Introduction To Adaptive Signal Processing

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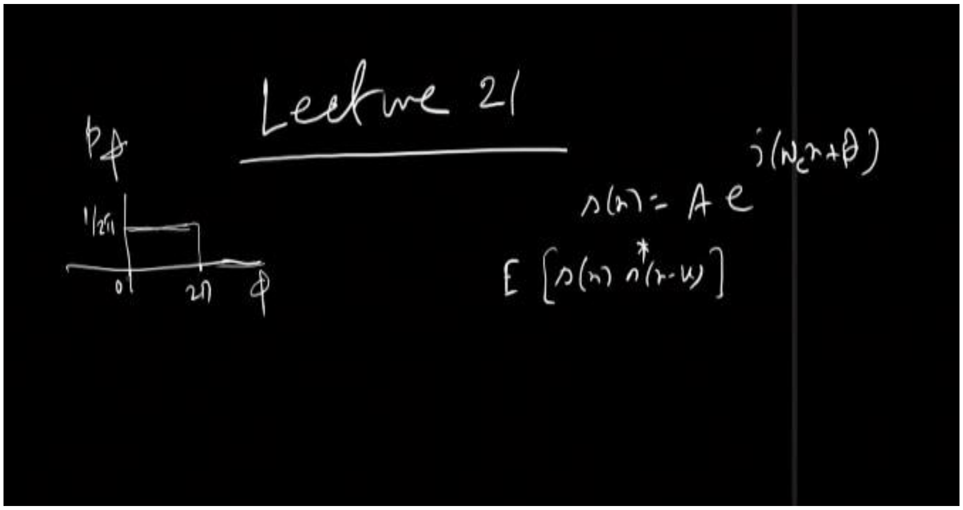
Lecture No # 21

Application of Adaptive Filter (Contd.)

Ok, yesterday I was considering the signal you know the Sn a e to the power j may be some omega cn plus phi I made a mistake there actually if I have to find the correlation since it is a complex signal it should not be Sn into Sn minus k. It should be a star minus k a star n minus k and their expected value. Ok, what is real the star has no meaning, but in general for a complex signal it is Sn into s conjugate n minus k. we have seen this earlier this is the definition of correlation. Ok, phi here is random and phi takes value between 0 to 2 pi only we can assume uniform distribution. Uniform distribution means probability density is constant within this period. So, any of this has to be 1 because if it is sorry this is p phi probability density this phi ok. This is the probability density of this phi in this phase phi this is you know constant over 0 to 2 pi and the 0 elsewhere.

Since overall area under the p phi curve should be 1 and area here is 0 here is 0 since area under this should be 1. So, this is 2 pi. So, height should be 1 by 2 pi. So, 1 by 2 pi into 2 pi the rectangular has area 1.

