Introduction To Adaptive Signal Processing

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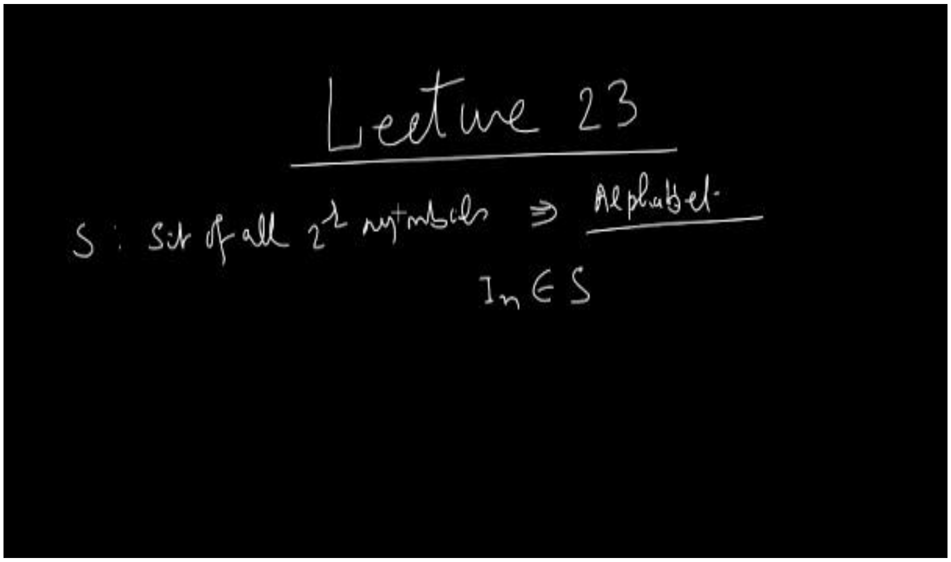
Lecture No # 22

Applications of Adaptive Filter

Okay, so we are considering channel equalization. I said that I mean there is a line through which binary bits are coming. You can take a block of may be small r number of bits, one block, r number of bits, next block another r number of bits and so and so. So, there are 2 to the power r possibilities and to every possibility you assign one voltage level for symbol. So, there are 2 to the power r symbols and the symbol set. All the 2 to the power r symbols it is called alphabet, okay alphabet.

So, now every time a symbol is there, we have to send it down a communication channel a media, but as I told yesterday all communication channels in the whole world are analog they can carry analog waveforms. If we can give an analog waveform on the input it will go through the analog channel and you will get some analog waveform on the output, but they are not I mean they cannot take discrete things, they are okay 5 volt, 10 volt, you know 4 volt not like that. So therefore, the symbol I have to have some pulse, okay which will be a basic pulse and that will carry on its shoulder this information about the symbols, okay. So, I got an e symbol sequence IN element of S that is I of 0 is the symbol at the 0th symbol period.



