Introduction To Adaptive Signal Processing

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Lecture No # 33

Introduction to RLS Algorithm (Contd.)

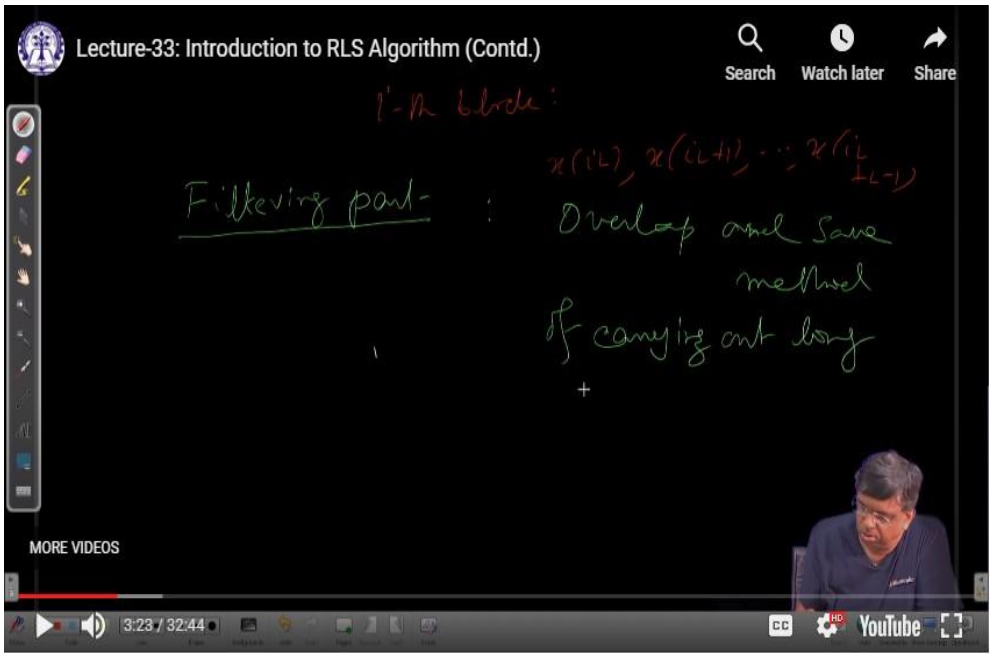
Okay, we have been discussing the block LMS algorithm block adapted filter. So, basic formulation of the block LMS algorithm the ambient equation etcetera were presented in the last class. And I told you said that you know I mean main motivation for going for this block-based approach block LMS algorithm is that you can have a very efficient implementation in real time using FFT algorithm. To that effect I also discussed certain properties of circular convolution what is made by circular convolution and how to calculate that in practice and all that all those are discussed last time. So, I will continue with that. So, I will discuss few more properties which are very useful in that FFT based implementation.

So, one is related to filtering part okay, one is related to filtering part that is this, we know that ith block ith block let me set things in order first. ith block it consists of data from xil xil plus 1 dot dot dot dot xil plus l minus 1 okay. So, ith block starts from the index il then il plus 1 goes up to il plus l minus 1. So, this l indices they form one block during this block the filter weight coefficient vector is fixed for this block it does not change from index to index only when this block is completed, we move to the next block then again filter weight vector is updated by an LMS type of algorithm called block LMS that is what we discussed last time.

But during this within this block I use the same filter coefficient vector as a fixed vector fixed coefficient vector and go on filtering at all these indices okay. I am not very sure how good this color is. So, maybe I will change over to some other color no the same problem filtering part and here I will be using something called overlap and save method of carrying

