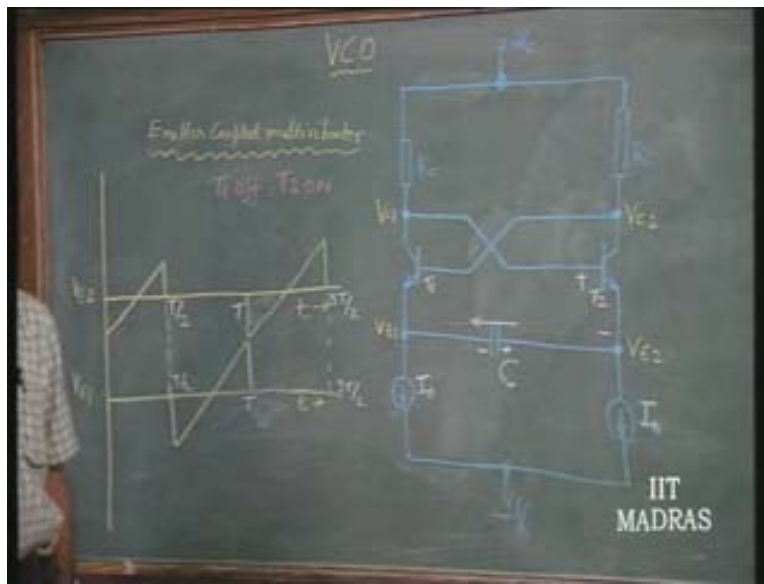


Analog ICs
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Indian Institute of Technology, Madras
Lecture - 22
Voltage Controlled Oscillator

Continuing with our discussion about voltage controlled oscillators last time we saw how we can come up with an integrator where voltage dependent current was charging the capacitor and this voltage was applied across a Schmitt trigger and how the Schmitt trigger in turn was controlling the discharging operations such that charging and discharging currents are the same.

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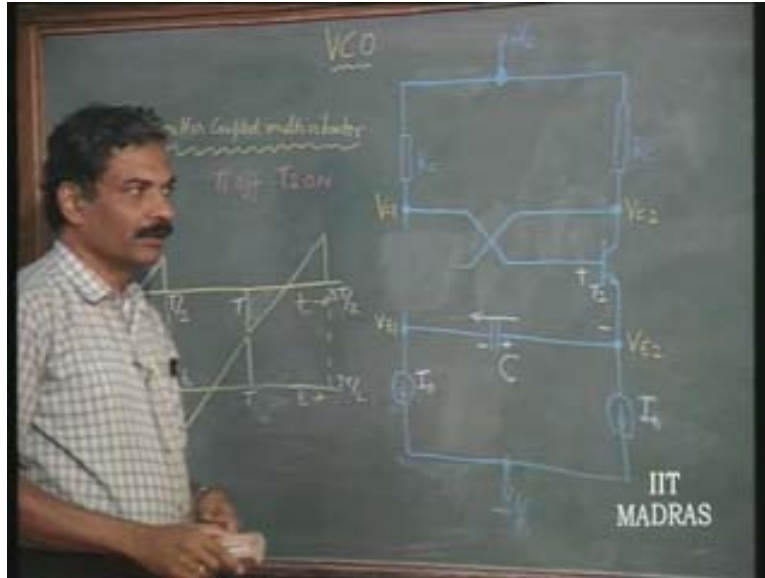


Today we are having another important circuit which is commonly used for voltage controlled oscillators in integrated circuits called emitter coupled multi vibrator. Let us go through the motions of analysis of this circuit. What is a multi vibrator where we have regenerative positive feedback where one inverter is cascaded on to another inverter and the output is connected back to the original inverter?

One inverter coupled on to another inverter and the output finally given back again to the original inverter results in complete regenerative positive feedback. What it means is, it is like the case of a Schmitt trigger, we can have only two states possible either T_1 can be ON and T_2 is OFF or T_1 can be OFF and T_2 can be ON. These are the only two possible states in which it can exist. While transiting it will go through the active region and there will be regenerative feedback and it will go quickly from ON to OFF or OFF to ON. So in this configuration where the base of one transistor is connected to the collector and the base of the other transistor is connected to the collector there is regenerative feed positive

feedback. Therefore either T_1 is OFF and T_2 is ON or T_1 is ON and T_2 is OFF. So let us assume that T_1 is OFF, so let us consider T_1 OFF and T_2 ON.

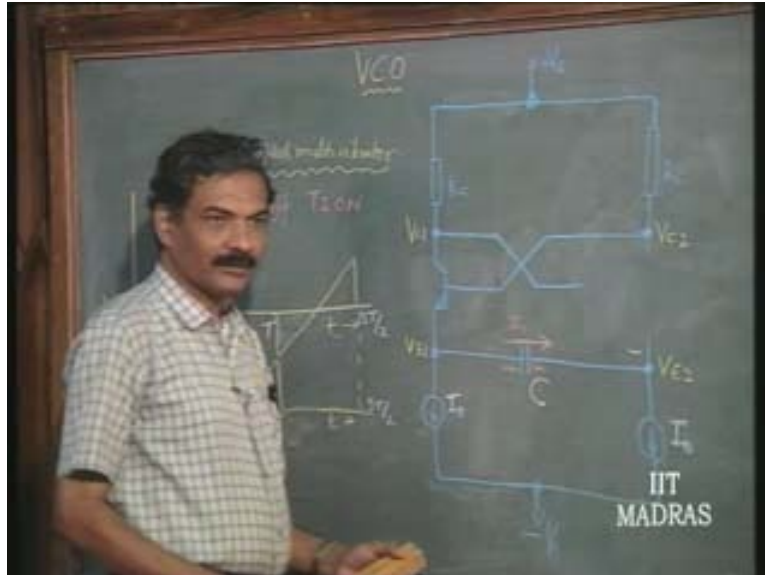
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T_1 is OFF that means for practical purposes this is open and T_2 is therefore in saturation or something, it just entered saturation because it is ON. Now what happens is, we can see that the capacitor is going to get charged by means of current source I_0 . So the constant current I_0 will be flowing through this in this direction. So what will be the voltage developed? It is I_0/C into T that means voltage will be increasing that is what is depicted here as V_{E2} . If you observe here the voltage will be linearly increasing at a rate which is given by I_0/C into T is plus plus here and minus minus here so the voltage will be increasing.

