Analog ICs

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Analog Multipliers
Lecture - 19

So, today we will start with one of the most important analog signal processing block that is multiplier. After discussing about amplifiers it is appropriate that we now go from linear to nonlinear operation. The most important nonlinear operation analog signal processing has is multiplication. So now we will consider the various means of multiplying using IC components or the basic techniques. First of all let us try to understand the basic definition of a multiplier.

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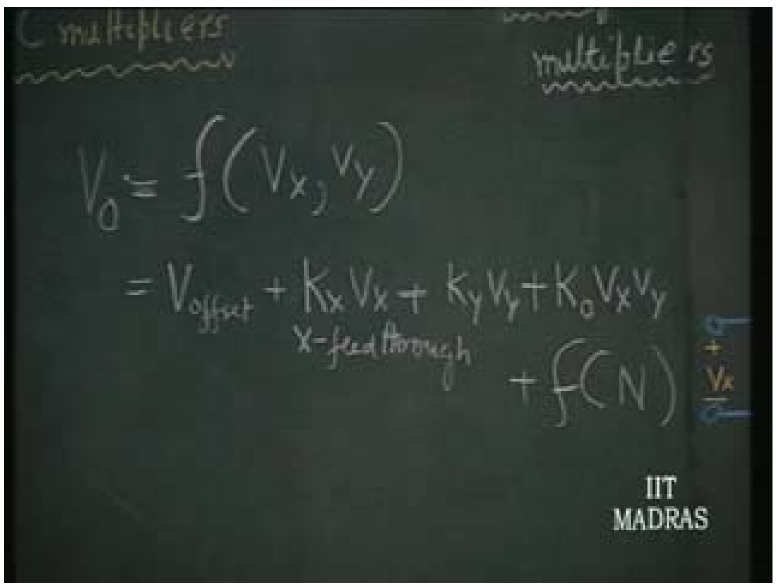

What is a multiplier? So V_0 is the output, this should be a function of two inputs V_X and V_Y, this is what a basic unit doing multiplication should do. That is, it should give an output which is a function of two inputs. Therefore in general any nonlinear device is going to give us an output which is independent of V_X and V_Y and we will call that as V, it is called offset because that is independent of V_X and V_Y. Then some term is purely dependent upon V_X, this in multiplier terminology is called x-feed through. It should not come but if it comes it is called x-feed through.

Then obviously what else will be there?
K_Y V_Y which is obviously called y-feed through and then we have K_0 into V_X V_Y and then we have obviously V_X square V_Y square V_X square V_Y V_Y square V_X these are called the nonlinear terms. That means this particular output is going to be there in almost all nonlinear functions in general. The only thing is in certain nonlinearity we may have V offset becoming naturally 0 and in another nonlinearity K_x K_Y may be naturally 0 and

fn naturally 0 etc. That means we must device a configuration wherein this is as small as possible and these two are going towards 0 and this is as small as possible. That is a good configuration for the operation of multiplication. That means any practical multiplier will be non-ideal obviously that means all these components are likely to be present in some measure.

Even if the best configuration is available to you these non-idealities will be small but they will be present. So in a precision multiplication operation it is your duty to see that there must be some possibility of adjusting this to go towards 0. If you select a configuration wherein this itself is going to be very small that is the proper configuration for multiplication so this has to be made as small as possible. Using the nonlinearity of the device you try to put it in a configuration that this is made as small as possible.

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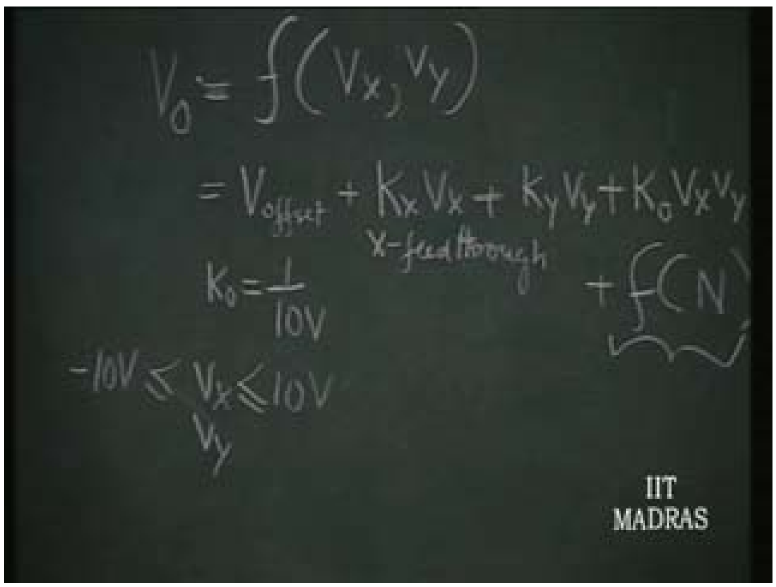

Then how do we make basic adjustments of a multiplier? We see to it that K_0 is equal to is standardized in most of precision multipliers as 1 by 10V so that the multiplication operation is possible by a dynamic range of less than or equal to 10V or greater than or equal to minus 10V. Similarly, the V_Y should have a dynamic range. This is the standard for precision multiplier design. That means the constant of proportionality has been adjusted to 1 by 10V and V_X and V_Y must have a dynamic range of minus 10 to plus 10.

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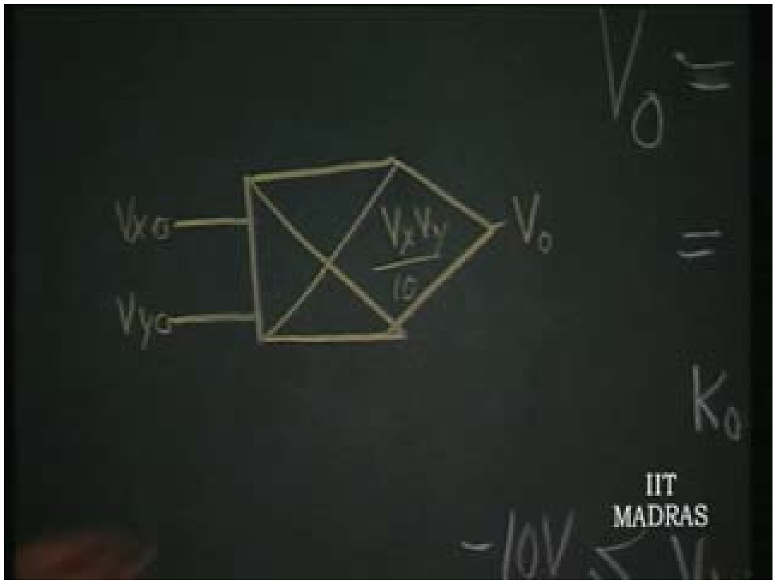

Then the symbol for the multiplier becomes, that is the block, this is V_X, this is V_Y and multiplication operation and this is V_0 and this is nothing but V_X V_Y by 10. So it is automatically assumed that if you are using a standard IC multiplier and these outputs are given you can take for granted that they are constant of proportionality and K0 has been adjusted to 1 by 10V. This is basic to the design of IC multipliers. So you can keep on connecting one multiplier to the other and it will be perfectly compatible because everything has been standardized. You can see that if K_0 is equal to 1 by 10V the maximum output ever of this multiplier will be 10V. These are compatible with one another therefore we can connect these multipliers in cascade without any problem of dynamic range.