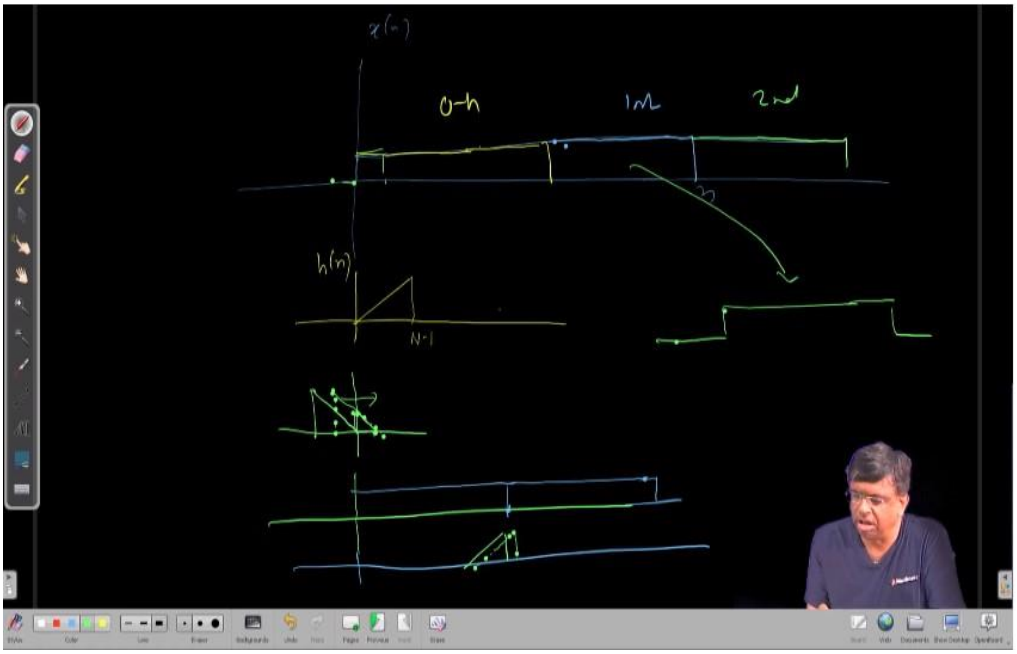
out convolution carrying out long convolution. This you have studied hopefully in your basic DSP course but nevertheless I will redo it here. Suppose this is my data xn I start from n equal to 0 but you can always extend my logic to the left side of origin also but for me it is easier.

So, suppose the data goes like this and you have got a filter hn okay length maybe n so 0 to n minus 1 you have to carry out convolution between xn and hn. What we do we can if we break this block partition into you know partition this entire data stream into blocks non overlapping blocks each of size L, L is of course greater than n something like this like you go up to this this is 0th block then you go up to this first block and like that then you go up to second block. Suppose I divide the data into blocks they are non-overlapping blocks and I carry out convolution between each block and this filtering this hn. Now you see how to how we carry out convolution you flip it in this direction and keep shifting to the right and left and see the overlap on the on the overlap portion you do multiplication by sample wise from here and here and add. Like if you shift it to the left, you have all zeros here so no point in carrying out any such convolution there is all reality is 0.



But now you will be shifting this to us right okay so maybe it will it will be like this it

after some time it will be like this. So, the tail still remains outside this so that will be get multiplied with 0 part so no calculation no consideration of that, but this part will multiply this part. So, I multiply sample wise you know terms here and terms here add I get the convolution at this point and like that so you keep sliding all right.



Point is when you are at the block boundary when you are at the block boundary this was one block this is one block all right and you are the block boundary so you have come here up to this there is no problem, but the moment you move to this side okay if you say that I will do block wise I will take this block only this block only and to the left I will take 0 to the right and 0 and then again this block only here to the right 0 to the left 0 that is I will then I will have a problem. Let me explain in the next page that is if you divide the data into non-overlapping blocks and hold them one at a time this is one block 0 to L minus 1 take another block L to L minus 1 take another block 2L to 3L minus 1 and dot dot dot.

