**Introduction To Adaptive Signal Processing**

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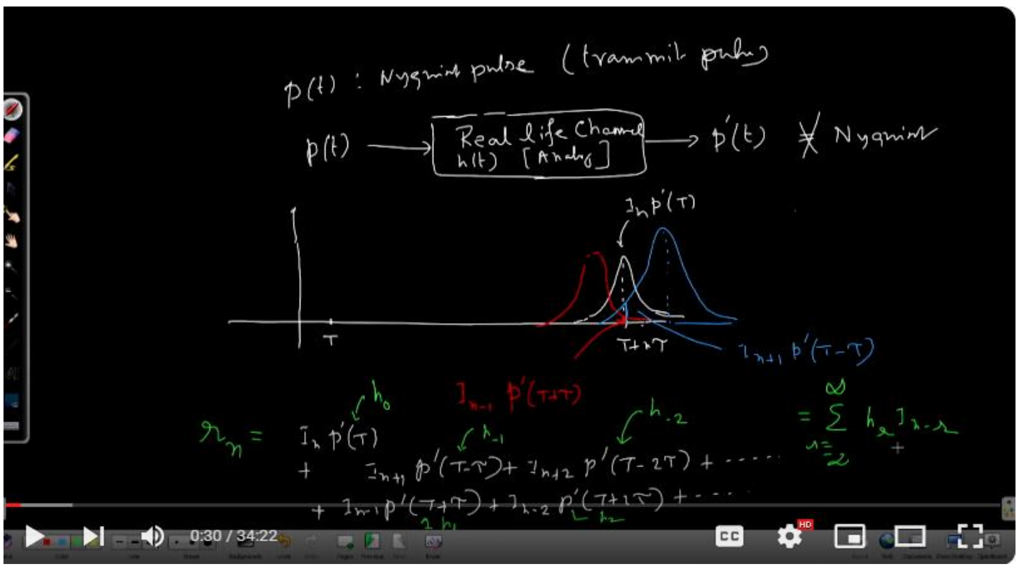
**Lecture No # 23**

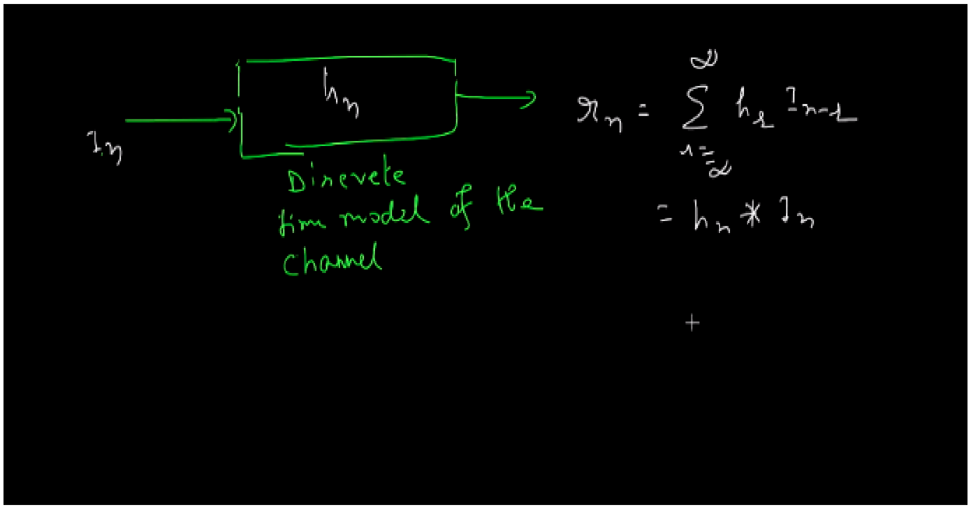
**Applications of Adaptive Filter**




So, in the previous class we are here Rn is a linear convolution between the symbol sequence In and channel model coefficients channel model sequence you can say unit impulse sequence given by Hn. So, I have this descriptive model this only model and that is why it is if it is non-causal non-causality is not in real time not for the analog case how the non-causality comes, I have already explained. So, you have got in sorry you have got In and Hn and you have got Yn not Yn, I called it Rn let me remove it Rn is the received sample sampled at the time capital T plus N tau that sample is called Rn that we have seen in the previous diagram and that is coming out to be this Hr In minus R there is a convolution with In. Therefore, if I take a Z transform Rz which is from here Z transform is Hz which is from here into Iz which is from here. So, this is the Z transform of Rn this is the Z transform of Hn this is from Iz. Now from Rn our purpose is to recover In exactly if possible because In is what was transmitted.