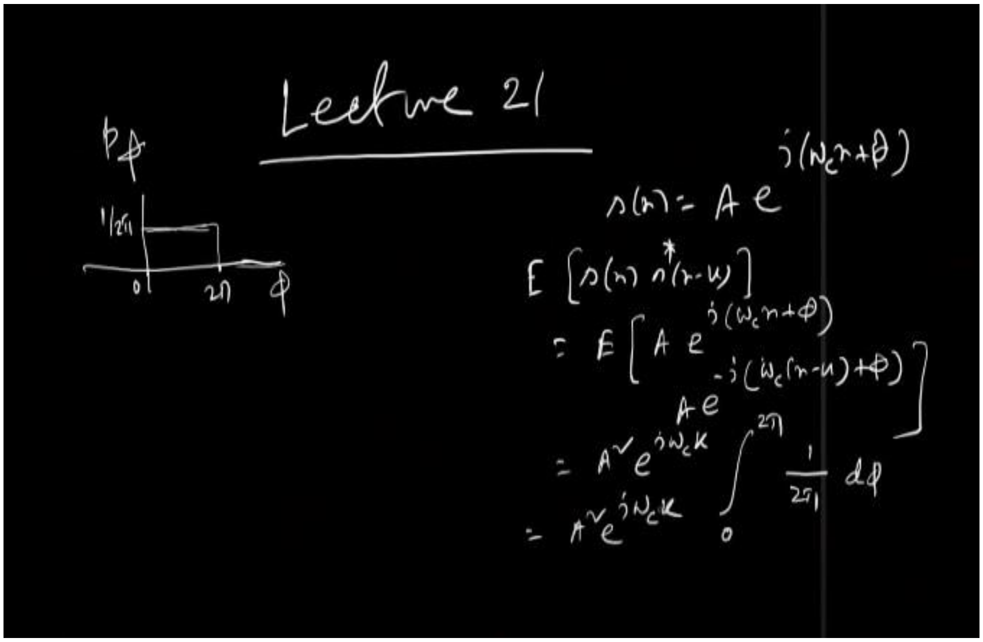


So, this is p phi is 1 by 2 pi from 0 to 2 pi otherwise 0 and in this case if you do Sn into s star n minus k, see omega cn cancels ok, j omega cn then there is a star. So, that will make it minus. So, j omega cn minus j omega cn goes ok j omega c j omega j phi and here again minus j phi that goes, but this in 1 case I have j omega cn another case I have got j omega cn minus k and minus of that. That is e of a e to the power j omega cn plus phi into a e to the power minus j star. So, you have to put a star here omega cn minus k plus phi. So, you can see j omega cn cancels j phi cancels and plus j omega k ok e to the power plus j omega k.

$$s(n) = A\, e^{j(\omega_c n + \phi_i)}$$

So, you have got a square expected value of e to the power j omega ck there is no phi there. So, that is again not random. So, it will be the entire thing ok e means the entire thing the product have to multiply by the joint density and integrate. So, when I multiply this entire thing by joint density and integrate this is constant independent of phi. So, it comes out of the integral.

So, it is 0 to 2 pi just the joint density comes here joint density is 1 by 2 pi ok. This factor was inside this factor is nothing, but this factor this was inside it has come out because this has there is no phi here, integral is with respect to that variable phi and 1 by 2 pi should come out 1 by 2 pi and integral will give you 2 pi integral of d phi is phi. So, 2 pi and 0. So, that makes it 2 pi. So, 2 pi and 2 pi cancels. So, it is a square e to the power j omega c k.



So, what will be the value of k for k equal to 0 only it is constant a square does not depend on omega c, but for any k, k equal to 1, k equal to 2, k equal to minus 1, k equal to minus 2 and all that this is a function of the frequency ok. So, R inverse p even if you have got only one coefficient R inverse p, R matrix ok, the diagonal terms will have k equal to 0 right and k equal to 0 will be a square. But all other terms will be function of omega c like a square e to the power j omega c into 1 1 such. So, here it will be a square e to the power j omega c, here it will be a square e to the power minus j omega c to your I mean stop this and like that ok.

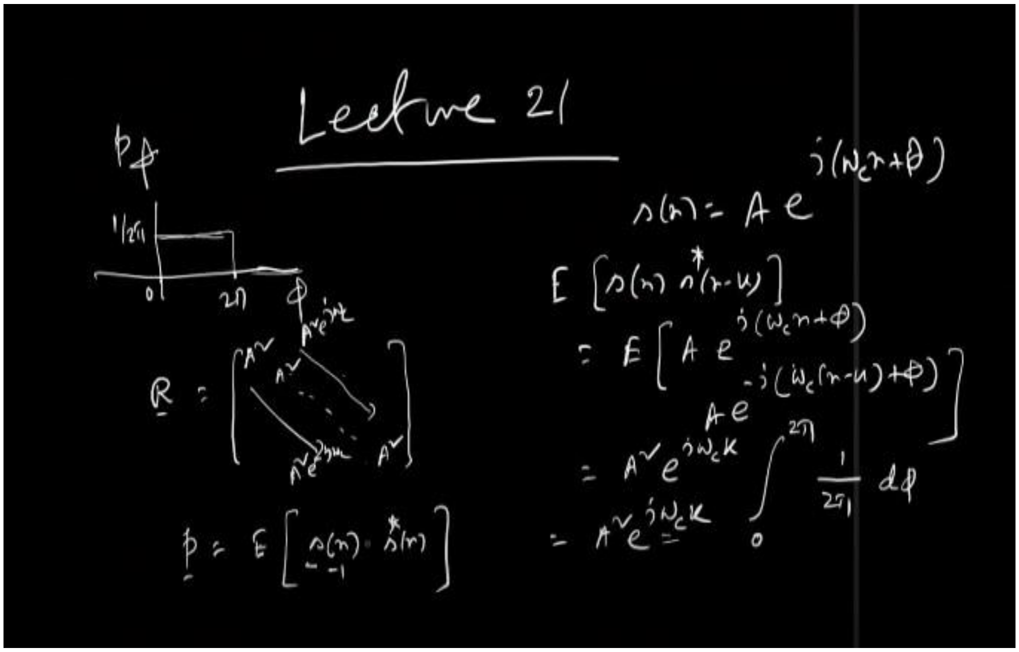
$$= E\left[Ae^{j(\omega_c n+\phi)}Ae^{-j(\omega_c(n-k)+\phi)}\right]$$