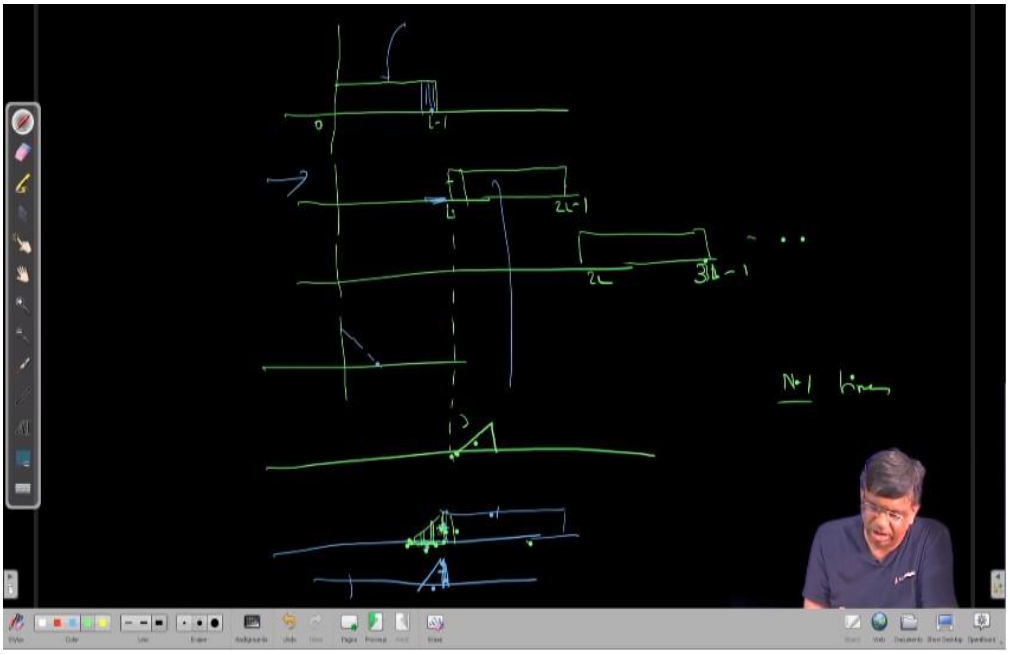
Naturally if we superimpose them we get back the original data and if you say that look I will be doing filtering of this block with my filter kernel that is that impulse response maybe this much and I will do the same here do the same here I will get various outputs and I will

superimpose them then there will be some error where the error is that we will see convolution means I will be flipping it right. If I keep moving it to the right okay I would not have a problem until and unless I reach the last point after that and as per this diagram I have to consider this the second block okay I have to consider consider that now that is I am reproducing here this block and this filter is moving here now moving here so this there is an overlap at this first sample but this previous samples previous values they find 0 here whereas in reality in actual data they would have found this this fellows this sample right because the two blocks they are side by side before this block I have got 0 here but actually it is 0 just before this block I have this so this filter would have taken this sample from this block and other samples from this block multiply term by term add that is missing here because now if I concentrate only on this I got 0 here 0

So this filter this part tail part it will multiply zeros not the actual samples I will get 0 region I will get wrong result then if I still move to the right I will keep getting wrong result till I reach a point till I reach this point where the whole filter block this kernel has come under this then there is no problem this is coming fully under this I do not require data from here and it is not multiplying the zeros here ok. So, initially there will be error for how many points well when I started here and there are N minus 1 total N samples so N minus 1 samples here and this one. So, N minus 1 samples they were present they are multiplying with zeros this one was multiplying correctly with this guy but this N minus 1 zeros were samples were the samples they are multiplying these zeros wrong result then I move to the right N minus 1 becomes N minus 2, 2 data then multiply correctly from data here but still N minus 2 then further shift to the right N minus 3 so finally, when I will be left with 0 samples 0 that is fully full of them will get in that will happen when all the N minus 1 fellows here have gone into this block under this block

That means I have shifted N minus 1 times I have shifted this N minus 1 times from that moment onwards convolution between this filter and this block will give giving me correct result that means initially first N minus 2 cases 0 shift when I am here this only one fellow multiplying this fellow shift then one shift two shift up to N minus 2 shift I will have wrong result from N minus 1 times N minus 1 number of shifts when the filter fully comes under