$$r_n = \sum_{r=-\infty}^{\infty} h_r \, I_{n-r}$$

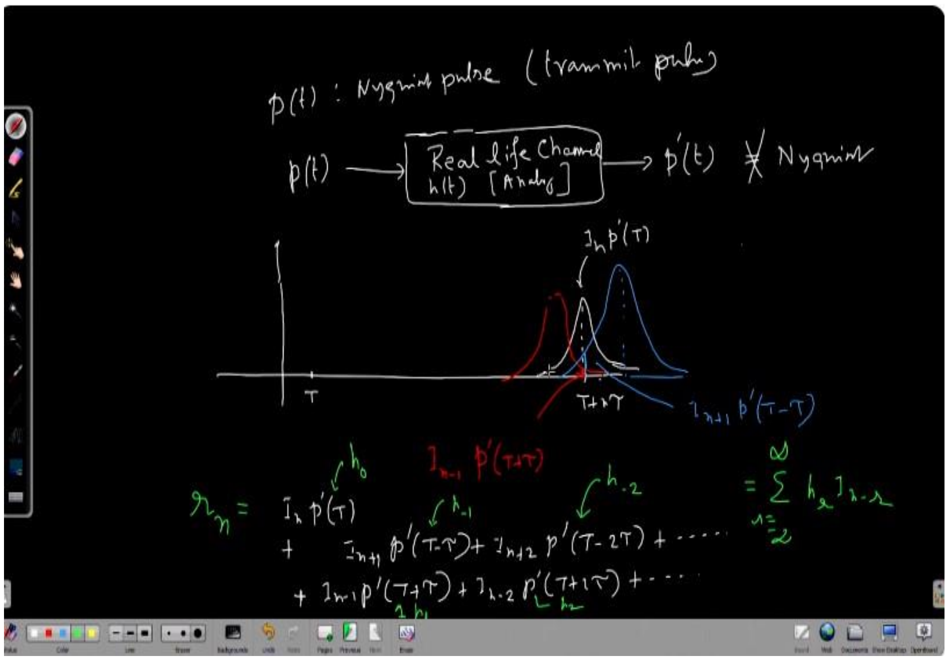
$$r_n = h_n * I_n$$




So, you have to get back from the received samples by doing some mathematics some algorithm some operation we have to get back In. So, Z transform it looks like this which means Iz if you want Iz you give this Rn Iz is what Iz is what you want in time domain you want In in Z domain it is Iz their equivalent, but Iz is what you want Iz is equal to Rz by Hz. So, Rz into 1 by Hz. So, if you can create a filter which is inverse of this channel this this thing, I call H I, I for inverse H Iz 1 by Hz is H Iz.

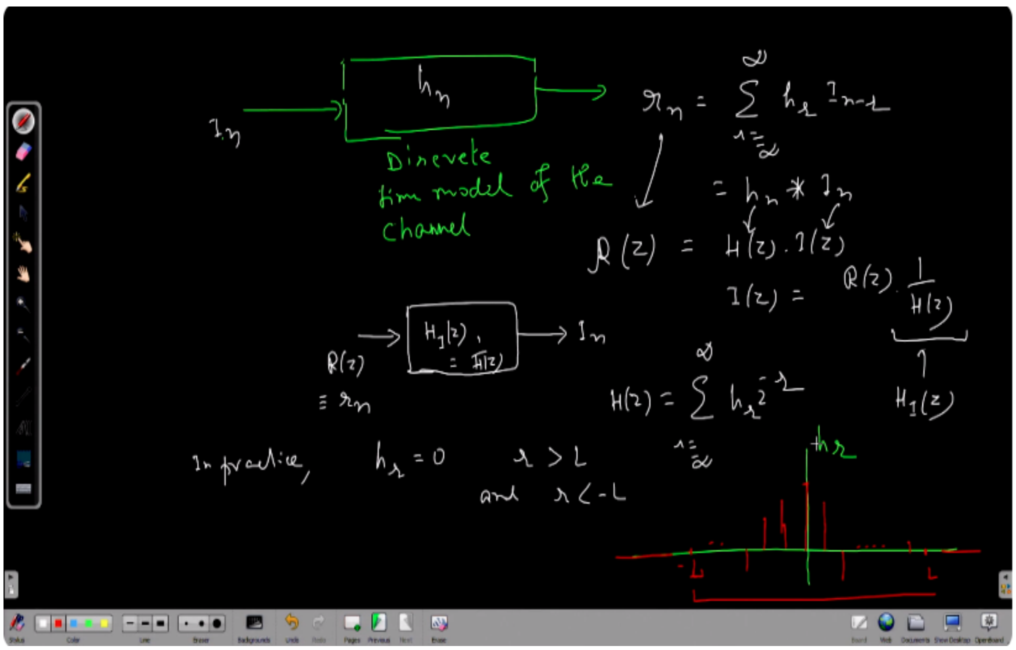
So, if I can create a filter H Iz pass Rz equivalent to Rn through this I will get In back here where H Iz is 1 by Hz. But remember what is Hz, Hz is actually the Z transform. So, in general in this model it is a non-causal system you have got H0, H1, H2, H minus 1, H minus 2 all that and also theoretically R goes from minus infinity to infinity, but this H R actually will not be non-zero or appreciable beyond certain range. If you go to the previous page, you see I called this height if I take the basic pulse only basic pulse only if you start from the center point, go tau to the left I call it H minus 1, 2 tau to the left I call it H minus 2 and dot dot dot. But you understand if a pulse is like this if you go tau to the left, you are here, 2 tau to the left you are here 3, tau to the left, but beyond a point there will be the pulse will become 0 because it is a time bound pulse only it will become 0.