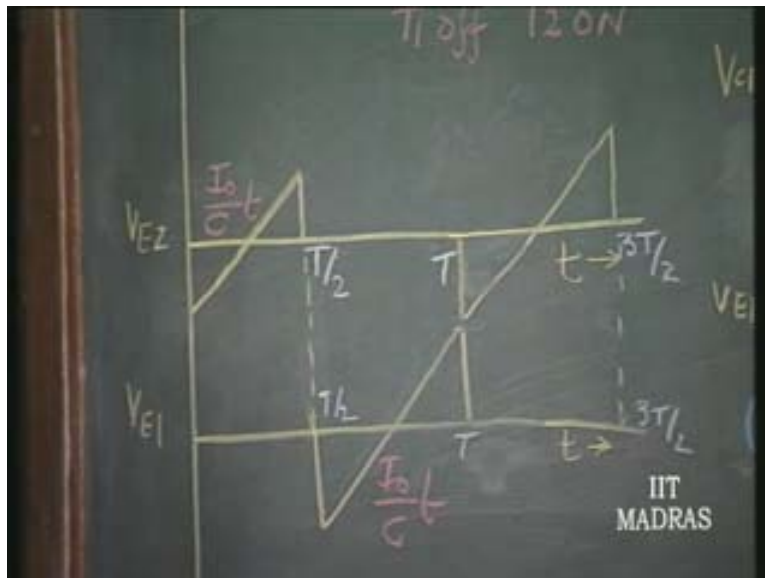
Now we have assumed T_1 to be OFF and T_2 is ON. This potential also will be linearly increasing above this here this potential is linearly increasing that means this will be above this by V_{gamma} and therefore this will be linearly increasing. In turn this also is going to **increase** because this is in saturation and this also is going to linearly increase. That means the base of this transistor is going to have its voltage linearly increasing. Originally it was assumed to be OFF. The moment this voltage between V_{E1} and this base reaches V_{gamma} it is brought into the active region. Now again regenerative feedback sets in and now T_1 will be ON and T_2 will be OFF immediately due to regenerative feedback.

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So what happens is that T_1 is going to be immediately ON and T_2 is going to be OFF. So, now the current in this is going to change direction and it is going to flow in this direction so the voltage is going to be plus plus here and minus minus here.

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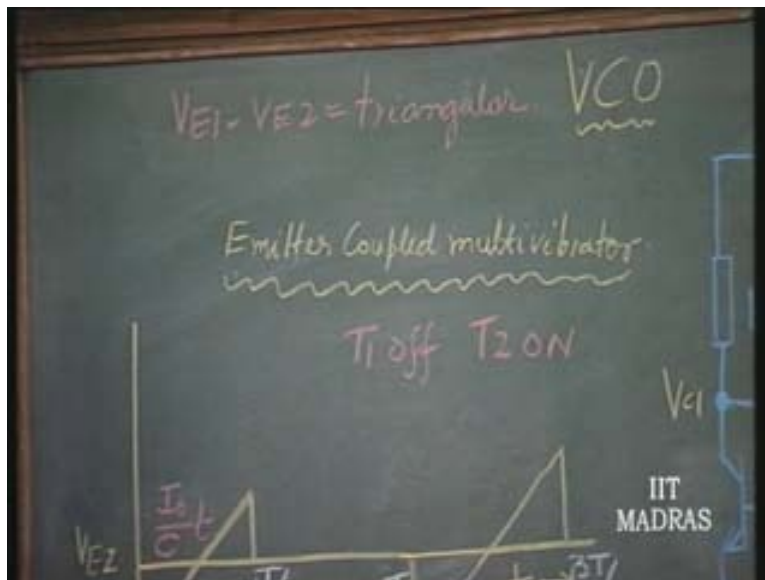


V_{E2} is going to remain at whatever constant value it had whereas V_{E1} is going to linearly increase at the same rate of I_0/C into t . Therefore we have here V_{E1} which is going to increase linearly. It will go on like this. Once again we have this voltage linearly increasing, this voltage linearly increasing and once again a time will come when this voltage reaches a sufficient value as to derive this to active region then again it gets

regenerative positive feedback and this will be ON and this will be OFF and this will go on.

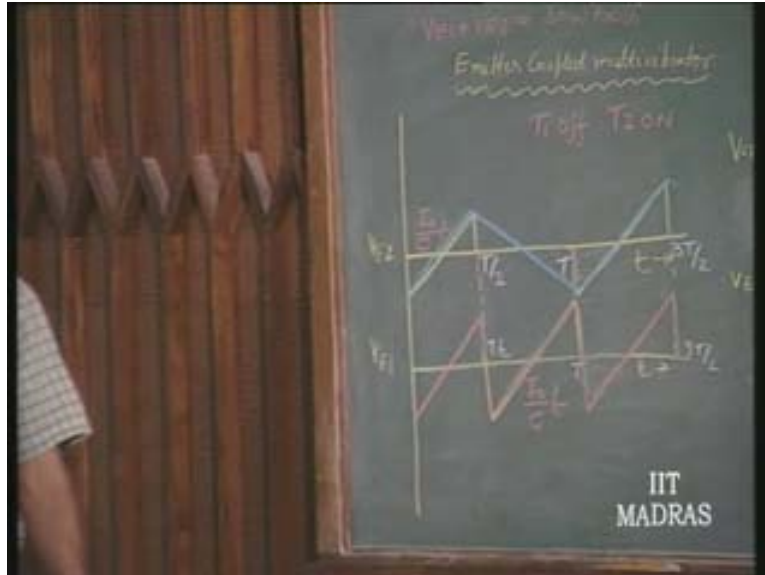
Actually speaking you will see a kind of waveform here and capacitor voltage is going to jump at every point from plus V_{γ} to minus V_{γ} . The moment the current switching occurs the capacitor voltage is going to jump. Therefore you see this kind of waveform repeated periodically. Now, if you want a triangular waveform you already get it. From this you can get a triangular waveform simply by taking V_{E1} minus V_{E2} because current is going to either go this way or this way.

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So V_{E1} minus V_{E2} will be nothing but a triangular waveform. This subtracted from that will give you this kind of a triangle and it completes the triangle. If you want a saw tooth waveform you superimpose this over this and you get a nice saw tooth.

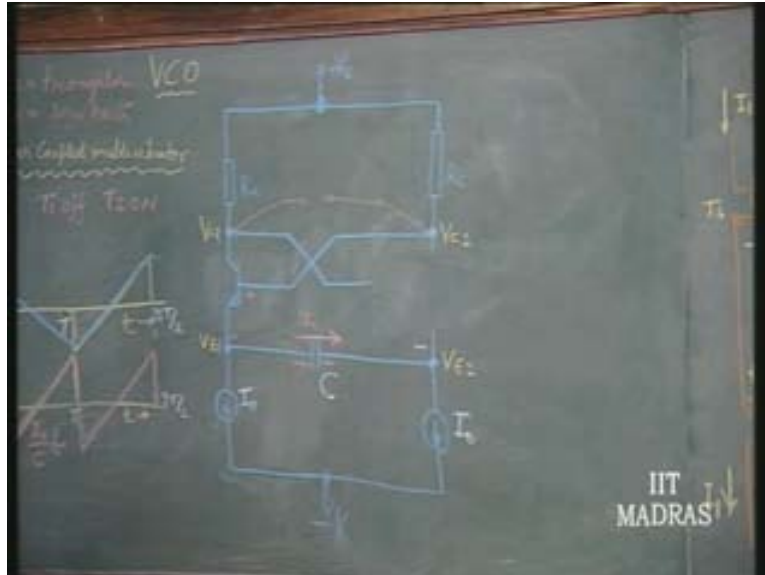
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By simply processing these voltages further we get two important waveforms, a triangle as well as a saw tooth. There is a triangle and the saw tooth is going to be like this. And if you take the waveform here, for example, V_{C1} when this is OFF and this is fully ON, this is OFF so this voltage is going towards V_S except for a small drop here. And this is ON that means a full current OFF I_0 plus I_0 it is these two I_0 going through this so this voltage is going to V_S minus $2I_0R_C$. This voltage is always going to change from V_C that is V_S minus $2I_0R_C$ that is V_S to V_S minus $2I_0R_C$.