$$= A^2 e^{j\omega_c k}\int_0^{2\pi}\frac{1}{2\pi}d\phi$$

Similarly, p is if you have just Sn vector, in fact Sn minus 1 vector into Sn. Here again we have seen that previous example the adaptive line head and side I am talking with regard to that. We had this R matrix plus a diagonal matrix coming from the noise ok this whole thing was the inverse of the whole thing into just p and p was expected value of Sn minus 1 vector into Sn. So, Sn into Sn minus 1, Sn into Sn minus 2, Sn into Sn minus 3 all that ok. Actually, there will be star because it is again complex. So, then again you will see this will be a vector which will be a function of every term will be a function of omega c ok, which means the optimal weight that varies ok the optimal value changes from frequency to frequency.

So, that is what I said if the input was having a constant frequency fine and I had this in the optimal filter which will eventually turn out to be very narrow passband filter band pass filter with center frequency at omega c. But if the input frequency changes the LMS algorithm will adapt itself the adaptive filter will lead to an error pressure. So, that the filter converges to a new optimal filter given by new value of R new value of p, R inverse p and omega c has changed from omega c to some new omega c ok. If it is adaptive filter then only it will happen, but if it is just a fixed coefficient filter you design a band pass filter once for all then there is no way the filter coefficients can be changed with time can be adapted with time. So, if the input frequency changes you are out of passband you are gone ok.

Here the optimal filter which is the optimal filter and the adaptive filter is you know takes you around that they depend on omega c. So, optimal filter changes because R is the function of omega c p is function of omega c ok. So, optimal filter changes, but it is an adaptive filter now. So, adaptive filter also will track the input that is s n or rather x n x n is s n plus z n and will adapt that will change the filter coefficients its omega c changes it will take you to a new optimal filter ok.

