v_{BE} by v_t -1 which is equal to $alpha_F$ I_F. Now $alpha_R$ I_R is equal to I_S exponential v_{BC} by v_t -1, now if both the voltages are applied the collector current is given by $alpha_F$ I_F – I_R.

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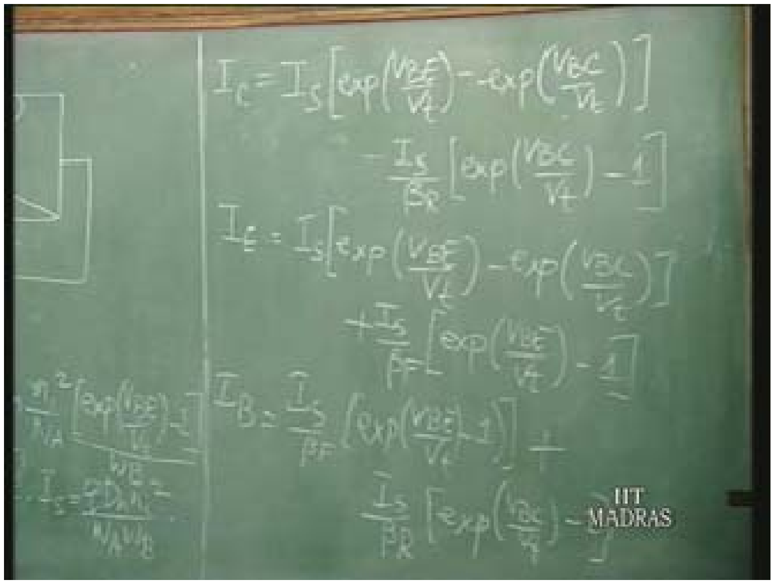

So you can express that now, alpha$_F$ I_F is I_S exponential V_{BE} by V_t -1and I_R is I_S by alpha$_R$ exponential V_{BC} by V_t -1. Similarly because you can now express the different currents in terms of I_S alpha$_R$ and alpha$_F$, these are now the three parameters nowhere here do we have I_{ES} and I_{CS} occurring, we have I_S alpha$_R$ and alpha$_F$, you can write the terminal currents when both the voltages are applied in terms of these three parameters and what you get finally, I shall write that down finally. Also you can express alpha in terms of beta because that is the more common used common emitter configuration, more commonly rather than the common base configurations. So beta is the common emitter refers to the common emitter configuration, current gain in the common emitter configuration, so we can write beta in terms of alpha and then the relations which we get I shall write them down.

This you can work out for yourself I think. I_C is equal to I_S exponential this is just plain manipulation basically you have to write I_C is equal to alpha$_F$ I_F minus I_R. So alpha$_F$ I_F you know from the forward configuration, when the current is flowing in the forward direction I_R expression also you have calculated and then you just write them in terms of I_S and then the two betas beta$_F$ and beta$_R$ and I_B is nothing but the difference of I_E and I_C is given by this. It looks very complicated but we can remember it very easily. If you look at these expressions one can look at them as three current sources. One is given by this I_S exponential V_{BE} by V_t - exponential V_{BC} by V_t which is occurring in these two and the other two is I_S by beta$_R$ exponential V_{BC} by V_t -1 and the third one is I_S by beta$_F$ exponential V_{BE} by V_t -1. So you can write them in terms of this is the collector terminal, this is the emitter terminal, this is the base terminal so this is I_S.

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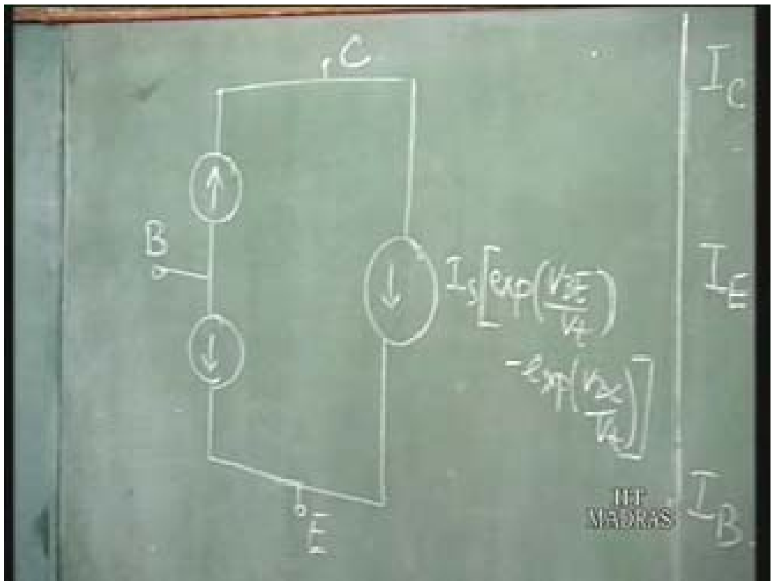

So what is the collector current? This current source minus this current source, this is flowing in this way so I_S exponential V_{BE} by V_t -1 - I_S by $beta_R$ exponential V_{BC} by V_t - 1. What is emitter current? It is the sum of these two current sources. This is the same this current source plus this. This is given by I_S by $beta_F$ exponential V_{BE} by V_t -1 and the base current is given by the sum of these two current sources. Again all the currents are expressed in terms of the three parameters I_S, $beta_F$ and $beta_R$. So now this is the final Ebers Moll model which is also available in spice, so you have three current sources in terms of which you express the terminal currents.