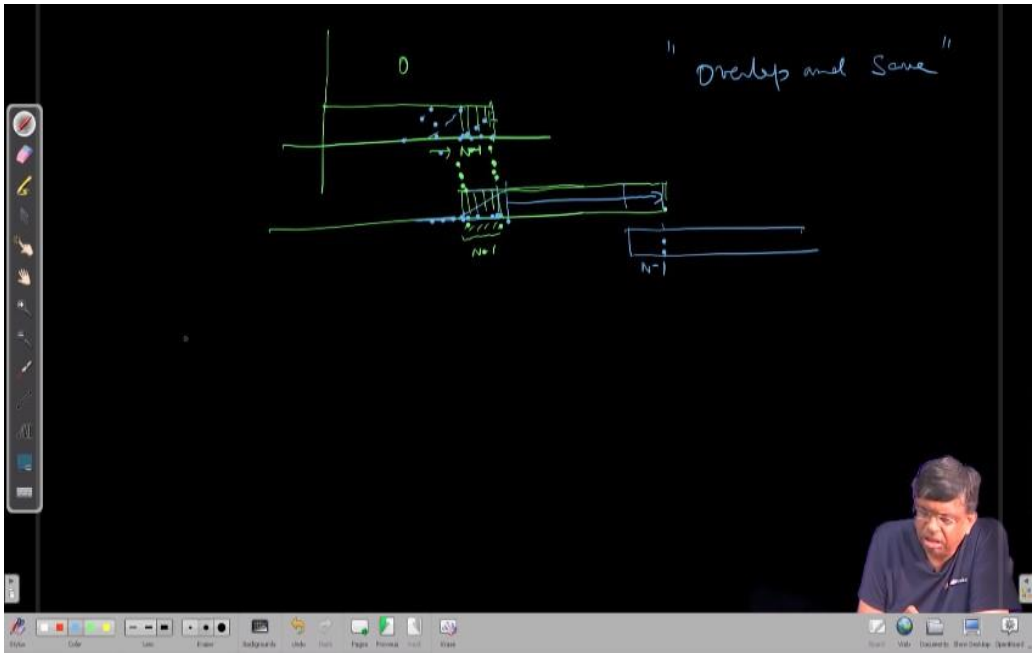
this block I will keep getting correct result that means 0 shift first shift second shift up to N minus 2 shifts I will get wrong result okay from N minus 1 shift when the whole filter block comes under this umbrella I left correct result that means if I just convolve this way that I will take one block at a time convolve with the filter then first and when I convolve with the filter I assume that all the data to the left of the block to the right of the block as zeros then obviously by first N minus 1, 0 to N minus 2 that is total N minus 1 first N minus 1 results will be wrong afterwards results will be correct okay



Therefore how to get correct result here so we go for what is called overlapping safe that is let there be some overlap between the blocks the 0th block up to here next block I don't start right after here I start little from behind so this length is N minus 1 it is overlapping in N minus 1 points from here to here okay and then this suppose it is this I am not considering other block same logic will work so I know when I take this block and do the convolution at these points first N minus 1 points I will get wrong result but that time the same data is common the same data is present here that time the result of convolution between this block and the filter at these points will be correct because the filter flipped impulses process come fully under this umbrella and now here it's like you know it will be somewhere here or here or here it is fully under this umbrella so I will get correct result at