So, that is why this coefficients H minus 1 H minus 2 they are progressively decaying value is becoming lesser lesser and then becoming 0. Similarly, for H1 H2 these coefficients are becoming 0 that is if you start from the H1 center of the pulse go tau to the right whatever is the contribution for the basic pulse you call it H1 then this 2 tau to the right you call it H2, but pulse is a decaying pulse as I told you it is a time bound decaying pulse. So, beyond the point of time it varies as a result H1 H2 they also go out.




So, therefore, theoretically the summation is from R minus infinity to infinity, but in practice in practice HR is 0 for R may be greater than some L and R less than minus L that is a range from minus into plus L that is where it is confined this is not equal to 0, otherwise it is 0 oh sorry this is ok, this is equal to 0 outside this range, which means HR could be something like this some value here some here some here some dot dot dot up to L its value is decreasing and here also my drawing is not correct maybe I redraw it it will be decaying function right. So, I should show that in my diagram this is L very small value.

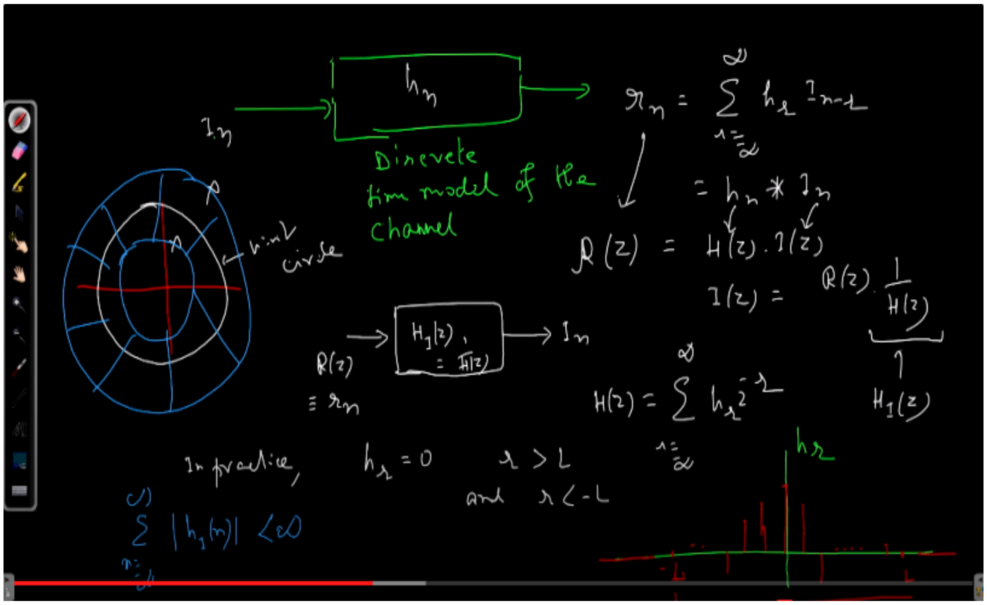
Similarly, on this side maybe and then 0s 0s. So, this is my range L2 minus L outside in practice it will be 0 after that. So, then that means, HR actually is an FIR filter finite impulse response filter HZ as only 0s right. So, 1 by HZ if I now substitute HZ here or from minus L to capital L this thing here ok 0s will become poles 0s of this is an FIR system it has 0s only ok, but now it is HZ comes in the denominator. So, 0s will become poles and therefore, HIZ is an IIR system HIIZ is an IIR system because IIR systems have poles ok, but again what will happen I do not know what the poles are because after all capital HZ is coming from this HR, HR is what, HR we have seen they are coming from this pulse heights at various sampling point, but this is a distorted pulse I have no knowledge about the distorted pulse I know about the transmit pulse that is what I had designed, but the channel is largely unknown to me its property there is impulse response also changes from time to time.