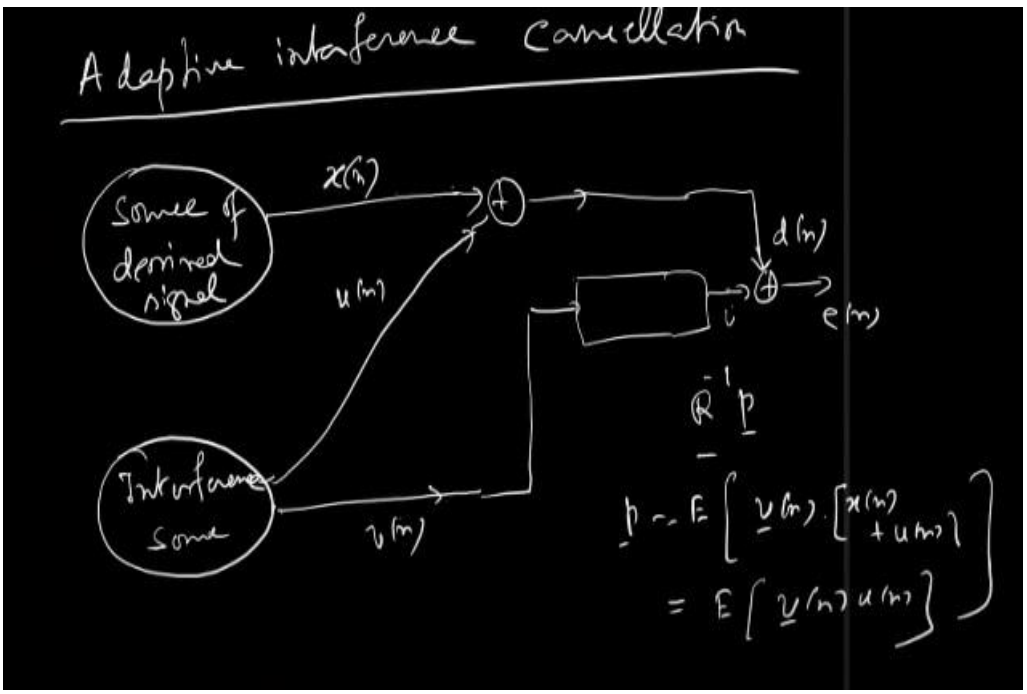


Alright next example adaptive interference cancellation. Here the general structure is like this, there is a source of some desired signal and this is interference source. So, this is generating a signal maybe x n and this interference it produces some interfering signal, some interfering signal maybe u n and so what I receive is a summation of the two x n plus u n this u n is a interference ok I have to remove it. So, what we do here is this from the same source we take another signal another interference signal. Obviously, these two are not same, but they are highly correlated because they are coming from the same source. Then what we do we first if you have an optimal filter, you give this here call it you can call it v n and take this as your desired response.

Then what will optimal filter do it will try to form a very good estimate of v n here. So, it will make R inverse I mean optimal filter will be R inverse p, R is the correlation here. R is the correlation here and it will estimate it has nothing to do with x n. So, x n and v n they are uncorrelated if you can even model them to be statistically independent. So, what will happen out in v n there are two components.

So, overall estimate here will be an estimate of x n and estimate of u n here, but x n and v n are uncorrelated. So, the estimate is 0 because R inverse p will have what is the p vector here R is the autocorrelation matrix of v n, but p is E of v n vector input to the filter into d n and d n is since all are real there is no conjugate down x n plus u n. Now, v n and x n all terms of v n and x n they are uncorrelated to each other they are 0 mean. So, the expected value will be equal to expected value of that v n term into x n expected value of the x n and then 0 mean.

So, that will be 0. So, it will be just v n u n as though x n is not present as though u n is directly given here and v n is here you are trying to estimate u n and since u n and v n they are highly correlated to correlated with each other because they are coming from the same interfering source a very good estimate of v n will be produced here.



So, you will have a u cap n here ok. So, what will come out x n plus u n minus u cap n. So, u n and u cap n they are very close to each other. So, the residual interference which is u n minus u cap n will be very small close to 0 and what will I will what we will have here largely is x n.