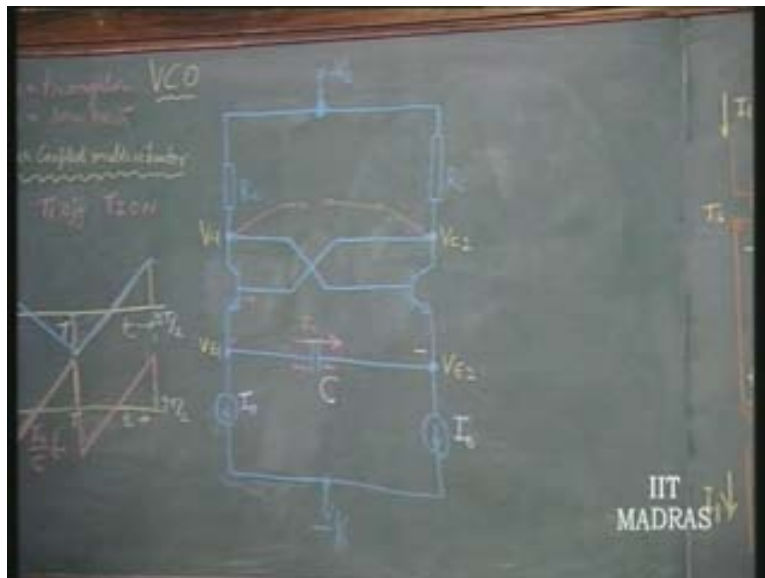
Similarly this voltage will be out of phase by 180 degrees. When this is going high this is going low when this is going low this is going high so they are going to be out of phase by 180 degrees. When this is V_S this is V_S minus $2I_0R_C$ and when this is V_S this is V_S minus $2I_0R_C$. If you take the voltage across this you will get a square wave of magnitude $2I_0R_C$ amplitude square wave.

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You can get square wave from here to ground or here to ground or if you take the voltage across this. Simultaneously you can get triangular wave here, square wave here and if you add this you can get saw tooth.

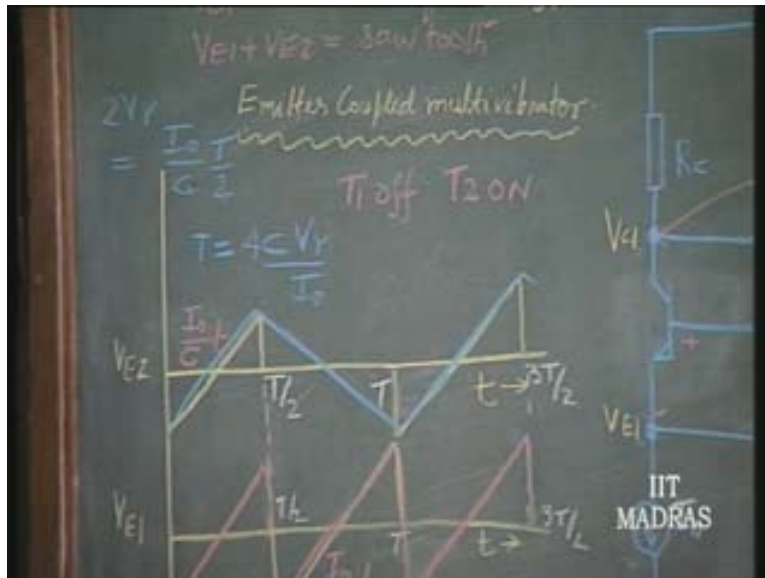
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This is a versatile function generator circuit, the frequency of oscillation of this can be easily found out because we know that I_0/C into t is the rate at which it is going on and it is going from minus V_{gamma} to plus V_{gamma} . So we have $2V_{\text{gamma}}$ is equal to I_0/C into $T/2$ so we get T is equal to $4C V_{\text{gamma}}/I_0$ or frequency of oscillation f is equal to $I_0/4C V_{\text{gamma}}$.

I_0 is a current source and can be made voltage control, we know the current here are extra we can use, so I can make I_0 voltage control using current mirror technique and therefore I can make this a linear voltage controlled oscillator. This is again one of the most common VCOs used. For example 560 also is a PLL which uses this kind of VCO for its application. Whether it is a function generator or a voltage controlled oscillator the basic concept used is a current source charging a capacitor and a regenerative feedback circuit. These are the two things mainly used in almost all ICs meant for function generation where the function is nothing but the triangle, sine wave etc, how do you get a sine wave from the same thing?