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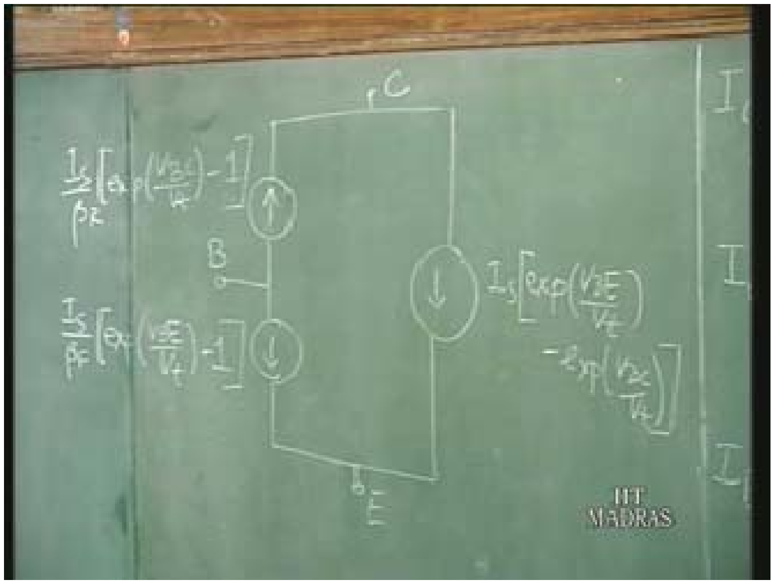

So you see that when the collector base voltage say is almost equal to the base emitter voltage or it is forward biased, collector base junction if you keep on forward biasing the collector base voltage these current source value reduces. So you have the current going down inside in saturation region. In saturation what happens is both the junctions are forward biased base emitter as well as base collector. So this is the difference, the collector current is mainly made up of this component which is the difference. If collector base junction is reverse biased this component is almost zero because it is very small compared to this component. In fact the current does not change much when you go on increasing the base collector voltage. When you have a constant V_{BE} and if we go on changing the base collector voltage so the current will almost be a constant. So this gives us the relations between the terminal voltages and the currents and this model is used in different circuit simulators.

An interesting thing before we wind up because we shall be coming to that again when circuits, I think it is the best time to discuss. If you plot the well known output characteristics of a transistor for different base currents, you have this type of characteristics. This is well known so this is for increasing I_B, this is the collector current versus V_{CE}.

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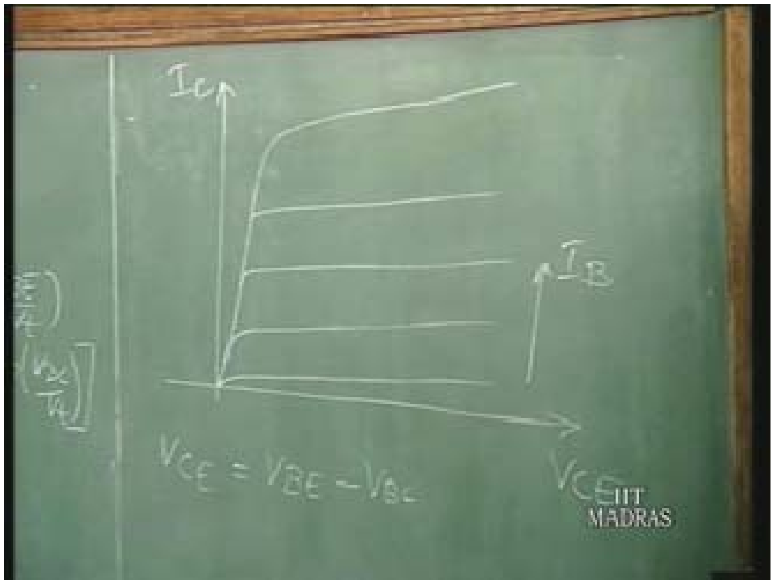

So what is happening when we come to this region? This is known as the saturation region, this is the active region. What happens when we go to the saturation region? What is v_{CE} given by? v_{CE} is equal to v_{BE} I should say, so when v_{CE} is going towards zero almost towards zero v_{BE} and v_{BC} are quite close to one another and so you can see from this model here when v_{BE} and v_{BC} are very close to one another this current is almost going to zero. When v_{CE} is very large it means v_{BC} is negative because v_{BE} has a forward voltage cannot be very large. So this is reverse biased which means the current becomes almost constant.

The third point I want to make is when v_{CE} is zero what does it mean? V_{BE} is equal to v_{BC} so v_{BE} is equal to v_{BC}, what happens to this current source? This current source is zero, is the collector current zero? Collector current is not zero because individually v_{BE} and v_{BC} are not zeroes, you have a current flowing here. So I_C is actually negative, there is a negative current collector current flowing, when v_{CE} is equal to zero.

In fact in order to make the collector current zero, v_{CE} should be slightly positive. That is v_{BE} must be slightly greater than v_{BC} to compensate for this current. These two current sources must be the same in order to the collector current is zero. So if you expand this region for a bipolar transistor, in fact if I draw it here this is I_C, this is v_{CE} it is actually like this around the zero point. This is called the offset voltage that is the collector emitter voltage at which i c is zero, it is slightly positive and at v_{CE} is equal to zero I_C is slightly negative. We shall see that and this transistor is actually operated in this point in when we discuss ttl circuits that is a particular region, I wanted to introduce this here itself. So we have seen the Ebers Moll model of the bipolar transistor which we shall be using sometimes later on in the course.

Thank you.