Analog ICs Prof. K. Radhakrishna Rao Department of Electrical Engineering Indian Institute of Technology, Madras Lecture - 20 Analog Multipliers (continued)

So, continuing with our lecture on transconductance type multiplier we saw that the Gilbert gain cell as depicted here has the two diodes connected in the following manner with currents I_1 and I_2 flowing through this.

(Refer Slide Time 03:12)

We saw, if I_1 and I_2 are the currents and I_3 and I_4 are the currents of the transistors pair that is connected to the voltage resulting due to the two diodes then by applying the Gilbert's the translinear principle in the Gilbert's gain cell we can say that I_1 into I_3 so I_1 into I_3 is equal to I_4 into I_2 this is what we wrote in the last class also. Therefore, I_1 by I_2 is equal to I_4 by I_3 and then by using the ratio principle I_1 minus I_2 by I_1 plus I_2 therefore in turn is equal to I_4 minus I_3 by I_4 plus I_3 .

Now, here we recognize the fact that I_1 and I_2 in turn are differential input currents generated by using transconductor wherein a voltage V_X is applied and this voltage V_X will generate a current in this which is V_X by R_X neglecting these effects V_X by R_X .

(Refer Slide Time 04:08)

And therefore if this current being I_0 and this current being I_0 this will be I_0 plus V_X by R_X and this will be I₀ minus V_X by R_X where a differential current is generated. So if you replace this I_1 and I_2 by these two currents now we can see here that I_1 minus I_2 is nothing but, let us say this is our I_1 and this is our I_2 .

(Refer Slide Time 05:21)

So we will see that I₀ plus V_X by R_X minus I₀ minus V_X by R_X that is this is $2V_X$ by R_X that is I_1 minus I_2 by I_1 plus I_2 this plus this is nothing but $2I_0$. This is now equal to the differential output current of this pair that is I_4 minus I_3 and divided by I_4 plus I_3 last time we called this current as I_0 dash. So this is going to be I_0 dash which is in this case I_0 plus

 V_Y by R_Y using the same principle of obtaining from a voltage a differential current by making a current of V_Y by R_Y flow through this. So once again the current in this is I_0 plus V_Y by R_Y and this is I_0 minus V_Y by R_Y . so we have now come to a conclusion that, by using a Gilbert gain cell one cell we can get this kind of a relationship.

(Refer Slide Time 06:56)

Similarly, we can show that this is also equal to by the same principle we have seen here that if you connect another transistor parallel to this call this T_3 which is the transistor connected here and similarly the same base is connected to the other base of T_6 , T_3 base and T_6 bases a are connected together and T_5 and T_4 bases are connected together. So it is just a parallel connection of another cell with another current flowing through this which is now going to be equal to, this was I_0 prime earlier let us call it, this is going to be I_0 double prime.

(Refer Slide Time 07:54)

So if this is the case by adopting a similar relationship here what will you get?

This voltage again remains the same so this diode current is I_1 that into 6 that is the current in this so I_1 into I_6 is equal to I_2 into I_5 by using the same principle. We can keep on connecting any number of such cells in parallel. The ratio relationship remains exactly similar to what we had earlier which means again I_1 by I_2 from this is going to be equal to I₅ by I₆. That means the ratio relationship I₁ by I₂ is equal to I₄ by I₃ is equal to I₅ by I₆ so on and so forth.

(Refer Slide Time 08:54)

Thus, adopting the same thing here we get I_4 minus I_3 by I_4 plus I_3 is same as I_5 minus I_6 by I_5 plus I_6 and we have obviously this differential output current of that transistor pair as I₅ by I₆ by I₅ plus I₆ is I₀ double prime. And I₅ plus I₆ is I₀ double prime which in turn is equal to in the diagram here I_0 minus V_Y by R_Y . So it is obvious that if you want to get rid of I_0 here the only thing you have to do is simply make this relationship equal to this plus this divided by this plus this. So, a by b is equal to c by d is same as a plus c by b plus d. so it tells us that in order to get rid of V_Y we can do this. If you want to get rid of I_0 you have to subtract this from this so a minus b by b minus d which is also same as a plus c by b plus d any of these relationships. So, if you want to get rid of V_Y we have to add and if you want to retain V_Y and get rid of I_0 we have to subtract.