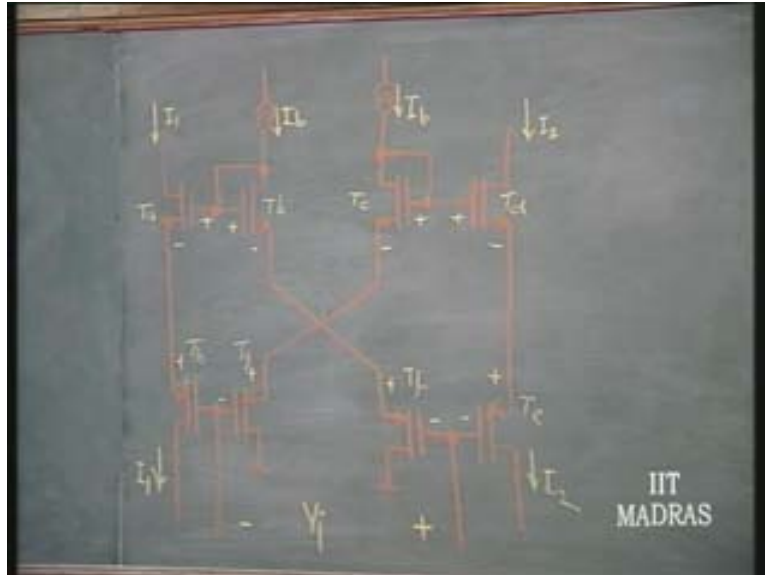
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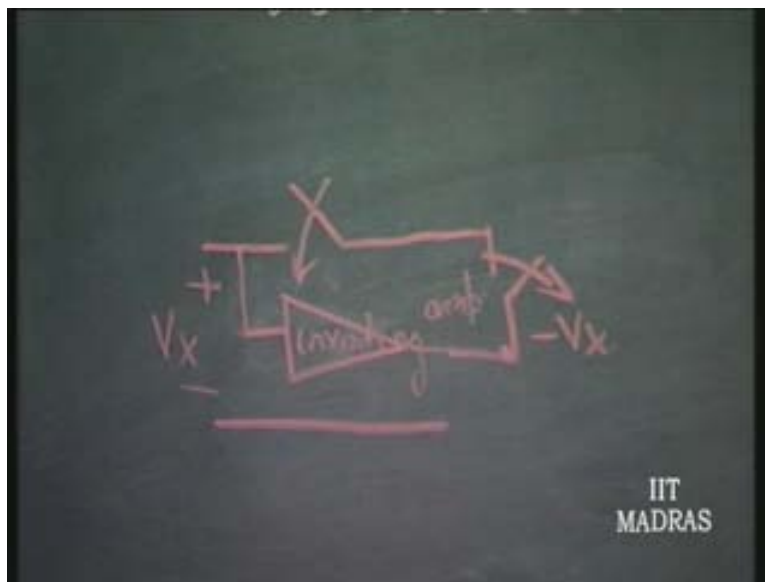
I apply this triangular waveform to a differential amplifier that means just this one is applied again to another differential amplifier so that that differential amplifier primes output is going to saturation and output of the differential amplifier is going to be nothing but a fairly good approximation to a sine wave. So you just simply apply these inputs to a differential amplifier and take the output across the collectors of the differential amplifier then you get an ideal sine wave here which is going to be an approximation of course to the sine wave. It can generate all these functions simply and that is why this is a very popular integrated circuit for VCO or function generation. Now let us see the other types of multipliers.

Let us now discuss about another important IC multiplier which uses the property of the active device as a switch rather than its exponential or square law relationship which was exploited earlier. We exploited the exponential relationship of the transistor in designing all these multipliers whereas in the case of this scheme we will now use the pulse width and amplitude modulation technique wherein the device transistor MOSFET or a bipolar is used only as a switch to switch in voltage.

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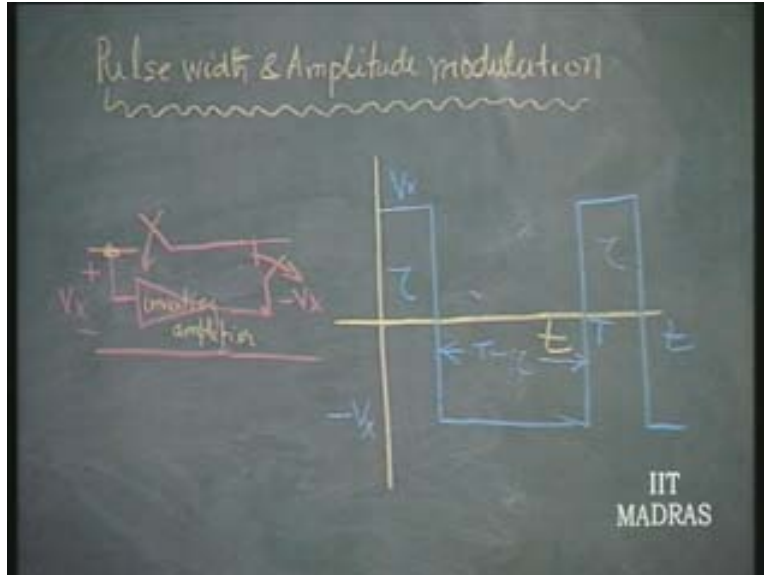


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Now let us consider the situation where a switch is being used in order to switch V_x , tau is connected to V_x and since it is a digital inverter we will say that it is coming through an inverting amplifier and therefore what is coming here is minus V_x . So it is either connected to V_x or connected to minus V_x and what happens here at the output is, if you now plot the specter time it is connected to V_x so this is the output for a duration of tau and then it is connected to minus V_x for a duration of T minus tau this being T .

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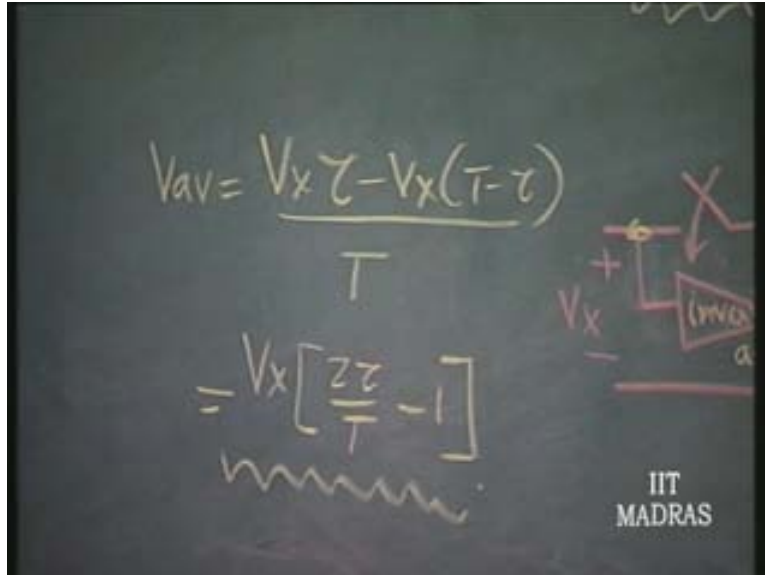


Here this is nothing but minus V_x . So, for duration of τ it is connected to V_x and for the rest of the time through this inverting amplifier it is connected to minus V_x .

What happens now?

If you take the average of this where the average is nothing but the total area under the curve so V_x into τ minus V_x into T minus τ/T is the average. Or we can rewrite this as V_x into $2\tau/T$ minus 1 this is the average of this. Now we have already understood how we can generate what is called as a DT cycle generator. This is modulating the amplitude, output amplitude is modulated by V_x . Now modulating the width we want another voltage which is dependent upon τ . Let us see how this can be done. We can consider a voltage V_C being applied and a triangle of peak amplitude V_p with time period T being applied to the other end.