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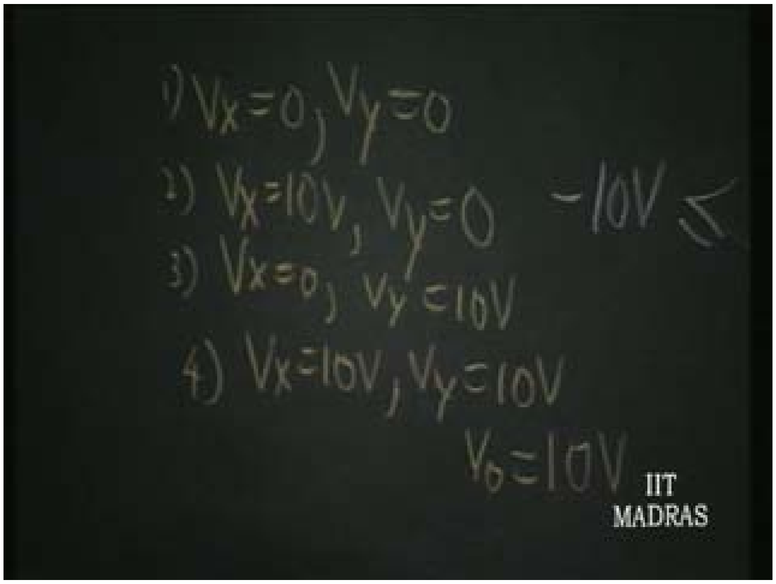

So obviously before you use any multiplier you must see to it that these non idealities go towards 0. Now how do you make the adjustment? The adjustment is done this way, V_X is equal to 0V, V_Y is equal to 0V then the only output that you can have is the offset and that is adjusted to be equal to 0 by what is called offset adjusting methods. So V offset is made equal to 0, this is made to go towards 0. Then we make V_X is equal to 10V and V_Y is equal to 0V then any output if it comes it must be due to x-feed through and that will be coming through as its maximum error output and that has to be adjusted to 0 not by adjusting V_X but by adjusting a means of making K_X is equal to 0. So Kx adjustment is done that way. Then V_X is equal to 0 V_Y is equal to 10V and the output that can now come is due to y feed through and therefore it is K_Y which is adjusted towards 0. Finally V_X is equal to 10V and V_Y is equal to 10V and V_0 should be adjusted to be equal to 10V.

That is, K_0 is going to be then adjusted to 1 by 10V. When V_X is equal to 10V and V_Y is equal to 10V V_0 should be 10V then only K_0 is going to be is equal to 1 by 10V. So, when all these measurement adjustments are made then the multiplier is ready for use, not before that if it is precision multiplication that you want. So you can see that unlike the amplifier wherein output is only a function of one input wherein you just have offset and K_X times V_X and then nonlinearity coming into the picture.

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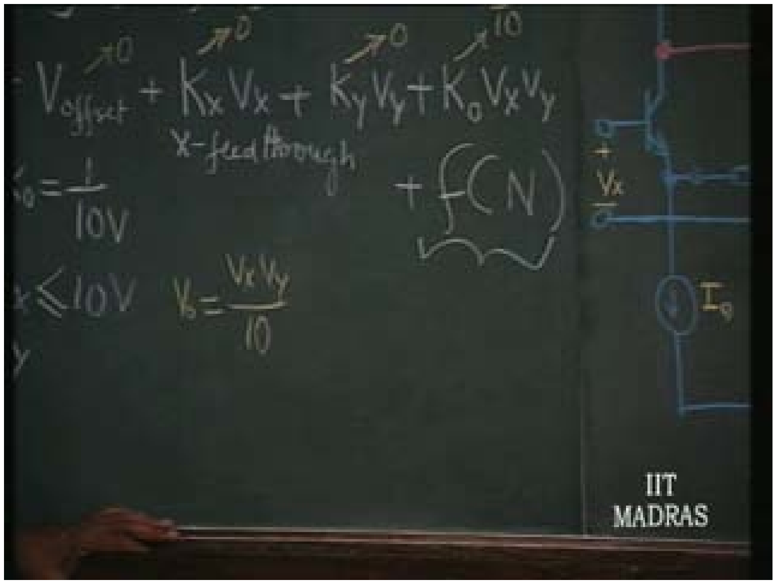

Here you have more number of adjustments to be made because it is a function of two inputs. So now after this adjustment is made the multiplier is ready for use and thereafter you can use the expression that V_0 is equal to V_X V_Y by 10 and this is going to be valid for V_X and V_Y both ranging from minus 10V to plus 10V. You should remember this as the property of a precision multiplier available in an IC form. Now this has to be implemented. Either the circuit implementation of this operation is what we are going to discuss henceforth for some time.

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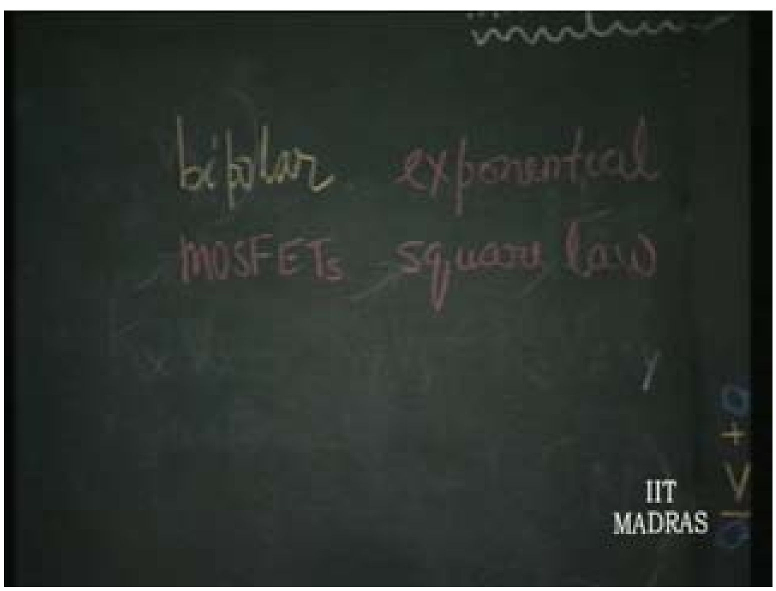

What are the basic methods of multiplication? Now we will consider the devices that are available to you. The nonlinear devices that are available to us are bipolar transistor and MOSFETS. Bipolar is essentially a nonlinear device which is using exponential relationship. The I is equal to IA_0 exponent Q_v by K_t minus 1.

So current and voltage are exponentially related and MOSFETS square law. In our previous lecture on voltage dependent transconductance and current dependent transconductance we have established that these are the complementary things available in nature. And these can evolve naturally as the devices that are necessary for an engineer to complete the whole process of voltage dependent transconductor, current dependent transconductor, that we have established.