$$\underline{p} = E[\,\underline{v}(n)[x(n) + u(n)]]$$

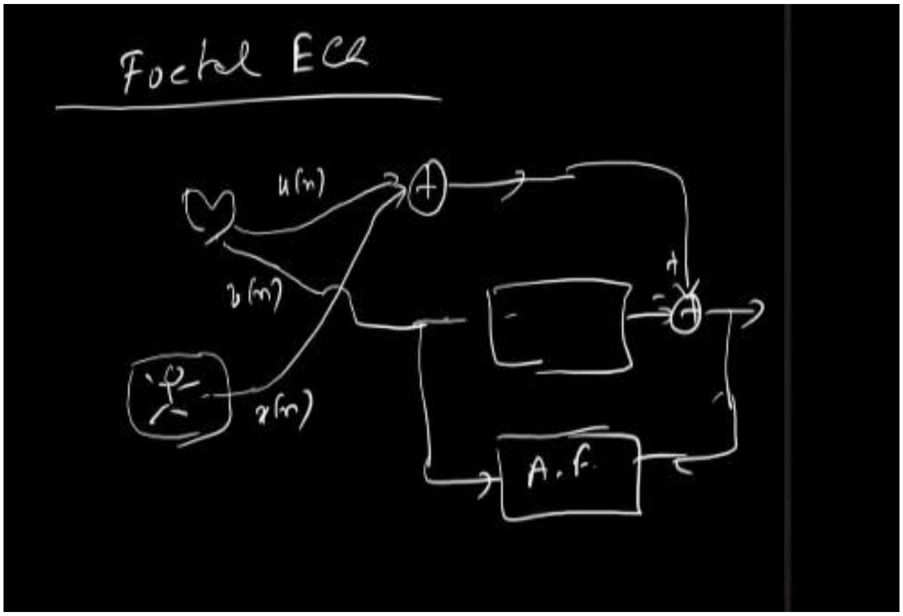
$$= E\,[\,\underline{v}(n)u(n)]$$

So, the interfering signal will go one very good example of this is fetal ECG. Imagine the pregnant mother ok. There is a baby she is carrying in a room there is a baby ok the baby and here is mother's heart. Now, what is happening the doctor will try to estimate try to analyze or assess the health of the baby by you know trying to find out the ECG the heart condition ECG of the baby. So, there will be a signal here ok that will try it will try to measure ok, but mother's heart will also give rise to I mean some interfering signal. So, because it is after all the same body.

So, it is not only child's heart mother's heart and together you will receive a signal. So, that has got your x n here and u n here. So, this is the interfering component this you have to remove. So, what they do they put this you know ECG probe somewhere around the heart of the mother and take another signal. So, this signal is coming in from the inside the body because inside the body we have mother's heart child's heart.

So, when if you put you know ECG probes around the child's body child's heart it will pick up signal from mother's heart and child's heart. So, together you will receive something here ok. To eliminate that interfering component, they use this you know ECG electrode, they use this near the and put them near the mother's heart and take signal from here and let that be v n. So, this is coming purely from mother's heart and here I have got a combined signal ok. And these are the interfering signals, but they are highly correlated.

So, I do like this optical filter I do not know what the optimal filter is. So, we use the adaptation adaptive filter ok. So, v n goes here it is just the previous block. So, what comes out you know this u n and v n they kind of get cancelled what comes out here is x n the interference is almost gone. So, this is fetal ECG.



Another example active noise control. Often you hear very annoying disturbing sound from you know I mean the silencer of a car the silencer is not functioning so properly. So, it creates tremendous noise or the duct of a you know or the machines in a factory or duct in an air conditioner and all that. So, these sounds are in this example they are cancelled in the acoustic domain only. I will somehow produce what is called anti-noise anti acoustic noise.

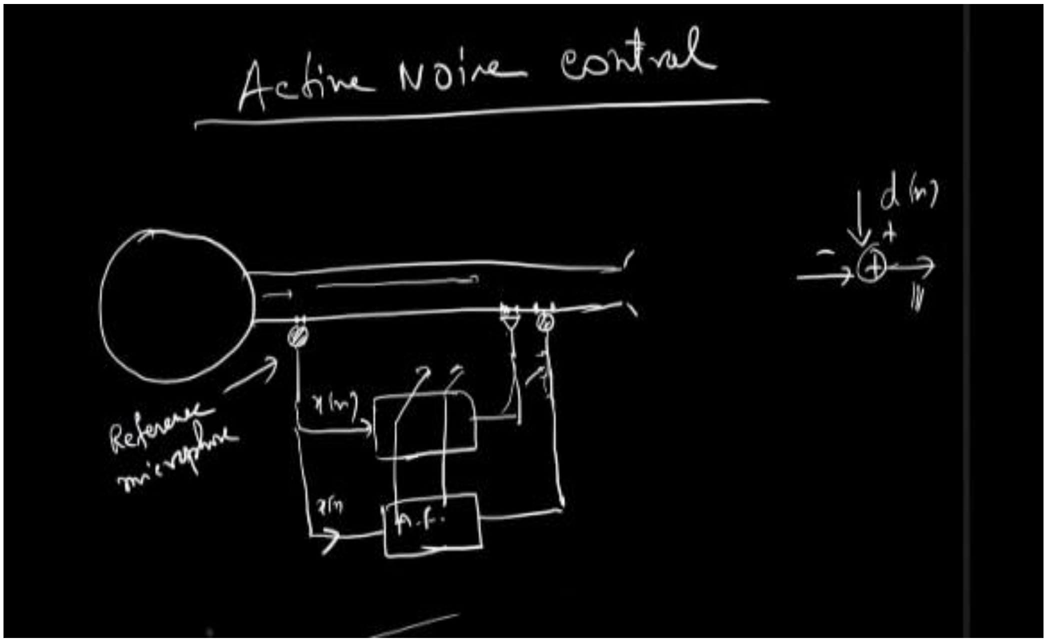
So, in the environment there will be original acoustic noise and I will create an anti-acoustic noise. So, they will cancel each other in the acoustic domain only and that mechanism is called active noise control because it is controlled in the it is eliminated or minimized the noise effect is minimized not by converting them into electrical signal in the not minimized not in electrical domain, but in acoustic domain. To explain suppose there is a source and there is a duct sound is coming out through this through this pipe or duct and we are tremendous you know very disturbing noise here acoustic noise here. To cancel it what they do you put you just cut a small hole here and put one microphone here it is called reference microphone. Microphone takes this noise acoustic noise and converts into electrical signal.