So, what is p prime t I do not know therefore, what are these coefficients I do not know remember ok. That is why in the end we will require adaptive filters because these coefficients are not only not known, they change with time all these H0 H minus 1 H minus 12 that H1 H2 all that right. Now, so this is typical HR you say FIR system finite impulse response it will have 0s if I have 1 by HZ it will be in general poles having poles because 0s become poles. So, HIZ which has poles will definitely be IIR ok. IIR means in the complex jet plane if it is a unit circle some poles may be here some poles may be outside.

So, you can take you can look for that solution which has which has region of convergence here at least that will guarantee one thing that region of convergence includes the unit circle and therefore, it is stable system, but this will be non-causal this will be non-causal because it is not it is within two circles outside one inner circle inside another outer circle. So, there is a non-causal, but stable system, but stability at least guaranteed non causal it is an issue ok. So, suppose we try to construct this HIZ and assuming suppose for a moment that HZ is known to us. So, HIZ which has various possibilities we try to construct this stable, but non causal one ok. Now, if it is stable then what happens if HIZ has impulse response HIN ok which is causal which is stable, but non-causal.

One thing is, there stable means if you take its absolute value of the samples and equate and sum this is finite this is the condition of stability absolutely solvable this is all from basic DSP and system linear time invariant system is causal if you take its impulse response take the mod on that and sample and sum over all if that is finite then it is stable. So, I am looking for this solution because you can have HIZ you can calculate this inverse Z transform for various ROC region of convergence one ROC is between these two circles, here the pole nearest to this unit circle, here the pole nearest to the unit circle in between there is no pole take that to be a region one region of convergence that includes the unit circle. So, if you take the inverse Z transform over this region of convergence you will get HIN which will be non-causal I agree, that is why N from minus infinity to infinity, but it is stable. Stable means there is one thing that HIN cannot remain high for all N as N increases it has to tie down finally.




It becomes 0 then only summation will be finite if it still remains appreciable even if N goes up or up to infinity and minus infinity then the summation will not converge summation will not be finite summation will not be finite which means for stable system since it is since we need this we need this it implies HIN tends to 0 as N tends to plus or minus infinity that is as we go towards plus infinity or minus infinity this HIN values have to go down and then only the summation you can work out it will converge, but if HIN does not go down suppose it remains constant at 1 or either side plus infinity and minus summation will not converge.

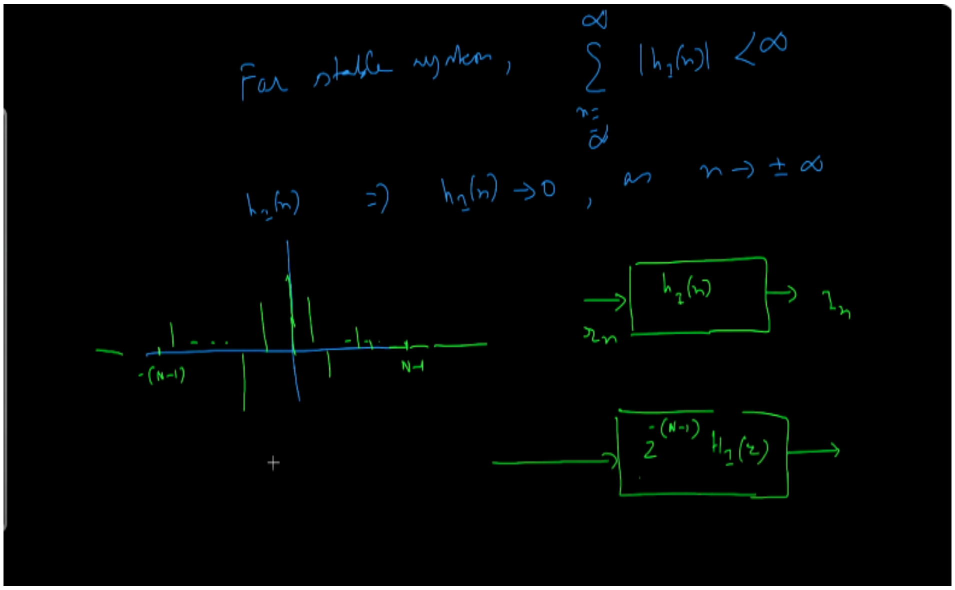
$$\sum_{r=-\infty}^{\infty} |h_I(n)| < \infty$$