Obviously a multiplier can be thought of as a voltage dependent or current dependent amplifier of a sort. Obviously we have to use these two relationships. Let us now see how we can use these relationships in formulating a multiplier.

First the basic multiplier application using bipolar device will naturally use the most commonly known knowledge about multiplication operation using log antilog tables you have been using for multiplication. Once we have a device which can do log operation and antilog operation obviously we can use it for multiplication by taking the logarithm of V_X, taking the logarithm of V_Y and taking the logarithm of 10V or V reference. Adding log V_X plus log V_Y minus log V_{ref} or log 10V is now going to be the multiplication operation of V_X V_Y by V_{ref} and log of that is what you have got, now you have to take the antilog to get the multiplication.

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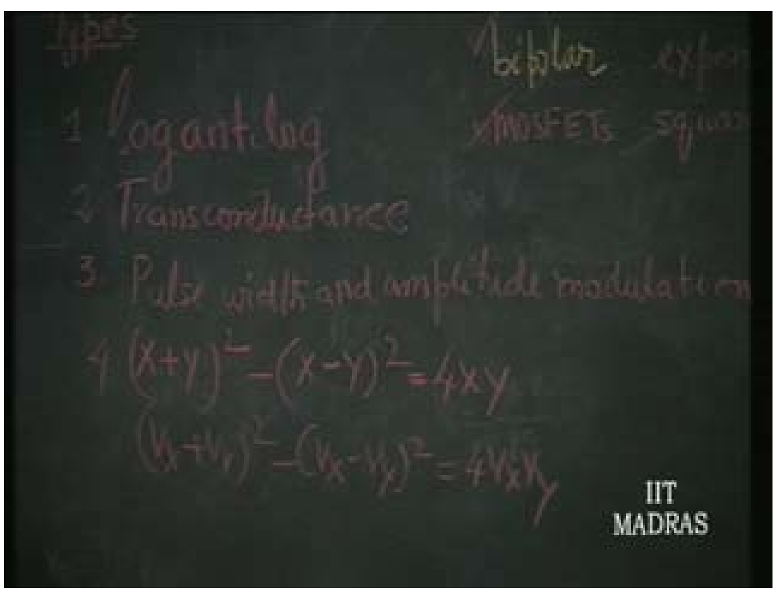

So this multiplication or multiplier is called log antilog multiplier. Then obviously we know that the bipolar transistor transconductance is dependent upon its operating current and this current in turn can be made to depend upon voltage. So you can now design a voltage dependent amplifier which can be a candidate for multiplication. So the second property that we will use is transconductance type of multiplier. These two essentially make use of the property of the bipolar transistor for its exponential function. The third one on the other hand is the basic principle that we had again learnt that when I have a switch which can connect V_X or disconnect V_X, connect V_X or connect minus V_X for respectively tau and t minus tau. I can get an average which is V_X tau by t which we have discussed in switched mode regulators and power supplies. Therefore by making tau dependent upon another voltage now tau by t by what is called designing a pulse width modulator I can design a precision multiplier. This is called pulse width and amplitude modulation type of multiplier.

Now, finally the other device that is the MOSFET basically uses square law relationship. And if you have square law relationship you can device multiplication and one simple technique is (x plus y) square minus(x minus y) square which is nothing but, this will be respectively V_X and V_Y so 4_{xy}. That means add two voltages (V_X plus V_Y) square, subtract these voltages minus (V_X minus V_Y) square which is equal to $4V_X V_Y$. So this is the basic principle of most of the transconductance types which can be used for multiplication in MOSFET devices which have a large dynamic range. So we will discuss all these types of multipliers one after the other and see how these can be implemented in an IC form.

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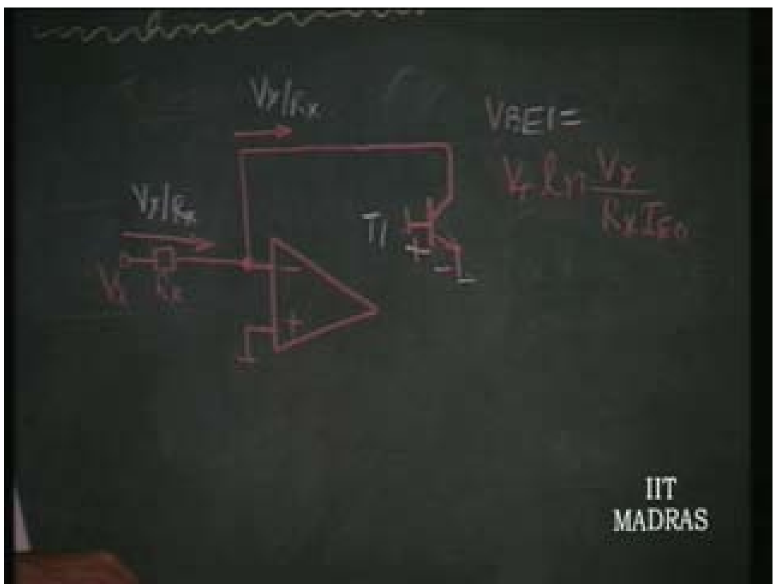

First let us therefore take this log antilog multiplier. So obviously we have to perform this log operation and the antilog operation and add. For performing the log operation we know that we can depend upon the operational amplifier to convert voltage into current because we want a relationship between voltage and voltage ultimately. So this V_X can be converted into current by making it go through this and as long as it is used in the negative feedback mode operation this is going to be virtual ground. There should be negative feedback from the output that way. Then the current in this is going to be V_X by

R_X. This current has to flow through the collector of a transistor that this will be forced through the collector of the transistor then if it is still a negative feedback configuration this will force itself through the collector in order to generate a voltage which is going to be Vt log, this voltage is going to be V_t log V_X by R_X I_{E0} because of the exponential relationship.

So V_X by R_X is going to be the current flowing through this and in order to develop that current the voltage here is going to be if this is T_1 then it is V_t log V_X by R_X. The output voltage will be negative here with respect to that. But there should a connection to the op amp. I can connect it here still it works. But if I connect it through a resistance here like this it can still work because this voltage will adjust itself so this voltage is the negative feedback.

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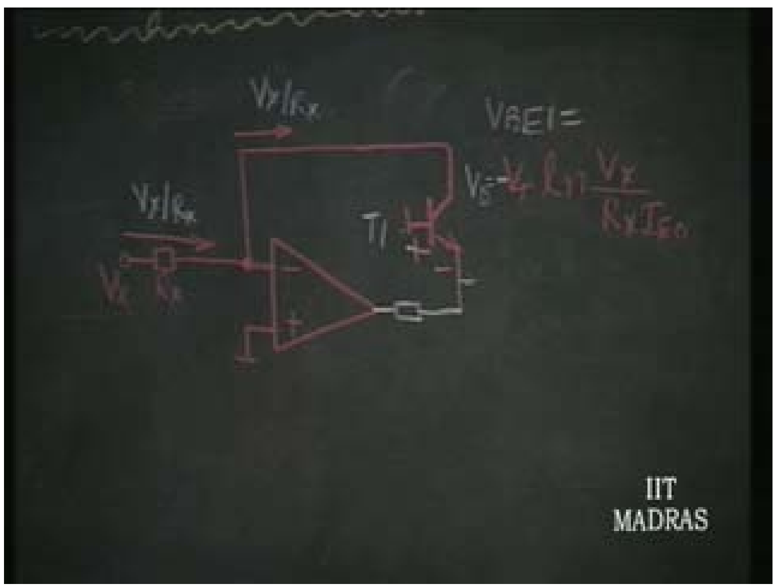

So why would I put a resistance like this?
I can even connect it like this. Here I am trying to discuss both, this called a log amplifier.