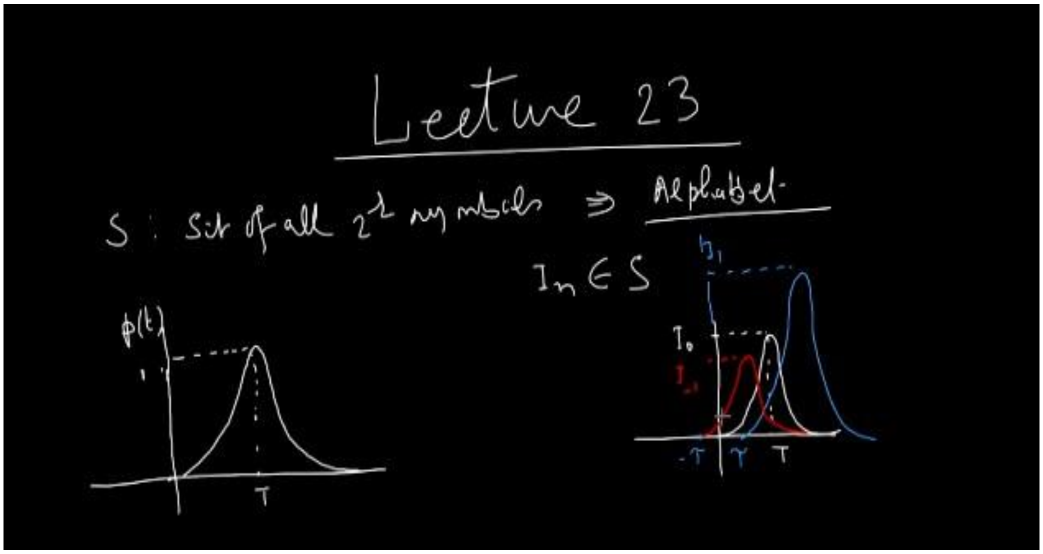
A symbol period means r number of bits forming a frame or a block which is a symbol period, okay. So, I 0 then comes I equal to I 1 then I 2, I 3 these are various voltage levels all belonging to S. So, there is a sequence of symbols every symbol value I have to send down the line through a pulse, okay through a pulse. So, the pulse is basic pulse is important because pulse is analog waveform and this is required because whole world all the communication channels the whole world you know actually is analog all the communication channels all the media, they are like you know free space or wire telephone wire optical fiber and all these are analog media analog channels, okay they can take only analog waveforms. So, that is why a pulse is necessary as I told you yesterday.

$$I_n \in S$$

So, suppose there is a basic pulse we design like this P of t pulse is triggered at this point, maybe it attains a peak something like this here. This height may be 1 and this center point is t, okay. So, you take this pulse multiply by say I 0, I 0 they give a symbol. So, pulse remain as it is since height will be multiplied by I 0, I 0 into 1 so I 0, alright. Then what happens you trigger the same pulse after another after sometime may be tau and then that will be multiplied by next symbol I 1, then after 2 tau another pulse will be triggered and again multiplied by I 2 and like that and through the channel the super imposition of this pulses will propagate, right.

So, you will have things like this one pulse its height is I 0 and I should measure the pulse when it is my highest. So, T then just after may be tau, tau amount of time next symbol comes. So, symbol period is tau every tau amount of time may be nanosecond or whatever a new symbol comes. So, at 0 one symbol came I 0, I trigger a pulse multiplied it by I 0. So, the pulse same remains same only amplitude changes from 1 here to I 0 here.

Then next symbol period when the next symbol comes after one symbol period that is tau same pulse is triggered multiplied by the new symbol which is I 1, I 0 is done now next is I 1. So, it could be something like this, this height is I 1. on the other hand, before this I 0 was triggered before that I had tau amount of time earlier and minus tau another pulse was triggered and that could have been you know multiplied by I minus 1, previous symbol something like this. So, this is I minus 1 and so on and so forth ok. There will be more pulses to the right more pulses to the left they all they may have some interference here ok.