$$h_I(n) \to 0 \qquad n \to \pm\infty$$

So, therefore, if you are looking for a non-causal, but stable system one thing is there that HIN it will be having values like this dot dot dot, but it will go down may be up to some N minus 1 and minus N minus 1 like this and after that 0 0 all right still this is not realizable in time because this is non causal this is non causal right, but one thing is there suppose I use this and on RIN you know RIN is to be passed through a filter whose impulse response is HIN see this HN that is the inverse filter always 1 by HZ I call it capital HIZ corresponding impulse response was HIN I for inverse.

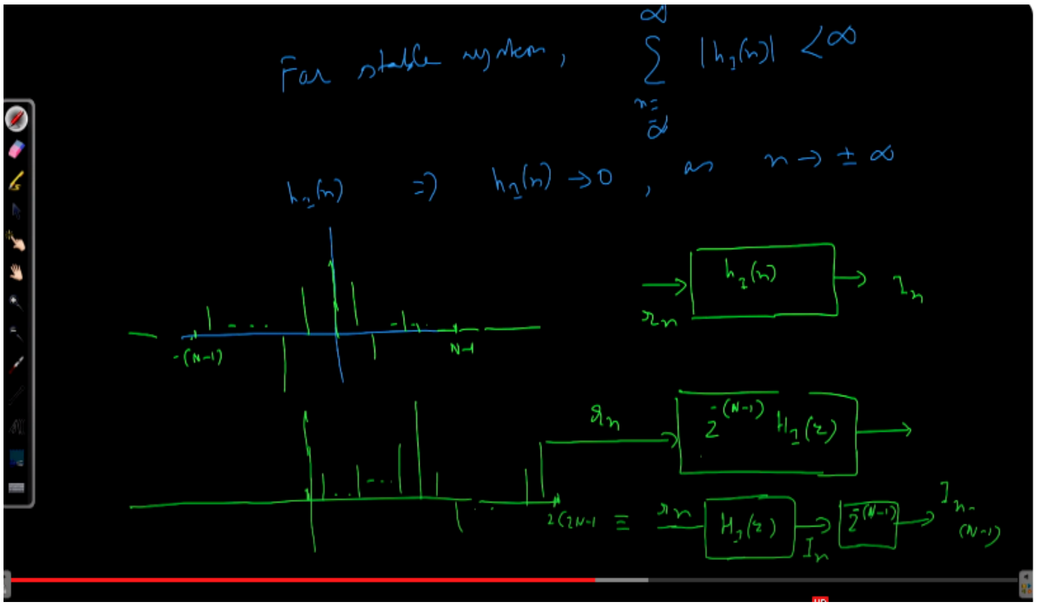
So, HIZ which is 1 by HZ corresponding impulse response is HIN. So, if I have to pass RIN through HIN to get back IN. So, that is here HIN, but this HIN I have done some approximation I have truncated it I am assuming values to be 0 to the right of capital N minus 1 is point and to the left of minus capital N minus 1 is point and through this RIN if I pass and if my approximation is not bad, I should still expect to get IN. So, I have to construct this filter, but here problem is if I have to run it in real time because I will be doing real time processing, I will be requiring filter symbols because of this non causality to get rid of this thing what I can do suppose instead of HIN I construct a filter suppose I construct a filter not HIN, but Z to the power minus N minus 1 HIN all right. Then what will happen this is a sequence in Z domain let me write it in Z domain only.