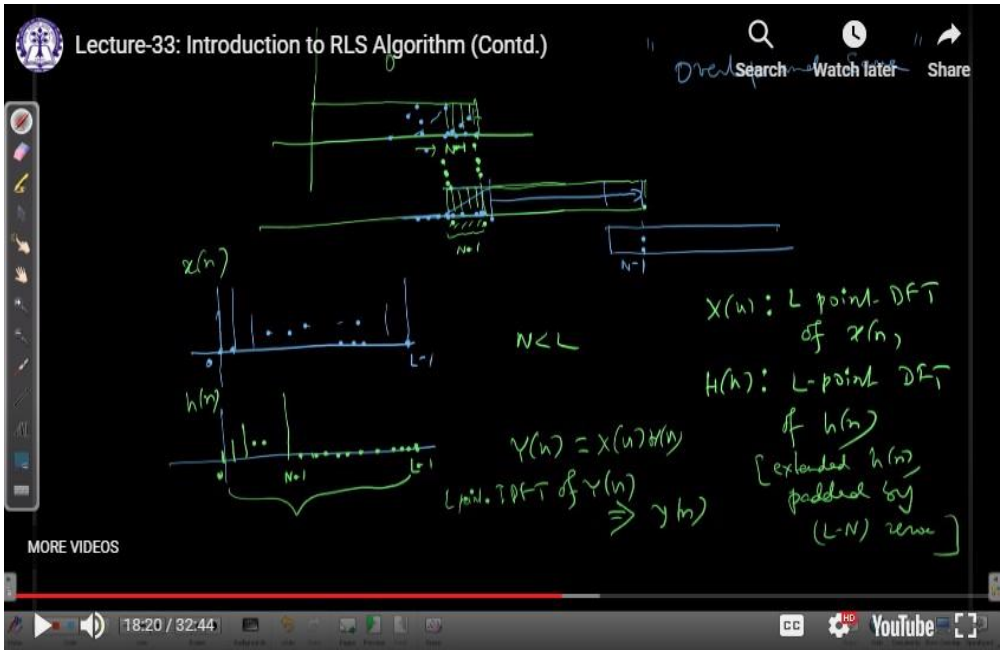
this point at this point all this point so I will pick up those results from processing of this zeroth block I will use them substitute correct result here I will not take the result of convolution from this part okay because they will give wrong result from here onwards till this I will take convolution output from here but from the first N minus 1 I know the result will be wrong because here I am taking to be 0 so my filter flip triple response will multiply zeros from here we keep giving wrong result as it goes on shifting finally when it gets shifted and fully comes under this block that happens after N minus 1 shifts from the N minus 1 is 6 so from that point onwards I will get correct result so from 0 to N minus 2 total N minus 1 these points will give me incorrect result so I will not take from here I will take from the upper block and again likewise so next block also I will have you know something like this there will be overlap with N minus 1 and I will take the convolution from processing of this block filtering of this block with the filter not here and so on and so forth okay this is called overlap and save method

Please follow the book by Oppenheim and Seifer either Discrete Type Single Processing or Digital Single Processing the very famous books they discuss it at length with good diagrams and all overlap and save now I have to relate this with DFT and circular convolution this much you remember I will do this kind of block partitioning and first N minus 1 results are wrong they will be discarded may be substituted from the previous block output that separate but here they will be discarded okay that is 0 to N minus 2 those many points total N minus 1 will give me wrong result from N minus 1 is point to the final L minus 1 I will get correct result okay this much you remember.