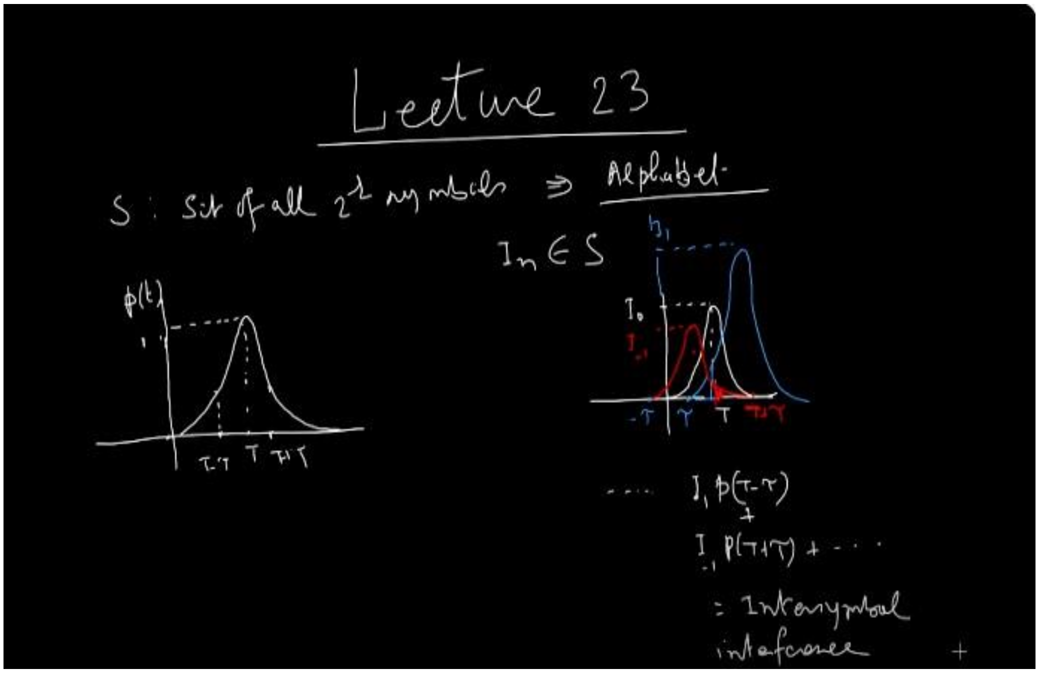


And theoretically all of them have, but practically as you go further to the right further to the left pulses situated further to the right, far left, they will not have overlap here they will not be able to you know I mean interfering here.

Nevertheless, now the superimposition of all these waveforms is a waveform which goes through the channel and then the receiver I first measure at measure the height of the waveform amplitude, at t assuming I get I 0 then at tau time to the I mean upper tau t plus tau t plus tau I measure again I expected to get I 1 or I at minus tau I sample again I expected to get minus 1. So, 0 at t, t plus tau, t minus tau here, tau to the left ok, because tau is the symbol period capital T is the peak of 1 pulse. Next pulse its peak will be shifted by tau because it was triggered tau second or tau nanosecond later, the pulse to the left it was triggered minus tau at minus tau time. So, obviously its peak will be not at capital D, but t minus tau. So, I will be sampling at t minus tau at, t at t plus tau t plus tau tau tau tau tau tau like that and I would expect to get I minus 1 I 0 I 1 I 2 I 3 dot dot dot, but that will not happen here because you can see even though I have I 0 the pulse from the right pulse from the left maybe some more pulses from the right some more pulses from the left they all will interfere here.

So, I will have not I will not only have I 0 I will have this much contribution and this much contribution and like that this this contribution from right pulse 1 minute. This much contribution from the right plus and this much from this left pulse they all get added as a result I will not get I 0, I will get other values here which is a called inter symbol interference. How much is the value? How much this blue colored height? And from the peak of the pulse, you go tau to the left whatever is the pulse width. So, t. So, if you go here t minus tau the same pulse, but now its height is multiplied by I 1.

So, we will get a component additional component I 1 and p capital T minus tau. Similarly, this pulse from its center point if you go tau to the right. So, if you go tau to the right and multiplied by I minus 1 you will get another component. So, from the center point tau to the right here because in the center point, next peak is tau to the right of this peak of this pulse they are separated by tau. So, they will they will all get added.



So, they are coming from other symbols I 1, I minus 1, I 2, I minus 2 and all that. So, this will give rise to error and that this is called ISI inter symbol interference component. To avoid this first we should design the pulse in a suitable way. We design so that when the see one pulse may be the white color one has the peak at time equal to T, the pulse to the right it should go through 0. So, it will have 0 interference pulse to the left it should also it should also go through 0 and so and so.