(Refer Slide Time 10:19)

So we get I_4 minus I_5 minus I_5 minus I_6 by V_Y by R_y . that means this relationship is same as this whole thing which is $2V_X$ by R_X by $2I_0$ equal to that is the final relationship I_4 minus I₃ minus I₅ plus I₆ if you remove the bracket so $(I_4$ plus I₆ minus bracket I₃ plus I₅) rewriting the same numerator we get this as $(2V_y$ by $R_y)$.

(Refer Slide Time 11:41)

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Now what does it say?

Whatever is the summation of all these currents I_4 plus I_6 minus I_3 plus I_5 that thing is uniquely equal to $4V_XV_Y$ by R_XR_Y into $2I_0$. You can see that we have to add current I_4 to 6, how do you add currents? They are current sources, so I_4 is added to I_6 by just connecting it to a node. Those two currents get automatically added so this is I_4 plus I_6 . And here you have to add 3 to 5 that is I_3 to I_5 and then this node current is going to be I_3 plus I5. Then develop a voltage which is proportional to this difference. That means you can therefore take the output voltage between these two points, then what will be the output voltage?

If you take this as the output voltage then this point is nothing but V_{cc} minus I₃ plus I₅ into RCy minus V_{cc} minus I₆ plus I₄ into R_C which is nothing but I₆ plus I₄ into R_C minus I_3 plus I_5 into R_C .

(Refer Slide Time 14:05)

That means if you just multiply this by R_C this is going to be nothing but my $V₀$. You see how neatly the output voltage is going to depend purely on, obviously you can cancel this, this two times so we get the output as V_xV_y by V_R we will call it wherein V_R is equal to I_0 into R_X into R_Y by $2R_C$.

(Refer Slide Time 14:37)

Let us say we make R_X is equal to R_Y is equal to R is equal to $2R_C$ then this will be simply R_X is equal to R_Y is equal to $2R_C$ is equal to R then this is going to be equal to I_0 into R.

(Refer Slide Time 15:07)

This is an important design equation. What we should do is, we have to make V_R is equal to 10V. Therefore if I_0 is given to you as 1 milliampere I_0 is the current source biasing current then automatically you know that R should be 10 kilo ohm because V_R has to be 10V. So, if I_0 is given as 1 milliampere R is coming out as 10 kilo ohms. That means R is 10 kilo ohms this is 10 kilo ohms R_X is equal to 10 kilo ohms R_Y is equal to 10 kilo ohm and RC will be is equal to 5 kilo ohms.

The design is over in the total sense because we have also fixed up the dynamic range now because we can see that if I now make I_0 into R as 10V then this V_X can keep on changing until V_X by R_X is such that I_0 minus V_X by R_X becomes equal to 0. That means the maximum V_X it can handle is automatically equal to I_0 into R_X or I_0 into R_Y and the maximum V_Y it can handle will be I_0 into R_Y at which point of time one transistor will become off. So the moment I select this design here for a typical current I_0 of 1 milliampere I make R is equal to 10k, R_X is equal to 10k, R_Y is equal to 10k, R_C is equal to 5k and my $V_X V_Y$ limits also gets automatically fixed at minus 10 to plus 10V range. So the entire design is neatly over and this system is now ready for use as a precision multiplier.

Obviously these resistors Rx , R_Y and R_C s have to be put externally if you want it to retain its precision property because all these dynamic ranges as well as the multiplication constant is determined by R_X , R_Y and R_C . So this is one of the most popular IC multiplier available today. The only thing is it can be used as a precise multiplication up to about 1 MHz retaining the whole dynamic range of plus minus 10V and beyond that range there will be frequency limitation problems. That means up to about megahertz range this is very easily usable. But suppose in actual communication application we do not normally have signal input which is of the order of 10V there is no question of that and it is going to be of the order of micro volts or millivolts. So there is no point in improving the dynamic range. Therefore for communication application we can simply say that there is

no need to put this that means this can be shorted and now there can be a single current source but the only thing is it will not be V_Y by R_Y now but it will be V_Y into gm that is 1 by $2R_E$ of the transistor.

(Refer Slide Time 19:19)

This is the modification for a communication IC. Therefore here also I do not have to do this modification of converting voltage into differential current like this, I get rid of the whole stuff because in any case for small signal the differential gain itself is getting multiplied by gm which is directly proportional to the current, so that is a multiplier.