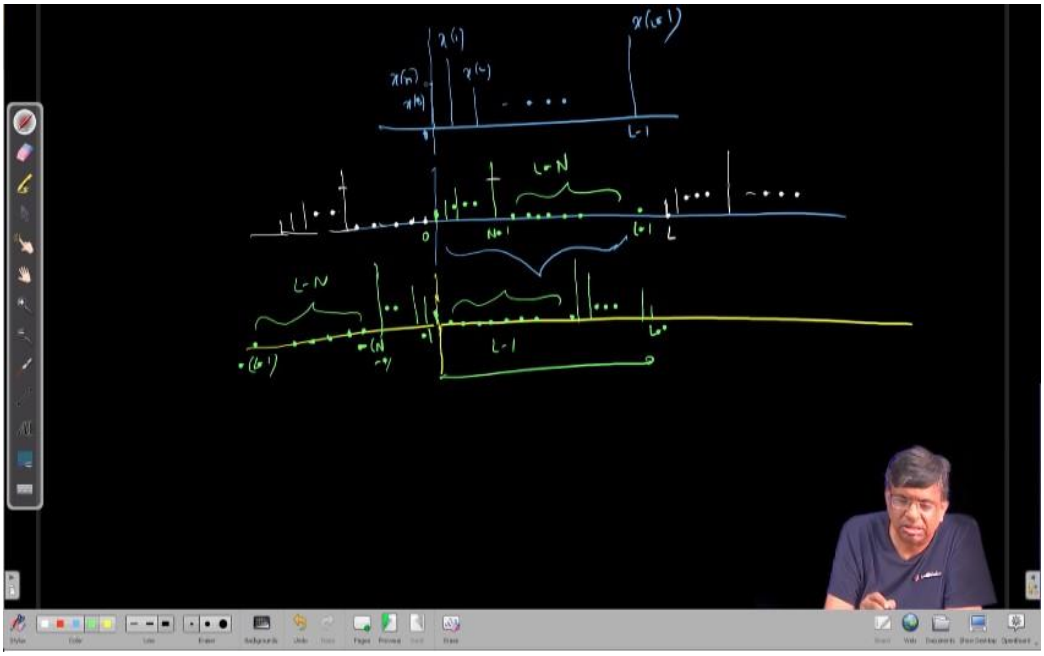
Now a separate result which I will be using here you know how to do circular convolution suppose I give you one block 0 1 dot dot dot up to L minus 1 some data and a filter up to N minus 1 and then 0s so length N length L and N less than L I have to carry out convolution between them suppose I am carrying out the circular convolution that is if it is x N and if it is h N I will do L point xk is L point DFT of x N here also I take from 0 to L minus 1 padding 0s here and so I take L point DFT of this extended sequence where I have brought in these 0s okay L minus N number of 0s total N here total L so difference is L minus N that many 0s I have brought in so the length of the two sequences are same I call it the extended sequence h N extended by 0s to make it of length L and then again I take L point DFT so hk L point DFT of h N alright actually it is extended h N padded by L minus N number of 0s and then I multiply term by term and calculate yk yk is xk hk and then I take inverse DFT IDFT of yk that gives us to y N and we have seen this y N is the circular convolution between these two we have to see how much of that y N is the correct linear convolution if I carry out the ordinary conventional linear convolution between them I get a sequence of course larger than L because after convolution length will go up so when I get the y N which is length L because L point L point IDFT sequence length is L send L point IDFT of y N will be yk will be y N only L point only so in this L point y N how much

will give how many points of which points will give you the correct convolution linear convolution between x N and h N when I have got L minus N 0s.



So, first we have to understand what is y N by carrying out the circular convolution between the two and we know how to do that right so we do that in the next page we hold one sequence as it is we hold one sequence as it is that is your x N x 0 x N x 0 x 1 x 2 dot dot dot dot so this is 0 to L minus 1 x L minus 1 other one you have to make periodic version of here 0 to N minus 1 then 0 0 0 0 up to 0 L minus N 0s alright this is L minus 1th point so this is your one one block but for carry out circular convolution to be it periodic you should have this version from Lth same thing again here also 0 0 0 0 0 and then same these two same and these, these, these all right it is a periodic version step 1 step 2 now you flip it

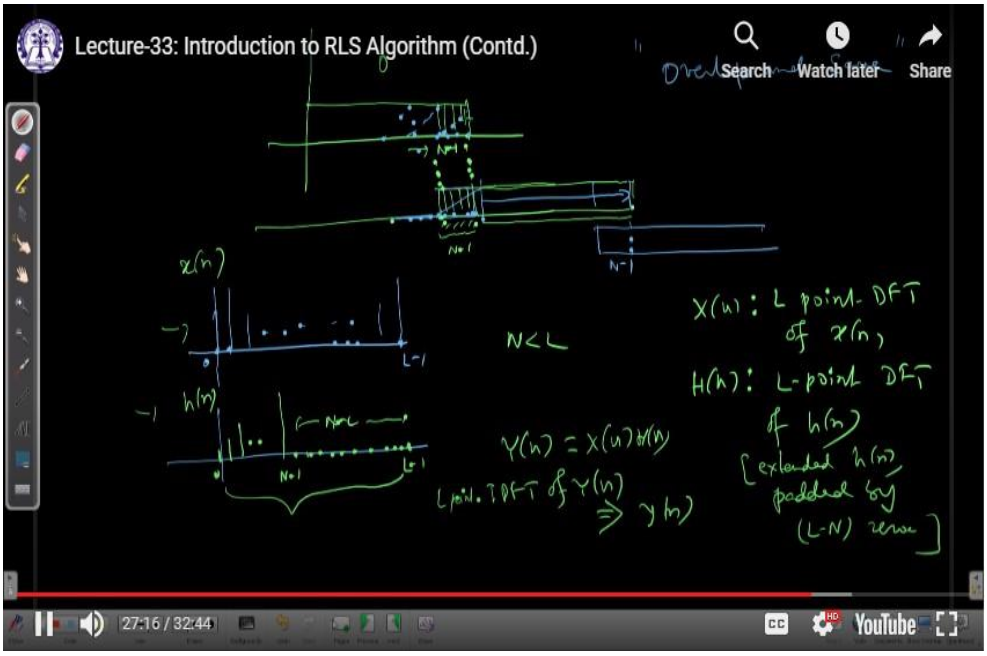
If you flip it this remains here so this remains here this guy then these fellows so minus 1 minus 2 up to minus N minus 1 then those 0s up to minus L minus 1 and this is your L minus N number of 0s this is a filter this is a filter so flipped it and again L minus N 0s will be here and this part will come over here this part from periodic right.