That means this pulse should be such that if I go tau to the left from the center point like from the center point of this you know the center point where the peak is located. If I go tau to the left this blue curve should go through 0. That means the basic pulse P t minus tau should be 0. Then if you have another pulse that is its peak will be shifted, I mean by 2 tau amounts because from this point capital T if you go tau to the right, you get the peak of the immediate right pulse. The next one its peak will be at capital T plus 2 tau.

If it that also should not interfere that is also go through 0 here and so and so forth. So, T plus tau T plus 2 tau T ok. So, this pulse what is here this from its center I go tau to the left it should go to 0. The one which is further to the right from its center if I go 2 tau to the left it should go through the 0 and like that. That means from capital T if I go tau to the left 2 tau to the left and all that there is P t minus any k tau ok that should be 0 and if I go tau by the same way if I go like what is the red color pulse from the center if I go tau to the right, I am here that should be 0.

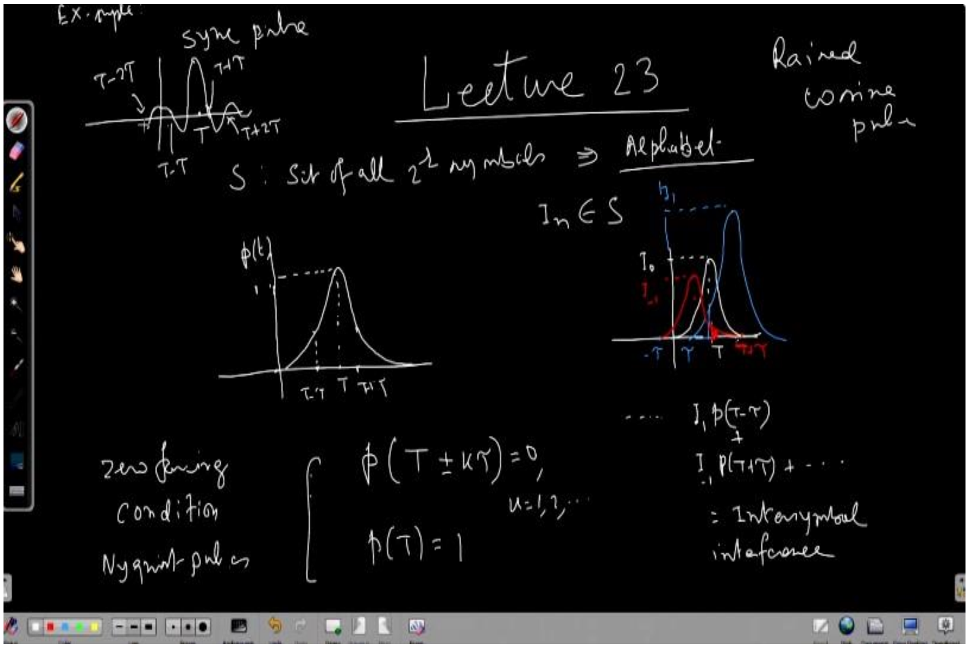
So, it should go through 0 basic pulse should go through 0 if from the center point if I go tau to the right it should go through 0 or if I take the one further to the left. So, its peak is at T minus 2 tau ok. So, from that peak if I go 2 tau to the right that pulse should go through 0. So, that is why we do like this ok and at the center point it is 1. So, this is called zero forcing condition pulses which satisfy this condition called Nyquist pulses ok.

So, sync pulse at capital T you have a peak. So, this is this point is T plus tau, this point is T plus 2 tau, this point is T minus tau I like that, next is this is T minus 2 tau like that. So, sync pulse a more popular waveform of this kind similar to this is called raised cosine and I am sure you have studied this in your undergraduate raised cosine pulse. Otherwise just take a basic book on communication then tell you about the pulse raised cosine pulse. So, all these pulses satisfy zero forcing condition, zero forcing means I am forcing the pulse to go through 0 after certain you know at certain points certain sampling points like a T plus tau T plus 2 tau, T plus 3 tau or T minus tau, T minus 2 tau and all that.