(Refer Slide Time 20:16)

So this circuit is called as balanced modulators by the communication engineers. This is separately available as another IC, so we have got rid of those linear transconductors which we had put to convert V_X and V_Y into differential currents and the circuit becomes pretty simple. And this is very commonly used by communication engineers obviously for multiplication purpose which is normally termed by communication terminology as balanced modulator ICs. This can be used up to hundreds of mega hertz. Therefore it is very popular for the purpose of fixing. All these operations are multiplication operations namely mixing, modulation, this is also called balanced modulator cum demodulator because the operation of modulation or demodulation both operations mean multiplication, balanced modulation and balanced demodulation.

So this particular IC is a very popular digital IC and the operation that is performed, if this is high or low and this is again high or low. That means now we are not talking of analog signal, this goes high or low and this also goes high or low independently. What is the output it is going to give? This circuit, when this goes high this goes high you find out the output and when this goes high and this goes low and you find out the output so do this and find out the truth table for this and you will see that this is called a gate Exclusive OR.

(Refer Slide Time 23:54)

If there are two inputs a and b what is the exclusive OR operation?

It is a b bar plus b a bar. So it is called exclusive OR. So if you look at a circuit you should be able to identify it straight away in ECL Emitter Coupler Logic. The fastest exclusive OR gate that you can think of for the purpose of this logic operation which is a b bar when this is a and that is b. You can find out the output corresponding to both high both low one high and one low etc and see how the output looks like. Output can in turn be limited by putting diodes across the output voltage so that the output does not swing as determined by the current but is limited to plus minus V_{gamma} . This circuit is also used as phase detector. Let us first understand the important communication application of this multiplier. Let us now look into the application to understand the difference between balance modulator, phase detector and precision multiplier.

(Refer Slide Time 26:16)

Application of multipliers:

We will assume that is a block obtained by anyone of those methods but it is a four quadrant multiplier as given here V_X and V_Y being input and the output is V_XV_Y by 10 and the dynamic range being plus minus 10V.

(Refer Slide Time 27:28)

Let us try to use this ideal multiplier for a variety of applications. The first application wherein it is used in large numbers is where V_X is going to be equal to the input voltage and V_c is going to be equal to the control voltage then the output is just used as a multiplier straight away without any additional paraphernalia being added here so V_c by 10 into V_i . So this is what is called as voltage controlled amplifier but it does not amplify in terms of voltage gain but in terms of power gain is what it really means when we say amplifier so still it can be called an amplifier. Therefore this is called voltage controlled amplifier.

In turn we can say that if you can design a linear voltage controlled amplifier it is a candidate for multiplier. The gain can be controlled from 0 to 1 it can change signs. That means it is acting both as inverting and non inverting amplifier depending upon the sign here. You have control over the magnitude of gain as well as the sign which is an important thing. So this voltage controlled amplifier is commonly used in what are called music synthesizers and things like that. In most of the music synthesizers the special effect generators etc this voltage controlled amplifier is used in large numbers.

(Refer Slide Time 29:31)

Voltage controlled amplifiers can in turn be used, this is an op amp integrator when this is V_i the current in this is V_i by R it will flow through this and develop a voltage which is V_0 is equal to minus V_i by SCR so it is an integrator. An integrator is one of the most important basic building blocks in analog signal processing particularly in what is called filtering. Therefore an integrator is an important building block.

(Refer Slide Time 31:13)

Therefore now if I want to convert this integrator to become voltage control I merely have to put this before that. Now you see that it becomes an integrator which is voltage controlled because originally we were feeding V_i here and now we are feeding V_i here so this becomes V_i into V_c by 10. So, wherever originally we had V_i we simply put V_i into V_c by 10 and we get the transfer function for this that is minus V_i by SCR into 10 by V_c because it is V_iV_c by 10 wherever V_i is there we put V_iV_c by 10. This means the time constant CR is merely changed to CR into 10 by V_c . That means in any circuit that was earlier using this integrator where the time constant was CR I simply replaced it by this indicator and replace the time constant CR by 10CR by Vc.

I get the performance factors of that particular circuit. Now let us say I use this integrator in what we are calling as a double integrator oscillator that we consider now. A double integrator oscillator is something in which two integrators are connected together and then an inverter is connected and if you now close the loop it becomes a double integrator oscillator or a harmonic oscillator. It just simulates a second order differential equation d square V_0 by dt square plus KV_0 is equal to 0.