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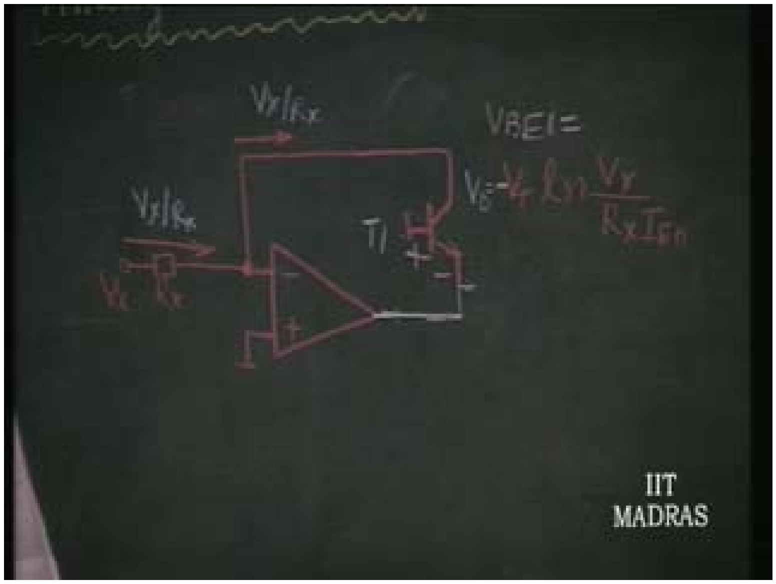

What we are now designing is nothing but a log amplifier. Given the input voltage V_X it is going to give an output here which is nothing but minus $V_t$log V_X by R_X I_{E0} whether you put a resistance or not. You do not put a resistance; nothing is going to happen because current is going to be limited to V_X by R_X.

The voltage will adjust such that this current is made to flow. You can look at this structure itself as a negative feedback amplifier now. This is an inverting amplifier here to here and then you have a transistor amplifier in common base if you ground this basically which has a gain of its gm which is 1 by R_E times R_X and that output is connected to the op amp. That means the open loop gain of this is A and that is to be multiplied by gm into R_X. That means the overall loop gain has increased in this negative feedback situation from A to A into gm into R_X.

So, if your op amp has been designed with just unity gain kind of operation being satisfactory for frequency compensation then it may feel because now the open loop gain is more that means the frequency compensation capacitor might have to be increased further because the open loop gain is increased. Therefore we do not want to put some other frequency compensation here or we would like to use same old frequency compensation then this kind of feedback must be given without the gain being increased.

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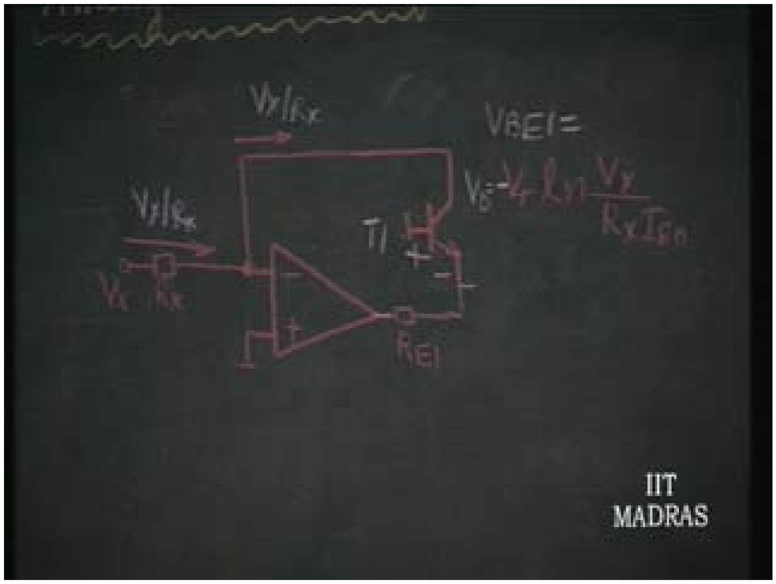

So purposely we put the emitter degeneration resistance. So we will increase the emitter resistance by putting additional resistance here so that the gain now is R_X by R_E plus R_{E1}. You can reduce the gain to almost nearly 1 and therefore resurrect this amplifier which was going to oscillate otherwise because of its inherent frequency compensation being valid for its maximum loop gain of A.

You can think about this as high as possible but that is not so. If I keep increasing this the voltage at this point will keep increasing to a value which is VX by R_X into R_E and that might go towards supply voltage. Therefore I should be very careful in limiting this to a value such that the output of the op amp does not go to its saturation value. These are the conditions for selecting the value of R_E. This is the case with even design of a log amplifier. Now we have got the required voltage. Now I should add the log of another voltage which is V_Y to this voltage.

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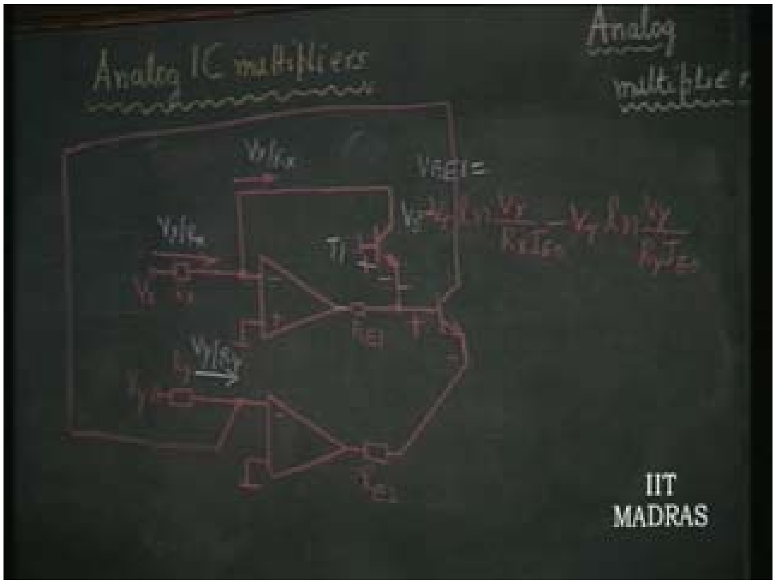

That again is going to be similar to this so I will take another voltage here that will be R_y and this you get as V_Y and the input current in this is going to be V_Y by R_Y and that again has to be converted into [.... 26:53]. You might think adding requires another amplifier but it does not require because we have a voltage here, we have to put another voltage in series with this which will correspond to the same thing but determined by this current. That means I will now have this coming like this, I am forcing V_Y by R_Y through the other collector then automatically this voltage gets developed but still I have to complete the negative feedback so once again I will put R_{E2} so that there is negative feedback for this.