So, I am forcing it to go through 0. So, zero forcing condition is imposed and if the pulse satisfies this it is called Nyquist pulse fine. So, I suppose have a Nyquist pulse ok and I have apparently no problem, but so suppose your Tt and Nyquist pulse, but the channel through which it will pass it is not an ideal channel. Channels are usually not ideal channel. So, ideal channel is such which will pass all frequencies present in the input signal equally and if it delays it will delete all frequencies you know by the same amount.

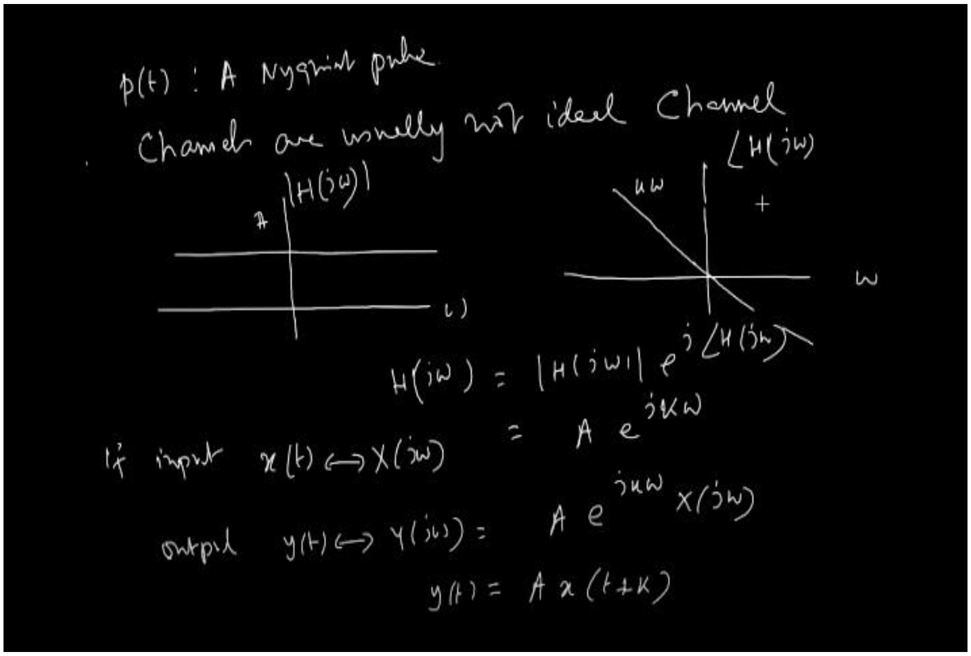
$$p(T \pm u\mathcal{T}) = 0, \qquad u = 1,2, \ldots.$$

$$P(T) = 1$$



So, it is actually something like this h e to the power j omega ideally it should be 0 and its angle you can take the magnitude its magnitude should be either 1 or some constant k for all frequency and phase angle will be constant the linear function of omega like some k omega maybe you know like this some k omega k is a negative number k omega. Then it is an ideal channel because if you pass a signal through it there will be no distortion you can easily see what is h e to the power j omega for any omega what is h e to the power j omega is a magnitude times e to the power j angle and this is constant may be a and this is e to the power j k omega right. So therefore, if input some xt they are all analog signals now xt which has Fourier transform x j omega or I am using sorry I have changed the notation where this is more used in DSP let me call it this because these are analog channels. So, I should have here also. So, this is k omega magnitude and angle.

So, this k omega sees if input is this output will be output, we know output y t that is at the output of the channel or at the input of the receiver you will receive away from y t which has Fourier transform this there is nothing but input Fourier transform times this. So, a e to the power j k omega and you know from your single layer system that if a Fourier transform is multiplied by e to the power j k omega in time domain it amounts to at shift by k delay by k. So, it will be so y t will be nothing but a as it is x as it is, but it will not be t it will be t plus k and k is a negative number that is why you give a downward slope.