(Refer Slide Time 32:37)

Therefore that is a double integrator oscillator and if you replace it by this the frequency of oscillation of the double integrator oscillator is going to be omega is equal to 1 by CR whereas in this case it will be omega is equal to V_c by 10CR. That means it becomes a voltage controlled oscillator. It becomes a linear voltage controlled oscillator or VCO. The frequency of oscillation can be linearly controlled by varying V_c .

What does this circuit mean in terms of communication?

You are having a FM generator. That means now if I have a carrier to be frequency modulated I simply apply the frequency the modulating voltage in series with some constant voltage. V_c will be some constant which will determine the carrier plus V_m sign omega mt then I can generate FM. So, voltage controlled oscillators are used for FM generators.. Communication engineers therefore build FM generators in order to build voltage controlled oscillators.

Now if the same block is used in a filter configuration which is called KHN filter, again it is nothing but the double integrator filter, the center frequency is going to be omega is equal to 1 by CR if it is band pass output and in the voltage controlled version you will have that being controlled by V_c linearly. So you can now design a voltage controlled filter. All these things are simple consequences of replacing the integrator by the voltage controlled integrator. So you can see how these variety of circuits get generated simply by obtaining this voltage controlled action of the multiplier.

(Refer Slide Time 36:06)

For example, here is something again used as an IC, this circuit is a very popular circuit. This circuit is called the function generator circuit. This gives you a square wave and this gives you a triangular wave. You have frequency omega or frequency f is equal to 1 by 4RC. That means now I want to convert it into t voltage controlled oscillator. Now it is enough if I just replace this one integrator by the multiplier. The advantage of this voltage controlled oscillator over the previous one is simply that it uses only one multiplier as against two multipliers needed in the other circuit.

(Refer Slide Time 37:59)

So this is presently being used in most of the ICs for the purpose of voltage controlled oscillator with facility for linear modulation of the frequency. So what happens now? The frequency of oscillation becomes, RC has to be replaced by 10 RC by V. This now becomes f is equal to V_c by 40RC. This whole thing is called a function generator IC with the multiplier, a Schmitt trigger and an integrator. It only gives you a triangle not a sine wave but for most of the communication applications we can use this itself as the FM output it does not matter. Hence it is perfectly linear.

Later on we will discuss this entire thing as a voltage controlled oscillator separately using transistors. These use op amps. Obviously the frequency up to which you can use this is highly limited not by the principle but by the op amp you are using. The op amp, slew rate etc will limit the performance of this IC. These are large signal oscillators so slew rate comes into picture. Therefore instead of realizing this using operational amplifiers and multipliers you can do this simply by using transistors.

Later on let us look into how transistor voltage controlled oscillator circuits can be generated which also uses the same principle of a Schmitt trigger a voltage controlled conductor charging a capacitor. But function generator IC is an important application of multiplier, the FM generator.

(Refer Slide Time 40:41)

Now, if I make one input equal to A sine omega 1t and the other input as B sine omega 2t, now the multiplier is still being used as multiplier but inputs change. In the earlier situation we had one as conventional input of any frequency but the other as DC then it became a voltage controlled amplifier. Here both the inputs are AC of two different frequencies. This operation of multiplication in communication terminology is called mixing. It is just mixing of two frequencies so it is called a mixer.

(Refer Slide Time 42:02)

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And obviously the output will be AB by 10 sine omega 1t sin omega 2t is equal to AB by 20 cos omega₁ minus omega₂ into t minus cos omega₁ plus omega₂ into t where these are called side bands. If omega₁ is the carrier and omega₂ is the modulating frequency then this application is called balanced modulation. These are the two sidebands omega_c minus omega_m and omega_c plus omega_m. So these are the two sidebands that you are getting. This is not the amplitude modulation but this is balanced modulation, the carrier is absent.

If you introduce the carrier, how do you introduce a carrier?

The only thing is, you should not multiply, this is omega ct but this is omega mt plus adc then automatically carrier will come or it will be having feed through components. That means actually for producing amplitude modulation you do not need an ideal multiplier but you need a non ideal multiplier. So, amplitude modulation means Vdc plus this and this is omega_m and this is omega c that is the amplitude modulated output.