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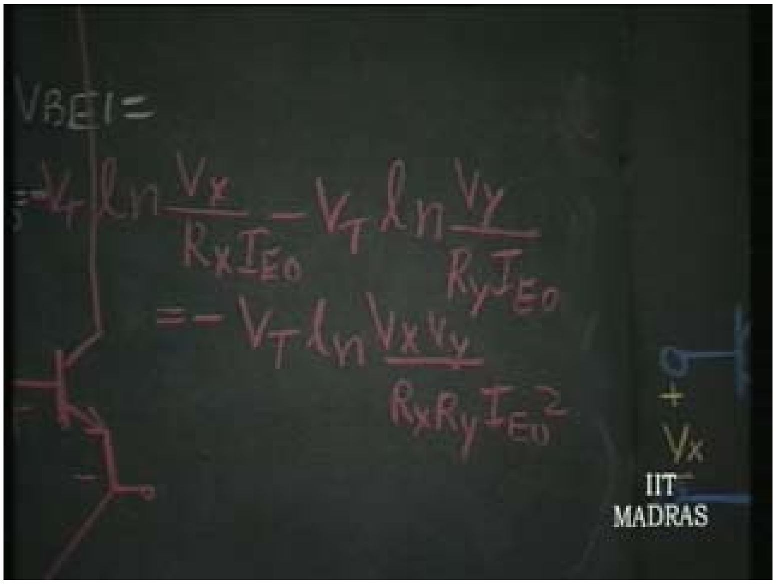

Ultimately now we are getting a voltage here at this point which is nothing but this point output we are taking which will be nothing but minus $V_t$log of this minus $V_t$log V_Y by $R_Y I_E$ this is the output at this point. Now we have got this such that now the output is going to be equal to minus $V_t$log V_X V_Y by R_X R_Y I_{E0} square. So we have multiplied V_X and V_Y. The only thing is, in our method of multiplication we have added these two negative voltages that means we have to subtract from this another similar term which is dependent upon V_R.

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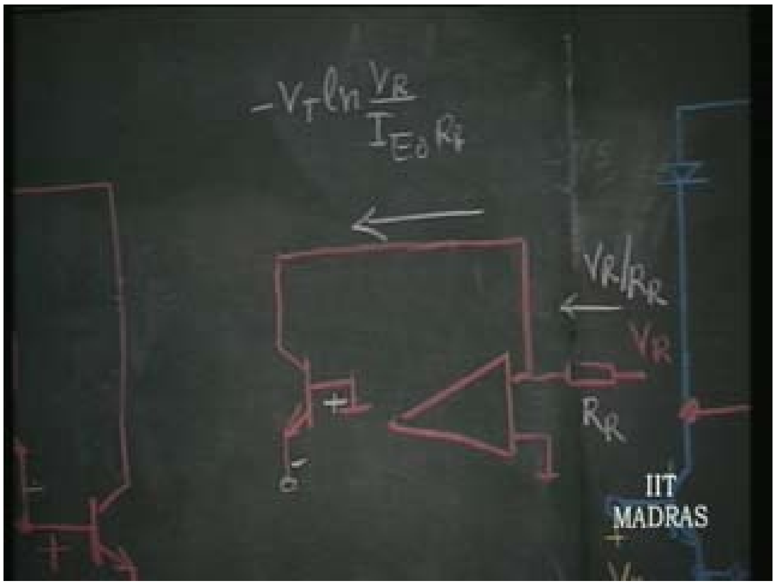

So, for that purpose again we will use a similar technique of converting. We are going to use another transistor V_R we call it, what will be the current in this? Let us call it R_R here so V_R by R_R which will flow through this and establish a voltage corresponding to a gain minus $V_t$log V_R by $I_{E0}R_R$. We have already developed a negative voltage with respect to this and we have developed another negative voltage that corresponds to minus $V_t$log V_X V_Y by R_X R_Y I_{E0} square. These are the two negative voltages we have developed with respect to ground.

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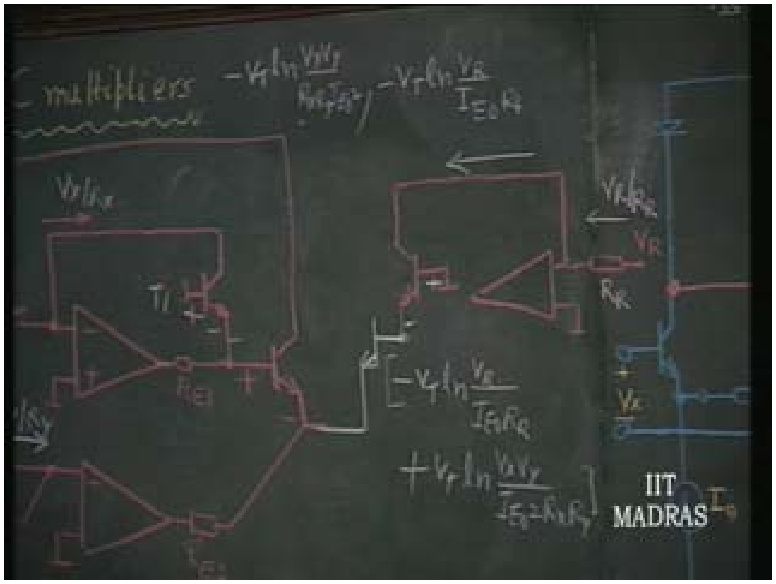

That means if you take this differential voltage it is the subtraction of this voltage from this voltage which will again form the input voltage to another transistor base emitter junction. This is more positive than this, this is minus $2V_{gamma}$ kind of thing and this is minus V gamma kind of thing. So now this will form an input voltage and what is the input voltage to this? Actually it is minus $V_t$log V_R by $I_{E0}R_R$ plus this minus this V_{tl}og V_X V_Y by I_{E0} square R_XR_Y.

Now the circuit is not complete because we have been able to establish this and obviously there should be negative feedback there which means there should be a resistance to this which we will call as R_{E4} and here we will call this as t4. As far as this is concerned it establishes the voltage.