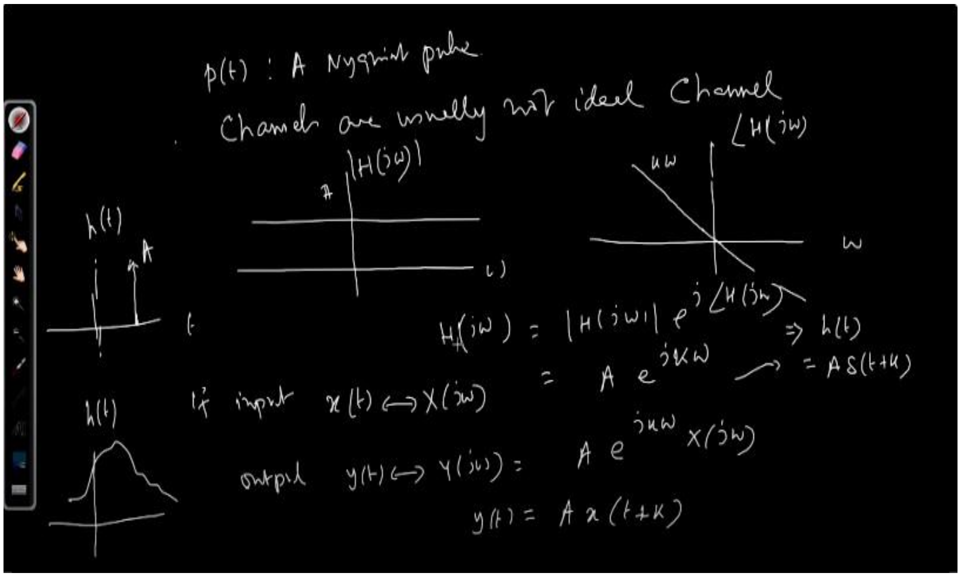


So, actually it will be delay which is fine if I give a positive slope say k equal to plus 2 then y t will be xt plus 2 which is not possible because that we make it non causal at standing at time t you have information about xt only not xt plus 2. So, you cannot generate a system whose output is xt plus 2 future output future input that is why in all practical circuits and systems you find phase response to be having a negative slope never positive slope because negative means it will be a negative number it will be non I mean it will be causal which is practical anyway these are some side inputs I try to give to my students as an additional knowledge item. anyway suppose this is the what is happening this kind of channels if it is x t input same xt just multiplied by capital A.

So, it does not distort the waveform also delayed by k. So, it just delayed and amplified, but basic waveform does not change and that is why it is an ideal channel all right. So, if the channel is ideal waveform does not lose its step maybe safe maybe it gets amplified a little bit, but from channel measurement I can find out what is A what is k and all that. So, I can make the corrections. So, no problem, but unfortunately in the whole world all practical communication channels between say from free space communication between antenna to antenna or say telephone wire or optical fiber or fluid like what are through which you know under water communication takes place all these have channel response which is far away from being ideal magnitude response will be very you know I mean zeta d it will be varying with omega phase also will not be linear.

So, altogether it will distort the waveform that is actually if you if you have h omega this for an ideal channel the impulse response will be you put I mean here impulse response will be A delta t plus k delta is the direct delta function you know delta t has got Fourier transform 1 if it is t plus k it will be e to the power j k omega times 1 and a remains as it is k is negative and therefore, in time domain if you have a signal x t that will be convolved with this, but x A will remain as it is x t convolved with delta t plus k will give you x t plus k. So, there is no change x remains x only delayed or adverse whatever delayed because k is negative, but in a practical channel h t will not be like that, it will be it will not be like you know A times an impulse this means an impulse d lambda delta, delta impulse at t equal to minus k, it will not be, so it will be h t will be rather I mean impulse was what I was trying to say is this ideally h t is maybe at k and minus k and k is negative. So, minus k is positive it will be like this. So, it is an impulse which is fine k is negative. So, that is why it is coming on this side because this impulse is at t equal to minus k, but k is negative means actually some plus value all right this is t this is ideal case, but this does not happen in practice rather h t will be some waveform like this.