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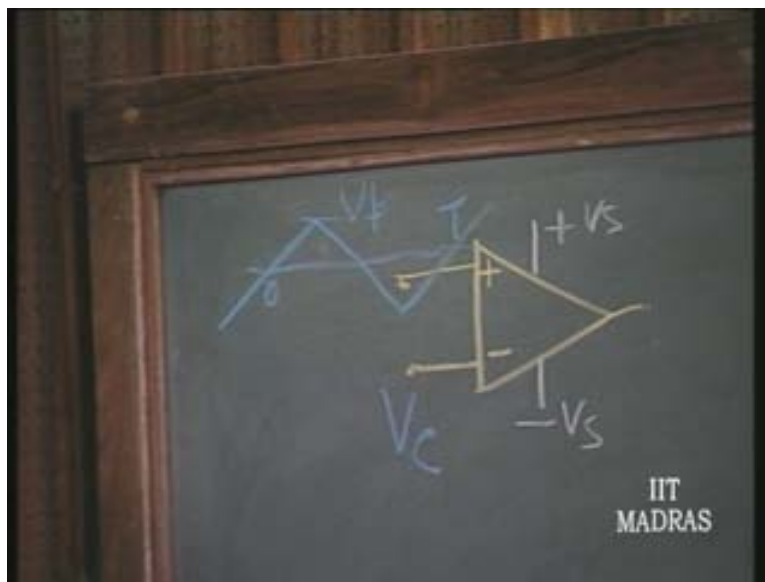


The chalkboard shows the derivation of the average voltage V_{av} for a half-wave rectifier. The equation is written as:

$$V_{av} = \frac{V_x \tau - V_x(T - \tau)}{T}$$
$$= V_x \left[\frac{\tau}{T} - 1 \right]$$

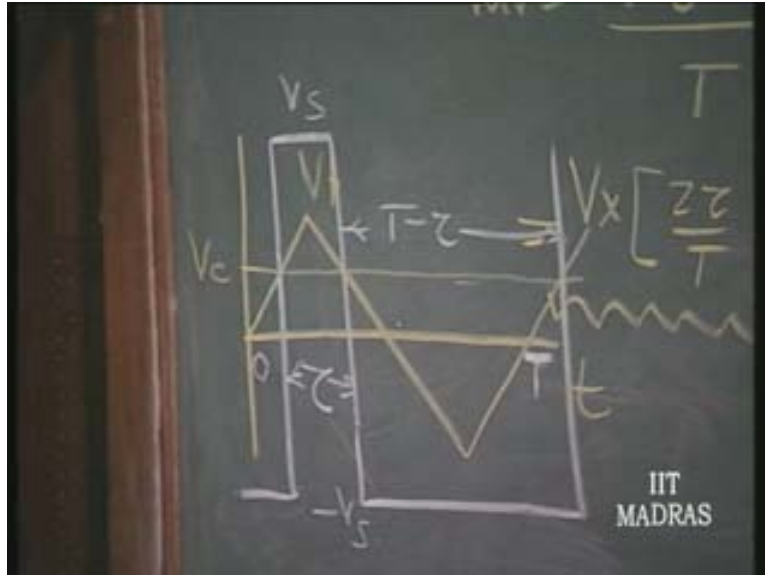
To the right of the equations is a simple circuit diagram of a half-wave rectifier. It consists of a diode connected in series with a load resistor. The input voltage is labeled V_x with a plus sign on the top terminal. The output voltage is labeled V_o . The diode is oriented such that it only allows current to flow during the positive half-cycle of the input. The IIT MADRAS logo is visible in the bottom right corner.

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If you do this we can show that this triangle is going to result in with the peak amplitude of V_p and if you assume that this is the control voltage V_c

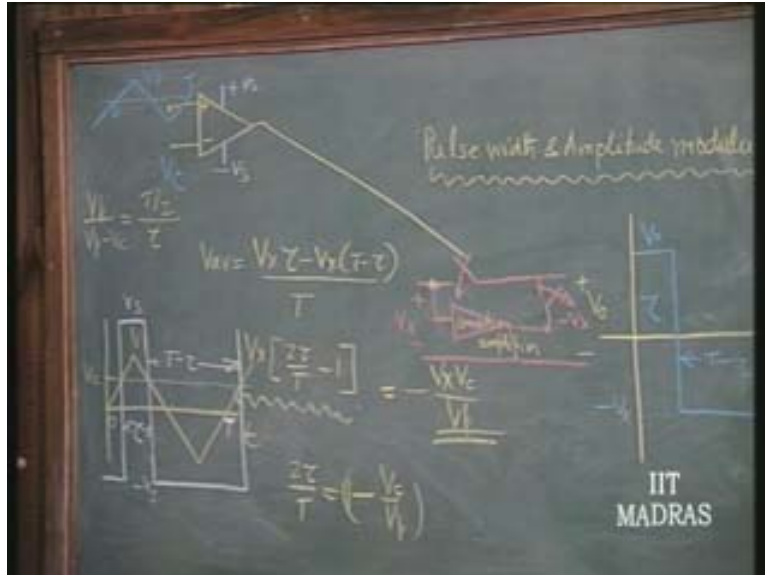
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The output of this comparator, let us say this is plus and this is minus, so whenever this triangular voltage is higher than V_c the output will go to plus V_s . So the output will go to plus V_s here whenever this is higher than this. And whenever it goes below V_c it will go to minus V_s so this is a very simple pulse width modulator circuit where just a comparator is used. And if you now find out the output is going to be just this, so assuming that this is plus V_s this is minus V_s so this will go to V_s and this to minus V_s . This width being τ and this is going to be T minus τ , this time period is T . Now using similar triangles here and here, these two triangles are similar triangles so using that property we know that this height divided by this height is nothing but this base divided by this base.

Let us again write that that height is V_p/V_p minus V_c is equal to V_p/V_p minus V_c is equal to $T/2/\tau$. Therefore from this we can get nothing but $2\tau/T$ where $2\tau/T$ is nothing but 1 minus V_c/V_p .

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Therefore if this is the controlling factor of this switch as well as the complementary switch here this gets Q and that gets Q bar so that it is complementary then we can say that $2\tau/T$ can be replaced by $1 - V_C/V_p$. So the output average is going to be nothing but $1 - 2\tau/T$ is V_C/V_p minus $V_x V_C/V_p$. So, by simply making use of switches appropriately, by using two simple circuits here, this is amplitude modulation and this is width modulation and combination of this is going to result in a very accurate precise multiplier which is $V_x V_C/V_p$ where V_C is our V_Y and V_p may be whatever you want like 10V or so. Therefore you can get a precise multiplication here just by using the property of the switch.

Therefore this is very popular as a very accurate multiplier. The whole thing is available in an IC chip, the width modulator, even the triangular waveform generator. The only thing is, you have to put a low pass filter outside so that you get the average. Therefore what it means is that this has to be working at only low frequencies. This is the clock at which the switching is done and the low pass filter should remove the clock frequency. That is what you mean by averaging. DC is now going to be the output. That means it can only be a low frequency output. Apart from this limitation there is no other limitation here. Both V_x and V_Y could be of any polarity. Therefore this is a four quadrant multiplier which is again one of the most popular ways of designing precision IC multipliers.