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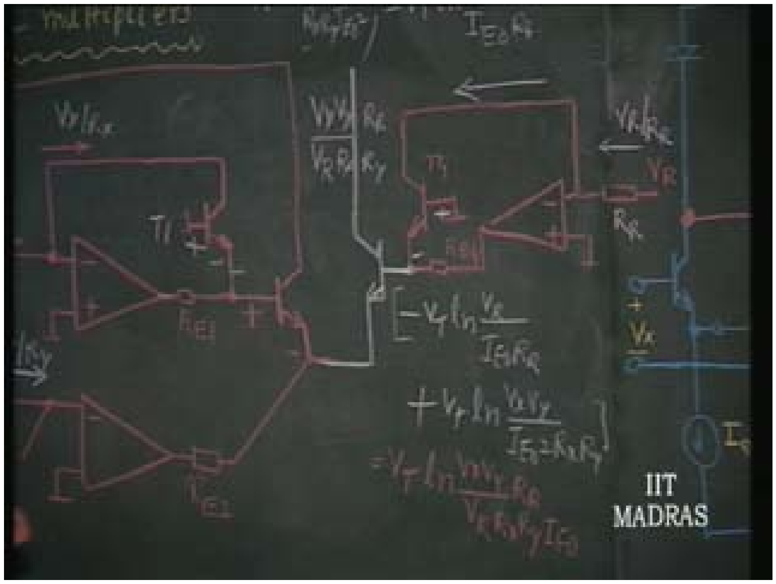

We have been able to apply this voltage to base the emitter junction of a fourth transistor and what will be the collector current dependent on? It will be dependent on this voltage, what is this voltage? This is $V_t$log you just take V_X V_Y by V_R and the I_{E0} gets cancelled, R_X R_Y R_R and there will be one I_{E0} and that is the voltage you are applying.

And what will be the collector current?
If that is the voltage applied then what will be the collector current? It is the exponent of this voltage divided by V_t, what is that? V_t and V_t will get cancelled and the exponential of log of that. So you will get $V_XV_YR_R$ by $V_RR_XR_y$ and this I_{E0} gets cancelled. So I_{E0} exponent this so we get here the current which is nothing but V_XV_Y by V_R R_XR_Y that is the current.

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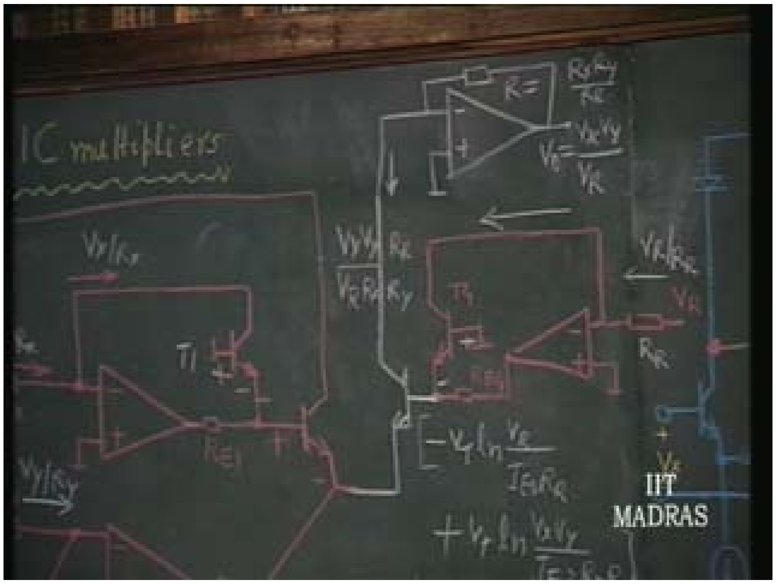

Now you see how neatly this is working. This current can be made to go through another resistor of an op amp which is connected in a similar fashion. This current will therefore be converted to a voltage, so what will be the output voltage now?

This is I and this I is flowing in this direction so it is I_r which is nothing but $V_X V_Y$ by V_R and I will make R_E is equal to $R_X R_Y$ by R_R. Then the output will be simply $V_X V_Y$ by V_R. So you get a precision multiplier which is totally independent of I_{E0}s as long as all the transistors have the same I_{E0}. That is why this is a very cheap precision multiplier but a cheap single quadrant because $V_X V_Y$ and V_R have been assumed to be positive so that the current is only in this direction. This is a single quadrant log antilog multiplier which can operate up to plus minus 10V if you design it very simply.

The design for its dynamic range is very simple because you can make this, for example V_X has to go up to 10V and V_Y has to go up to 10V or V_X has go up to 20V because it is a single quadrant multiplier we might like to go up to 20V it is simple and we want the operation of this to be limited to 1 milliampere then we automatically know the resistance as 20 kilo ohm and if it is 10V we know that the resistance has to be 10 kilo ohm. So we can limit the dynamic range of operation of this by limiting the current into this, this is important.

This current actually fixes up the dynamic range, whatever you want 20 Ohm so 20, if you want you can even make it 50V even if the op amps are not working at so called 50V because it is always getting converted into a current. That is why this is also called data compressor. Here the voltage can vary by any amount but the voltage variation here is compressed to something around V_{gamma}. So this is the data compressor or a compander that is the log amplifier. So this is also used for coding purposes because it can exactly compress the data in a very small range of voltage variation. It can be used straight away for multiplication as also division because if I make this V_X and this V_R and the other one

V_Y will straight away do the division. This is what is called multiplier cum divider; the V_R is what we are calling as your reference which is going to be maintained constant. So if you make V_R as V_Y and V_Y as your V_R you are getting a divider. And once again division by 0 etc is not allowed because you cannot go to the current which is very low here because our assumption is that I_E is equal to I_{E0} exponent Q_v by k_t and not minus 1 and that minus 1 has been ignored. So the exponential operation is necessary. That means current should not be going down to values of the order of I_{E0}. So you are not allowed to make V_X or V_Y or V_R is equal to 0 in this even for multiplication operation not just even for division. Even for multiplication operation it will go wrong if the forward current is of the order of I_{E0} or lesser. So this is a very versatile thing. The only disadvantage of this is, it is using op amps to convert voltage to current. Therefore the frequency of operation is going to depend upon the op amp compensation.

Therefore it is going to be limited to low frequencies which are the limiting thing for the op amp. So its limitation is only for the op amp and not for the basic idea of multiplication. We have just now seen how the idea of log antilog can be very neatly implemented with the help of a matched pair of transistors. This is obviously a single quadrant multiplier.

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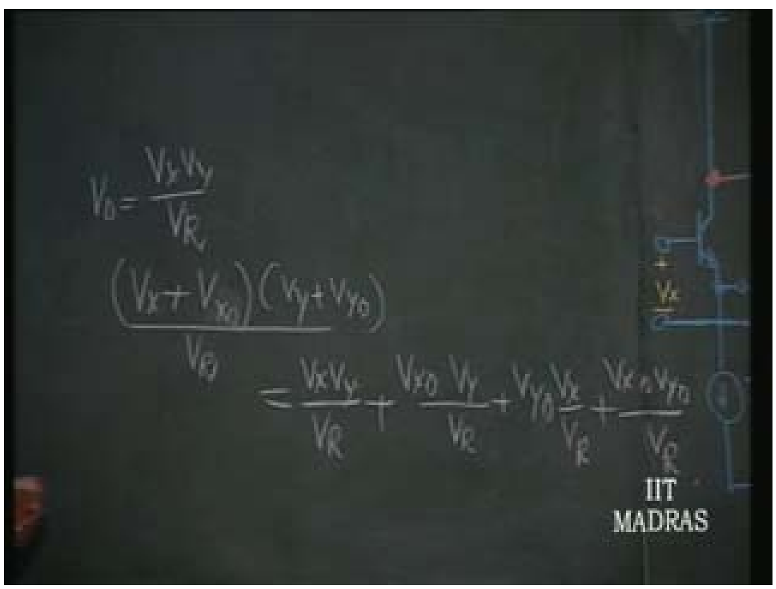

In fact we will see later how any single quadrant multiplier can be converted into four quadrant by simply biasing it. For example, I have achieved a single quadrant multiplication which is V_XV_Y by V_R that is what we achieved today using log antilog. But we said V_X has to be positive, V_Y has to be positive and V_R has also to be positive or of a single polarity. If you want you can make all them work for negative polarity or something like that but it should be of single polarity.