So, convolution of this with x t will distort your x t heavily it will no longer have the same shape as x t. Therefore, even if you take lot of care to design your Nyquist pulses which satisfy the zero forcing criteria as it goes through the channel it will lose its shape. So, it will not follow the zero forcing criteria correctly, it might be close to that if the channel is not that bad, but it will not and as a result inter symbol interference will still come back at the receiver input and that is what we have to remove and it is removed by a device called equalizer ok.



So, to carry it further suppose p t Nyquist pulse, transmit pulse. p t as you go through the channel analog channel real life. There is analog this basic pulse which was satisfying Nyquist condition that is these zero forcing conditions it will go through this, after convolution it will be distorted it might become p prime t and it will be not Nyquist because it is distorted.

So, zero forcing criteria will not be satisfied in general though it might be close to this because channel may not be that bad, but therefore, you inter symbol interference will come back ok. So, what will happen is this you will receive something like this at capital T you know you will be sampling the received waveform this is the received waveform sampling capital T at capital T to look for symbol i 0, there at capital T plus tau to look for symbol i 1, at T plus 2 tau to look for symbol i 2 dot dot dot. So, in general at T plus n tau you know you have this pulse which is no longer Nyquist it might be like this some pulse ok. So, it will be its height will be what the basic this pulse p prime at capital T multiplied by i n, this is the nth symbol n tau. So, i n times p prime because of the basic pulse its center point is at capital T where is peak and I am measuring there.

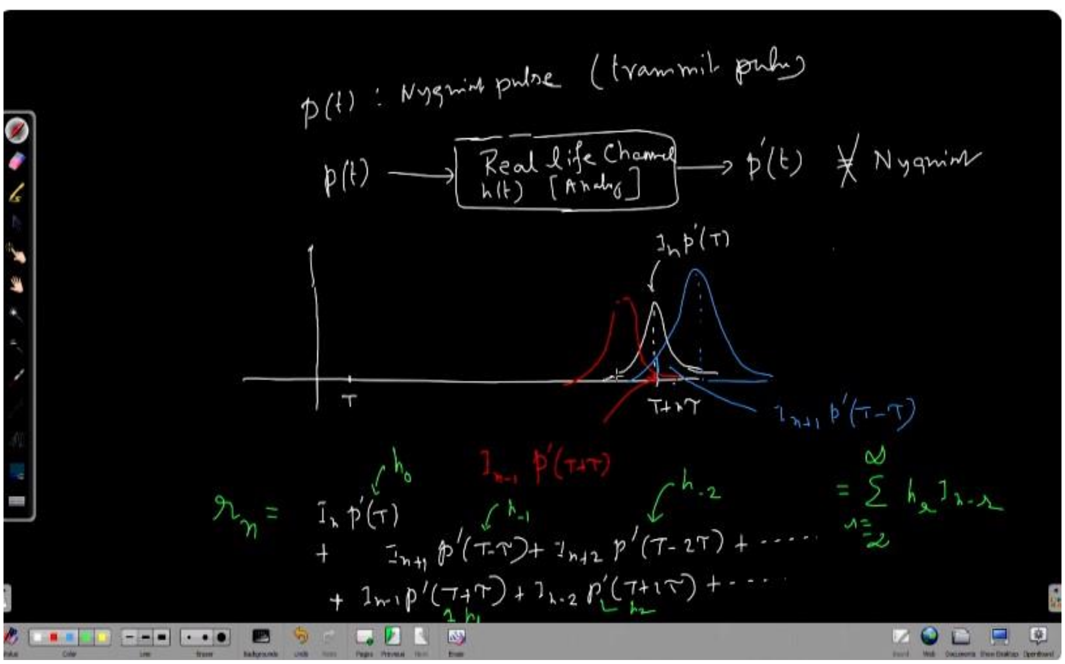
So, p prime capital T value of this distorted pulse which is not 1 anymore that times i n. So, this is what I will have here then maybe from the left-hand side. So, there will be a component here ok what is that component again the same p prime T pulse center point here ok. And if I go tau to the right. So, this much is the basic pulse p prime from T to tau to the right this much height times that symbol here which is i minus 1 this much will come as interference.