So far we have discussed the transconductance type multiplier, log anti log multiplier and the modulation pulse width and amplitude modulation type of multiplier. The only one that is now left is how to use the MOSFET property which is the square law property in designing multipliers. There are different ways by which the square law relationship can be really used for the purpose of designing multipliers. Therefore before we go to that let us see one of the important active units which can be finally converted into a multiplier unit which is based on the square law property of the MOSFET or FET.

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n-channel MOSFET

$$I = K(V_{GS} - V_T)^2 \quad \text{for } V_{DS} \geq V_{GS} - V_T \quad \text{current saturation}$$
$$I = 2k \left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2} \right] \quad \text{for } V_{DS} < V_{GS} - V_T \quad \text{triode region}$$

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The basic property of the MOSFET is the current I which is K times $(V_{GS}$ minus V_T) square in the current saturation region. That is for V_{GS} V_{DS} greater than V_{GS} minus V_T that is called current saturation region. In the other region it is $2K$ times V_{GS} minus V_T into V_{DS} dependent upon V_{DS} and there is a square law relationship in the triode region where V_{DS} is less than or equal to V_{GS} minus V_T . This is applicable for the n-channel enhancement type of MOSFET.

Now let us look at this relationship first and then this relationship. Can we use this relationship to obtain a fairly linear relationship between voltage and current and make it either current control or voltage control? It is possible. Let us assume that we have some way of using two MOSFETs so the general idea of all circuits using this concept, suppose you get I_1 is equal to $K_1(V_{GS1}$ minus $V_{T1})$ square and I_2 is equal to $K_1(V_{GS2}$ minus $V_{T1})$ square then I can say that I use V_{GS1} as V_C plus V_i and V_{GS2} as V_C minus V_i

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$$I_1 = k_1 (V_{GS1} - V_{T1})^2$$
$$I_2 = k_1 (V_{GS2} - V_{T1})^2$$
$$V_{GS1} = V_C + V_i'$$
$$V_{GS2} = V_C - V_i'$$

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I can now get I_1 is equal to $K_1(V_C \text{ minus } V_{T1} \text{ plus } V_i)$ square and I_2 as $K_1(V_C \text{ minus } V_{T1} \text{ minus } V_i)$ square. So now you know that $I_1 \text{ minus } I_2$ is equal to $4K_1(V_C \text{ minus } V_{T1})(V_i)$. Therefore it is $I_1 \text{ minus } I_2$ but this square minus this square and therefore a square minus b square is a plus b into a minus b and you can see that I am getting a differential output current here which is dependent upon input voltage V_i and then a constant factor which is controllable by means of a voltage V_C .

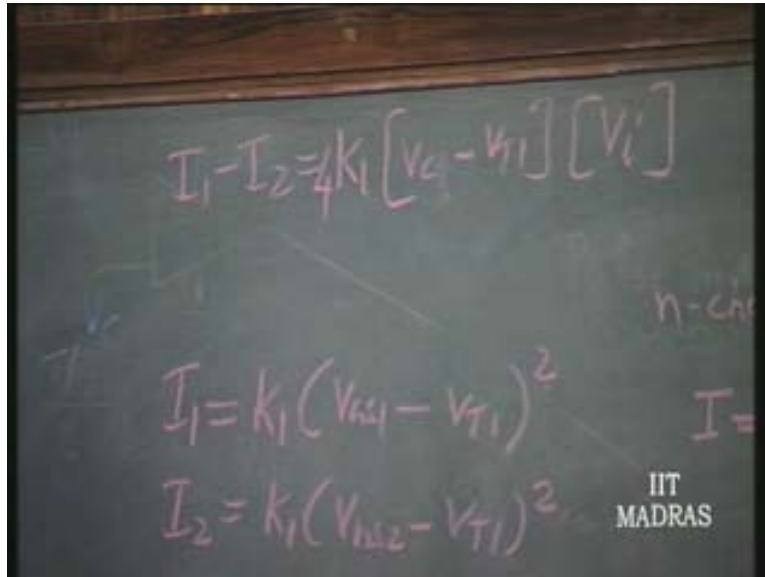
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$$I_2 = k_1 (V_{GS2} - V_{T1})^2$$
$$V_{GS1} = V_C + V_i'$$
$$V_{GS2} = V_C - V_i'$$
$$I_1 = k_1 (V_C - V_{T1} + V_i')^2$$
$$I_2 = k_1 (V_C - V_{T1} - V_i')^2$$

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So this is a voltage controlled transconductor. We have so far not discussed about how to make V_{GS1} is equal to V_C plus V_i , our V_{GS2} has a function of V_C minus V_i . If we are capable of doing this, then this is possible.

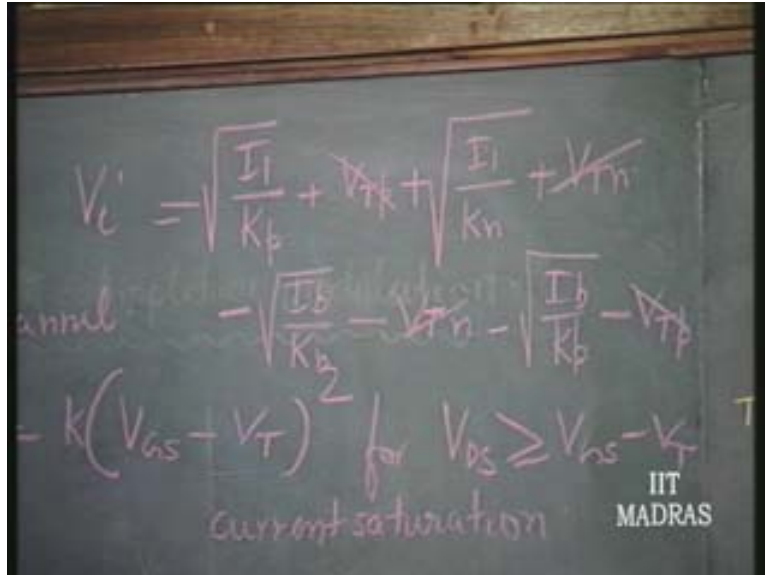
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Hence almost all linear transconductors which are voltage controlled can be generated by using this principle with FET operating always in current saturation region. Now you can generate a pair of MOSFET using n-channel and p-channel both of which control I_1 and another n-channel and p-channel both of which control I_2 through both of which we have I_2 then again we can come up with the CMOS transconductor which is having exactly the same relationship if obviously we can get V_{GS1} and V_{GS2} in the following manner.