Now, if I want to operate with dual polarity input obviously I will put V_X plus some V_1dc and V_Y instead of Vdc$_1$ we will call it as V_{X0} V_Y plus V_{Y0} this is a DC and then make it a multiplier which can work with V_X going both plus and minus 10V maximum and V_Y going both plus and minus 10V. For that I have to make V_{X0} equal to, how much? V_X should be capable of going both plus 10V and minus 10V and still the total has to be positive. So, if you make V_{X0} is equal to 10V and V_{Y0} is equal to 10V means basically you have to make the log amplifier work for maximum of 20V both of them and apply a DC offset V_{X0} as an additional current.

Later you will get an output which is going to be V_XV_Y by V_R wherein V_X and V_Y can go both plus and minus 10V plus V_{X0} into V_Y by V_R plus V_{Y0} into V_X by V_R plus V_{X0} V_{Y0} by V_R. Obviously these are unwanted terms. These are the two feed through components which have come out because I introduced a DC offset where we have unnecessarily got a V_Y feed through and V_X feed through and DC offset.

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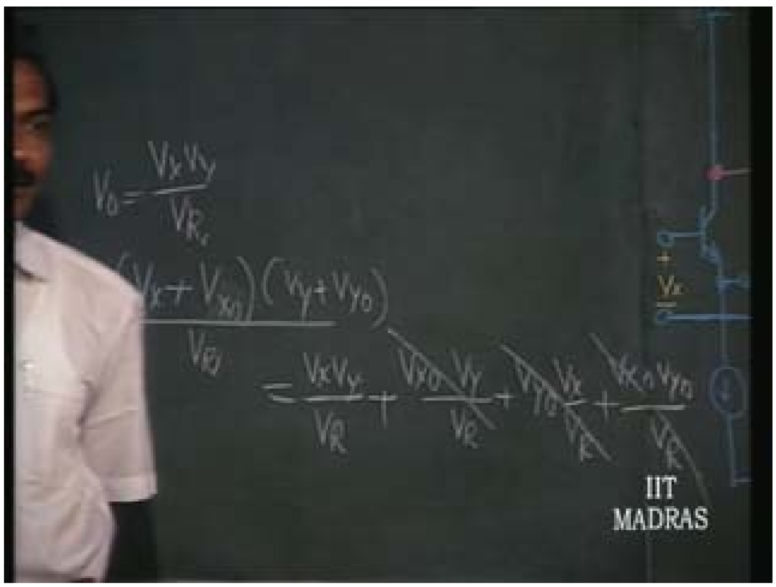

Now this is behaving like a non ideal output in the case of a multiplier. So what should you do is you should subtract DC. Subtracting that DC is not a problem, it is only a DC and you already have V_Y and V_X with you. So some portion of V_X and some portion of V_Y have to be subtracted from this. Using op amp I want you to obtain a generalized single quadrant to four quadrant block.

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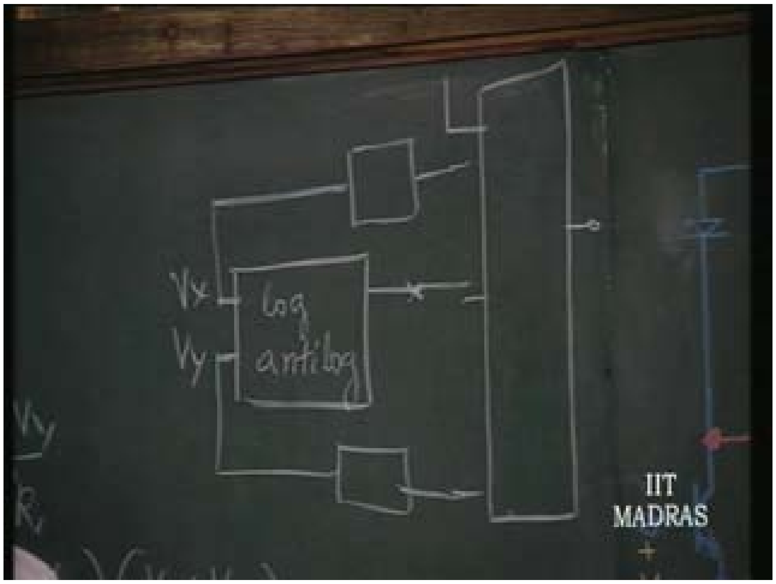

How do you convert using op amp?
You have V_X and V_Y, you have the log antilog multiplier, you have the output here then some portion of V_X, you have some portion of V_Y to take, now you have to use all these information and also DC and then device an op amp circuit to get an output which is just V_XV_Y by V_R.

Using a simple op amp, adder or subtracter you can obtain this kind of a conversion from single quadrant to four quadrants. Nobody bothers whether the initial precision multiplication is a cure using transistors in such a manner that you have only single quadrant operation. Any single quadrant multiplier can always be converted by subtracting the feed through components as well as deducting the offset voltage.

Now let us go to the next type which is one of the most popular multiplier the IC multipliers for precision operations called as transconductance type of multiplier. So this is Gilbert's multiplier.

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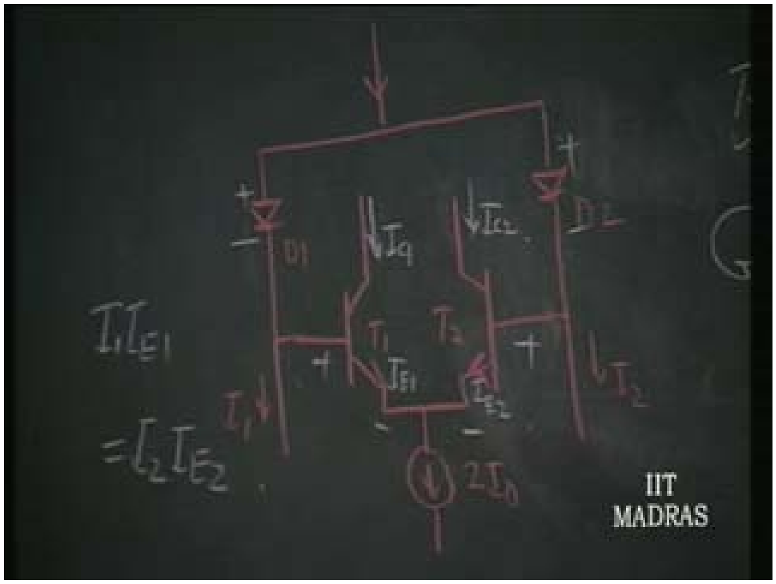

And we have already discussed about Gilbert's gain cell. What does it comprise?
It is made up of transistor cell with input coming from two diodes through which currents are flowing. So this is $2I_0$. Basically we will have I_0 and I_0 flowing through this when these two are connected to ground. But let us say we have two diodes connected to this which are forward biased in such a way that the input currents to these diodes d_1 and d_2 are respectively I_1 and I_2. Then these are T_1 and T_2 transistors.