That means this entire sequence will be shifted to the right by capital N minus 1 point because I am multiplying HIZ in Z domain by Z to the power minus N minus 1. So, in time domain it is just a shift operation by N minus 1 what we will do this will become this will come at 0 the entire thing will be shifted you know it will go up to some point. So, just twice N minus 1 up to this it will go, but this is a causal system. So, if instead of this filter I construct this filter then there is no problem with by practical realization because it will be causal, but R N what it will give now this is a system which is equivalent to having R N here you can say first I put HIZ and then Z to the power this Z to the power minus N minus 1, but I know from here from this diagram if R N is passed through HIZ I get my IN and now IN delayed by capital N minus 1. So, it will be IN minus.




So, this filter will give me an output which is transmitted sequence, but a delayed version and if I know this capital N or N minus 1 I know amount of delay. So, from the received from this waveform at the output I can make the correct decision I know what it is because if I know how much this capital N minus 1 is then I know what is the delay on ideal IN. So, that way I can get back my IN. So, this is the filter that we have to design unfortunately we cannot design the filter because it requires knowledge of HIZ. HIZ is an inverse system from here is a inverse system right 1 by HZ if I have to know H I Z I must know HZ, but HZ is for the channel and channel impulse response is not known and they change from time to time as a result H I Z is not known and therefore, this filter is not known this overall filter is not known.

Therefore, we have to use our adaptive filter technique this is not only not known this changes from time to time. So, therefore, we have to use an adaptive filter technique to obtain this all right. So, that will lead to what is called adaptive equalizer.



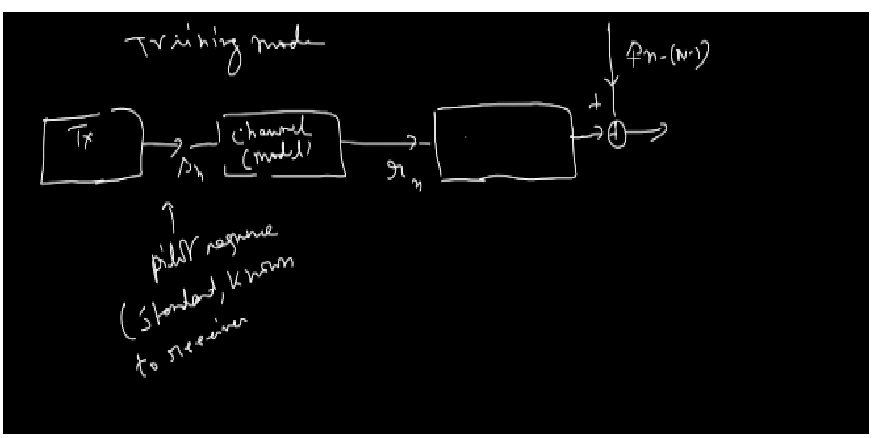
In an adaptive equalizer there is a one mode called training mode, adaptation mode during that time transmitter, transmits universally standard training sequence called pilot sequence transmitter TX it transmits some sequence some training sequence all right maybe you can call it SN some pilot sequence standard known to receiver. So, receiver knows what is transmitted it will go through channel, channel means the channel model channel model because I am considering the descriptive thing let me use this notation, I am using the index and the subscript not with the bracket SN.

So, what you receive is RN, but you know what SN is. So, you construct a filter and this SN which is known to you, you delay it by this amount. So, this is a filter you design optimal filter you design. So, that output is a good estimate of this. So, in this case we know the model if RN is passed through this and then delay then you get back this thing right.

So, therefore, I am giving the same RN and this is what was transmitted. So, I want the filter to produce that or something very close to that a good estimate of that. So, this I give to be the this is known to me because during training mode this is called training mode, I know I the receiver know what was transmitted there was some S sequence. So, SN minus within bracket capital N minus 1 that I give as a desired response for that I derive the optimal filter with RN input. So, that should give me this because generally optimal filter output and this transmitted sequence this thing they are same and therefore, error will be 0 that is why.