Now next is to start with 0 shift so just take out ignore the left of origin and ignore the point to the right of L minus 1 just consider this part and do term by term multiplication and add so this times this that is okay that will give me the if that is the only thing then I get correct linear convolution there because in the case of linear convolution all these periodicity goes this is flipped so flip the tail part to the left of origin they find 0s here so forget them only this guy at the origin this multiplies x0 same thing would be happening here this guy and this x0 but here the problem is this is there is an additional part that will be multiplying data from here and get added with the product between this sample and x0 so it will give me wrong result so 0 shift wrong result there you shift it to the right by 1 this will come here this will come here still some part will be there in only one fellow will go out okay only one fellow will go out I have in my total out of N samples one was here and then N minus 1 now one goes out and this fellow comes in so but still I have N minus 2 so they will be multiplying data from here will get added with the term by term multiplication between this guy and this guy that is this sample at x1 and this sample at x0 okay and we will contribute the error so again ignore it so this will continue till all this chunk goes out and that will happen when this entire filter block comes over here so from N minus 1 shift onwards okay

It is not 0 sorry so now here I have got 0s those L by dash 0s just before this they multiply you know so there is no problem the entire filter length has come and here I have got those again 0s alright so from this point onwards this point I got the entire filter block under this there is all 0 to the right and now it will be term by term multiplication and addition so I will get correct result that this will continue you keep moving this to the right till you bring it here and this L minus N 0s from behind the N this is L minus 1th point okay till this you get correct result this is a header that means from this point onwards to this point you get correct result