The currents I_{c1} and I_{c2} or for that matter current I_{E1} and I_{E2} will be related to I_1 and I_2 in the following manner because of the translinear principle. What does it say? We go through the currents and now this and this will add, here these two are again getting subtracted so I_1 into I_{E1} is equal to I_2 into I_{E2} this is from the translinear principle that we have obtained.

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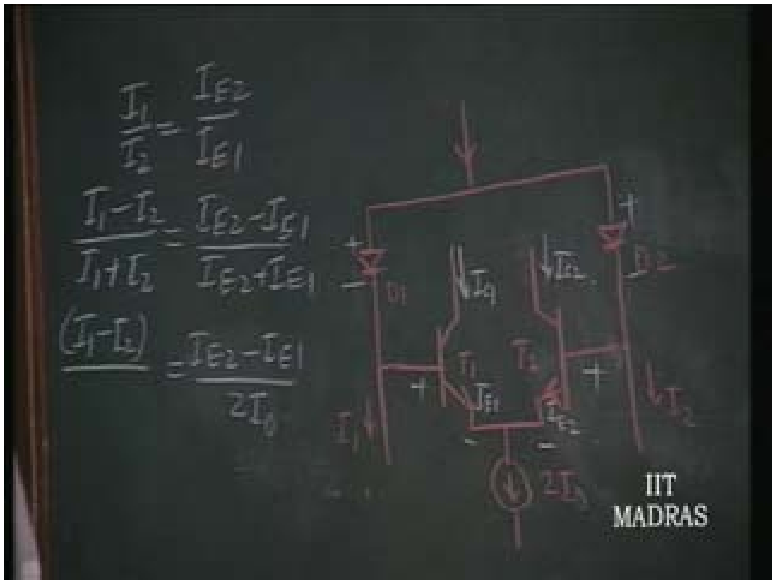

So already we know that I_1 by I_2 is equal to I_{E2} by I_{E1} from this. Or I_1 minus I_2 by I_1 plus I_2 is equal to I_{E2} minus I_{E1} by I_{E2} plus I_{E1} which is nothing but at all times equal to $2I_0$.

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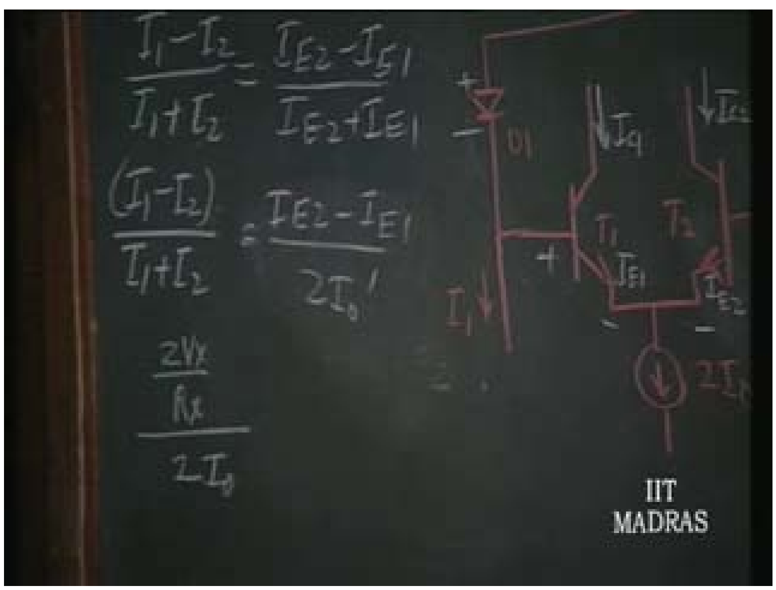

So we are able to obtain a differential output current I_1 minus I_2 by if we have this signal itself coming as a differential current signal. How do we achieve that? It is by using a transconductor. If I can make I_1 equal to some DC current plus V_X by R_X and I_2 equal to same DC current minus V_X by R_X then I can get I_1 minus I_2 which is twice V_X by R_X and I_1 by I_2 which is a DC current. For that I use a voltage to current converter with emitters being connected by Rx. here this current is going to be I_0 plus V_X by R_X and this is I_0 minus V_X by R_X and these are flowing through these two diodes which is similar to this.

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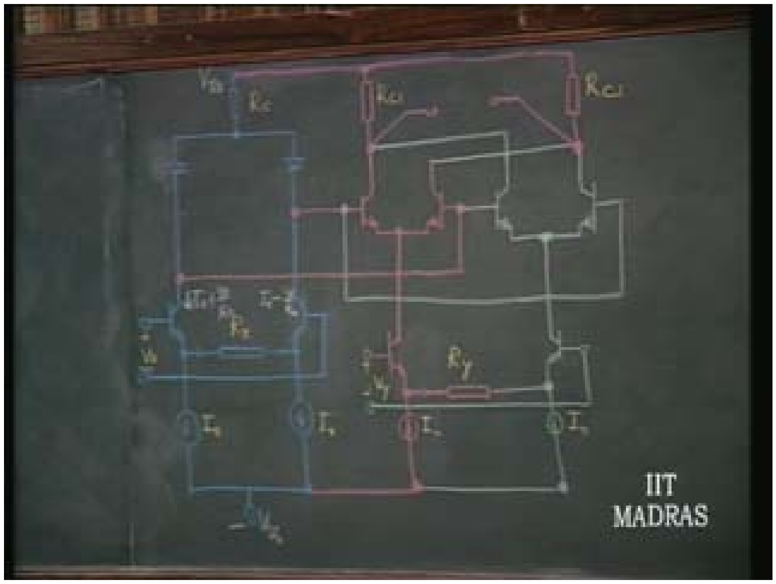

So what you get here?
Here I_1 minus I_2 by I_1 plus I_2 is going to be essentially equal to, I_1 minus I_2 is $2V_X$ by R_X and I_1 plus I_2 is $2I_0$. So this is going to be equal to, we will call this as I_0 prime, I_{E2} minus I_{E1} by $2I_0$ because of this current the bias current flowing through this which may be different from the bias current that is flowing through this I_0 prime. So this is important, that we are now able to multiply a current with another voltage and that is appearing at the differential output current. We will repeatedly use this idea of converting voltage to current and doing the current multiplication using Gilbert's gain cell. We have to also now see how V_Y can be converted to corresponding current. That is also done by using voltage to current converter. So, now I_0 prime is going to depend upon V_Y and therefore output differential current is going to be comprising of a product of V_X and V_Y.

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