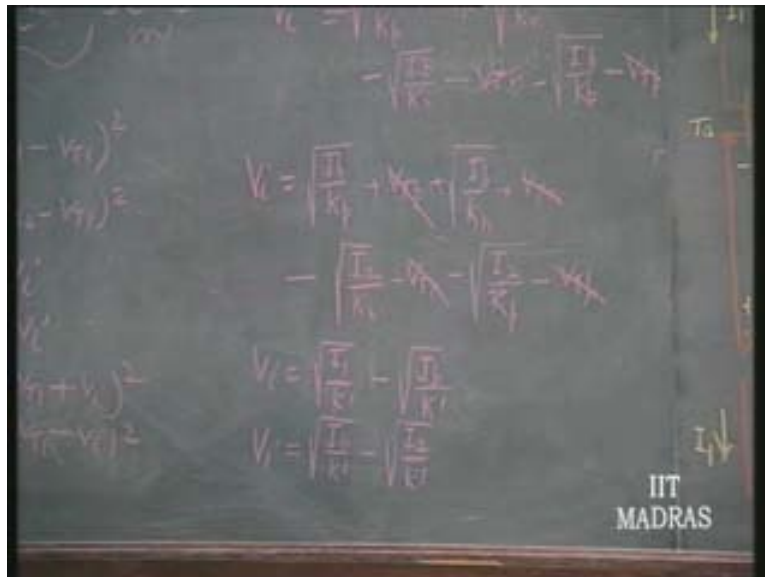
Therefore analyze this based on similar lines. Instead of this being voltage controlled this particular thing is going to be current controlled. The transconductor will be controlled by square root of I_b . Therefore V_i is this voltage here in this transconductor. So it is this voltage which is due to p-channel MOSFET which will be nothing but root of current I_1/K_p of this plus the magnitude of the threshold voltage of this. So it is minus plus root of again I_1/K of n-channel plus V_{Tn} . And again this one is the root of I_b/K_n plus V_{Tn} that has to be detected, this voltage has to be detected from this voltage.

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So we can write down these voltages in a neat manner in the following way. This voltage is nothing but root of $I_1/K(p)$ plus $V_{T(p)}$ plus root of I_1/K_n plus $V_{T(n)}$ minus root of I_b/K_n minus V_{Tn} so that it gets cancelled with this. In this case I am only considering the magnitude here. And then plus minus again minus root of I_b/K_p minus V_{Tp} is equal to V_i , and this gets cancelled with this.

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Now the same thing coming from here minus plus we have again minus plus, let us write down that, V_i is equal to minus plus potential rise here root of I_b/K_p plus V_{Tp} then we are coming here, root of I_b/K_n plus V_{Tn} minus root of I_2/K_n T_n minus root of $I_2/K_p/V_{Tp}$. So

this is also equal to V_i , so what do I get here? I can write V_i from this as root of I_1/K prime where K prime is a combination of K_p and K_n . So root of I_1/K prime minus root of I_b/K prime simplifies to that and the same thing is also equal to root of I_b/K prime minus root of I_2/K prime. So what do you get?

Let us take this V_i to this side, so V_i plus root of I_b/K prime is equal to root of I_1/K prime.

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$$(V_i + \sqrt{\frac{I_b}{K}}) = \sqrt{\frac{I_1}{K}}$$

$$\left(\sqrt{\frac{I_b}{K}} - V_i\right) = \sqrt{\frac{I_2}{K}}$$

And what is root of I_2/K prime?

The root of I_2/K prime is nothing but root of I_b/K prime minus V_i .

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$$(V_i + \sqrt{\frac{I_b}{K}})^2 = \frac{I_1}{K}$$

$$\left(\sqrt{\frac{I_b}{K}} - V_i\right)^2 = \frac{I_2}{K}$$

So now square this, square this and what do you get?

The root will go, root will go so now take I_1 minus I_2 is equal to K' prime into this square minus this square which is $4V_i$ root of I_b/K' prime or this is actually $4V_i$ root of I_b K' prime.

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The image shows a chalkboard with handwritten mathematical equations in red chalk. The equations are:

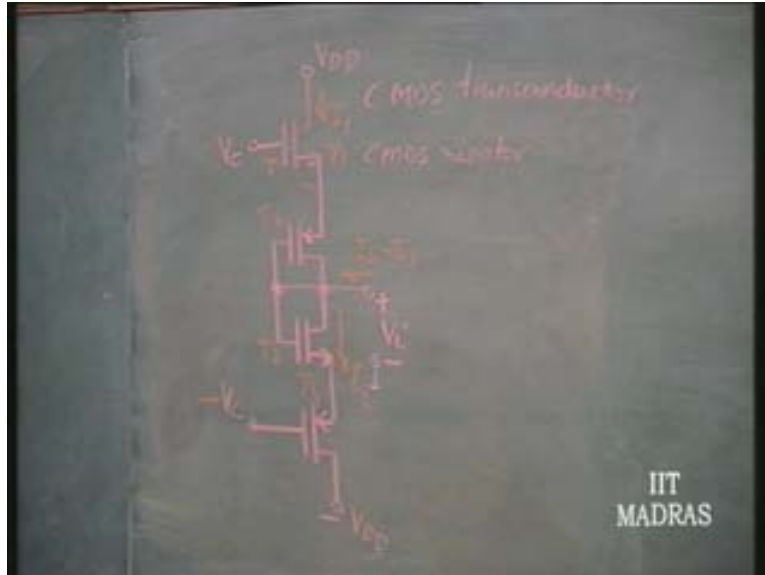
$$\left(V_i + \sqrt{\frac{I_b}{K'}}\right)^2 = \frac{I_1}{K'}$$
$$\left(\sqrt{\frac{I_b}{K'}} - V_i\right)^2 = \frac{I_2}{K'}$$
$$I_1 - I_2 = 4K' V_i \sqrt{\frac{I_b}{K'}}$$
$$= 4V_i \sqrt{I_b K'}$$

The chalkboard also has the text "IIT MADRAS" written in the bottom right corner.

This is what I wanted you to derive and yesterday we put it as g times V_i and g is now recognized as 4 root of I_b K' prime. It is now a powerful active cell which can be used for any application where current controlled transconductors will be needed as tune filters or V_{COS} or multipliers etc.

We just finished discussing about one active block which we called as CMOS transconductor. Here we have another one as simple as the earlier one which is also a CMOS transconductor or CMOS resistor. That means if the voltage to current relationship is a two terminal thing it becomes a resistor otherwise it is a transconductor. We have to have control voltage plus V_i and control voltage minus V_i which can be easily got here if you can make this minus V_C . This is V_C , this is minus V_C and now this is a CMOS arrangement. Again we can see that these things are arranged in a series manner so that this voltage V_i now comes here, this is V_C , this is V_i

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So this voltage again by the same distinct letters call this as I_1 and this current as I_2 because we are connecting voltage here this current can be I_1 this current is I_2 and this current here is I_2 minus I_1 .

What is I_1 ?

Now we can get this relationship this voltage V_{GS} and call it as T_1 , this is T_2 , this is T_3 , this is T_4 . So once again root of I_1/K_n plus V_{Tn} is that voltage V_{GS1} .