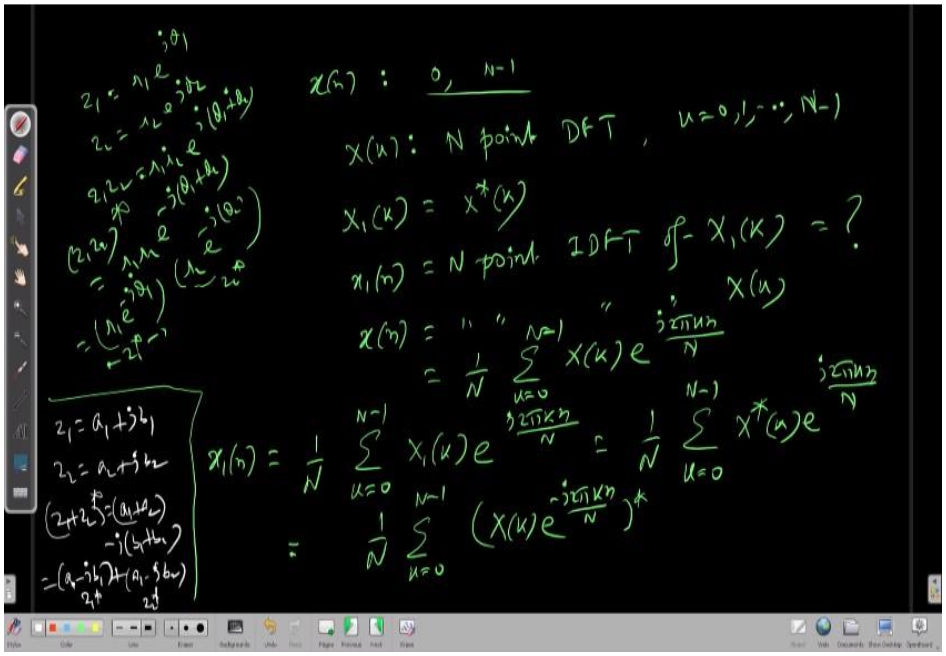
So from N minus 1th this is a N minus 1th point because before that I shifted I had 0 shift 1 shift 2 shift up to N minus 2 shift there was wrong result only after N minus 1 number of shifts from the left entire block has come below this and there to the right side 0 to the left side 0 now term by term multiplication addition is nothing but ordinary convolution you get correct result from this point onwards that means here also first N minus 2 results wrong correct results from N minus 1 to L minus 1 therefore going back to this overlap and safe if I take this block any particular block but there is an overlap part of N minus 1 samples okay 0 to N minus 2 so total N minus 1 sample between this block and this block I take this I carry out circular convolution between this block and this filter after appending this N minus L 0s circular convolution then again first N minus 1 results there is 0 to N minus 2 those points real should be incorrect which match here because first N minus 1 means 0 1 2 up to N minus 2 I discard here because I get them from here they give me wrong result because they are taking 0 from the left but actually data here from the previous block okay in either case I forget that because I am discarding I was discarding those result here also I am discarding because of circular convolution thing so I simply carry out DFT of this DFT of this okay DFT of this data and DFT of this extended impulse response multiply inverse DFT which is nothing but circular convolution between this sequence and this sequence and there I discard the first N minus 1 output that is from 0 to N minus 2 and then I take from here to here correct I will get the correct convolution and this part I replace the result by the correct real from top from the previous block okay so I can apply DFT here alright I can apply DFT here by this blockwise partitioning where there is an overlap between the blocks okay



There is one more result which we will be using suppose xn from 0 to N minus 1 xk in point sorry there is k equal to 0 1 dot dot dot dot N minus 1. Alright now let me define some x1k by taking xk and putting a star on them conjugate then x1n if I take the inverse DFT of that is N point IDFT of x1k what is that okay I know what is this xn, there is N point IDFT of xk, which is nothing but 1 by capital N summation summation is over k inverse DFT so similarly x1n will be 1 by N, x1k, next one case x star k so put that here and now I take the star out if I take the product and push the star outside if I take product between xk first and I put a minus here take the conjugate and z star I will get this because if you have two complex number z1 and z2, z1 z2 star is z1 star z2 star say z star k and star of this which is e to the power plus j2 pi k N by N alright and the summation of star there is suppose summation of you have got z1 star plus z2 star you have got capital N in general z1 star plus z2 star is z1 plus z2 N star.

If you have forgotten these things z1 is suppose r1 e to the power j theta 1 z2 r2 e to the power j theta 2 then z1 z2 r1 r2 e to the power j theta 1 plus theta 2 so this is the magnitude this is the phase so conjugate of z2 is r1 r2 remains as it is e to the power minus j theta 1 plus theta 2 you can take r1 e to the power minus j theta 1 which is z1 star and r2 e to the power minus j theta 2 which is z2 star that is what I use here z1 z2 star z1 z2 star is this z1

star z2 star. On the other hand, if you have z1 I write in the rectangular from now j b1 z2 a2 j b2 then z1 plus z2 star z1 plus z2 is a1 plus a2 plus j b1 plus b2 if you put a star there a1 plus a2 minus j b1 plus b2 and now we take out a1 minus j b1 plus a2 minus j b2 which is z1 star z2 star that is a z1 plus z2 star is z1 star plus z2 star is the very basic if you have forgotten. So, there I can put the star outside.



Now I can complete the rest, but let me do an exercise let me I mean I ask you to go through this and try yourself to get further result is and let us see how many of you can do of course, I am not going to take a check on that, but just this gives you a practice. So, I will complete this part in the next class. Thank you very much.