So, electrical signal comes here and then here again it there is a speaker and there is a filter

I will tell you everything there is a filter it generates an output which is fed to a speaker and the speaker loudspeaker pumps new noise into the system, but this noise and the original noise our objective is to see that they are in opposite phase. So, they cancel each other, for that very near to this we need you know come dig another hole and put another microphone here this. So, this microphone takes the signal and again converts into electrical signal. So, it is like so noise coming in and I am generating new noise their difference produces a signal here which is my desired response and I want to minimize this. To minimize this what I have to do I will use this adaptation loop, this is my input xn and this.

So, you see the subtraction between the original signal or say desired signal if you take it a desired signal it is not desired, but this signal and the filter output that difference is this thing as we do in optimal filter or adaptive filter. This subtraction this is done in acoustic domain, how? you have original noise and this their difference come through the speaker this microphone because the optimal filter the or adaptive filter here we try to minimize the you know norm square the energy the power of this, error power of this in this case we will try to minimize the power of this. So, this will be minimum only when these are opposite in phase. So, they cancel each other or try to minimize both then only as we do more and more of that there is original and these are they subtract each other this gets subtracted from this there is because they are of opposite phase. If such a kind of noise is generated by my filter, then only the net noise which goes into this microphone will be having much less power.

So, if I have to minimize the power here because it is optimal filter only we can do is to produce anti-noise here. So, that these two are in opposite phase and the difference which the error signal is very less in power and that is used like by LMS algorithm that is en and there is a exchange that is used to adapt filter, adaptive filter this is called active noise control. On these tones of work have gone you know lots and many, many important applications have become out there are semester courses not just one course two semester courses on active noise control, but that you know I mean it is not purpose of this topic.



I am just showing you various examples of adaptive filters and there are still plenty, plenty more. I will now come to another important application which is which will take lot of time some more time it has details.

So, I will not be able to cover it in this lecture it will roll into next and. So, it is actually so long we have been using the forward modeling there is an unknown system, there is an input and output, both input and output are known to us, we are trying to model that system by a refer filter the using that model only, we went for acoustic, we went for this you know I mean echo cancellation and all those things even all these active noise control and all that or that adaptive line agents are just a forward model input going in and output coming both are given to us you have to identify the system and as I identify the unknown system by that process my job is done,

But there is the other side of the story is inverse modeling. Here it is not the unknown system that we are interested into the unknown system is assumed to be invertible that is if we have the unknown system output and that we face to the inverse that we fit to the inverse systems input I will get that the original input and the output inverse modeling like if there is a system, they take t, it works out some input xn here yn which is t of xn and we are assuming t is an invertible system. So, then that inverse is t inverse that is t inverse yn comes here which is xn then this is called inverse of this we assume that inverse exists and ideal inverse we work on these systems output as its input and we will give you these systems input as its output and we will now be targeting modeling this or identifying the inverse system rather than this direct system.