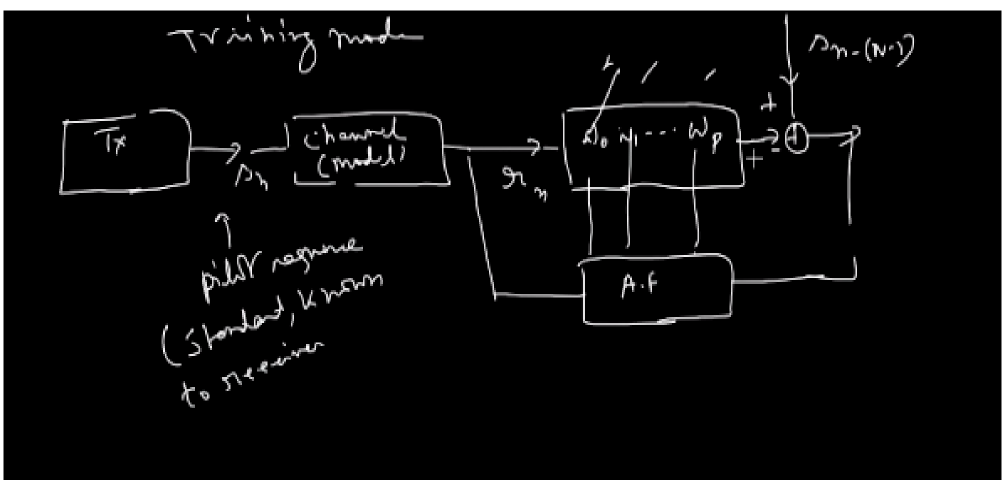
So, this and this will be close to each other and they will be 0. So, this is the optimal filter the problem is optimal filter requires knowledge of the statistics of these and all that, but this again requires channel information because statistics of this depends not only on the statistics of this autocorrelation it will also depend on the channel because SN goes through channel then only becomes RN and those are not known to me or they vary from time to time. So, I make it adaptive. So, there is a training mode this is called training mode what happens is from time to time may be after every half an hour training mode takes place during that time transmitter transmits one known sequence of symbols called pilot sequence which is standard and known to receiver. So, receiver knows now training is going on.

So, as SN goes through this channel you receive RN SN is known to you. So, you take you want to make an optimal filter of the right order. So, that output is a good estimate of this like I mean if it is a correct inverse system then the output will be just delayed version of what was transmitted.



So, this delayed version delay by capital N minus 1 delayed version of SN which was transmitted is taken as the desired response and then I try to find out the optimal filter and that I do using the adaptive filter route because I cannot directly compute the adaptive filter because a optimal filter because it will require autocorrelation matrix knowledge of the input that I do not know because input comes through channel I mean by the comes not only from SN it uses information about it has information about the channel which is not known to us. So, you have to make it adaptive.

So, what I do during the training mode ok, during the training mode we use it adaptive filter and we have got w 0 w 1 up to some appropriate p and all these coefficients I add up. So, I run this adaptation for that for the overall training mode. So, some time is given to me some nanoseconds during that time training is given because and then again whatever values I get I remain happy with them for the next half an hour maybe I use them I assume that channel is not changing during that half an hour. So, they will do pretty well. So, for now you transmit your real sequences music or speech or whatever you receive distorted sequence R N, but using this inverse filter you get back here a very good estimate of what was transmitted delayed of course, but delay does not make any difference to me ok.




This delay is hardly appreciable in real time. So, this is what happens during training ok. This is what happens during training and this training mode is called DC, I mean supervised mode ok. Supervised because this is a supervising signal the desired response is already known to receiver is stored in receiver because this is a kind of a constant universal knowledge it knows what was transmitted ok. From its own storage it uses this and this is a kind of supervisor which can you helps in carrying out the supervision I mean adaptation.

Now, after the training mode still we can do something. So, we use a switch during training mode this desired response come as it is S N minus N minus 1, but once the training mode is over this comes from 1 minute. Let me explain what this is suppose training mode is over now you are transmitting your real stuffs be music whatever they get distorted by and then using these inverse filters which I believe they model by H i N correctly I get back values which are close to what was transmitted. So, I may get not S N I may get S N cap here, but S N cap will then be pass through a quantizer. Quantizer means I know what was transmitted where discrete levels either this value of voltage or that value or that value or that value nothing in between ok.

So, on this scale maybe I have got either this or this or this or this like that. So, around each I have a band around this I have a band. So, suppose S N cap is somewhere here this is actual S N and S N cap is here. So, this quantizer will correctly quantize it to this value because this is known ok. So, this quantizer output I take a strainer which of course, this much delay.

So, during training mode I take it from this I take this to be my supervisor training sequence. Once training mode is over still I carry out some kind of adaptation by using this and a delay this delay is required from here. So, assuming it was correct ok and this is not S cap N sorry ok I am making a mistake here. So, let me correct this does not produce S cap N it produces S cap N minus it produces an estimate of this which means I would not require this delay. Because it you this optimal filter which is basically adaptive filter here it will produce a good estimate of this desired response during the training.