(Refer Slide Time 42:50)

So, by nearly changing the input I am getting different applications. For AM generation the input will be this and for balanced modulation the input will be this. If on the other hand, omega₁ is omega_C plus or minus omega_m and omega₂ is omega_c plus omega_{IF} what happens?

(Refer Slide Time 44:28)

In the difference component you will get omega_c going and it will be omega_{IF} plus or minus omega_m. There will be of course two omega_c components plus omega_{IF} plus or minus omega_m component the other one. That can be got rid of by using a band pass filter which is called IF filter, this is called IF filter. That means at that point of time this is taking the role of a true mixer. And this generated by what is called local oscillator that is the basic principle of super heterodyne receiver. The radio receiver or for that matter even television receiver. That means you are having an IF.

For example, it is like different people carrying the same luggage may be from the railway station to the taxi stand a porter is carrying thereafter once it comes to the house you have to carry it there is no other go, so that is the thing. Therefore basically we can design these local things to operate at a fixed frequency. Therefore we can design better amplifiers at fixed frequency than at variable frequency. That means RF amplifiers are more difficult to design than IF amplifiers. that is the reason we go in for amplification at a fixed frequency and transfer the entire responsibility of amplification or what is called sensitivity of the radio receiver or a television receiver to what is called an IF stage. That design is done at fixed frequency and there is no problem whereas most of the carriers will be coming at different frequencies. These are selected and the entire information is transferred, it is like merely acting as a porter carrying the information but a common porter.

As soon as I come to my house instead of me carrying I can always call may be call my servant who is going to carry this and any number of times I come the same person will be carrying whereas from the railway station to the house it will be different persons for carrying. Then there might be some trouble in dealing with such people whereas I have trained my servant and have given him enough rewards so there is no likelihood of any complaint. The same thing is with the design of IF stages. It has been optimized and therefore it is going to do the business without much of a problem. This particular thing is going to be then called mixer. So we have to now see how inputs can change the application of the particular multiplier. The name also is different.

(Refer Slide Time 48:53)

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So omega₁ is omegaI_F plus or minus omega_m and omega₂ is omega_{IF} which is easily generated locally and the output will be 2 omega_{IF} plus or minus omega m but it can be got rid of by using a low pass filter and the only component you get will be omega m component which is the modulating frequency component. That is nothing but demodulation of the balanced modulator. This demodulation technique is not adopted in the conventional receivers simply because it requires the generation of omega_{IF}. It can be done simpler if you adopt amplitude modulation that is adopted only for the fact that it can be simply detected by chopping of one portion and doing envelope detection using a simple diode. Therefore balanced demodulation becomes imperative if balanced modulation is there, carrier is not there and you are able generate equal stability carrier at the receiving point which is a tough affair. So this is the balanced demodulator. Now, if omega₁ is omega and omega₂ is also omega but then if they are two same frequencies they can be differing in phase.

(Refer Slide Time 49:36)

Then what do you get as the output here? This will be phi and then that will be 2 omega_t plus phi. Once again there are two applications. So AB by 20 cos phi that means it develops a DC voltage which is dependent upon phase shift. So it is defined now as a phase detector. It is not a linear phase detector but it is a cosine FM. Again you can see you have to put a low pass filter in order to get rid of 2 omega_t component. You put a high pass filter you get rid of the DC and you will get frequency doubling. The same thing can be used as a phase detector or frequency multiplier depending upon whether you connect a low pass filter or a high pass filter.

Now again, if A is a voltage which is proportional to line voltage and B is a voltage proportional to line current then it is nothing but power, it is like an electronic watt meter. So it is again used for such applications. I mean you have to use a current transformer and a voltage transformer respectively in order to generate these input voltages and feed it to this multiplier at the proper level of voltage. Then you can get power factor or power etc.

So you can use your 2 watt meter method and instead of 2 watt meter method of power measurement in three phase circuit you can use two multiplier method of power measurement, active or reactive whatever it is. So a variety of such applications are possible with the help of the versatile tool which is the multiplier. You can multiply frequency, if you can multiply frequency you can generate harmonics,

if you can generate harmonics you can synthesize waveform, you can synthesize any periodic waveform in terms of t square, t cube and all these things can be done. So a nonlinear functional block like a multiplier is used for function synthesis. Any non linear function can be generated to any accuracy you want if you are prepared to use any number of multipliers. These are the powerful applications of this multiplier. In the next class we will continue further with other varieties of multiplier that are available in integrated circuit form.