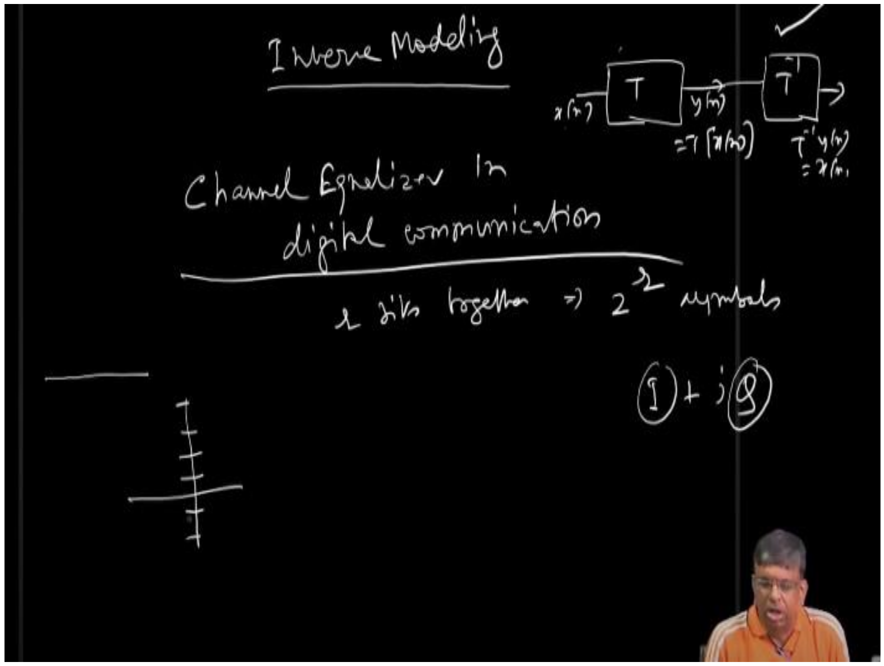
So, we will get this job. So, there are many there are some applications most important application is channel equalizer in digital communication. This is bit lengthy, but I will cover in details. Suppose there is a line, through these bits are coming bits 1 or 0, 1 or 0, 1 or 0 you take odd number of bits together. So, there are 2 to the power r possibilities each r bit block or number of bits each block of such r number of bits will be mapped to a particular voltage level which is called the symbol.

So, we will have 2 to the power r symbols. So, that in a simplistic scenario there you know they can be like this like these various levels positive or negative all right. Actually, in today's communication I mean there are you know every symbol has 2 components one is a real component which is the inface just a minute. I and another is jq. So, depending on the r bits you will have this. So, it will be mapped to one real part, one imaginary part and we transmit both and all that, but the for our treatment I will just consider this case.

This can be easily extended to this. So, as though every r block of r number of bits you take and depending on what that block is, you map that to a particular voltage level called a symbol. So, there are 2 to the power r symbols. So, these symbols have to be transmitted. Now this is discrete. So, one symbol period there is r blocks r bits a block of r bits coming over certain period of time that is called symbol period.

So, that period maybe there is one symbol next symbol period may be another symbol. So, it is discrete. So, I have to transmit, but my problem is I cannot do like this and look now it is this much and it goes and next I say now this much this goes it cannot because even if all the manipulations here all my you know processing everything is done in digital form which makes it discrete in time that may be discrete in amplitude real world real world is actually analog it only accepts analog waveforms real world means. So, when you communicate between say antenna to antenna that is called free space communication or if you when you communicate through a telephone line, line communication or optical fiber or through fluid like you know underwater there can be communication from one underwater you know entity like submarine to another submarine and like that.

So, there is this fluid water. So, all such media real world media they are analog and they can only accept analog waveform, they cannot just take a discrete value. you cannot take a telephone line and just give 5 the figure 5 to its input and 5 goes no you have to convert that information 5 volt or 5 micro volt or whatever milli volt into a waveform analog waveform. Then only it will be able to pass through that media that channel. So, very first thing we do in communication is this before we transmit the symbol, we have to you know first trigger a waveform basic pulse and that basic pulse will be multiplied the basic pulse is limited over the symbol period say it will be multiplied by the particular symbol level, it will carry on its holder the symbol level at the receiver. So, it will be able to go through the media that the channel because channel is your analog but you are also sending an analog pulse analog waveform. So, it goes through at the receiver side again I sample that received pulse or received waveform and from that I to find out I will try to estimate what was the symbol.



So, this is the mechanism. So, both the transmitter receiver they are digital, but in between I have got analog world and that is why when I transmit and then when I receive you know I need analog waveform. So, we will start with that design of analog waveform and limitations of that leading to inter symbol interference all those things in the next class and how to cancel that by adaptive equalizer and the receiver front end. So, this we will consider in the next class. Thank you very much.