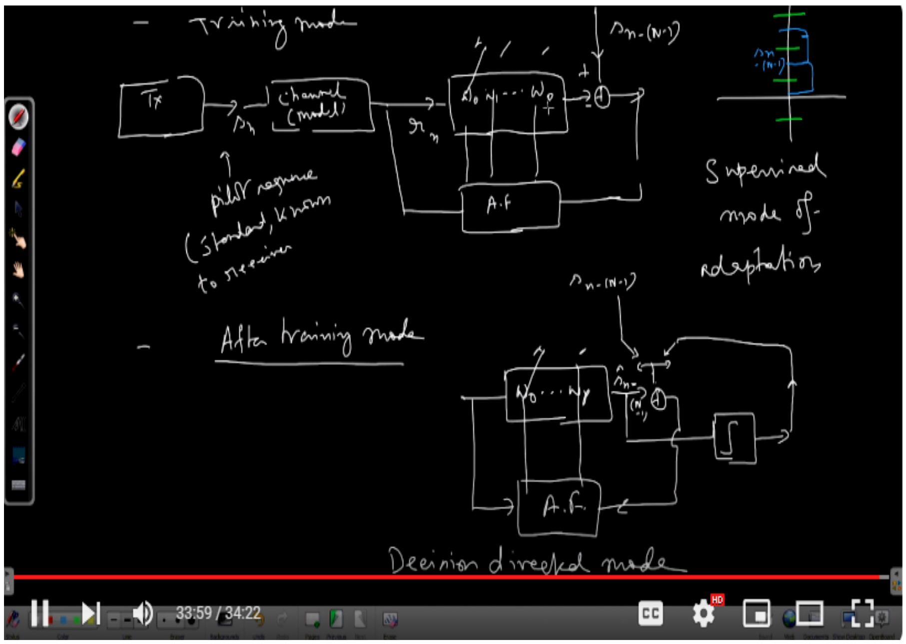
So, it was not N it is N minus a delayed version capital N minus 1. So, it will be an estimate.

So, just S cap of N minus capital N minus 1 that and that will go through this and hopefully it will be quantized to the right value. So, that will come here as desired response that will come here as desired response. Because what we are targeting then is this first let me make it adaptive adaptation you know the error has to from here during training mode this switch moves here.

So, what happens this is already what was discussed here this is my stored sequence delayed version this is taken as a supervisor that is training sequence that minus the filter output filter output is a good estimate of this. Because that minus the filter output is a error that error variance is minimized by this adaptation loop and I get a good estimate of this that is fine that happens during your training mode. Once the training mode is over and you are transmitting your speech and music and all those signals still, I carry out some kind of adaptation what happens whatever comes out here that will be passed through quantizer and even if this has some error because this may not be exactly equal to this S N minus within bracket N minus 1 that is the transmitted thing with a delay of N minus 1. Because of all the approximations and all and maybe some noise effects it will be there will be some difference, but the difference is not much it still remains in the correct band then after quantization it will come back to the correct level.

So, correct level will come here. So, correct level minus its estimated thing that error also we want to minimize that is why this adaptation loop. So, if I minimize the error that means, S N cap and its true value there is a quantized value they will remain in the same band it will not fly off to another band. So, I will make correct decisions what does this loop do it tries to minimize the error variance here that means, S N cap and its true value there will be which is coming through this path they will be close to each other which means this will not fly off to other band and therefore, I will not make error after quantization I will quantize it to the correct value that is what is achieved here and this is called decision this adaptive filter this is called decision directed mode. Again, after half an hour you switch to training mode carry out this kind of training then once that is over then again switch this way. So, estimated value of transmitted symbol with a delay that quantized and if it is quantized to the current value that minus this error, I want to always keep minimum.

So, that they are very close to each other and this fellow does not fly off to another band that is done by this optimal filter which is adaptive filter here.




So, that is a very good example of this thing modeling inverse system because of all equalizer is that inverse model H i n not H N. So, I covered several important applications now again in the next class I will come back to analysis some analysis of that elevation algorithm convergence we proved convergence in mean but you have to show see that around mean if it fluctuates how to control the range of fluctuation. So, that it remains close to the optimal value that is what we will target in the next class. Thank you very much.