Then this one from the center point if I go tau to the left whatever is the value, that is multiplied I am talking the basic pulse that is multiplied by the symbol i n plus 1 sorry this is n I am sorry, this is not i 1, i minus 1 it was i n. So, it will be the previous symbol i n minus 1, i n previous is i n minus 1 and now this is i n plus 1, i n plus 1 times, to this much this much is i n plus 1 times p prime from the center point you are going to the left by tau. So, p prime T minus tau and dot dot dot dot so and so forth. So, typically you will have. So, when you sample here the net received that sample if I call it r for received r n. r n that will be having one term from the correct symbol i n, p prime p there is a distorted pulse which looks like this its value at the center point capital T where it is peak is supposed to be peak because my sampling point in any case at that point, I sample to measure i n.

So, therefore, whatever be the value no longer 1 because of distortion now. So, this is this then I have got one set of terms i n plus 1 p prime T minus tau then further to the right it will be i n plus 2, p prime T minus if you have another pulse here from the center point, I go 2 tau to the left to reach this point. So, the value of that pulse now will be from center point if you go 2 tau to the left that value times this and dot dot dot dot. And theoretically up to all the pulses up to infinity there is all the pulses to the right whatever be the value at this point that much, but actually in practice these pulses are time bound pulses. So, you do not have to go too far after some time only I mean the pulse is situated to the right we no longer contribute here they will disappear all right.

And then there is one more term from the left-hand side i n minus 1, p prime t plus tau, i n minus 2, p prime t plus 2 tau, plus dot dot dot. Let me give some name to them let me call this to be well this is a constant right once you have a chosen p T this is known to you if you pass it through a channel, you know what is p prime T it is a fixed wave form. So, its values are different sampling points are known to you. So, value at T only thing is the value is not known when the channel is not known and that is most often the case, but still, it is a constant wave form. So, this is a constant let us call it channel constant let us call it A is 0 just I am giving a new name nothing else.

Let me call this H minus 1 let me call this H minus 2 and dot dot dot and let me call this H 1 let me call this H 2 in this case the whole summation turns out to be this. So, R n will be you can see H R i n minus R, R from minus infinity to plus infinity, very easily if you start with R equal to 0. So, H 0 i n there is a term you have H 0 i n if you take R equal to 1 i n minus 1, H 1, here H 1 i n minus 1 if you take R equal to 2 i n minus 2, H 2, here and dot dot dot. On the other hand, if you take R equal to minus 1, H minus 1, i n plus 1 you have here if you take R equal to minus 2, H minus 2, i n plus 2 you have here and like that, but this is nothing, but a discrete convolution. So, this is a model after you sample the wave form whatever value you get this is the equation these are mathematical model then of the received symbol in terms of the future and past symbols.



Future symbol is coming, but do not worry this is not real life in the sense you know how the causality is bridged here it is because causality is bridged because when I sample the current wave form to find out i n by the time, I already triggered the wave forms of the future wave form. So, they pass through the sampling point I mean at the sampling point they already exist. So, they contribute with their values that is why contribution from future pulses come that is I can trigger a wave form here, but I measure it after some time and within that time I have triggered several such wave forms future wave forms. So, they carry information about i n plus 1 i n plus 2 i n plus 3 and they all contribute that is why contribution from future terms is coming that is I am telling it is not a real-life analog system it is a mathematical equation which we get which can be said a discrete time model discrete model of the channel and that model can be non-causal, but physical vision of the non-causality I have already explained. So, I stop here and we carry on from here in the next class. Thank you very much.