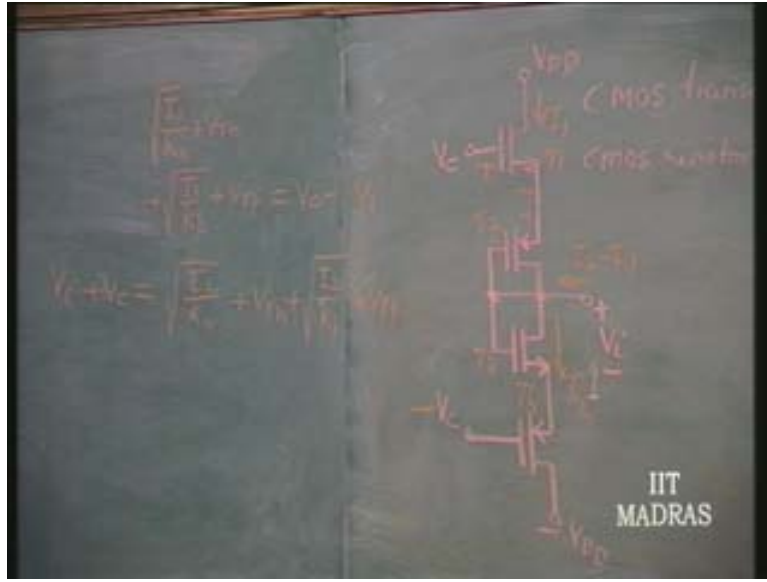
And what is this relationship again?

It is plus root of I_1/K_p plus V_{Tp} is equal to V_C minus V_i . V_C minus V_i is equal to V_{GS1} plus V_{GS2} . Coming to this again V_i minus V_C is V_i plus V_C is equal to V_{GS3} plus V_{GS4} .

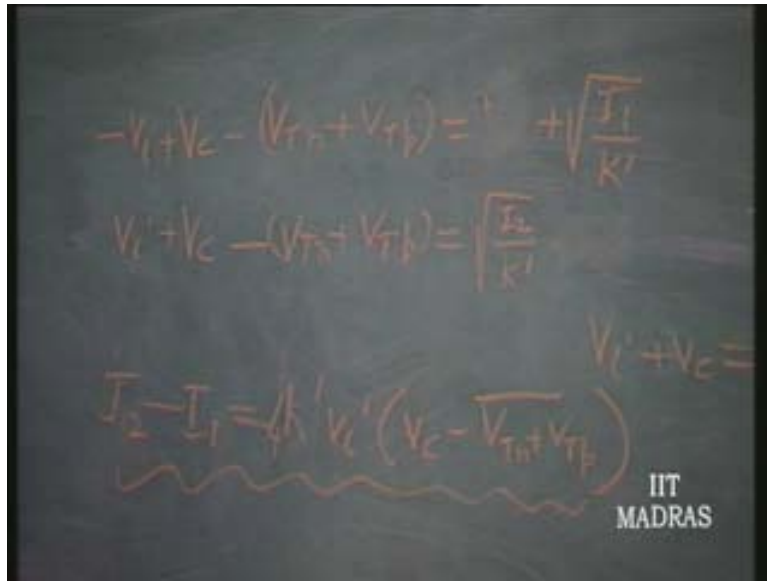
Now what is V_{GS3} ?

It is root of I_2/K_n plus V_{Tn} plus root of I_2/K_p plus V_{Tp} . So you have again got this kind of a relationship. And we rewrite this now as V_C minus $(V_{Tn}$ plus $V_{Tp})$ is equal to V_i plus root of I_1/K prime where K prime is a combination of K_p and K_n , $K_p K_n / K_p$ plus K_n .

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Then the other case is V_C minus V_{Tn} plus V_{Tp} is equal to root of I_2/K prime minus V_i . Actually I can now take V_i that side, minus V_i and here plus V_i . Again square and subtract. So I_2 minus I_1 is equal to $4K$ prime $V_i(V_C$ minus V_{Tn} bar plus $V_{Tp})$. So you get a differential current that is actually the input current you can call it I_i . This is nothing but I_i .

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$$V_i = V_C - (V_{Tn} + V_{Tp}) = \frac{I_i}{k'}$$
$$I_i = I_2 - I_1 = k' V_C (V_C - V_{Tn} + V_{Tp})$$
$$\frac{I_i}{V_i} = \frac{1}{R_i} = 4k' (V_C - V_{Tn} + V_{Tp})$$

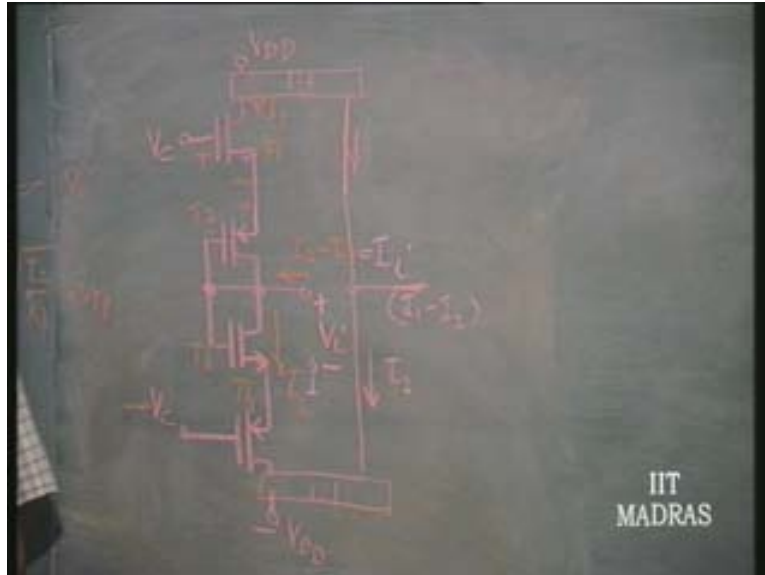
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So V_i/I_i is nothing but a linear resistor. Therefore we get this as I_i so I_i/V_i is nothing but $1/R_i$ the linear resistance which is nothing but $4K$ prime (V_C minus V_{Tn} bar plus V_{Tp}). Therefore you have a voltage controlled linear resistor using MOSFETs in current saturation. If you put V_{DD} and minus V_{DD} sufficiently large then all these transistors will automatically be in current saturation region. These are anyway all the time in current saturation region so these will also be in current saturation region. So you can get a wide dynamic range for this MOS resistor which is perfectly linear and voltage controlled.

Now if you want this to become a transconductor what should you do?

You have to convert this I_i into a current by putting a current mirror here, another current mirror here and then we can take the current difference in the following manner.

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So you can put current mirrors at these points and at this point another current mirror one using n-channel MOSFET and another using p-channel MOSFET and therefore current I_1 can be pumped here like this and current I_2 can be taken here from this and then you will get I_1 minus I_2 as the output current. This is now going to be a current transconductor, CMOS transconductor. This is the way you can synthesis large number of this type of CMOS transconductors which are usable in a variety of analog applications.